- 1. Preface to Physics
- 2. Preface
- 3. Introduction: The Nature of Science and Physics
  - 1. <u>Introduction to Science and the Realm of Physics</u>, <u>Physical Quantities</u>, and <u>Units</u>
  - 2. Physics: An Introduction
  - 3. Physical Quantities and Units
  - 4. Accuracy, Precision, and Significant Figures
  - 5. <u>Approximation</u>
- 4. Kinematics
  - 1. Introduction to One-Dimensional Kinematics
  - 2. <u>Displacement</u>
  - 3. Vectors, Scalars, and Coordinate Systems
  - 4. <u>Time, Velocity, and Speed</u>
  - 5. Acceleration
  - 6. <u>Motion Equations for Constant Acceleration in One Dimension</u>
  - 7. Problem-Solving Basics for One-Dimensional Kinematics
  - 8. Falling Objects
  - 9. Graphical Analysis of One-Dimensional Motion
- 5. Two-Dimensional Kinematics
  - 1. <u>Introduction to Two-Dimensional Kinematics</u>
  - 2. Kinematics in Two Dimensions: An Introduction
  - 3. <u>Vector Addition and Subtraction: Graphical Methods</u>
  - 4. Vector Addition and Subtraction: Analytical Methods
  - 5. Projectile Motion
- 6. Dynamics: Force and Newton's Laws of Motion
  - 1. Introduction to Dynamics: Newton's Laws of Motion
  - 2. <u>Development of Force Concept</u>
  - 3. Newton's First Law of Motion: Inertia
  - 4. Newton's Second Law of Motion: Concept of a System

- 5. Newton's Third Law of Motion: Symmetry in Forces
- 6. Normal, Tension, and Other Examples of Forces
- 7. Friction
- 8. <u>Problem-Solving Strategies</u>
- 9. Further Applications of Newton's Laws of Motion
- 10. The First Condition for Equilibrium
- 7. Work, Energy, and Power
  - 1. Introduction to Work, Energy, and Energy Resources
  - 2. Work: The Scientific Definition
  - 3. <u>Kinetic Energy and the Work-Energy Theorem</u>
  - 4. <u>Gravitational Potential Energy</u>
  - 5. Conservative Forces and Potential Energy
  - 6. Nonconservative Forces
  - 7. Conservation of Energy
  - 8. Power
  - 9. Work, Energy, and Power in Humans
  - 10. World Energy Use
- 8. Linear Momentum and Collisions
  - 1. <u>Introduction to Linear Momentum and Collisions</u>
  - 2. <u>Linear Momentum and Force</u>
  - 3. <u>Impulse</u>
  - 4. Conservation of Momentum
  - 5. Elastic Collisions in One Dimension
  - 6. Inelastic Collisions in One Dimension
  - 7. <u>Introduction to Rocket Propulsion</u>
- 9. Circular Motion
  - 1. Introduction to Circular Motion
  - 2. Rotation Angle and Angular Velocity
  - 3. Angular Acceleration
  - 4. <u>Introduction to Uniform Circular Motion</u>
  - 5. <u>Centripetal Acceleration</u>
  - 6. <u>Centripetal Force</u>

# 10. Static Electricity

- 1. Electric Charge and Electric Field
  - 1. Introduction to Electric Charge and Electric Field
  - 2. <u>Static Electricity and Charge: Conservation of Charge</u>
  - 3. Conductors and Insulators
  - 4. Coulomb's Law
  - 5. <u>Electric Field: Concept of a Field Revisited</u>
  - 6. Electric Field Lines: Multiple Charges
  - 7. Electric Forces in Biology
  - 8. Conductors and Electric Fields in Static Equilibrium
  - 9. Applications of Electrostatics
- 2. Electric Potential and Electric Field
  - 1. Introduction to Electric Potential and Electric Energy
  - 2. <u>Electric Potential Energy: Potential Difference</u>
  - 3. Electric Potential in a Uniform Electric Field
  - 4. <u>Electrical Potential Due to a Point Charge</u>
  - 5. Equipotential Lines
  - 6. Capacitors and Dielectrics
- 11. Electric Current, Resistance, and Ohm's Law
  - 1. <u>Introduction to Electric Current, Resistance, and Ohm's</u>
    Law
  - 2. Current
  - 3. Ohm's Law: Resistance and Simple Circuits
  - 4. Resistance and Resistivity
  - 5. Resistors in Series and Parallel
  - 6. <u>Electromotive Force: Terminal Voltage</u>
  - 7. Kirchhoff's Rules
  - 8. Electric Power and Energy
  - 9. Electric Hazards and the Human Body
  - 10. Nerve Conduction—Electrocardiograms
- 12. Magnetism
  - 1. <u>Introduction to Magnetism</u>

- 2. Magnets
- 3. Ferromagnets and Electromagnets
- 4. Magnetic Fields and Magnetic Field Lines
- 5. <u>Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field</u>
- 6. <u>Force on a Moving Charge in a Magnetic Field: Examples and Applications</u>
- 7. Magnetic Force on a Current-Carrying Conductor
- 8. <u>Torque on a Current Loop: Motors and Meters</u>
- 9. Magnetic Fields Produced by Currents: Ampere's Law
- 10. Magnetic Force between Two Parallel Conductors
- 11. More Applications of Magnetism

# 13. Light as a Wave

- 1. Waves
  - 1. Introduction to Waves
  - 2. Period and Frequency in Oscillations
  - 3. Waves
  - 4. Superposition and Interference
- 2. Geometric Optics
  - 1. Introduction to Geometric Optics
  - 2. The Ray Aspect of Light
  - 3. The Law of Reflection
  - 4. The Law of Refraction
  - 5. Total Internal Reflection
- 3. Wave Optics
  - 1. Introduction to Wave Optics
  - 2. The Wave Aspect of Light: Interference
  - 3. <u>Huygens's Principle: Diffraction</u>
  - 4. Young's Double Slit Experiment
  - 5. Multiple Slit Diffraction
  - 6. Single Slit Diffraction

# 14. Modern Physics

# 1. Introduction to Quantum Physics

- 1. Introduction to Quantum Physics
- 2. Quantization of Energy
- 3. The Photoelectric Effect
- 4. Photon Energies and the Electromagnetic Spectrum
- 5. Photon Momentum
- 6. The Particle-Wave Duality
- 7. The Wave Nature of Matter
- 8. Probability: The Heisenberg Uncertainty Principle
- 9. The Particle-Wave Duality Reviewed

# 2. Atomic Physics

- 1. Introduction to Atomic Physics
- 2. <u>Discovery of the Atom</u>
- 3. <u>Discovery of the Parts of the Atom: Electrons and Nuclei</u>
- 4. Bohr's Theory of the Hydrogen Atom
- 5. X Rays: Atomic Origins and Applications
- 6. <u>Applications of Atomic Excitations and De-</u> Excitations
- 7. The Wave Nature of Matter Causes Quantization
- 8. Patterns in Spectra Reveal More Quantization
- 9. Quantum Numbers and Rules
- 10. The Pauli Exclusion Principle
- 15. Atomic Masses
- 16. <u>Useful Information</u>
- 17. Glossary of Key Symbols and Notation

# Preface to Physics This is a preface to the collection Physics

Thanks to OpenStax College, now we have a Physics textbook that is affordable and customizable. This textbook is used in the first year MS2127 Physics module in Singapore Polytechnic. The main changes from the College Physics textbook is the number of contents and the arrangement of the contents. MS2127 deals mainly with classical mechanics, with some topics in electricity, magnetism, wave optics, and introduction to modern physics. It is hoped that with this textbook, students will have more opportunity to own a textbook and get familiar with Physics.

### **Preface**

Welcome to *College Physics*, an OpenStax resource. This textbook was written to increase student access to high-quality learning materials, maintaining highest standards of academic rigor at little to no cost.

# **About OpenStax**

OpenStax is a nonprofit based at Rice University, and it's our mission to improve student access to education. Our first openly licensed college textbook was published in 2012, and our library has since scaled to over 20 books for college and AP courses used by hundreds of thousands of students. Our adaptive learning technology, designed to improve learning outcomes through personalized educational paths, is being piloted in college courses throughout the country. Through our partnerships with philanthropic foundations and our alliance with other educational resource organizations, OpenStax is breaking down the most common barriers to learning and empowering students and instructors to succeed.

# **About OpenStax Resources**

### **Customization**

College Physics is licensed under a Creative Commons Attribution 4.0 International (CC BY) license, which means that you can distribute, remix, and build upon the content, as long as you provide attribution to OpenStax and its content contributors.

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Instructors also have the option of creating a customized version of their OpenStax book. The custom version can be made available to students in low-cost print or digital form through their campus bookstore. Visit your book page on openstax.org for more information.

### **Errata**

All OpenStax textbooks undergo a rigorous review process. However, like any professional-grade textbook, errors sometimes occur. Since our books are web based, we can make updates periodically when deemed pedagogically necessary. If you have a correction to suggest, submit it through the link on your book page on openstax.org. Subject matter experts review all errata suggestions. OpenStax is committed to remaining transparent about all updates, so you will also find a list of past errata changes on your book page on openstax.org.

### **Format**

You can access this textbook for free in web view or PDF through openstax.org, and in low-cost print and iBooks editions.

# **About College Physics**

College Physics meets standard scope and sequence requirements for a two-semester introductory algebra-based physics course. The text is grounded in real-world examples to help students grasp fundamental physics concepts. It requires knowledge of algebra and some trigonometry, but not calculus. College Physics includes learning objectives, concept questions, links to labs and simulations, and ample practice opportunities for traditional physics application problems.

# **Coverage and Scope**

College Physics is organized such that topics are introduced conceptually with a steady progression to precise definitions and analytical applications. The analytical aspect (problem solving) is tied back to the conceptual before moving on to another topic. Each introductory chapter, for example, opens with an engaging photograph relevant to the subject of the chapter and interesting applications that are easy for most students to visualize.

Chapter 1: Introduction: The Nature of Science and Physics

Chapter 2: Kinematics

Chapter 3: Two-Dimensional Kinematics

Chapter 4: Dynamics: Force and Newton's Laws of Motion

Chapter 5: Further Applications of Newton's Laws: Friction, Drag, and

Elasticity

Chapter 6: Uniform Circular Motion and Gravitation

Chapter 7: Work, Energy, and Energy Resources

Chapter 8: Linear Momentum and Collisions

Chapter 9: Statics and Torque

Chapter 10: Rotational Motion and Angular Momentum

Chapter 11: Fluid Statics

Chapter 12: Fluid Dynamics and Its Biological and Medical Applications

Chapter 13: Temperature, Kinetic Theory, and the Gas Laws

Chapter 14: Heat and Heat Transfer Methods

Chapter 15: Thermodynamics

Chapter 16: Oscillatory Motion and Waves

Chapter 17: Physics of Hearing

Chapter 18: Electric Charge and Electric Field

Chapter 19: Electric Potential and Electric Field

Chapter 20: Electric Current, Resistance, and Ohm's Law

Chapter 21: Circuits and DC Instruments

Chapter 22: Magnetism

Chapter 23: Electromagnetic Induction, AC Circuits, and Electrical

Technologies

Chapter 24: Electromagnetic Waves

Chapter 25: Geometric Optics

Chapter 26: Vision and Optical Instruments

Chapter 27: Wave Optics

Chapter 28: Special Relativity

Chapter 29: Introduction to Quantum Physics

Chapter 30: Atomic Physics

Chapter 31: Radioactivity and Nuclear Physics

Chapter 32: Medical Applications of Nuclear Physics

Chapter 33: Particle Physics

Chapter 34: Frontiers of Physics

Appendix A: Atomic Masses

Appendix B: Selected Radioactive Isotopes

Appendix C: Useful Information

Appendix D: Glossary of Key Symbols and Notation

### **Concepts and Calculations**

The ability to calculate does not guarantee conceptual understanding. In order to unify conceptual, analytical, and calculation skills within the learning process, we have integrated Strategies and Discussions throughout the text.

# **Modern Perspective**

The chapters on modern physics are more complete than many other texts on the market, with an entire chapter devoted to medical applications of nuclear physics and another to particle physics. The final chapter of the text, "Frontiers of Physics," is devoted to the most exciting endeavors in physics. It ends with a module titled "Some Questions We Know to Ask."

# **Key Features**

**Modularity** 

This textbook is organized as a collection of modules that can be rearranged and modified to suit the needs of a particular professor or class. That being said, modules often contain references to content in other modules, as most topics in physics cannot be discussed in isolation.

### **Learning Objectives**

Every module begins with a set of learning objectives. These objectives are designed to guide the instructor in deciding what content to include or assign, and to guide the student with respect to what he or she can expect to learn. After completing the module and end-of-module exercises, students should be able to demonstrate mastery of the learning objectives.

#### **Call-Outs**

Key definitions, concepts, and equations are called out with a special design treatment. Call-outs are designed to catch readers' attention, to make it clear that a specific term, concept, or equation is particularly important, and to provide easy reference for a student reviewing content.

### **Key Terms**

Key terms are in bold and are followed by a definition in context. Definitions of key terms are also listed in the Glossary, which appears at the end of the module.

### **Worked Examples**

Worked examples have four distinct parts to promote both analytical and conceptual skills. Worked examples are introduced in words, always using some application that should be of interest. This is followed by a Strategy section that emphasizes the concepts involved and how solving the problem

relates to those concepts. This is followed by the mathematical Solution and Discussion.

Many worked examples contain multiple-part problems to help the students learn how to approach normal situations, in which problems tend to have multiple parts. Finally, worked examples employ the techniques of the problem-solving strategies so that students can see how those strategies succeed in practice as well as in theory.

### **Problem-Solving Strategies**

Problem-solving strategies are first presented in a special section and subsequently appear at crucial points in the text where students can benefit most from them. Problem-solving strategies have a logical structure that is reinforced in the worked examples and supported in certain places by line drawings that illustrate various steps.

### **Misconception Alerts**

Students come to physics with preconceptions from everyday experiences and from previous courses. Some of these preconceptions are misconceptions, and many are very common among students and the general public. Some are inadvertently picked up through misunderstandings of lectures and texts. The Misconception Alerts feature is designed to point these out and correct them explicitly.

#### **Take-Home Investigations**

Take Home Investigations provide the opportunity for students to apply or explore what they have learned with a hands-on activity.

#### **Things Great and Small**

In these special topic essays, macroscopic phenomena (such as air pressure) are explained with submicroscopic phenomena (such as atoms bouncing off walls). These essays support the modern perspective by describing aspects of modern physics before they are formally treated in later chapters. Connections are also made between apparently disparate phenomena.

#### **Simulations**

Where applicable, students are directed to the interactive PHeT physics simulations developed by the University of Colorado. There they can further explore the physics concepts they have learned about in the module.

### **Summary**

Module summaries are thorough and functional and present all important definitions and equations. Students are able to find the definitions of all terms and symbols as well as their physical relationships. The structure of the summary makes plain the fundamental principles of the module or collection and serves as a useful study guide.

#### Glossary

At the end of every module or chapter is a Glossary containing definitions of all of the key terms in the module or chapter.

#### **End-of-Module Problems**

At the end of every chapter is a set of Conceptual Questions and/or skills-based Problems & Exercises. Conceptual Questions challenge students' ability to explain what they have learned conceptually, independent of the mathematical details. Problems & Exercises challenge students to apply both concepts and skills to solve mathematical physics problems. Online,

every other problem includes an answer that students can reveal immediately by clicking on a "Show Solution" button.

In addition to traditional skills-based problems, there are three special types of end-of-module problems: Integrated Concept Problems, Unreasonable Results Problems, and Construct Your Own Problems. All of these problems are indicated with a subtitle preceding the problem.

### **Integrated Concept Problems**

In Integrated Concept Problems, students are asked to apply what they have learned about two or more concepts to arrive at a solution to a problem. These problems require a higher level of thinking because, before solving a problem, students have to recognize the combination of strategies required to solve it.

#### **Unreasonable Results**

In Unreasonable Results Problems, students are challenged to not only apply concepts and skills to solve a problem, but also to analyze the answer with respect to how likely or realistic it really is. These problems contain a premise that produces an unreasonable answer and are designed to further emphasize that properly applied physics must describe nature accurately and is not simply the process of solving equations.

#### **Construct Your Own Problem**

These problems require students to construct the details of a problem, justify their starting assumptions, show specific steps in the problem's solution, and finally discuss the meaning of the result. These types of problems relate well to both conceptual and analytical aspects of physics, emphasizing that physics must describe nature. Often they involve an integration of topics from more than one chapter. Unlike other problems, solutions are not provided since there is no single correct answer.

Instructors should feel free to direct students regarding the level and scope of their considerations. Whether the problem is solved and described correctly will depend on initial assumptions.

### **Additional Resources**

### Student and Instructor Resources

We've compiled additional resources for both students and instructors, including Getting Started Guides, an instructor solution manual, and PowerPoint slides. Instructor resources require a verified instructor account, which can be requested on your openstax.org log-in. Take advantage of these resources to supplement your OpenStax book.

### **Partner Resources**

OpenStax Partners are our allies in the mission to make high-quality learning materials affordable and accessible to students and instructors everywhere. Their tools integrate seamlessly with our OpenStax titles at a low cost. To access the partner resources for your text, visit your book page on openstax.org.

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# Introduction to Science and the Realm of Physics, Physical Quantities, and Units

class="introduction"

```
Galaxies are
as immense
as atoms are
small. Yet the
same laws of
  physics
  describe
both, and all
 the rest of
 nature—an
indication of
     the
 underlying
unity in the
universe. The
   laws of
 physics are
surprisingly
   few in
  number,
implying an
 underlying
simplicity to
  nature's
  apparent
complexity.
   (credit:
NASA, JPL-
 Caltech, P.
  Barmby,
  Harvard-
Smithsonian
```

Center for



What is your first reaction when you hear the word "physics"? Did you imagine working through difficult equations or memorizing formulas that seem to have no real use in life outside the physics classroom? Many people come to the subject of physics with a bit of fear. But as you begin your exploration of this broad-ranging subject, you may soon come to realize that physics plays a much larger role in your life than you first thought, no matter your life goals or career choice.

For example, take a look at the image above. This image is of the Andromeda Galaxy, which contains billions of individual stars, huge clouds of gas, and dust. Two smaller galaxies are also visible as bright blue spots in the background. At a staggering 2.5 million light years from the Earth, this galaxy is the nearest one to our own galaxy (which is called the Milky Way). The stars and planets that make up Andromeda might seem to be the furthest thing from most people's regular, everyday lives. But Andromeda is a great starting point to think about the forces that hold together the universe. The forces that cause Andromeda to act as it does are the same forces we contend with here on Earth, whether we are planning to send a rocket into space or simply raise the walls for a new home. The same gravity that causes the stars of Andromeda to rotate and revolve also causes water to flow over hydroelectric dams here on Earth. Tonight, take a moment to look up at the stars. The forces out there are the same as the ones here on Earth. Through a study of physics, you may gain a greater

understanding of the interconnectedness of everything we can see and know in this universe.

Think now about all of the technological devices that you use on a regular basis. Computers, smart phones, GPS systems, MP3 players, and satellite radio might come to mind. Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above tracks, "invisibility cloaks" that bend light around them, and microscopic robots that fight cancer cells in our bodies. All of these groundbreaking advancements, commonplace or unbelievable, rely on the principles of physics. Aside from playing a significant role in technology, professionals such as engineers, pilots, physicians, physical therapists, electricians, and computer programmers apply physics concepts in their daily work. For example, a pilot must understand how wind forces affect a flight path and a physical therapist must understand how the muscles in the body experience forces as they move and bend. As you will learn in this text, physics principles are propelling new, exciting technologies, and these principles are applied in a wide range of careers.

In this text, you will begin to explore the history of the formal study of physics, beginning with natural philosophy and the ancient Greeks, and leading up through a review of Sir Isaac Newton and the laws of physics that bear his name. You will also be introduced to the standards scientists use when they study physical quantities and the interrelated system of measurements most of the scientific community uses to communicate in a single mathematical language. Finally, you will study the limits of our ability to be accurate and precise, and the reasons scientists go to painstaking lengths to be as clear as possible regarding their own limitations.

### Physics: An Introduction

- Explain the difference between a principle and a law.
- Explain the difference between a model and a theory.



The flight formations of migratory birds such as Canada geese are governed by the laws of physics. (credit: David Merrett)

The physical universe is enormously complex in its detail. Every day, each of us observes a great variety of objects and phenomena. Over the centuries, the curiosity of the human race has led us collectively to explore and catalog a tremendous wealth of information. From the flight of birds to the colors of flowers, from lightning to gravity, from quarks to clusters of galaxies, from the flow of time to the mystery of the creation of the universe, we have asked questions and assembled huge arrays of facts. In the face of all these details, we have discovered that a surprisingly small and unified set of physical laws can explain what we observe. As humans, we make generalizations and seek order. We have found that nature is remarkably cooperative—it exhibits the *underlying order and simplicity* we so value.

It is the underlying order of nature that makes science in general, and physics in particular, so enjoyable to study. For example, what do a bag of chips and a car battery have in common? Both contain energy that can be

converted to other forms. The law of conservation of energy (which says that energy can change form but is never lost) ties together such topics as food calories, batteries, heat, light, and watch springs. Understanding this law makes it easier to learn about the various forms energy takes and how they relate to one another. Apparently unrelated topics are connected through broadly applicable physical laws, permitting an understanding beyond just the memorization of lists of facts.

The unifying aspect of physical laws and the basic simplicity of nature form the underlying themes of this text. In learning to apply these laws, you will, of course, study the most important topics in physics. More importantly, you will gain analytical abilities that will enable you to apply these laws far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any professional career you choose to pursue. This module discusses the realm of physics (to define what physics is), some applications of physics (to illustrate its relevance to other disciplines), and more precisely what constitutes a physical law (to illuminate the importance of experimentation to theory).

# **Science and the Realm of Physics**

Science consists of the theories and laws that are the general truths of nature as well as the body of knowledge they encompass. Scientists are continually trying to expand this body of knowledge and to perfect the expression of the laws that describe it. **Physics** is concerned with describing the interactions of energy, matter, space, and time, and it is especially interested in what fundamental mechanisms underlie every phenomenon. The concern for describing the basic phenomena in nature essentially defines the *realm of physics*.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone ([link]). Physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and

circuit layout when building the smart phone. Next, consider a GPS system. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics equations to determine the travel time from one location to another.



The Apple "iPhone" is a common smart phone with a GPS function. **Physics** describes the way that electricity flows through the circuits of this device. Engineers use their knowledge of physics to construct an

iPhone with features that consumers will enjoy. One specific feature of an iPhone is the **GPS** function. GPS uses physics equations to determine the driving time between two locations on a map. (credit: @gletham GIS, Social, Mobile Tech Images)

# **Applications of Physics**

You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. It can help you understand how microwave ovens work, why metals should not be put into them, and why they might affect pacemakers. (See [link] and [link].) Physics allows you to understand the hazards of radiation and rationally evaluate these hazards more easily. Physics also explains the reason why a black car radiator helps remove heat in a car engine, and it explains why a white roof helps keep the inside of a house cool. Similarly, the operation of a car's ignition system as well as the transmission of electrical signals through our body's nervous system are

much easier to understand when you think about them in terms of basic physics.

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry, for example—since it deals with the interactions of atoms and molecules—is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability, and is involved in the acoustics, heating, lighting, and cooling of buildings. Parts of geology rely heavily on physics, such as radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes ([link]] and [link]). On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics; for example, cancer radiotherapy uses ionizing radiation. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

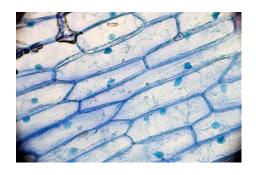
It is not necessary to formally study all applications of physics. What is most useful is knowledge of the basic laws of physics and a skill in the analytical methods for applying them. The study of physics also can improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences, and the study of physics makes other sciences easier to understand.



The laws of physics help us understand how common appliances work. For example, the laws of physics can help explain how microwave ovens heat up food, and they also help us understand why it is dangerous to place metal objects in a microwave oven. (credit: MoneyBlogNewz)

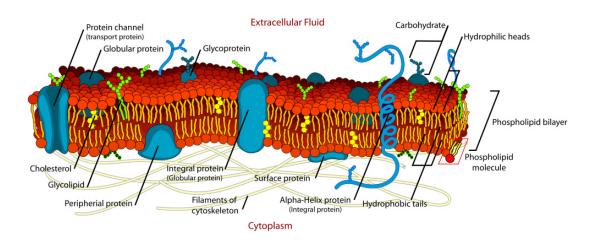


These two applications of physics have more in common than meets the eye. Microwave ovens use electromagnetic waves to heat food. Magnetic resonance imaging (MRI) also uses electromagnetic waves to yield an image of the brain, from which the exact location of tumors can be determined. (credit: Rashmi Chawla, Daniel Smith, and Paul E. Marik)



Physics, chemistry,

and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (credit: Umberto Salvagnin)



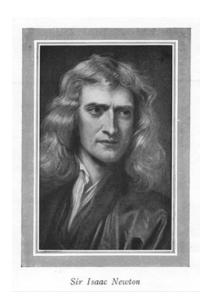
An artist's rendition of the the structure of a cell membrane.

Membranes form the boundaries of animal cells and are complex in structure and function. Many of the most fundamental properties of life, such as the firing of nerve cells, are related to membranes. The disciplines of biology, chemistry, and physics all help us understand the membranes of animal cells. (credit: Mariana Ruiz)

# Models, Theories, and Laws; The Role of Experimentation

The laws of nature are concise descriptions of the universe around us; they are human statements of the underlying laws or rules that all natural processes follow. Such laws are intrinsic to the universe; humans did not

create them and so cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. (See [link] and [link].) The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be.



Isaac Newton
(1642–1727) was
very reluctant to
publish his
revolutionary
work and had to
be convinced to
do so. In his later
years, he stepped
down from his
academic post and
became
exchequer of the
Royal Mint. He
took this post

seriously, inventing reeding (or creating ridges) on the edge of coins to prevent unscrupulous people from trimming the silver off of them before using them as currency. (credit: Arthur Shuster and Arthur E. Shipley: Britain's Heritage of Science. London, 1917.)



Marie Curie (1867–1934) sacrificed

monetary assets to help finance her early research and damaged her physical wellbeing with radiation exposure. She is the only person to win Nobel prizes in both physics and chemistry. One of her daughters also won a Nobel Prize. (credit: Wikimedia Commons)

We all are curious to some extent. We look around, make generalizations, and try to understand what we see—for example, we look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how the data may be organized and unified. We then formulate models, theories, and laws based on the data we have collected and analyzed to generalize and communicate the results of these experiments.

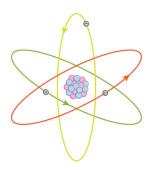
A **model** is a representation of something that is often too difficult (or impossible) to display directly. While a model is justified with experimental proof, it is only accurate under limited situations. An example is the planetary model of the atom in which electrons are pictured as orbiting the

nucleus, analogous to the way planets orbit the Sun. (See [link].) We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases (atomic spectra). Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation. A **theory** is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. Some theories include models to help visualize phenomena, whereas others do not. Newton's theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, is a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

A law uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation *law* is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton's second law of motion, which relates force, mass, and acceleration by the simple equation  ${\bf F}=m{\bf a}$ . A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a theory is much more complex and dynamic. A law describes a single action, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Less broadly applicable statements are usually called principles (such as Pascal's principle, which is applicable only in fluids), but the distinction

between laws and principles often is not carefully made.



What is a model? This planetary model of the atom shows electrons orbiting the nucleus. It is a drawing that we use to form a mental image of the atom that we cannot see directly with our eyes because it is too small.

### Note:

### Models, Theories, and Laws

Models, theories, and laws are used to help scientists analyze the data they have already collected. However, often after a model, theory, or law has been developed, it points scientists toward new discoveries they would not otherwise have made.

The models, theories, and laws we devise sometimes *imply the existence of objects or phenomena as yet unobserved*. These predictions are remarkable triumphs and tributes to the power of science. It is the underlying order in the universe that enables scientists to make such spectacular predictions. However, if *experiment* does not verify our predictions, then the theory or law is wrong, no matter how elegant or convenient it is. Laws can never be known with absolute certainty because it is impossible to perform every imaginable experiment in order to confirm a law in every possible scenario. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

The study of science in general and physics in particular is an adventure much like the exploration of uncharted ocean. Discoveries are made; models, theories, and laws are formulated; and the beauty of the physical universe is made more sublime for the insights gained.

### Note:

### The Scientific Method

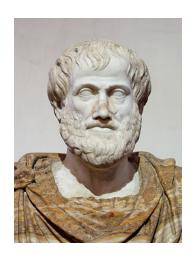
As scientists inquire and gather information about the world, they follow a process called the **scientific method**. This process typically begins with an observation and question that the scientist will research. Next, the scientist

typically performs some research about the topic and then devises a hypothesis. Then, the scientist will test the hypothesis by performing an experiment. Finally, the scientist analyzes the results of the experiment and draws a conclusion. Note that the scientific method can be applied to many situations that are not limited to science, and this method can be modified to suit the situation.

Consider an example. Let us say that you try to turn on your car, but it will not start. You undoubtedly wonder: Why will the car not start? You can follow a scientific method to answer this question. First off, you may perform some research to determine a variety of reasons why the car will not start. Next, you will state a hypothesis. For example, you may believe that the car is not starting because it has no engine oil. To test this, you open the hood of the car and examine the oil level. You observe that the oil is at an acceptable level, and you thus conclude that the oil level is not contributing to your car issue. To troubleshoot the issue further, you may devise a new hypothesis to test and then repeat the process again.

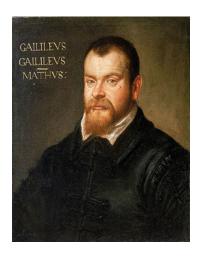
# The Evolution of Natural Philosophy into Modern Physics

Physics was not always a separate and distinct discipline. It remains connected to other sciences to this day. The word *physics* comes from Greek, meaning nature. The study of nature came to be called "natural philosophy." From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, physics, mathematics, and medicine. Over the last few centuries, the growth of knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. (See [link], [link], and [link].) Physics as it developed from the Renaissance to the end of the 19th century is called **classical physics**. It was transformed into modern physics by revolutionary discoveries made starting at the beginning of the 20th century.



Over the centuries, natural philosophy has evolved into more specialized disciplines, as illustrated by the contributions of some of the greatest minds in history. The Greek philosopher Aristotle (384-322 B.C.) wrote on a broad range of topics including physics, animals, the soul, politics, and poetry. (credit: Jastrow

# (2006)/Ludovisi Collection)



Galileo Galilei
(1564–1642) laid
the foundation of
modern
experimentation
and made
contributions in
mathematics,
physics, and
astronomy.
(credit:
Domenico
Tintoretto)



Niels Bohr
(1885–1962)
made
fundamental
contributions to
the development
of quantum
mechanics, one
part of modern
physics. (credit:
United States
Library of
Congress Prints
and Photographs
Division)

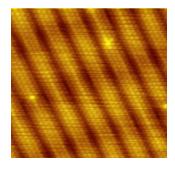
Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields, such as the field generated by the Earth, can be involved. Because humans live under such circumstances, classical physics seems intuitively reasonable, while many aspects of modern physics seem bizarre. This is why models are so useful in modern physics—they let us

conceptualize phenomena we do not ordinarily experience. We can relate to models in human terms and visualize what happens when objects move at high speeds or imagine what objects too small to observe with our senses might be like. For example, we can understand an atom's properties because we can picture it in our minds, although we have never seen an atom with our eyes. New tools, of course, allow us to better picture phenomena we cannot see. In fact, new instrumentation has allowed us in recent years to actually "picture" the atom.

# Note:

# Limits on the Laws of Classical Physics

For the laws of classical physics to apply, the following criteria must be met: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields (such as the field generated by the Earth) can be involved.



Using a scanning tunneling microscope (STM), scientists can see the individual atoms that

compose this sheet of gold. (credit: Erwinrossen)

Some of the most spectacular advances in science have been made in modern physics. Many of the laws of classical physics have been modified or rejected, and revolutionary changes in technology, society, and our view of the universe have resulted. Like science fiction, modern physics is filled with fascinating objects beyond our normal experiences, but it has the advantage over science fiction of being very real. Why, then, is the majority of this text devoted to topics of classical physics? There are two main reasons: Classical physics gives an extremely accurate description of the universe under a wide range of everyday circumstances, and knowledge of classical physics is necessary to understand modern physics.

**Modern physics** itself consists of the two revolutionary theories, relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. **Relativity** must be used whenever an object is traveling at greater than about 1% of the speed of light or experiences a strong gravitational field such as that near the Sun. **Quantum mechanics** must be used for objects smaller than can be seen with a microscope. The combination of these two theories is *relativistic quantum mechanics*, and it describes the behavior of small objects traveling at high speeds or experiencing a strong gravitational field. Relativistic quantum mechanics is the best universally applicable theory we have. Because of its mathematical complexity, it is used only when necessary, and the other theories are used whenever they will produce sufficiently accurate results. We will find, however, that we can do a great deal of modern physics with the algebra and trigonometry used in this text.

**Exercise:** 

**Check Your Understanding** 

# **Problem:**

A friend tells you he has learned about a new law of nature. What can you know about the information even before your friend describes the law? How would the information be different if your friend told you he had learned about a scientific theory rather than a law?

# **Solution:**

Without knowing the details of the law, you can still infer that the information your friend has learned conforms to the requirements of all laws of nature: it will be a concise description of the universe around us; a statement of the underlying rules that all natural processes follow. If the information had been a theory, you would be able to infer that the information will be a large-scale, broadly applicable generalization.

## Note:

PhET Explorations: Equation Grapher

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. y = bx) to see how they add to generate the polynomial curve. https://phet.colorado.edu/sims/equation-grapher/equation-grapher en.html

# Summary

- Science seeks to discover and describe the underlying order and simplicity in nature.
- Physics is the most basic of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.

# **Conceptual Questions**

# **Exercise:**

# **Problem:**

Models are particularly useful in relativity and quantum mechanics, where conditions are outside those normally encountered by humans. What is a model?

# **Exercise:**

**Problem:** How does a model differ from a theory?

# **Exercise:**

# **Problem:**

If two different theories describe experimental observations equally well, can one be said to be more valid than the other (assuming both use accepted rules of logic)?

## **Exercise:**

**Problem:** What determines the validity of a theory?

## **Exercise:**

## **Problem:**

Certain criteria must be satisfied if a measurement or observation is to be believed. Will the criteria necessarily be as strict for an expected result as for an unexpected result?

## **Exercise:**

# **Problem:**

Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?

## **Exercise:**

# **Problem:**

Classical physics is a good approximation to modern physics under certain circumstances. What are they?

## **Exercise:**

**Problem:** When is it *necessary* to use relativistic quantum mechanics?

# **Exercise:**

## **Problem:**

Can classical physics be used to accurately describe a satellite moving at a speed of 7500 m/s? Explain why or why not.

# Glossary

# classical physics

physics that was developed from the Renaissance to the end of the 19th century

# physics

the science concerned with describing the interactions of energy, matter, space, and time; it is especially interested in what fundamental mechanisms underlie every phenomenon

## model

representation of something that is often too difficult (or impossible) to display directly

# theory

an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

## law

a description, using concise language or a mathematical formula, a generalized pattern in nature that is supported by scientific evidence

# and repeated experiments

# scientific method

a method that typically begins with an observation and question that the scientist will research; next, the scientist typically performs some research about the topic and then devises a hypothesis; then, the scientist will test the hypothesis by performing an experiment; finally, the scientist analyzes the results of the experiment and draws a conclusion

# modern physics

the study of relativity, quantum mechanics, or both

# relativity

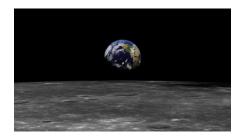
the study of objects moving at speeds greater than about 1% of the speed of light, or of objects being affected by a strong gravitational field

# quantum mechanics

the study of objects smaller than can be seen with a microscope

### Physical Quantities and Units

- Perform unit conversions both in the SI and English units.
- Explain the most common prefixes in the SI units and be able to write them in scientific notation.

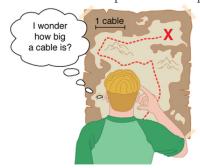


The distance from Earth to the Moon may seem immense, but it is just a tiny fraction of the distances from Earth to other celestial bodies. (credit: NASA)

The range of objects and phenomena studied in physics is immense. From the incredibly short lifetime of a nucleus to the age of the Earth, from the tiny sizes of sub-nuclear particles to the vast distance to the edges of the known universe, from the force exerted by a jumping flea to the force between Earth and the Sun, there are enough factors of 10 to challenge the imagination of even the most experienced scientist. Giving numerical values for physical quantities and equations for physical principles allows us to understand nature much more deeply than does qualitative description alone. To comprehend these vast ranges, we must also have accepted units in which to express them. And we shall find that (even in the potentially mundane discussion of meters, kilograms, and seconds) a profound simplicity of nature appears—all physical quantities can be expressed as combinations of only four fundamental physical quantities: length, mass, time, and electric current.

We define a **physical quantity** either by *specifying how it is measured* or by *stating how it is calculated* from other measurements. For example, we define distance and time by specifying methods for measuring them, whereas we define *average speed* by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way. (See [link].)



Distances given in unknown units are maddeningly useless.

There are two major systems of units used in the world: **SI units** (also known as the metric system) and **English units** (also known as the customary or imperial system). **English units** were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard; the metric system is also the standard system agreed upon by scientists and mathematicians. The acronym "SI" is derived from the French *Système International*.

#### SI Units: Fundamental and Derived Units

[link] gives the fundamental SI units that are used throughout this textbook. This text uses non-SI units in a few applications where they are in very common use, such as the measurement of blood pressure in millimeters of mercury (mm Hg). Whenever non-SI units are discussed, they will be tied to SI units through conversions.

Length	Mass	Time	Electric Current
meter (m)	kilogram (kg)	second (s)	ampere (A)

## Fundamental SI Units

It is an intriguing fact that some physical quantities are more fundamental than others and that the most fundamental physical quantities can be defined *only* in terms of the procedure used to measure them. The units in which they are measured are thus called **fundamental units**. In this textbook, the fundamental physical quantities are taken to be length, mass, time, and electric current. (Note that electric current will not be introduced until much later in this text.) All other physical quantities, such as force and electric charge, can be expressed as algebraic combinations of length, mass, time, and current (for example, speed is length divided by time); these units are called **derived units**.

## Units of Time, Length, and Mass: The Second, Meter, and Kilogram

## The Second

The SI unit for time, the **second**(abbreviated s), has a long history. For many years it was defined as 1/86,400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a non-varying, or constant, physical phenomenon (because the solar day is getting longer due to very gradual slowing of the Earth's rotation). Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. In 1967 the second was redefined as the time required for 9,192,631,770 of these vibrations. (See [link].) Accuracy in the fundamental units is essential, because all measurements are ultimately expressed in terms of fundamental units and can be no more accurate than are the fundamental units themselves.



An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic fountain nearly 30 feet tall! (credit: Steve Jurvetson/Flickr)

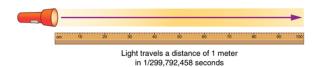
## The Meter

The SI unit for length is the **meter** (abbreviated m); its definition has also changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar now kept near Paris. By 1960, it had become possible to define the meter even more accurately in terms of the wavelength of light, so it was again redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second. (See [link].) This change defines the speed of light to be exactly 299,792,458 meters per second. The length of the meter will change if the speed of light is someday measured with greater accuracy.

## The Kilogram

The SI unit for mass is the **kilogram** (abbreviated kg); it is defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. Exact replicas of the standard kilogram are also kept at the United States' National Institute of Standards

and Technology, or NIST, located in Gaithersburg, Maryland outside of Washington D.C., and at other locations around the world. The determination of all other masses can be ultimately traced to a comparison with the standard mass.



The meter is defined to be the distance light travels in 1/299,792,458 of a second in a vacuum. Distance traveled is speed multiplied by time.

Electric current and its accompanying unit, the ampere, will be introduced in <u>Introduction to Electric Current, Resistance, and Ohm's Law</u> when electricity and magnetism are covered. The initial modules in this textbook are concerned with mechanics, fluids, heat, and waves. In these subjects all pertinent physical quantities can be expressed in terms of the fundamental units of length, mass, and time.

## **Metric Prefixes**

SI units are part of the **metric system**. The metric system is convenient for scientific and engineering calculations because the units are categorized by factors of 10. [<u>link</u>] gives metric prefixes and symbols used to denote various factors of 10.

Metric systems have the advantage that conversions of units involve only powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as the system of U.S. customary units, the relationships are not as simple—there are 12 inches in a foot, 5280 feet in a mile, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by using an appropriate metric prefix. For example, distances in meters are suitable in construction, while distances in kilometers are appropriate for air travel, and the tiny measure of nanometers are convenient in optical design. With the metric system there is no need to invent new units for particular applications.

The term **order of magnitude** refers to the scale of a value expressed in the metric system. Each power of 10 in the metric system represents a different order of magnitude. For example,  $10^1$ ,  $10^2$ ,  $10^3$ , and so forth are all different orders of magnitude. All quantities that can be expressed as a product of a specific power of 10 are said to be of the *same* order of magnitude. For example, the number 800 can be written as  $8 \times 10^2$ , and the number 450 can be written as  $4.5 \times 10^2$ . Thus, the numbers 800 and 450 are of the same order of magnitude:  $10^2$ . Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of  $10^{-9}$  m, while the diameter of the Sun is on the order of  $10^9$  m.

#### Note:

The Quest for Microscopic Standards for Basic Units

The fundamental units described in this chapter are those that produce the greatest accuracy and precision in measurement. There is a sense among physicists that, because there is an underlying microscopic substructure to matter, it would be most satisfying to base our standards of measurement on microscopic objects and fundamental physical phenomena such as the speed of light. A microscopic standard has been accomplished for the standard of time, which is based on the oscillations of the cesium atom.

The standard for length was once based on the wavelength of light (a small-scale length) emitted by a certain type of atom, but it has been supplanted by the more precise measurement of the speed of light. If it becomes possible to measure the mass of atoms or a particular arrangement of atoms such as a silicon sphere to greater precision than the kilogram standard, it may become possible to base mass measurements on the small scale. There are also possibilities that electrical phenomena on the small scale may someday allow us to base a unit of charge on the charge of electrons and protons, but at present current and charge are related to large-scale currents and forces between wires.

Prefix	Symbol	Value[footnote] See Appendix A for a discussion of powers of 10.	Example (some are approximate)			
exa	E	$10^{18}$	exameter	Em	$10^{18}\mathrm{m}$	distance light travels in a century
peta	P	$10^{15}$	petasecond	Ps	$10^{15}\mathrm{s}$	30 million years
tera	Т	$10^{12}$	terawatt	TW	$10^{12}\mathrm{W}$	powerful laser output
giga	G	10 <sup>9</sup>	gigahertz	GHz	$10^9\mathrm{Hz}$	a microwave frequency
mega	M	$10^6$	megacurie	MCi	$10^6\mathrm{Ci}$	high radioactivity
kilo	k	$10^3$	kilometer	km	$10^3\mathrm{m}$	about 6/10 mile
hecto	h	$10^2$	hectoliter	hL	$10^2\mathrm{L}$	26 gallons

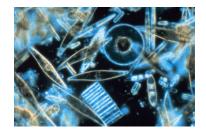
Prefix	Symbol	Value[footnote] See Appendix A for a discussion of powers of 10.	Example (sor	ne are apj	proximate)	
deka	da	$10^1$	dekagram	dag	$10^1\mathrm{g}$	teaspoon of butter
_	_	10 <sup>0</sup> (=1)				
deci	d	$10^{-1}$	deciliter	dL	$10^{-1}\mathrm{L}$	less than half a soda
centi	С	$10^{-2}$	centimeter	cm	$10^{-2}\mathrm{m}$	fingertip thickness
milli	m	$10^{-3}$	millimeter	mm	$10^{-3}\mathrm{m}$	flea at its shoulders
micro	μ	$10^{-6}$	micrometer	μm	$10^{-6}\mathrm{m}$	detail in microscope
nano	n	$10^{-9}$	nanogram	ng	$10^{-9}\mathrm{g}$	small speck of dust
pico	p	$10^{-12}$	picofarad	pF	$10^{-12}{ m F}$	small capacitor in radio
femto	f	$10^{-15}$	femtometer	fm	$10^{-15}{ m m}$	size of a proton
atto	a	$10^{-18}$	attosecond	as	$10^{-18}{ m s}$	time light crosses an atom

Metric Prefixes for Powers of 10 and their Symbols

# **Known Ranges of Length, Mass, and Time**

The vastness of the universe and the breadth over which physics applies are illustrated by the wide range of examples of known lengths, masses, and times in [link]. Examination of this table will give you some

feeling for the range of possible topics and numerical values. (See [link] and [link].)



Tiny phytoplankton swims among crystals of ice in the Antarctic Sea. They range from a few micrometers to as much as 2 millimeters in length. (credit: Prof. Gordon T. Taylor, Stony Brook University; NOAA Corps Collections)



Galaxies collide 2.4
billion light years away
from Earth. The
tremendous range of
observable phenomena in
nature challenges the
imagination. (credit:
NASA/CXC/UVic./A.
Mahdavi et al.
Optical/lensing:
CFHT/UVic./H. Hoekstra
et al.)

## **Unit Conversion and Dimensional Analysis**

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you will need to convert units of feet to miles.

Let us consider a simple example of how to convert units. Let us say that we want to convert 80 meters (m) to kilometers (km).

The first thing to do is to list the units that you have and the units that you want to convert to. In this case, we have units in *meters* and we want to convert to *kilometers*.

Next, we need to determine a **conversion factor** relating meters to kilometers. A conversion factor is a ratio expressing how many of one unit are equal to another unit. For example, there are 12 inches in 1 foot, 100 centimeters in 1 meter, 60 seconds in 1 minute, and so on. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor so that the units cancel out, as shown:

## **Equation:**

$$80\,\mathrm{m} imes rac{1\ \mathrm{km}}{1000\,\mathrm{m}} = 0.080\ \mathrm{km}.$$

Note that the unwanted m unit cancels, leaving only the desired km unit. You can use this method to convert between any types of unit.

Click [link] for a more complete list of conversion factors.

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
$10^{-18}$	Present experimental limit to smallest observable detail	$10^{-30}$	Mass of an electron $\left(9.11  imes 10^{-31} \; \mathrm{kg} \right)$	$10^{-23}$	Time for light to cross a proton
$10^{-15}$	Diameter of a proton	$10^{-27}$	Mass of a hydrogen atom $\left(1.67 \times 10^{-27} \; \mathrm{kg}\right)$	$10^{-22}$	Mean life of an extremely unstable nucleus

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
$10^{-14}$	Diameter of a uranium nucleus	$10^{-15}$	Mass of a bacterium	$10^{-15}$	Time for one oscillation of visible light
$10^{-10}$	Diameter of a hydrogen atom	$10^{-5}$	Mass of a mosquito	$10^{-13}$	Time for one vibration of an atom in a solid
$10^{-8}$	Thickness of membranes in cells of living organisms	$10^{-2}$	Mass of a hummingbird	$10^{-8}$	Time for one oscillation of an FM radio wave
$10^{-6}$	Wavelength of visible light	1	Mass of a liter of water (about a quart)	$10^{-3}$	Duration of a nerve impulse
$10^{-3}$	Size of a grain of sand	$10^2$	Mass of a person	1	Time for one heartbeat
1	Height of a 4-year- old child	$10^3$	Mass of a car	$10^5$	One day $\left(8.64  imes 10^4  \mathrm{s}  ight)$
$10^2$	Length of a football field	108	Mass of a large ship	$10^7$	One year (y) $\left(3.16 \times 10^7  \mathrm{s} \right)$
$10^4$	Greatest ocean depth	$10^{12}$	Mass of a large iceberg	$10^9$	About half the life expectancy of a human
$10^7$	Diameter of the Earth	$10^{15}$	Mass of the nucleus of a comet	$10^{11}$	Recorded history
$10^{11}$	Distance from the Earth to the Sun	$10^{23}$	Mass of the Moon $\left(7.35  imes 10^{22} \;  ext{kg} \right)$	$10^{17}$	Age of the Earth
$10^{16}$	Distance traveled by light in 1 year (a light year)	$10^{25}$	Mass of the Earth $\left(5.97  imes 10^{24} \;  ext{kg}  ight)$	$10^{18}$	Age of the universe
$10^{21}$	Diameter of the Milky Way galaxy	$10^{30}$	Mass of the Sun $\left(1.99  imes 10^{30} \;  ext{kg}  ight)$		

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
$10^{22}$	Distance from the Earth to the nearest large galaxy (Andromeda)	$10^{42}$	Mass of the Milky Way galaxy (current upper limit)		
$10^{26}$	Distance from the Earth to the edges of the known universe	$10^{53}$	Mass of the known universe (current upper limit)		

Approximate Values of Length, Mass, and Time

## **Example:**

## **Unit Conversions: A Short Drive Home**

Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)

### Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

## Solution for (a)

(1) Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

## **Equation:**

average speed 
$$=\frac{\text{distance}}{\text{time}}$$
.

(2) Substitute the given values for distance and time.

## **Equation:**

average speed = 
$$\frac{10.0 \text{ km}}{20.0 \text{ min}} = 0.500 \frac{\text{km}}{\text{min}}$$
.

(3) Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is 60 min/hr. Thus,

### **Equation:**

average speed = 
$$0.500 \frac{\text{km}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ h}} = 30.0 \frac{\text{km}}{\text{h}}$$
.

### Discussion for (a)

To check your answer, consider the following:

(1) Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows:

## **Equation:**

$$\frac{\mathrm{km}}{\mathrm{min}} \times \frac{1 \; \mathrm{hr}}{60 \; \mathrm{min}} = \frac{1}{60} \frac{\mathrm{km} \cdot \mathrm{hr}}{\mathrm{min}^2},$$

which are obviously not the desired units of km/h.

- (2) Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.
- (3) Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/hr does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 minutes, so the precision of the conversion factor is perfect.
- (4) Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

### Solution for (b)

There are several ways to convert the average speed into meters per second.

- (1) Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.
- (2) Multiplying by these yields

## **Equation:**

$$\label{eq:average speed} \text{Average speed} = 30.0 \frac{\text{km}}{\text{h}} \times \frac{1 \text{ h}}{3,\!600 \text{ s}} \times \frac{1,\!000 \text{ m}}{1 \text{ km}},$$

### **Equation:**

Average speed = 
$$8.33 \frac{\text{m}}{\text{s}}$$
.

## Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits. Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces? The module <u>Accuracy, Precision, and Significant Figures</u> will help you answer these questions.

#### Note:

### Nonstandard Units

While there are numerous types of units that we are all familiar with, there are others that are much more obscure. For example, a **firkin** is a unit of volume that was once used to measure beer. One firkin equals about 34 liters. To learn more about nonstandard units, use a dictionary or encyclopedia to research different "weights and measures." Take note of any unusual units, such as a barleycorn, that are not listed in the text. Think about how the unit is defined and state its relationship to SI units.

Exercise:

## **Check Your Understanding**

#### **Problem:**

Some hummingbirds beat their wings more than 50 times per second. A scientist is measuring the time it takes for a hummingbird to beat its wings once. Which fundamental unit should the scientist use to describe the measurement? Which factor of 10 is the scientist likely to use to describe the motion precisely? Identify the metric prefix that corresponds to this factor of 10.

### **Solution:**

The scientist will measure the time between each movement using the fundamental unit of seconds. Because the wings beat so fast, the scientist will probably need to measure in milliseconds, or  $10^{-3}$  seconds. (50 beats per second corresponds to 20 milliseconds per beat.)

Exercise:

### **Check Your Understanding**

### **Problem:**

One cubic centimeter is equal to one milliliter. What does this tell you about the different units in the SI metric system?

### **Solution:**

The fundamental unit of length (meter) is probably used to create the derived unit of volume (liter). The measure of a milliliter is dependent on the measure of a centimeter.

## **Summary**

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- Units are standards for expressing and comparing the measurement of physical quantities. All units can be expressed as combinations of four fundamental units.
- The four fundamental units we will use in this text are the meter (for length), the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.
- The four fundamental units are abbreviated as follows: meter, m; kilogram, kg; second, s; and ampere, A. The metric system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.

## **Conceptual Questions**

## Exercise:

**Problem:** Identify some advantages of metric units.

### **Problems & Exercises**

## **Exercise:**

### **Problem:**

The speed limit on some interstate highways is roughly 100 km/h. (a) What is this in meters per second? (b) How many miles per hour is this?

### **Solution:**

a. 27.8 m/s b. 62.1 mph

#### **Exercise:**

#### **Problem:**

A car is traveling at a speed of 33 m/s. (a) What is its speed in kilometers per hour? (b) Is it exceeding the 90 km/h speed limit?

#### **Exercise:**

### **Problem:**

Show that  $1.0~\rm m/s=3.6~\rm km/h$ . Hint: Show the explicit steps involved in converting  $1.0~\rm m/s=3.6~\rm km/h$ .

### **Solution:**

$$\begin{split} &\frac{1.0\,\mathrm{m}}{\mathrm{s}} = \frac{1.0\,\mathrm{m}}{\mathrm{s}} \times \frac{3600\,\mathrm{s}}{1\,\mathrm{hr}} \times \frac{1\,\mathrm{km}}{1000\,\mathrm{m}} \\ &= 3.6\,\mathrm{km/h}. \end{split}$$

#### **Exercise:**

### **Problem:**

American football is played on a 100-yd-long field, excluding the end zones. How long is the field in meters? (Assume that 1 meter equals 3.281 feet.)

#### **Exercise:**

### **Problem:**

Soccer fields vary in size. A large soccer field is 115 m long and 85 m wide. What are its dimensions in feet and inches? (Assume that 1 meter equals 3.281 feet.)

### **Solution:**

length: 377 ft;  $4.53 \times 10^3$  in. width: 280 ft;  $3.3 \times 10^3$  in.

### **Exercise:**

### **Problem:**

What is the height in meters of a person who is 6 ft 1.0 in. tall? (Assume that 1 meter equals 39.37 in.)

### **Exercise:**

## **Problem:**

Mount Everest, at 29,028 feet, is the tallest mountain on the Earth. What is its height in kilometers? (Assume that 1 kilometer equals 3,281 feet.)

### **Solution:**

8.847 km

#### **Exercise:**

**Problem:** The speed of sound is measured to be 342 m/s on a certain day. What is this in km/h?

## **Exercise:**

#### **Problem:**

Tectonic plates are large segments of the Earth's crust that move slowly. Suppose that one such plate has an average speed of 4.0 cm/year. (a) What distance does it move in 1 s at this speed? (b) What is its speed in kilometers per million years?

### **Solution:**

- (a)  $1.3 \times 10^{-9}$  m
- (b) 40 km/My

#### **Exercise:**

## **Problem:**

(a) Refer to [link] to determine the average distance between the Earth and the Sun. Then calculate the average speed of the Earth in its orbit in kilometers per second. (b) What is this in meters per second?

## **Glossary**

### physical quantity

a characteristic or property of an object that can be measured or calculated from other measurements

#### units

a standard used for expressing and comparing measurements

### SI units

the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams

## **English** units

system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds

### fundamental units

units that can only be expressed relative to the procedure used to measure them

```
derived units
```

units that can be calculated using algebraic combinations of the fundamental units

### second

the SI unit for time, abbreviated (s)

### meter

the SI unit for length, abbreviated (m)

## kilogram

the SI unit for mass, abbreviated (kg)

## metric system

a system in which values can be calculated in factors of 10

## order of magnitude

refers to the size of a quantity as it relates to a power of 10

## conversion factor

a ratio expressing how many of one unit are equal to another unit

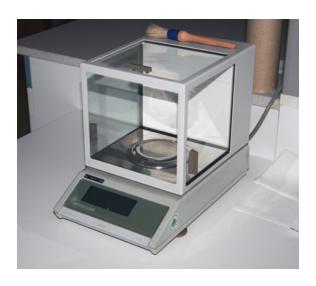
# Accuracy, Precision, and Significant Figures

- Determine the appropriate number of significant figures in both addition and subtraction, as well as multiplication and division calculations.
- Calculate the percent uncertainty of a measurement.



A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The "known masses" are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams.

(credit: Serge Melki)



Many mechanical balances, such as double-pan balances, have been replaced by digital scales, which can typically measure the mass of an object more precisely. Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, many digital scales can measure the mass of an object up to the nearest thousandth of a gram. (credit: Karel Jakubec)

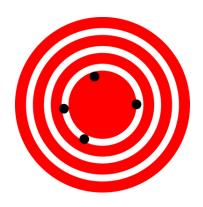
# **Accuracy and Precision of a Measurement**

Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard computer paper. The packaging in which you purchased the paper states that it is 11.0 inches long. You measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in.

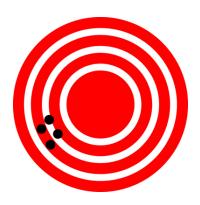
These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate.

The **precision** of a measurement system is refers to how close the agreement is between repeated measurements (which are repeated under the same conditions). Consider the example of the paper measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements would be to determine the range, or difference, between the lowest and the highest measured values. In that case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by at most 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9, 11.1, and 11.9, then the measurements would not be very precise because there would be significant variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider an example of a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull's-eye target, and think of each GPS attempt to locate the restaurant as a black dot. In [link], you can see that the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in [link], the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system.



A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (credit: Dark Evil)



In this figure, the dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (credit: Dark Evil)

# **Accuracy, Precision, and Uncertainty**

The degree of accuracy and precision of a measuring system are related to the **uncertainty** in the measurements. Uncertainty is a quantitative measure of how much your measured values deviate from a standard or expected value. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 in., plus or minus 0.2 in. The uncertainty in a measurement, A, is often denoted as  $A \pm \delta A$ . In our paper example, the length of the paper could be expressed as  $A \pm \delta A$ . In our paper example, the length of the paper could be expressed as  $A \pm \delta A$ .

The factors contributing to uncertainty in a measurement include:

- 1. Limitations of the measuring device,
- 2. The skill of the person making the measurement,
- 3. Irregularities in the object being measured,
- 4. Any other factors that affect the outcome (highly dependent on the situation).

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 in., the person using the ruler has bad eyesight, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

## Note:

Making Connections: Real-World Connections – Fevers or Chills?

Uncertainty is a critical piece of information, both in physics and in many other real-world applications. Imagine you are caring for a sick child. You suspect the child has a fever, so you check his or her temperature with a thermometer. What if the uncertainty of the thermometer were 3.0°C? If the child's temperature reading was 37.0°C (which is normal body temperature), the "true" temperature could be anywhere from a

hypothermic 34.0°C to a dangerously high 40.0°C. A thermometer with an uncertainty of 3.0°C would be useless.

# **Percent Uncertainty**

One method of expressing uncertainty is as a percent of the measured value. If a measurement A is expressed with uncertainty,  $\delta A$ , the **percent uncertainty** (%unc) is defined to be

# **Equation:**

$$\%~{
m unc}=rac{\delta A}{A} imes 100\%.$$

# **Example:**

# **Calculating Percent Uncertainty: A Bag of Apples**

A grocery store sells 5-lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

Week 1 weight: 4.8 lb Week 2 weight: 5.3 lb Week 3 weight: 4.9 lb Week 4 weight: 5.4 lb

You determine that the weight of the 5-lb bag has an uncertainty of  $\pm 0.4$  lb. What is the percent uncertainty of the bag's weight?

# Strategy

First, observe that the expected value of the bag's weight, A, is 5 lb. The uncertainty in this value,  $\delta A$ , is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight:

# **Equation:**

$$\%~{
m unc}=rac{\delta A}{A} imes 100\%.$$

# Solution

Plug the known values into the equation:

# **Equation:**

$$\%~{
m unc} = rac{0.4~{
m lb}}{5~{
m lb}} imes 100\% = 8\%.$$

## Discussion

We can conclude that the weight of the apple bag is  $5 \text{ lb} \pm 8\%$ . Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100%. If you do not do this, you will have a decimal quantity, not a percent value.

# **Uncertainties in Calculations**

There is an uncertainty in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements going into the calculation have small uncertainties (a few percent or less), then the **method of adding percents** can be used for multiplication or division. This method says that the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation. For example, if a floor has a length of 4.00 m and a width of 3.00 m, with uncertainties of 2% and 1%, respectively, then the area of the floor is 12.0 m<sup>2</sup> and has an uncertainty of 3%. (Expressed as an area this is 0.36 m<sup>2</sup>, which we round to 0.4 m<sup>2</sup> since the area of the floor is given to a tenth of a square meter.)

# **Exercise:**

# **Check Your Understanding**

# **Problem:**

A high school track coach has just purchased a new stopwatch. The stopwatch manual states that the stopwatch has an uncertainty of  $\pm 0.05$  s. Runners on the track coach's team regularly clock 100-m sprints of 11.49 s to 15.01 s. At the school's last track meet, the first-place sprinter came in at 12.04 s and the second-place sprinter came in at 12.07 s. Will the coach's new stopwatch be helpful in timing the sprint team? Why or why not?

# **Solution:**

No, the uncertainty in the stopwatch is too great to effectively differentiate between the sprint times.

# **Precision of Measuring Tools and Significant Figures**

An important factor in the accuracy and precision of measurements involves the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, a standard ruler can measure length to the nearest millimeter, while a caliper can measure length to the nearest 0.01 millimeter. The caliper is a more precise measuring tool because it can measure extremely small differences in length. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool. For example, if you use a standard ruler to measure the length of a stick, you may measure it to be 36.7 cm. You could not express this value as 36.71 cm because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between 36.6 cm and 36.7 cm, and he or she must estimate the value of the last digit. Using the

method of **significant figures**, the rule is that *the last digit written down in a measurement is the first digit with some uncertainty*. In order to determine the number of significant digits in a value, start with the first measured value at the left and count the number of digits through the last digit written on the right. For example, the measured value 36.7 cm has three digits, or significant figures. Significant figures indicate the precision of a measuring tool that was used to measure a value.

## Zeros

Special consideration is given to zeros when counting significant figures. The zeros in 0.053 are not significant, because they are only placekeepers that locate the decimal point. There are two significant figures in 0.053. The zeros in 10.053 are not placekeepers but are significant—this number has five significant figures. The zeros in 1300 may or may not be significant depending on the style of writing numbers. They could mean the number is known to the last digit, or they could be placekeepers. So 1300 could have two, three, or four significant figures. (To avoid this ambiguity, write 1300 in scientific notation.) *Zeros are significant except when they serve only as placekeepers*.

## **Exercise:**

# **Check Your Understanding**

## **Problem:**

Determine the number of significant figures in the following measurements:

- a. 0.0009
- b. 15,450.0
- c.  $6 \times 10^3$
- d. 87.990
- e. 30.42

# **Solution:**

- (a) 1; the zeros in this number are placekeepers that indicate the decimal point
- (b) 6; here, the zeros indicate that a measurement was made to the 0.1 decimal point, so the zeros are significant
- (c) 1; the value  $10^3$  signifies the decimal place, not the number of measured values
- (d) 5; the final zero indicates that a measurement was made to the 0.001 decimal point, so it is significant
- (e) 4; any zeros located in between significant figures in a number are also significant

# **Significant Figures in Calculations**

When combining measurements with different degrees of accuracy and precision, the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value. There are two different rules, one for multiplication and division and the other for addition and subtraction, as discussed below.

**1. For multiplication and division:** The result should have the same number of significant figures as the quantity having the least significant figures entering into the calculation. For example, the area of a circle can be calculated from its radius using  $A=\pi r^2$ . Let us see how many significant figures the area has if the radius has only two—say,  $r=1.2~\mathrm{m}$ . Then,

# **Equation:**

$$A=\pi r^2=(3.1415927...) imes(1.2~ ext{m})^2=4.5238934~ ext{m}^2$$

is what you would get using a calculator that has an eight-digit output. But because the radius has only two significant figures, it limits the calculated

quantity to two significant figures or

# **Equation:**

$$A = 4.5 \text{ m}^2$$

even though  $\pi$  is good to at least eight digits.

**2. For addition and subtraction:** *The answer can contain no more decimal places than the least precise measurement.* Suppose that you buy 7.56-kg of potatoes in a grocery store as measured with a scale with precision 0.01 kg. Then you drop off 6.052-kg of potatoes at your laboratory as measured by a scale with precision 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with precision 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:

# **Equation:**

$$7.56~{
m kg} \ -~6.052~{
m kg} \ rac{+13.7~{
m kg}}{15.208~{
m kg}} = 15.2~{
m kg}.$$

Next, we identify the least precise measurement: 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer is rounded to the tenths place, giving us 15.2 kg.

# **Significant Figures in this Text**

In this text, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits, for example. If the input has fewer significant figures, the answer will also have fewer significant

figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, particularly in optics, more accurate numbers are needed and more than three significant figures will be used. Finally, if a number is *exact*, such as the two in the formula for the circumference of a circle,  $c=2\pi r$ , it does not affect the number of significant figures in a calculation.

# **Exercise:**

# **Check Your Understanding**

# **Problem:**

Perform the following calculations and express your answer using the correct number of significant digits.

- (a) A woman has two bags weighing 13.5 pounds and one bag with a weight of 10.2 pounds. What is the total weight of the bags?
- (b) The force F on an object is equal to its mass m multiplied by its acceleration a. If a wagon with mass 55 kg accelerates at a rate of  $0.0255 \,\mathrm{m/s}^2$ , what is the force on the wagon? (The unit of force is called the newton, and it is expressed with the symbol N.)

## **Solution:**

- (a) 37.2 pounds; Because the number of bags is an exact value, it is not considered in the significant figures.
- (b) 1.4 N; Because the value 55 kg has only two significant figures, the final value must also contain two significant figures.

# Note:

PhET Explorations: Estimation

Explore size estimation in one, two, and three dimensions! Multiple levels of difficulty allow for progressive skill improvement. <a href="https://phet.colorado.edu/sims/estimation/estimation">https://phet.colorado.edu/sims/estimation/estimation en.html</a>

# **Summary**

- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- The precision of a *measuring tool* is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.
- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.

# **Conceptual Questions**

# **Exercise:**

## **Problem:**

What is the relationship between the accuracy and uncertainty of a measurement?

## Exercise:

## **Problem:**

Prescriptions for vision correction are given in units called *diopters* (D). Determine the meaning of that unit. Obtain information (perhaps by calling an optometrist or performing an internet search) on the minimum uncertainty with which corrections in diopters are determined and the accuracy with which corrective lenses can be produced. Discuss the sources of uncertainties in both the prescription and accuracy in the manufacture of lenses.

#### **Problems & Exercises**

Express your answers to problems in this section to the correct number of significant figures and proper units.

#### **Exercise:**

#### **Problem:**

Suppose that your bathroom scale reads your mass as 65 kg with a 3% uncertainty. What is the uncertainty in your mass (in kilograms)?

#### **Solution:**

2 kg

#### **Exercise:**

#### **Problem:**

A good-quality measuring tape can be off by 0.50 cm over a distance of 20 m. What is its percent uncertainty?

#### **Exercise:**

#### **Problem:**

(a) A car speedometer has a 5.0% uncertainty. What is the range of possible speeds when it reads  $90~\rm km/h?$  (b) Convert this range to miles per hour.  $(1~\rm km=0.6214~mi)$ 

#### **Solution:**

```
a. 85.5 to 94.5 km/hb. 53.1 to 58.7 mi/h
```

### **Exercise:**

#### **Problem:**

An infant's pulse rate is measured to be  $130 \pm 5$  beats/min. What is the percent uncertainty in this measurement?

#### **Exercise:**

#### **Problem:**

(a) Suppose that a person has an average heart rate of 72.0 beats/min. How many beats does he or she have in 2.0 y? (b) In 2.00 y? (c) In 2.000 y?

### **Solution:**

- (a)  $7.6 \times 10^7$  beats
- (b)  $7.57 \times 10^7$  beats
- (c)  $7.57 \times 10^7$  beats

#### **Exercise:**

### **Problem:**

A can contains 375 mL of soda. How much is left after 308 mL is removed?

#### **Exercise:**

#### **Problem:**

State how many significant figures are proper in the results of the following calculations: (a) (106.7)(98.2)/(46.210)(1.01) (b)  $(18.7)^2$  (c)  $(1.60 \times 10^{-19})(3712)$ .

# **Solution:**

- a. 3
- b. 3
- c. 3

#### **Exercise:**

#### **Problem:**

(a) How many significant figures are in the numbers 99 and 100? (b) If the uncertainty in each number is 1, what is the percent uncertainty in each? (c) Which is a more meaningful way to express the accuracy of these two numbers, significant figures or percent uncertainties?

#### **Exercise:**

#### **Problem:**

(a) If your speedometer has an uncertainty of 2.0 km/h at a speed of 90 km/h, what is the percent uncertainty? (b) If it has the same percent uncertainty when it reads 60 km/h, what is the range of speeds you could be going?

### **Solution:**

- a) 2.2%
- (b) 59 to 61 km/h

#### **Exercise:**

#### **Problem:**

(a) A person's blood pressure is measured to be  $120\pm2$  mm Hg. What is its percent uncertainty? (b) Assuming the same percent uncertainty, what is the uncertainty in a blood pressure measurement of 80 mm Hg?

#### **Exercise:**

#### **Problem:**

A person measures his or her heart rate by counting the number of beats in 30 s. If  $40 \pm 1$  beats are counted in  $30.0 \pm 0.5 \text{ s}$ , what is the heart rate and its uncertainty in beats per minute?

#### **Solution:**

 $80 \pm 3 \text{ beats/min}$ 

#### **Exercise:**

**Problem:** What is the area of a circle 3.102 cm in diameter?

**Exercise:** 

#### **Problem:**

If a marathon runner averages 9.5 mi/h, how long does it take him or her to run a 26.22-mi marathon?

#### **Solution:**

2.8 h

#### **Exercise:**

#### **Problem:**

A marathon runner completes a 42.188-km course in 2 h, 30 min, and 12 s. There is an uncertainty of 25 m in the distance traveled and an uncertainty of 1 s in the elapsed time. (a) Calculate the percent uncertainty in the distance. (b) Calculate the uncertainty in the elapsed time. (c) What is the average speed in meters per second? (d) What is the uncertainty in the average speed?

#### **Exercise:**

#### **Problem:**

The sides of a small rectangular box are measured to be  $1.80 \pm 0.01$  cm,  $2.05 \pm 0.02$  cm, and  $3.1 \pm 0.1$  cm long. Calculate its volume and uncertainty in cubic centimeters.

#### **Solution:**

 $11\pm1~\mathrm{cm}^3$ 

#### **Exercise:**

#### **Problem:**

When non-metric units were used in the United Kingdom, a unit of mass called the *pound-mass* (lbm) was employed, where 1 lbm = 0.4539 kg. (a) If there is an uncertainty of 0.0001 kg in the pound-mass unit, what is its percent uncertainty? (b) Based on that percent uncertainty, what mass in pound-mass has an uncertainty of 1 kg when converted to kilograms?

#### **Exercise:**

#### **Problem:**

The length and width of a rectangular room are measured to be  $3.955 \pm 0.005~\mathrm{m}$  and  $3.050 \pm 0.005~\mathrm{m}$ . Calculate the area of the room and its uncertainty in square meters.

#### **Solution:**

 $12.06 \pm 0.04 \,\mathrm{m}^2$ 

#### **Exercise:**

#### **Problem:**

A car engine moves a piston with a circular cross section of  $7.500 \pm 0.002~\mathrm{cm}$  diameter a distance of  $3.250 \pm 0.001~\mathrm{cm}$  to compress the gas in the cylinder. (a) By what amount is the gas decreased in volume in cubic centimeters? (b) Find the uncertainty in this volume.

# Glossary

### accuracy

the degree to which a measured value agrees with correct value for that measurement

method of adding percents

the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation

### percent uncertainty

the ratio of the uncertainty of a measurement to the measured value, expressed as a percentage

# precision

the degree to which repeated measurements agree with each other

# significant figures

express the precision of a measuring tool used to measure a value

# uncertainty

a quantitative measure of how much your measured values deviate from a standard or expected value

# Approximation

• Make reasonable approximations based on given data.

On many occasions, physicists, other scientists, and engineers need to make **approximations** or "guesstimates" for a particular quantity. What is the distance to a certain destination? What is the approximate density of a given item? About how large a current will there be in a circuit? Many approximate numbers are based on formulae in which the input quantities are known only to a limited accuracy. As you develop problem-solving skills (that can be applied to a variety of fields through a study of physics), you will also develop skills at approximating. You will develop these skills through thinking more quantitatively, and by being willing to take risks. As with any endeavor, experience helps, as well as familiarity with units. These approximations allow us to rule out certain scenarios or unrealistic numbers. Approximations also allow us to challenge others and guide us in our approaches to our scientific world. Let us do two examples to illustrate this concept.

# **Example:**

# Approximate the Height of a Building

Can you approximate the height of one of the buildings on your campus, or in your neighborhood? Let us make an approximation based upon the height of a person. In this example, we will calculate the height of a 39-story building.

# Strategy

Think about the average height of an adult male. We can approximate the height of the building by scaling up from the height of a person.

#### Solution

Based on information in the example, we know there are 39 stories in the building. If we use the fact that the height of one story is approximately equal to about the length of two adult humans (each human is about 2-m tall), then we can estimate the total height of the building to be

# **Equation:**

$$rac{2 ext{ m}}{1 ext{ person}} imes rac{2 ext{ person}}{1 ext{ story}} imes 39 ext{ stories} = 156 ext{ m}.$$

#### Discussion

You can use known quantities to determine an approximate measurement of unknown quantities. If your hand measures 10 cm across, how many hand lengths equal the width of your desk? What other measurements can you approximate besides length?

# Example: Approximating Vast Numbers: a Trillion Dollars



A bank stack contains one-hundred \$100 bills, and is worth \$10,000. How many bank stacks make up a trillion dollars? (credit: Andrew Magill)

The U.S. federal deficit in the 2008 fiscal year was a little greater than \$10 trillion. Most of us do not have any concept of how much even one trillion actually is. Suppose that you were given a trillion dollars in \$100 bills. If you made 100-bill stacks and used them to evenly cover a football field (between the end zones), make an approximation of how high the money pile would become. (We will use feet/inches rather than meters here

because football fields are measured in yards.) One of your friends says 3 in., while another says 10 ft. What do you think?

### Strategy

When you imagine the situation, you probably envision thousands of small stacks of 100 wrapped \$100 bills, such as you might see in movies or at a bank. Since this is an easy-to-approximate quantity, let us start there. We can find the volume of a stack of 100 bills, find out how many stacks make up one trillion dollars, and then set this volume equal to the area of the football field multiplied by the unknown height.

### Solution

(1) Calculate the volume of a stack of 100 bills. The dimensions of a single bill are approximately 3 in. by 6 in. A stack of 100 of these is about 0.5 in. thick. So the total volume of a stack of 100 bills is:

# **Equation:**

volume of stack = length 
$$\times$$
 width  $\times$  height, volume of stack = 6 in.  $\times$  3 in.  $\times$  0.5 in., volume of stack = 9 in.<sup>3</sup>.

(2) Calculate the number of stacks. Note that a trillion dollars is equal to  $$1 \times 10^{12}$ , and a stack of one-hundred \$100 bills is equal to \$10,000, or  $$1 \times 10^4$ . The number of stacks you will have is:

# **Equation:**

$$1 \times 10^{12} (a trillion dollars) / 1 \times 10^4 per stack = 1 \times 10^8 stacks.$$

(3) Calculate the area of a football field in square inches. The area of a football field is  $100 \text{ yd} \times 50 \text{ yd}$ , which gives  $5{,}000 \text{ yd}^2$ . Because we are working in inches, we need to convert square yards to square inches:

# **Equation:**

$$\begin{split} \text{Area} = 5,\!000 \ \text{yd}^2 \times \tfrac{3 \ \text{ft}}{1 \ \text{yd}} \times \tfrac{3 \ \text{ft}}{1 \ \text{yd}} \times \tfrac{12 \ \text{in.}}{1 \ \text{ft}} \times \tfrac{12 \ \text{in.}}{1 \ \text{ft}} = 6,\!480,\!000 \ \text{in.}^2, \\ \text{Area} \approx 6 \times 10^6 \ \text{in.}^2. \end{split}$$

This conversion gives us  $6 \times 10^6$  in.<sup>2</sup> for the area of the field. (Note that we are using only one significant figure in these calculations.)

- (4) Calculate the total volume of the bills. The volume of all the \$100-bill stacks is  $9 \text{ in.}^3/\text{stack} \times 10^8 \text{ stacks} = 9 \times 10^8 \text{ in.}^3$ .
- (5) Calculate the height. To determine the height of the bills, use the equation:

# **Equation:**

volume of bills = area of field  $\times$  height of money:

Height of money  $= \frac{\text{volume of bills}}{\text{area of field}}$ ,

Height of money =  $\frac{9 \times 10^8 \text{in.}^3}{6 \times 10^6 \text{in.}^2} = 1.33 \times 10^2 \text{in.},$ 

Height of money  $\approx 1 \times 10^2 \text{in.} = 100 \text{ in.}$ 

The height of the money will be about 100 in. high. Converting this value to feet gives

# **Equation:**

$$100 ext{ in.} imes rac{1 ext{ ft}}{12 ext{ in.}} = 8.33 ext{ ft} pprox 8 ext{ ft.}$$

#### Discussion

The final approximate value is much higher than the early estimate of 3 in., but the other early estimate of 10 ft (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise tell you in terms of rough "guesstimates" versus carefully calculated approximations?

#### **Exercise:**

# **Check Your Understanding**

#### **Problem:**

Using mental math and your understanding of fundamental units, approximate the area of a regulation basketball court. Describe the process you used to arrive at your final approximation.

#### **Solution:**

An average male is about two meters tall. It would take approximately 15 men laid out end to end to cover the length, and about 7 to cover the width. That gives an approximate area of  $420 \text{ m}^2$ .

# **Summary**

Scientists often approximate the values of quantities to perform calculations and analyze systems.

### **Problems & Exercises**

#### **Exercise:**

**Problem:** How many heartbeats are there in a lifetime?

### **Solution:**

Sample answer:  $2 \times 10^9$  heartbeats

#### **Exercise:**

#### **Problem:**

A generation is about one-third of a lifetime. Approximately how many generations have passed since the year 0 AD?

#### **Exercise:**

#### **Problem:**

How many times longer than the mean life of an extremely unstable atomic nucleus is the lifetime of a human? (Hint: The lifetime of an unstable atomic nucleus is on the order of  $10^{-22}$  s.)

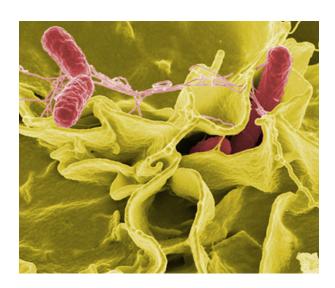
#### **Solution:**

Sample answer:  $2\times 10^{31}\, \text{if an average human lifetime is taken to be about 70 years.}$ 

### **Exercise:**

# **Problem:**

Calculate the approximate number of atoms in a bacterium. Assume that the average mass of an atom in the bacterium is ten times the mass of a hydrogen atom. (Hint: The mass of a hydrogen atom is on the order of  $10^{-27}$  kg and the mass of a bacterium is on the order of  $10^{-15}$  kg.)



This color-enhanced photo shows *Salmonella typhimurium* (red) attacking human cells. These bacteria are commonly known for causing foodborne illness. Can you estimate the number of atoms in each bacterium? (credit: Rocky Mountain Laboratories, NIAID, NIH)

### **Exercise:**

#### **Problem:**

Approximately how many atoms thick is a cell membrane, assuming all atoms there average about twice the size of a hydrogen atom?

#### **Solution:**

Sample answer: 50 atoms

### **Exercise:**

#### **Problem:**

(a) What fraction of Earth's diameter is the greatest ocean depth? (b) The greatest mountain height?

#### **Exercise:**

#### **Problem:**

(a) Calculate the number of cells in a hummingbird assuming the mass of an average cell is ten times the mass of a bacterium. (b) Making the same assumption, how many cells are there in a human?

### **Solution:**

Sample answers:

- (a)  $10^{12}$  cells/hummingbird
- (b)  $10^{16}$  cells/human

### **Exercise:**

### **Problem:**

Assuming one nerve impulse must end before another can begin, what is the maximum firing rate of a nerve in impulses per second?

# Glossary

approximation an estimated value based on prior experience and reasoning

# Introduction to One-Dimensional Kinematics class="introduction"

The motion of an American kestrel through the air can be described by the bird's displacement , speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Vince Maidens, Wikimedia Commons)



Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: How long will it take for a space probe to get to Mars? Where will a football land if it is thrown at a certain angle? But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Our formal study of physics begins with **kinematics** which is defined as the *study of motion without considering its causes*. The word "kinematics" comes from a Greek term meaning motion and is related to other English words such as "cinema" (movies) and "kinesiology" (the study of human motion). In one-dimensional kinematics and <u>Two-Dimensional Kinematics</u> we will study only the *motion* of a football, for example, without worrying about what forces cause or change its motion. Such considerations come in other chapters. In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion. In <u>Two-Dimensional Kinematics</u>, we apply concepts developed here to study motion along curved paths (two- and three-dimensional motion); for example, that of a car rounding a curve.

# Displacement

- Define position, displacement, distance, and distance traveled.
- Explain the relationship between position and displacement.
- Distinguish between displacement and distance traveled.
- Calculate displacement and distance given initial position, final position, and the path between the two.



These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Suzan Black, Fotopedia)

### **Position**

In order to describe the motion of an object, you must first be able to describe its **position**—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For

example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. (See [link].) In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame. (See [link].)

# **Displacement**

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object's position changes. This change in position is known as **displacement**. The word "displacement" implies that an object has moved, or has been displaced.

### **Note:**

Displacement

Displacement is the *change in position* of an object:

# **Equation:**

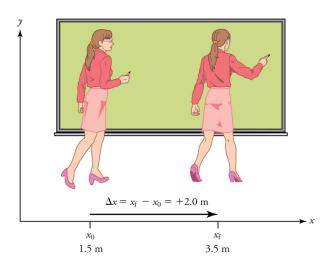
$$\Delta x = x_{
m f} - x_0,$$

where  $\Delta x$  is displacement,  $x_{\rm f}$  is the final position, and  $x_0$  is the initial position.

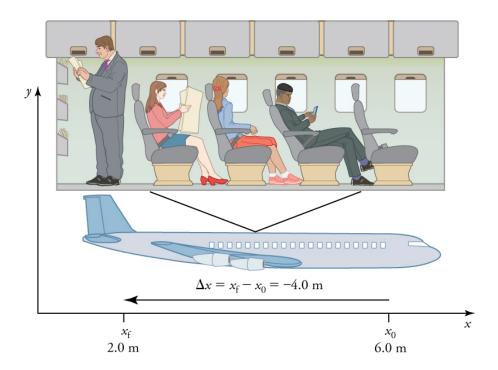
In this text the upper case Greek letter  $\Delta$  (delta) always means "change in" whatever quantity follows it; thus,  $\Delta x$  means *change in position*. Always solve for displacement by subtracting initial position  $x_0$  from final position  $x_0$ .

Note that the SI unit for displacement is the meter (m) (see <u>Physical</u> <u>Quantities and Units</u>), but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that when units other than the meter are

used in a problem, you may need to convert them into meters to complete the calculation.



A professor paces left and right while lecturing. Her position relative to Earth is given by x. The +2.0 m displacement of the professor relative to Earth is represented by an arrow pointing to the right.



A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4.0-m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far) in [link].

Note that displacement has a direction as well as a magnitude. The professor's displacement is 2.0 m to the right, and the airline passenger's displacement is 4.0 m toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is  $x_0 = 1.5$  m and her final position is  $x_1 = 3.5$  m. Thus her displacement is

# **Equation:**

$$\Delta x = x_{\rm f} - x_0 = 3.5 \text{ m} - 1.5 \text{ m} = +2.0 \text{ m}.$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is  $x_0 = 6.0$  m and his final position is  $x_f = 2.0$  m, so his displacement is **Equation:** 

$$\Delta x = x_{\rm f} - x_0 = 2.0 \text{ m} - 6.0 \text{ m} = -4.0 \text{ m}.$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative x direction in our coordinate system.

#### **Distance**

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

#### Note:

Misconception Alert: Distance Traveled vs. Magnitude of Displacement It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The

displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

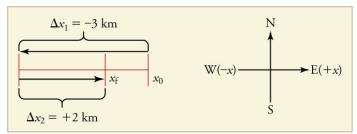
#### **Exercise:**

### **Check Your Understanding**

#### **Problem:**

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?

#### **Solution:**



- (a) The rider's displacement is  $\Delta x = x_{\rm f} x_0 = -1$  km. (The displacement is negative because we take east to be positive and west to be negative.)
- (b) The distance traveled is 3 km + 2 km = 5 km.
- (c) The magnitude of the displacement is 1 km.

# **Section Summary**

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.

• In symbols, displacement  $\Delta x$  is defined to be **Equation:** 

$$\Delta x = x_{\rm f} - x_0$$

where  $x_0$  is the initial position and  $x_f$  is the final position. In this text, the Greek letter  $\Delta$  (delta) always means "change in" whatever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.

- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

#### **Exercise:**

#### Problem:

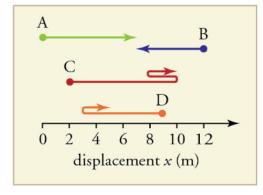
Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?

#### **Exercise:**

#### **Problem:**

Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to  $50~\mu m/s~\left(50\times10^{-6}~m/s\right)$  have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

# **Problems & Exercises**



### **Exercise:**

# **Problem:**

Find the following for path A in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

# **Solution:**

- (a) 7 m
- (b) 7 m
- (c) + 7 m

### **Exercise:**

#### **Problem:**

Find the following for path B in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

### **Exercise:**

#### **Problem:**

Find the following for path C in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

#### **Solution:**

- (a) 13 m
- (b) 9 m
- (c) + 9 m

#### **Exercise:**

#### **Problem:**

Find the following for path D in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

# **Glossary**

#### kinematics

the study of motion without considering its causes

# position

the location of an object at a particular time

# displacement

the change in position of an object

#### distance

the magnitude of displacement between two positions

#### distance traveled

the total length of the path traveled between two positions

# Vectors, Scalars, and Coordinate Systems

- Define and distinguish between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.



The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the xcoordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr)

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A **vector** is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

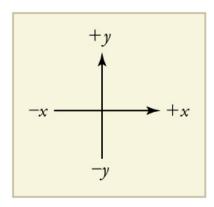
The direction of a vector in one-dimensional motion is given simply by a plus (+) or minus (-) sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A **scalar** is any quantity that has a magnitude, but no direction. For example, a  $20^{\circ}$ C temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a  $-20^{\circ}$ C temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

# **Coordinate Systems for One-Dimensional Motion**

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in [link], it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are

running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.



It is usually convenient to consider motion upward or to the right as positive (+) and motion downward or to the left as negative (-)

### **Exercise:**

# **Check Your Understanding**

### **Problem:**

A person's speed can stay the same as he or she rounds a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

#### **Solution:**

Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

# **Section Summary**

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

# **Conceptual Questions**

#### **Exercise:**

### **Problem:**

A student writes, "A bird that is diving for prey has a speed of -m/s." What is wrong with the student's statement? What has the student actually described? Explain.

#### **Exercise:**

**Problem:** What is the speed of the bird in [link]?

#### **Exercise:**

#### **Problem:**

Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

#### **Exercise:**

### **Problem:**

A weather forecast states that the temperature is predicted to be  $-5^{\circ}\mathrm{C}$  the following day. Is this temperature a vector or a scalar quantity? Explain.

# Glossary

scalar

a quantity that is described by magnitude, but not direction

vector

a quantity that is described by both magnitude and direction

# Time, Velocity, and Speed

- Explain the relationships between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial time, final position, and final time.
- Derive a graph of velocity vs. time given a graph of position vs. time.
- Interpret a graph of velocity vs. time.



The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr)

There is more to motion than distance and displacement. Questions such as, "How long does a foot race take?" and "What was the runner's speed?" cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

### Time

As discussed in <u>Physical Quantities and Units</u>, the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in

some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of time is simple—**time** is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time**  $\Delta t$  is the difference between the ending time and beginning time,

# **Equation:**

$$\Delta t = t_{
m f} - t_0,$$

where  $\Delta t$  is the change in time or elapsed time,  $t_{\rm f}$  is the time at the end of the motion, and  $t_0$  is the time at the beginning of the motion. (As usual, the delta symbol,  $\Delta$ , means the change in the quantity that follows it.)

Life is simpler if the beginning time  $t_0$  is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If  $t_0 = 0$ , then  $\Delta t = t_{\rm f} \equiv t$ .

In this text, for simplicity's sake,

- motion starts at time equal to zero  $(t_0 = 0)$
- ullet the symbol t is used for elapsed time unless otherwise specified  $(\Delta t = t_{
  m f} \equiv t)$

# **Velocity**

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

### Note:

Average Velocity

**Average velocity** is displacement (change in position) divided by the time of travel,

# **Equation:**

$$ar{v} = rac{\Delta x}{\Delta t} = rac{x_{
m f} - x_0}{t_{
m f} - t_0},$$

where  $\overline{v}$  is the *average* (indicated by the bar over the v) velocity,  $\Delta x$  is the change in position (or displacement), and  $x_{\rm f}$  and  $x_{\rm 0}$  are the final and beginning positions at times  $t_{\rm f}$  and  $t_{\rm 0}$ , respectively. If the starting time  $t_{\rm 0}$  is taken to be zero, then the average velocity is simply

# **Equation:**

$$\bar{v} = \frac{\Delta x}{t}$$
.

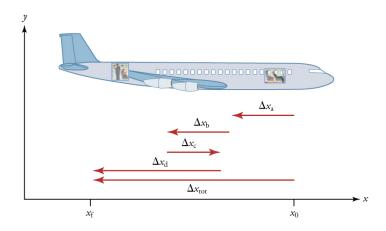
Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move –4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be

# **Equation:**

$$\bar{v} = \frac{\Delta x}{t} = \frac{-4 \text{ m}}{5 \text{ s}} = -0.8 \text{ m/s}.$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.



A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity* or the *velocity at a specific instant*. A car's speedometer, for example, shows the magnitude (but not the

direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) **Instantaneous velocity** v is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity, v, at a precise instant t can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

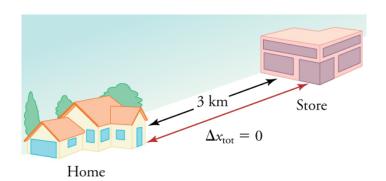
# **Speed**

In everyday language, most people use the terms "speed" and "velocity" interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

**Instantaneous speed** is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

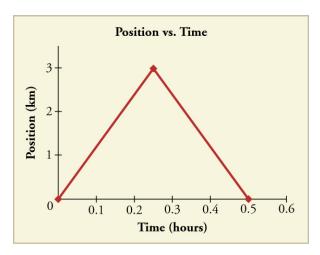
We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero.

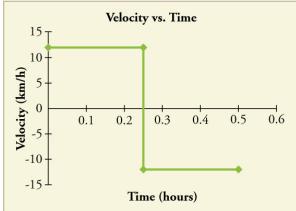
(Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.

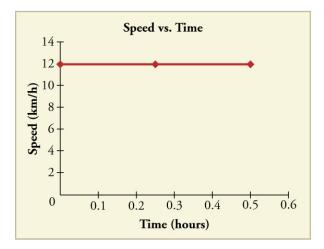


During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in [link]. (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we'll probably stop at the store. But for simplicity's sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)







Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

### Note:

Making Connections: Take-Home Investigation—Getting a Sense of Speed If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

### **Exercise:**

### **Check Your Understanding**

### **Problem:**

A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 40 miles. What is (a) the average velocity of the train, and (b) the average speed of the train in m/s?

#### **Solution:**

- (a) The average velocity of the train is zero because  $x_{\rm f}=x_0$ ; the train ends up at the same place it starts.
- (b) The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

### **Equation:**

$$\frac{\text{distance}}{\text{time}} = \frac{80 \text{ miles}}{105 \text{ minutes}}$$

# **Equation:**

$$\frac{80 \text{ miles}}{105 \text{ minutes}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ meter}}{3.28 \text{ feet}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 20 \text{ m/s}$$

# **Section Summary**

• Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is

## **Equation:**

$$\Delta t = t_{
m f} - t_0,$$

where  $t_f$  is the final time and  $t_0$  is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just t.

• Average velocity  $\overline{v}$  is defined as displacement divided by the travel time. In symbols, average velocity is **Equation:** 

$$ar{v} = rac{\Delta x}{\Delta t} = rac{x_{
m f} - x_0}{t_{
m f} - t_0}.$$

- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity v is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

# **Conceptual Questions**

### **Exercise:**

### **Problem:**

Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

### **Exercise:**

### **Problem:**

There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

#### **Exercise:**

### **Problem:**

Does a car's odometer measure position or displacement? Does its speedometer measure speed or velocity?

#### **Exercise:**

#### **Problem:**

If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

#### **Exercise:**

#### **Problem:**

How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

### **Problems & Exercises**

(a) Calculate Earth's average speed relative to the Sun. (b) What is its average velocity over a period of one year?

### **Solution:**

- (a)  $3.0 \times 10^4 \, {\rm m/s}$
- (b) 0 m/s

### **Exercise:**

### **Problem:**

A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation. (a) Calculate the average speed of the blade tip in the helicopter's frame of reference. (b) What is its average velocity over one revolution?

### **Exercise:**

#### **Problem:**

The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?

### **Solution:**

$$2 \times 10^7 \, \mathrm{years}$$

Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?

#### **Exercise:**

#### **Problem:**

On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

### **Solution:**

34.689 m/s = 124.88 km/h

#### **Exercise:**

#### **Problem:**

Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by  $3.84 \times 10^6$  m (1%)?

A student drove to the university from her home and noted that the odometer reading of her car increased by 12.0 km. The trip took 18.0 min. (a) What was her average speed? (b) If the straight-line distance from her home to the university is 10.3 km in a direction 25.0° south of east, what was her average velocity? (c) If she returned home by the same path 7 h 30 min after she left, what were her average speed and velocity for the entire trip?

### **Solution:**

- (a) 40.0 km/h
- (b) 34.3 km/h,  $25^{\circ}$  S of E.
- (c) average speed = 3.20 km/h,  $\overline{v} = 0$ .

### **Exercise:**

#### **Problem:**

The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?

Conversations with astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light  $(3.00 \times 10^8 \, \text{m/s})$ .

### **Solution:**

384,000 km

#### **Exercise:**

### **Problem:**

A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.

#### **Exercise:**

#### **Problem:**

The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit  $1.06 \times 10^{-10}$  m in diameter. (a) If the average speed of the electron in this orbit is known to be  $2.20 \times 10^6$  m/s, calculate the number of revolutions per second it makes about the nucleus. (b) What is the electron's average velocity?

### **Solution:**

(a) 
$$6.61 \times 10^{15}~\mathrm{rev/s}$$

(b) 0 m/s

# Glossary

average speed

distance traveled divided by time during which motion occurs

average velocity

displacement divided by time over which displacement occurs

instantaneous velocity

velocity at a specific instant, or the average velocity over an infinitesimal time interval

instantaneous speed

magnitude of the instantaneous velocity

time

change, or the interval over which change occurs

model

simplified description that contains only those elements necessary to describe the physics of a physical situation

elapsed time

the difference between the ending time and beginning time

### Acceleration

- Define and distinguish between instantaneous acceleration, average acceleration, and deceleration.
- Calculate acceleration given initial time, initial velocity, final time, and final velocity.



A plane decelerates, or slows down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. (credit: Steve Conry, Flickr)

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

#### Note:

Average Acceleration
Average Acceleration is the rate at which velocity changes,
Equation:

$$ar{a} = rac{\Delta v}{\Delta t} = rac{v_{
m f} - v_0}{t_{
m f} - t_0},$$

where  $\bar{a}$  is average acceleration, v is velocity, and t is time. (The bar over the a means average acceleration.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are  $m/s^2$ , meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

### Note:

### Acceleration as a Vector

Acceleration is a vector in the same direction as the *change* in velocity,  $\Delta v$ . Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object slows down, its acceleration is opposite to the direction of its motion. This is known as **deceleration**.

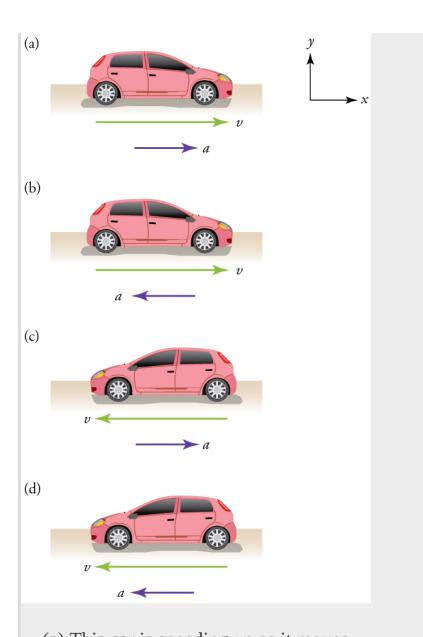


A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)

### Note:

# Misconception Alert: Deceleration vs. Negative Acceleration

Deceleration always refers to acceleration in the direction opposite to the direction of the velocity. Deceleration always reduces speed. Negative acceleration, however, is acceleration *in the negative direction in the chosen coordinate system*. Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. For example, consider [link].



(a) This car is speeding up as it moves toward the right. It therefore has positive acceleration in our coordinate system.
(b) This car is slowing down as it moves toward the right.
Therefore, it has negative acceleration in our coordinate system, because its acceleration is toward the left. The car is also decelerating: the direction of its acceleration is opposite to its direction of motion.
(c) This car is moving

toward the left, but slowing down over time. Therefore, its acceleration is positive in our coordinate system because it is toward the right. However, the car is decelerating because its acceleration is opposite to its motion. (d) This car is speeding up as it moves toward the left. It has negative acceleration because it is accelerating toward the left. However, because its acceleration is in the same direction as its motion, it is speeding up (not decelerating).

# **Example:**

### **Calculating Acceleration: A Racehorse Leaves the Gate**

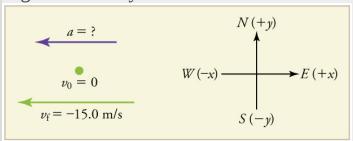
A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?



(credit: Jon Sullivan, PD Photo.org)

### Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.



We can solve this problem by identifying  $\Delta v$  and  $\Delta t$  from the given information and then calculating the average acceleration directly from the equation  $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_{\rm f} - v_0}{t_{\rm f} - t_0}$ .

### Solution

- 1. Identify the knowns.  $v_0 = 0$ ,  $v_{\rm f} = -15.0 \, {\rm m/s}$  (the negative sign indicates direction toward the west),  $\Delta t = 1.80 \, {\rm s}$ .
- 2. Find the change in velocity. Since the horse is going from zero to  $-15.0~\mathrm{m/s}$ , its change in velocity equals its final velocity:  $\Delta v = v_\mathrm{f} = -15.0~\mathrm{m/s}$ .
- 3. Plug in the known values ( $\Delta v$  and  $\Delta t$ ) and solve for the unknown  $\overline{a}$ . **Equation:**

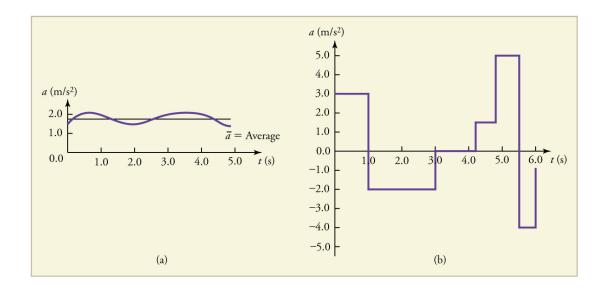
$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0 \text{ m/s}}{1.80 \text{ s}} = -8.33 \text{ m/s}^2.$$

### Discussion

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of  $8.33~\mathrm{m/s^2}$  due west means that the horse increases its velocity by  $8.33~\mathrm{m/s}$  due west each second, that is,  $8.33~\mathrm{meters}$  per second per second, which we write as  $8.33~\mathrm{m/s^2}$ . This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

### **Instantaneous Acceleration**

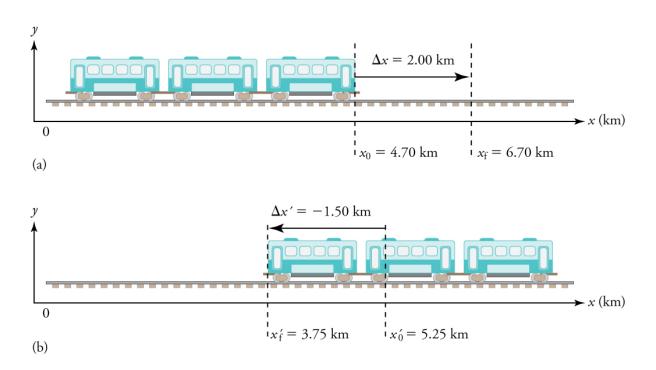
**Instantaneous acceleration** a, or the acceleration at a specific instant in *time*, is obtained by the same process as discussed for instantaneous velocity in Time, Velocity, and Speed—that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. [link] shows graphs of instantaneous acceleration versus time for two very different motions. In [link](a), the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in this case about  $1.8 \mathrm{\ m/s}^2$ ). In [link](b), the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of  $+3.0 \text{ m/s}^2$  and  $-2.0 \text{ m/s}^2$ , respectively.



Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the

acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.

The next several examples consider the motion of the subway train shown in [link]. In (a) the shuttle moves to the right, and in (b) it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.



One-dimensional motion of a subway train considered in [link], [link], [link], [link], [link], and [link]. Here we have chosen the x-axis so that + means to the right and — means to the left for displacements, velocities, and accelerations. (a) The subway train moves to the right from  $x_0$  to  $x_f$ . Its displacement  $\Delta x$  is +2.0 km. (b) The train moves to the left from  $x_0$  to  $x_f$ . Its displacement  $\Delta x_f$  is

 $-1.5~\mathrm{km}$ . (Note that the prime symbol (') is used simply to distinguish between displacement in the two different situations. The distances of travel and the size of the cars are on different scales to fit everything into the diagram.)

# **Example:**

### Calculating Displacement: A Subway Train

What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of [link]?

### Strategy

A drawing with a coordinate system is already provided, so we don't need to make a sketch, but we should analyze it to make sure we understand what it is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation  $\Delta x = x_{\rm f} - x_{\rm 0}$ . This is straightforward since the initial and final positions are given.

### Solution

- 1. Identify the knowns. In the figure we see that  $x_{\rm f}=6.70~{\rm km}$  and  $x_0=4.70~{\rm km}$  for part (a), and  $x_{\rm f}=3.75~{\rm km}$  and  $x_0=5.25~{\rm km}$  for part (b).
- 2. Solve for displacement in part (a).

# **Equation:**

$$\Delta x = x_{\rm f} - x_0 = 6.70 \text{ km} - 4.70 \text{ km} = +2.00 \text{ km}$$

3. Solve for displacement in part (b).

# **Equation:**

$$\Delta x' = x'_{\rm f} - x'_{\rm 0} = 3.75 \text{ km} - 5.25 \text{ km} = -1.50 \text{ km}$$

#### Discussion

The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and thus has a negative sign.

### **Example:**

## Comparing Distance Traveled with Displacement: A Subway Train

What are the distances traveled for the motions shown in parts (a) and (b) of the subway train in [link]?

### Strategy

To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in [link]. Distance traveled is the total length of the path traveled between the two positions. (See <u>Displacement</u>.) In the case of the subway train shown in [link], the distance traveled is the same as the distance between the initial and final positions of the train.

### Solution

- 1. The displacement for part (a) was +2.00 km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.
- 2. The displacement for part (b) was -1.5 km. Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.

#### Discussion

Distance is a scalar. It has magnitude but no sign to indicate direction.

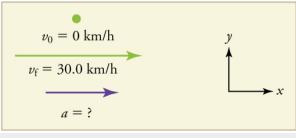
# **Example:**

# Calculating Acceleration: A Subway Train Speeding Up

Suppose the train in [link](a) accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?

# **Strategy**

It is worth it at this point to make a simple sketch:



This problem involves three steps. First we must determine the change in velocity, then we must determine the change in time, and finally we use these values to calculate the acceleration.

### Solution

- 1. Identify the knowns.  $v_0=0$  (the trains starts at rest),  $v_{
  m f}=30.0~{
  m km/h}$ , and  $\Delta t=20.0~{
  m s}$ .
- 2. Calculate  $\Delta v$ . Since the train starts from rest, its change in velocity is  $\Delta v = +30.0 \text{ km/h}$ , where the plus sign means velocity to the right.
- 3. Plug in known values and solve for the unknown,  $\bar{a}$ .

### **Equation:**

$$ar{a}=rac{\Delta v}{\Delta t}=rac{+30.0 ext{ km/h}}{20.0 ext{ s}}$$

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds. (See <a href="Physical Quantities and Units">Physical Quantities and Units</a> for more guidance.)

### **Equation:**

$$ar{a} = igg(rac{+30 ext{ km/h}}{20.0 ext{ s}}igg)igg(rac{10^3 ext{ m}}{1 ext{ km}}igg)igg(rac{1 ext{ h}}{3600 ext{ s}}igg) = 0.417 ext{ m/s}^2$$

### Discussion

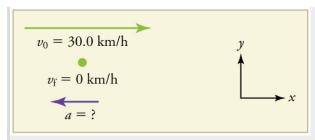
The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the *change* in velocity, as is always the case.

# **Example:**

# Calculate Acceleration: A Subway Train Slowing Down

Now suppose that at the end of its trip, the train in [link](a) slows to a stop from a speed of 30.0 km/h in 8.00 s. What is its average acceleration while stopping?

# Strategy



In this case, the train is decelerating and its acceleration is negative because it is toward the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

### Solution

- 1. Identify the knowns.  $v_0 = 30.0 \text{ km/h}$ ,  $v_f = 0 \text{ km/h}$  (the train is stopped, so its velocity is 0), and  $\Delta t = 8.00 \text{ s}$ .
- 2. Solve for the change in velocity,  $\Delta v$ .

## **Equation:**

$$\Delta v = v_{
m f} - v_0 = 0 - 30.0 \ {
m km/h} = -30.0 \ {
m km/h}$$

3. Plug in the knowns,  $\Delta v$  and  $\Delta t$ , and solve for  $\bar{a}$ .

### **Equation:**

$$ar{a} = rac{\Delta v}{\Delta t} = rac{-30.0 ext{ km/h}}{8.00 ext{ s}}$$

4. Convert the units to meters and seconds.

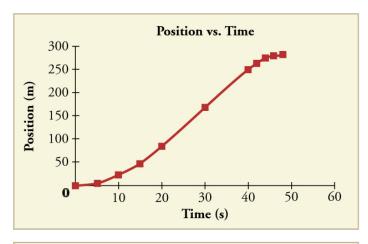
# **Equation:**

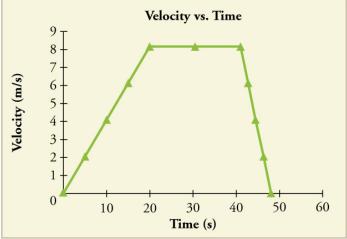
$$ar{a} = rac{\Delta v}{\Delta t} = igg(rac{-30.0 ext{ km/h}}{8.00 ext{ s}}igg)igg(rac{10^3 ext{ m}}{1 ext{ km}}igg)igg(rac{1 ext{ h}}{3600 ext{ s}}igg) = -1.04 ext{ m/s}^2.$$

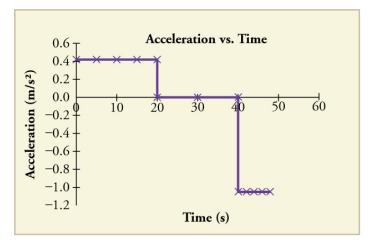
### **Discussion**

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion. Again, acceleration is in the same direction as the *change* in velocity, which is negative here. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

The graphs of position, velocity, and acceleration vs. time for the trains in [link] and [link] are displayed in [link]. (We have taken the velocity to remain constant from 20 to 40 s, after which the train decelerates.)





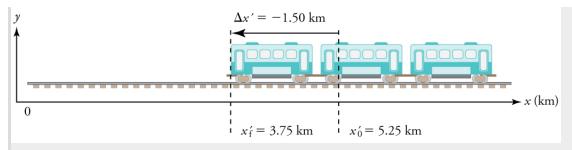


(a) Position of the train over time. Notice that the train's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity of the train over time. The train's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train decelerates at the end of the journey. (c) The acceleration of the train over time. The train has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.

# **Example:**

### Calculating Average Velocity: The Subway Train

What is the average velocity of the train in part b of [link], and shown again below, if it takes 5.00 min to make its trip?



### Strategy

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

### Solution

- 1. Identify the knowns.  $x'_{\rm f}=3.75$  km,  $x'_{\rm 0}=5.25$  km,  $\Delta t=5.00$  min.
- 2. Determine displacement,  $\Delta x'$ . We found  $\Delta x'$  to be -1.5 km in [link].
- 3. Solve for average velocity.

### **Equation:**

$$ar{v} = rac{\Delta x \prime}{\Delta t} = rac{-1.50 ext{ km}}{5.00 ext{ min}}$$

4. Convert units.

### **Equation:**

$$ar{v} = rac{\Delta x\prime}{\Delta t} = igg(rac{-1.50 ext{ km}}{5.00 ext{ min}}igg)igg(rac{60 ext{ min}}{1 ext{ h}}igg) = -18.0 ext{ km/h}$$

### **Discussion**

The negative velocity indicates motion to the left.

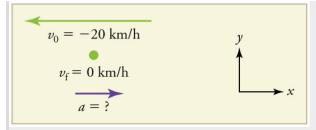
### **Example:**

# **Calculating Deceleration: The Subway Train**

Finally, suppose the train in [link] slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

# Strategy

Once again, let's draw a sketch:



As before, we must find the change in velocity and the change in time to calculate average acceleration.

### Solution

- 1. Identify the knowns.  $v_0 = -20 \ \mathrm{km/h}$ ,  $v_\mathrm{f} = 0 \ \mathrm{km/h}$ ,  $\Delta t = 10.0 \ \mathrm{s}$ .
- 2. Calculate  $\Delta v$ . The change in velocity here is actually positive, since **Equation:**

$$\Delta v = v_{
m f} - v_0 = 0 - (-20 \ {
m km/h}) = +20 \ {
m km/h}.$$

3. Solve for  $\bar{a}$ .

### **Equation:**

$$ar{a} = rac{\Delta v}{\Delta t} = rac{+20.0 ext{ km/h}}{10.0 ext{ s}}$$

4. Convert units.

### **Equation:**

$$ar{a} = igg(rac{+20.0 ext{ km/h}}{10.0 ext{ s}}igg)igg(rac{10^3 ext{ m}}{1 ext{ km}}igg)igg(rac{1 ext{ h}}{3600 ext{ s}}igg) = +0.556 ext{ m/s}^2$$

#### Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right). Again, acceleration is in the same direction as the *change* in velocity, which is positive here. As in [link], this acceleration can be called a deceleration since it is in the direction opposite to the velocity.

# **Sign and Direction**

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. This was not the case in [link], where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will *increase* a negative velocity. For example, the train moving to the left in [link] is sped up by an acceleration to the left. In that case, both v and a are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down.

#### **Exercise:**

## **Check Your Understanding**

### **Problem:**

An airplane lands on a runway traveling east. Describe its acceleration.

#### **Solution:**

If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also decelerating: its acceleration is opposite in direction to its velocity.

#### Note:

# PhET Explorations: Moving Man Simulation

Learn about position, velocity, and acceleration graphs. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you. <a href="https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/">https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/</a>

# **Section Summary**

• Acceleration is the rate at which velocity changes. In symbols, average acceleration  $\bar{a}$  is Equation:

$$ar{a} = rac{\Delta v}{\Delta t} = rac{v_{
m f} - v_0}{t_{
m f} - t_0}.$$

- The SI unit for acceleration is  $m/s^2$ .
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration a is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

# **Conceptual Questions**

### **Exercise:**

#### **Problem:**

Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.

#### **Exercise:**

#### **Problem:**

Is it possible for velocity to be constant while acceleration is not zero? Explain.

### **Exercise:**

### **Problem:**

Give an example in which velocity is zero yet acceleration is not.

### **Exercise:**

### **Problem:**

If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

### **Exercise:**

### **Problem:**

Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

### **Problems & Exercises**

#### **Exercise:**

### **Problem:**

A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

### **Solution:**

$$4.29 \text{ m/s}^2$$

### **Exercise:**

# **Problem: Professional Application**

Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration.

Express each in multiples of g (9.80 m/s<sup>2</sup>) by taking its ratio to the acceleration of gravity.

### **Exercise:**

### **Problem:**

A commuter backs her car out of her garage with an acceleration of  $1.40~\rm{m/s}^2$ . (a) How long does it take her to reach a speed of 2.00 m/s? (b) If she then brakes to a stop in 0.800 s, what is her deceleration?

### **Solution:**

- (a)  $1.43 \, \mathrm{s}$
- (b)  $-2.50 \text{ m/s}^2$

### **Exercise:**

### **Problem:**

Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in  $m/s^2$  and in multiples of g (9.80  $m/s^2$ )?

# **Glossary**

acceleration

the rate of change in velocity; the change in velocity over time

average acceleration

the change in velocity divided by the time over which it changes

instantaneous acceleration

acceleration at a specific point in time

deceleration

acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

## Motion Equations for Constant Acceleration in One Dimension

- Calculate displacement of an object that is not accelerating, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.



Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. (credit: Barry Skeates, Flickr)

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

Notation: t, x, v, a

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is  $\Delta t = t_{\rm f} - t_0$ , taking  $t_0 = 0$  means that  $\Delta t = t_{\rm f}$ , the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is,  $x_0$  is the initial position and  $v_0$  is the initial velocity. We put no subscripts on the final values. That is, t is the final time, x is the final position, and v is the final velocity. This gives a simpler expression for elapsed time—now,  $\Delta t = t$ . It also simplifies the expression for displacement, which is now  $\Delta x = x - x_0$ . Also, it simplifies the expression for change in velocity, which is now  $\Delta v = v - v_0$ . To summarize, using the simplified notation, with the initial time taken to be zero,

### **Equation:**

$$egin{array}{lll} \Delta t &=& t \ \Delta x &=& x-x_0 \ \Delta v &=& v-v_0 \end{array}$$

where the subscript 0 denotes an initial value and the absence of a subscript denotes a final value in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant*. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

# **Equation:**

$$\bar{a} = a = \text{constant},$$

so we use the symbol a for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration is constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in

motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

#### Note:

Solving for Displacement  $(\Delta x)$  and Final Position (x) from Average Velocity when Acceleration (a) is Constant

To get our first two new equations, we start with the definition of average velocity:

### **Equation:**

$$ar{v} = rac{\Delta x}{\Delta t}.$$

Substituting the simplified notation for  $\Delta x$  and  $\Delta t$  yields

# **Equation:**

$$\overline{v} = rac{x - x_0}{t}$$
.

Solving for *x* yields

# **Equation:**

$$x=x_0+ar{v}t,$$

where the average velocity is

# **Equation:**

$$ar{v} = rac{v_0 + v}{2} \; ext{(constant } a ext{)}.$$

The equation  $\overline{v} = \frac{v_0 + v}{2}$  reflects the fact that, when acceleration is constant, v is just the simple average of the initial and final velocities. For example, if

you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your average velocity during this steady increase is 45 km/h. Using the equation  $\bar{v} = \frac{v_0 + v}{2}$  to check this, we see that

### **Equation:**

$$ar{v} = rac{v_0 + v}{2} = rac{30 ext{ km/h} + 60 ext{ km/h}}{2} = 45 ext{ km/h},$$

which seems logical.

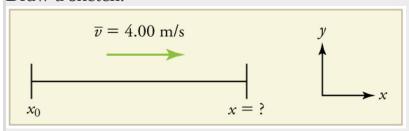
### **Example:**

### Calculating Displacement: How Far does the Jogger Run?

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

### Strategy

Draw a sketch.



The final position x is given by the equation

# **Equation:**

$$x=x_0+ar{v}t.$$

To find x, we identify the values of  $x_0$ ,  $\overline{v}$ , and t from the statement of the problem and substitute them into the equation.

### Solution

- 1. Identify the knowns.  $\overline{v}=4.00~\mathrm{m/s}$ ,  $\Delta t=2.00~\mathrm{min}$ , and  $x_0=0~\mathrm{m}$ .
- 2. Enter the known values into the equation.

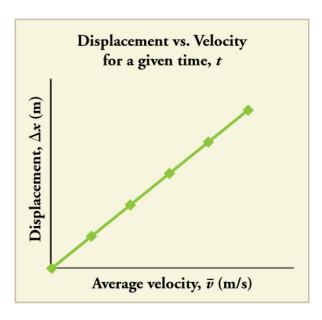
# **Equation:**

$$x = x_0 + \overline{v}t = 0 + (4.00 \text{ m/s})(120 \text{ s}) = 480 \text{ m}$$

#### Discussion

Velocity and final displacement are both positive, which means they are in the same direction.

The equation  $x=x_0+v t$  gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on v rather than on v raised to some other power, such as v. When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.



There is a linear relationship between displacement and average velocity. For a given time t, an object moving twice as fast as another object will

move twice as far as the other object.

### Note:

### Solving for Final Velocity

We can derive another useful equation by manipulating the definition of acceleration.

### **Equation:**

$$a=rac{\Delta v}{\Delta t}$$

Substituting the simplified notation for  $\Delta v$  and  $\Delta t$  gives us

# **Equation:**

$$a = \frac{v - v_0}{t}$$
 (constant a).

Solving for v yields

# **Equation:**

$$v = v_0 + at \text{ (constant } a).$$

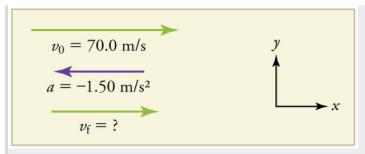
# **Example:**

# Calculating Final Velocity: An Airplane Slowing Down after Landing

An airplane lands with an initial velocity of 70.0 m/s and then decelerates at  $1.50~\mathrm{m/s}^2$  for 40.0 s. What is its final velocity?

# **Strategy**

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is decelerating.



#### Solution

- 1. Identify the knowns.  $v_0 = 70.0 \text{ m/s}$ ,  $a = -1.50 \text{ m/s}^2$ , t = 40.0 s.
- 2. Identify the unknown. In this case, it is final velocity,  $v_{
  m f}$
- 3. Determine which equation to use. We can calculate the final velocity using the equation  $v = v_0 + at$ .
- 4. Plug in the known values and solve.

### **Equation:**

$$v = v_0 + {
m at} = 70.0 \ {
m m/s} + \Big( -1.50 \ {
m m/s}^2 \Big) (40.0 \ {
m s}) = 10.0 \ {
m m/s}$$

#### Discussion

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.



The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note that the acceleration is negative because its direction is opposite to its velocity, which is positive.

In addition to being useful in problem solving, the equation  $v = v_0 + at$  gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity ( $v=v_0$ ), as expected (i.e., velocity is constant)
- if *a* is negative, then the final velocity is less than the initial velocity

(All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.)

#### Note:

Making Connections: Real-World Connection



The Space Shuttle *Endeavor* blasts off from the Kennedy Space Center in February 2010. (credit: Matthew Simantov, Flickr)

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the

first minute or two of flight (actual ICBM burn times are classified—short-burn-time missiles are more difficult for an enemy to destroy). But the Space Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

#### Note:

### Solving for Final Position When Velocity is Not Constant ( $a \neq 0$ )

We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

### **Equation:**

$$v = v_0 + at$$
.

Adding  $v_0$  to each side of this equation and dividing by 2 gives

# **Equation:**

$$\frac{v_0+v}{2}=v_0+\frac{1}{2}\mathrm{at}.$$

Since  $\frac{v_0+v}{2} = \overline{v}$  for constant acceleration, then

## **Equation:**

$$ar{v}=v_0+rac{1}{2}{
m at}.$$

Now we substitute this expression for  $\overline{v}$  into the equation for displacement,  $x=x_0+\overline{v}t$ , yielding

# **Equation:**

$$x=x_0+v_0t+rac{1}{2}at^2 ext{ (constant } a).$$

### **Example:**

# Calculating Displacement of an Accelerating Object: Dragsters

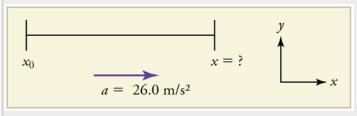
Dragsters can achieve average accelerations of  $26.0 \text{ m/s}^2$ . Suppose such a dragster accelerates from rest at this rate for 5.56 s. How far does it travel in this time?



U.S. Army Top Fuel pilot
Tony "The Sarge"
Schumacher begins a race
with a controlled burnout.
(credit: Lt. Col. William
Thurmond. Photo
Courtesy of U.S. Army.)

# **Strategy**

Draw a sketch.



We are asked to find displacement, which is x if we take  $x_0$  to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation  $x = x_0 + v_0 t + \frac{1}{2} a t^2$  once we identify  $v_0$ , a, and t from the statement of the problem.

#### Solution

- 1. Identify the knowns. Starting from rest means that  $v_0 = 0$ , a is given as  $26.0 \text{ m/s}^2$  and t is given as 5.56 s.
- 2. Plug the known values into the equation to solve for the unknown x:

### **Equation:**

$$x = x_0 + v_0 t + rac{1}{2} a t^2.$$

Since the initial position and velocity are both zero, this simplifies to **Equation:** 

$$x = \frac{1}{2}at^2.$$

Substituting the identified values of a and t gives

### **Equation:**

$$x = rac{1}{2} \Big( 26.0 ext{ m/s}^2 \Big) (5.56 ext{ s})^2,$$

yielding

### **Equation:**

$$x = 402 \text{ m}.$$

### Discussion

If we convert 402 m to miles, we find that the distance covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation  $x = x_0 + v_0 t + \frac{1}{2}at^2$ ? We see that:

• displacement depends on the square of the elapsed time when acceleration is not zero. In [link], the dragster covers only one fourth of the total distance in the first half of the elapsed time

• if acceleration is zero, then the initial velocity equals average velocity  $(v_0=\bar{v})$  and  $x=x_0+v_0t+\frac{1}{2}at^2$  becomes  $x=x_0+v_0t$ 

#### Note:

Solving for Final Velocity when Velocity Is Not Constant ( $a \neq 0$ )

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve  $v = v_0 +$ at for t, we get

### **Equation:**

$$t = rac{v - v_0}{a}$$
.

Substituting this and  $\overset{-}{v}=\frac{v_0+v}{2}$  into  $x=x_0+\overset{-}{v}t$ , we get

# **Equation:**

$$v^2 = v_0^2 + 2a(x - x_0)$$
 (constant a).

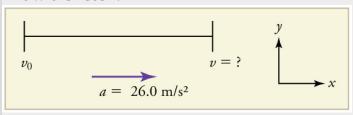
# **Example:**

### **Calculating Final Velocity: Dragsters**

Calculate the final velocity of the dragster in [link] without using information about time.

### **Strategy**

Draw a sketch.



The equation  $v^2 = v_0^2 + 2a(x - x_0)$  is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

#### **Solution**

- 1. Identify the known values. We know that  $v_0=0$ , since the dragster starts from rest. Then we note that  $x-x_0=402~\mathrm{m}$  (this was the answer in [link]). Finally, the average acceleration was given to be  $a=26.0~\mathrm{m/s}^2$
- 2. Plug the knowns into the equation  $v^2 = v_0^2 + 2a(x x_0)$  and solve for v.

### **Equation:**

$$v^2 = 0 + 2 \Big( 26.0 \ \mathrm{m/s}^2 \Big) (402 \ \mathrm{m}).$$

Thus

### **Equation:**

$$v^2 = 2.09 \times 10^4 \,\mathrm{m}^2/\mathrm{s}^2.$$

To get v, we take the square root:

### **Equation:**

$$v = \sqrt{2.09 imes 10^4 ext{ m}^2/ ext{s}^2} = 145 ext{ m/s}.$$

#### Discussion

145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation  $v^2 = v_0^2 + 2a(x - x_0)$  can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts
- For a fixed deceleration, a car that is going twice as fast doesn't simply stop in twice the distance—it takes much further to stop. (This is why

# **Putting Equations Together**

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

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Summary of Kinematic Equations (constant *a*)

**Equation:** 

$$x=x_0+ar{v}t$$

**Equation:** 

$$ar{v}=rac{v_0+v}{2}$$

**Equation:** 

$$v = v_0 + at$$

**Equation:** 

$$x=x_0+v_0t+\frac{1}{2}at^2$$

**Equation:** 

$$v^2 = v_0^2 + 2a(x-x_0)$$

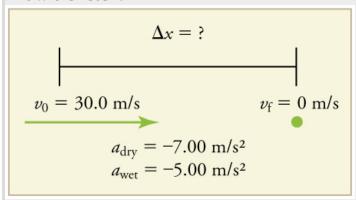
# **Example:**

# Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, a car can decelerate at a rate of  $7.00~\mathrm{m/s^2}$ , whereas on wet concrete it can decelerate at only  $5.00~\mathrm{m/s^2}$ . Find the distances necessary to stop a car moving at  $30.0~\mathrm{m/s}$  (about  $110~\mathrm{km/h}$ ) (a) on dry concrete and (b) on wet concrete. (c) Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of  $0.500~\mathrm{s}$  to get his foot on the brake.

### Strategy

Draw a sketch.



In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off. **Solution for (a)** 

- 1. Identify the knowns and what we want to solve for. We know that  $v_0 = 30.0 \text{ m/s}$ ; v = 0;  $a = -7.00 \text{ m/s}^2$  (a is negative because it is in a direction opposite to velocity). We take  $x_0$  to be 0. We are looking for displacement  $\Delta x$ , or  $x x_0$ .
- 2. Identify the equation that will help up solve the problem. The best equation to use is

# **Equation:**

$$v^2 = v_0^2 + 2a(x - x_0).$$

This equation is best because it includes only one unknown, x. We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for x, but they require us to know

the stopping time, t, which we do not know. We could use them but it would entail additional calculations.)

3. Rearrange the equation to solve for x.

### **Equation:**

$$x-x_0=rac{v^2-v_0^2}{2a}$$

4. Enter known values.

### **Equation:**

$$x-0 = rac{0^2 - (30.0 ext{ m/s})^2}{2 \Big( -7.00 ext{ m/s}^2 \Big)}$$

Thus,

# **Equation:**

x = 64.3 m on dry concrete.

### Solution for (b)

This part can be solved in exactly the same manner as Part A. The only difference is that the deceleration is  $-5.00 \text{ m/s}^2$ . The result is

# **Equation:**

$$x_{\rm wet} = 90.0 \,\mathrm{m}$$
 on wet concrete.

### Solution for (c)

Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver's reaction time.

- 1. Identify the knowns and what we want to solve for. We know that
- $\overline{v}=30.0~\mathrm{m/s}$ ;  $t_{\mathrm{reaction}}=0.500~\mathrm{s}$ ;  $a_{\mathrm{reaction}}=0$ . We take  $x_{0-\mathrm{reaction}}$  to be
- 0. We are looking for  $x_{\text{reaction}}$ .
- 2. Identify the best equation to use.

 $x = x_0 + \overline{v}t$  works well because the only unknown value is x, which is what we want to solve for.

3. Plug in the knowns to solve the equation.

### **Equation:**

$$x = 0 + (30.0 \text{ m/s})(0.500 \text{ s}) = 15.0 \text{ m}.$$

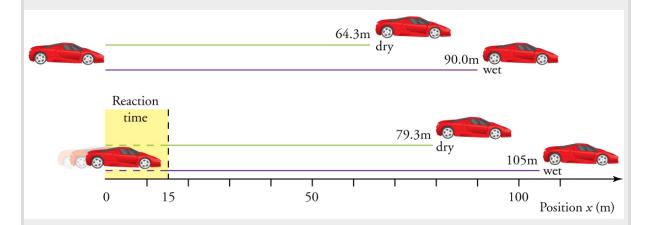
This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.

4. Add the displacement during the reaction time to the displacement when braking.

### **Equation:**

$$x_{
m braking} + x_{
m reaction} = x_{
m total}$$

a. 
$$64.3 \text{ m} + 15.0 \text{ m} = 79.3 \text{ m}$$
 when dry b.  $90.0 \text{ m} + 15.0 \text{ m} = 105 \text{ m}$  when wet



The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time.

#### Discussion

The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.

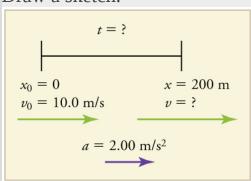
### **Example:**

### Calculating Time: A Car Merges into Traffic

Suppose a car merges into freeway traffic on a 200-m-long ramp. If its initial velocity is 10.0 m/s and it accelerates at  $2.00 \text{ m/s}^2$ , how long does it take to travel the 200 m up the ramp? (Such information might be useful to a traffic engineer.)

### Strategy

Draw a sketch.



We are asked to solve for the time t. As before, we identify the known quantities in order to choose a convenient physical relationship (that is, an equation with one unknown, t).

#### Solution

- 1. Identify the knowns and what we want to solve for. We know that  $v_0=10~\mathrm{m/s}$ ;  $a=2.00~\mathrm{m/s}^2$ ; and  $x=200~\mathrm{m}$ .
- 2. We need to solve for t. Choose the best equation.  $x = x_0 + v_0 t + \frac{1}{2}at^2$  works best because the only unknown in the equation is the variable t for which we need to solve.

3. We will need to rearrange the equation to solve for t. In this case, it will be easier to plug in the knowns first.

### **Equation:**

$$200~ ext{m} = 0~ ext{m} + (10.0~ ext{m/s})t + rac{1}{2} \Big( 2.00~ ext{m/s}^2 \Big) \, t^2$$

4. Simplify the equation. The units of meters (m) cancel because they are in each term. We can get the units of seconds (s) to cancel by taking t=t s, where t is the magnitude of time and s is the unit. Doing so leaves

### **Equation:**

$$200 = 10t + t^2$$
.

- 5. Use the quadratic formula to solve for t.
- (a) Rearrange the equation to get 0 on one side of the equation.

### **Equation:**

$$t^2 + 10t - 200 = 0$$

This is a quadratic equation of the form

### **Equation:**

$$at^2 + bt + c = 0,$$

where the constants are a = 1.00, b = 10.0, and c = -200.

(b) Its solutions are given by the quadratic formula:

## **Equation:**

$$t=rac{-b\pm\sqrt{b^2-4{
m ac}}}{2a}.$$

This yields two solutions for t, which are

### **Equation:**

$$t = 10.0 \text{ and } -20.0.$$

In this case, then, the time is t = t in seconds, or

# **Equation:**

$$t = 10.0 \text{ s and} - 20.0 \text{ s}.$$

A negative value for time is unreasonable, since it would mean that the event happened 20 s before the motion began. We can discard that solution. Thus,

### **Equation:**

$$t = 10.0 \text{ s}.$$

#### Discussion

Whenever an equation contains an unknown squared, there will be two solutions. In some problems both solutions are meaningful, but in others, such as the above, only one solution is reasonable. The 10.0 s answer seems reasonable for a typical freeway on-ramp.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships. <a href="Problem-Solving Basics">Problem-Solving Basics</a> discusses problem-solving basics and outlines an approach that will help you succeed in this invaluable task.

#### Note:

Making Connections: Take-Home Experiment—Breaking News We have been using SI units of meters per second squared to describe some examples of acceleration or deceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking deceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration,  $\bar{a} = \Delta v/\Delta t$ . While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the deceleration in miles per hour per second. Convert this to meters per second squared and compare with other decelerations mentioned in this chapter. Calculate the distance traveled in braking.

#### **Exercise:**

# **Check Your Understanding**

#### **Problem:**

A manned rocket accelerates at a rate of  $20 \text{ m/s}^2$  during launch. How long does it take the rocket to reach a velocity of 400 m/s?

#### **Solution:**

To answer this, choose an equation that allows you to solve for time t, given only a,  $v_0$ , and v.

### **Equation:**

$$v = v_0 + at$$

Rearrange to solve for t.

### **Equation:**

$$t = rac{v - v_0}{a} = rac{400 ext{ m/s} - 0 ext{ m/s}}{20 ext{ m/s}^2} = 20 ext{ s}$$

# **Section Summary**

- To simplify calculations we take acceleration to be constant, so that  $\bar{a}=a$  at all times.
- We also take initial time to be zero.
- Initial position and velocity are given a subscript 0; final values have no subscript. Thus,

## **Equation:**

$$\Delta t = t 
\Delta x = x - x_0 
\Delta v = v - v_0$$



**Equation:** 

$$x=x_0+ar{v}t$$

**Equation:** 

$$ar{v}=rac{v_0+v}{2}$$

**Equation:** 

$$v = v_0 + at$$

**Equation:** 

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

**Equation:** 

$$v^2 = v_0^2 + 2a(x - x_0)$$

• In vertical motion, y is substituted for x.

### **Problems & Exercises**

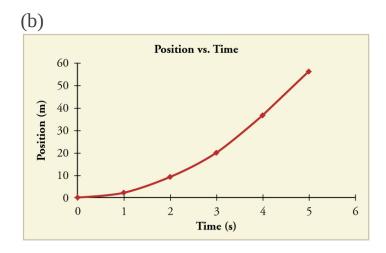
**Exercise:** 

**Problem:** 

An Olympic-class sprinter starts a race with an acceleration of  $4.50~{\rm m/s}^2$ . (a) What is her speed 2.40 s later? (b) Sketch a graph of her position vs. time for this period.

# **Solution:**

(a) 10.8 m/s



### **Exercise:**

### **Problem:**

A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is  $2.10 \times 10^4 \, \mathrm{m/s^2}$ , and 1.85 ms (1 ms =  $10^{-3}$  s) elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?

#### **Solution:**

38.9 m/s (about 87 miles per hour)

#### **Exercise:**

### **Problem:**

A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of  $6.20 \times 10^5~\mathrm{m/s^2}$  for  $8.10 \times 10^{-4}~\mathrm{s}$ . What is its muzzle velocity (that is, its final velocity)?

#### **Exercise:**

#### **Problem:**

(a) A light-rail commuter train accelerates at a rate of  $1.35 \text{ m/s}^2$ . How long does it take to reach its top speed of 80.0 km/h, starting from rest? (b) The same train ordinarily decelerates at a rate of  $1.65 \text{ m/s}^2$ . How long does it take to come to a stop from its top speed? (c) In emergencies the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency deceleration in  $\text{m/s}^2$ ?

#### **Solution:**

- (a) 16.5 s
- (b) 13.5 s
- (c)  $-2.68 \text{ m/s}^2$

#### **Exercise:**

#### **Problem:**

While entering a freeway, a car accelerates from rest at a rate of  $2.40~\mathrm{m/s^2}$  for 12.0 s. (a) Draw a sketch of the situation. (b) List the knowns in this problem. (c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable. (d) What is the car's final velocity? Solve for this unknown in the same manner as in part (c), showing all steps explicitly.

#### **Exercise:**

#### **Problem:**

At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of  $2.00 \text{ m/s}^2$ . (a) How far does she travel in the next 5.00 s? (b) What is her final velocity? (c) Evaluate the result. Does it make sense?

#### **Solution:**

- (a) 20.0 m
- (b) -1.00 m/s
- (c) This result does not really make sense. If the runner starts at 9.00 m/s and decelerates at  $2.00 \, \mathrm{m/s}^2$ , then she will have stopped after 4.50 s. If she continues to decelerate, she will be running backwards.

#### **Exercise:**

## **Problem:**Professional Application:

Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?

#### **Exercise:**

#### **Problem:**

In a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes  $3.33 \times 10^{-2} \text{ s}$ , calculate the distance over which the puck accelerates.

#### **Solution:**

 $0.799 \; \text{m}$ 

#### **Exercise:**

#### **Problem:**

A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) How far does it travel in that time?

#### **Exercise:**

#### **Problem:**

Freight trains can produce only relatively small accelerations and decelerations. (a) What is the final velocity of a freight train that accelerates at a rate of  $0.0500~\mathrm{m/s}^2$  for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of  $0.550~\mathrm{m/s}^2$ , how long will it take to come to a stop from this velocity? (c) How far will it travel in each case?

#### **Solution:**

- (a) 28.0 m/s
- (b) 50.9 s
- (c) 7.68 km to accelerate and 713 m to decelerate

#### **Exercise:**

#### **Problem:**

A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) How long did the acceleration last? (b) Calculate the acceleration.

#### **Exercise:**

#### **Problem:**

A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of  $0.350 \text{ m/s}^2$ , how far will it travel before becoming airborne? (b) How long does this take?

### **Solution:**

- (a) 51.4 m
- (b) 17.1 s

#### **Exercise:**

### **Problem: Professional Application:**

A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in  $m/s^2$  and in multiples of  $g(g=9.80~m/s^2)$ . (b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of g?

#### **Exercise:**

#### Problem:

An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is his deceleration? (b) How long does the collision last?

#### **Solution:**

(a) 
$$-80.4 \text{ m/s}^2$$

(b) 
$$9.33 \times 10^{-2} \text{ s}$$

#### **Exercise:**

#### **Problem:**

In World War II, there were several reported cases of airmen who jumped from their flaming airplanes with no parachute to escape certain death. Some fell about 20,000 feet (6000 m), and some of them survived, with few life-threatening injuries. For these lucky pilots, the tree branches and snow drifts on the ground allowed their deceleration to be relatively small. If we assume that a pilot's speed upon impact was 123 mph (54 m/s), then what was his deceleration? Assume that the trees and snow stopped him over a distance of 3.0 m.

#### **Exercise:**

#### **Problem:**

Consider a grey squirrel falling out of a tree to the ground. (a) If we ignore air resistance in this case (only for the sake of this problem), determine a squirrel's velocity just before hitting the ground, assuming it fell from a height of 3.0 m. (b) If the squirrel stops in a distance of 2.0 cm through bending its limbs, compare its deceleration with that of the airman in the previous problem.

#### **Solution:**

- (a) 7.7 m/s
- (b)  $-15 \times 10^2$  m/s<sup>2</sup>. This is about 3 times the deceleration of the pilots, who were falling from thousands of meters high!

#### **Exercise:**

#### **Problem:**

An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of  $0.150 \, \mathrm{m/s^2}$  as it goes through. The station is 210 m long. (a) How long is the nose of the train in the station? (b) How fast is it going when the nose leaves the station? (c) If the train is 130 m long, when does the end of the train leave the station? (d) What is the velocity of the end of the train as it leaves?

#### **Exercise:**

#### **Problem:**

Dragsters can actually reach a top speed of 145 m/s in only 4.45 s—considerably less time than given in [link] and [link]. (a) Calculate the average acceleration for such a dragster. (b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for 402 m (a quarter mile) without using any information on time. (c) Why is the final velocity greater than that used to find the average acceleration? *Hint*: Consider whether the assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.

#### **Solution:**

- (a)  $32.6 \text{ m/s}^2$
- (b) 162 m/s
- (c)  $v>v_{\rm max}$ , because the assumption of constant acceleration is not valid for a dragster. A dragster changes gears, and would have a greater acceleration in first gear than second gear than third gear, etc. The acceleration would be greatest at the beginning, so it would not be accelerating at  $32.6~{\rm m/s}^2$  during the last few meters, but substantially less, and the final velocity would be less than  $162~{\rm m/s}$ .

#### **Exercise:**

### **Problem:**

A bicycle racer sprints at the end of a race to clinch a victory. The racer has an initial velocity of 11.5 m/s and accelerates at the rate of  $0.500 \text{ m/s}^2$  for 7.00 s. (a) What is his final velocity? (b) The racer continues at this velocity to the finish line. If he was 300 m from the finish line when he started to accelerate, how much time did he save? (c) One other racer was 5.00 m ahead when the winner started to accelerate, but he was unable to accelerate, and traveled at 11.8 m/s until the finish line. How far ahead of him (in meters and in seconds) did the winner finish?

#### **Exercise:**

#### **Problem:**

In 1967, New Zealander Burt Munro set the world record for an Indian motorcycle, on the Bonneville Salt Flats in Utah, with a maximum speed of 183.58 mi/h. The one-way course was 5.00 mi long. Acceleration rates are often described by the time it takes to reach 60.0 mi/h from rest. If this time was 4.00 s, and Burt accelerated at this rate until he reached his maximum speed, how long did it take Burt to complete the course?

#### **Solution:**

 $104 \, s$ 

#### **Exercise:**

#### **Problem:**

(a) A world record was set for the men's 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt "coasted" across the finish line with a time of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration. (b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s. Using the same assumptions as for the 100-m dash, what was his maximum speed for this race?

### **Solution:**

(a) 
$$v = 12.2 \text{ m/s}$$
;  $a = 4.07 \text{ m/s}^2$ 

(b) 
$$v = 11.2 \text{ m/s}$$

### Problem-Solving Basics for One-Dimensional Kinematics

- Apply problem-solving steps and strategies to solve problems of onedimensional kinematics.
- Apply strategies to determine whether or not the result of a problem is reasonable, and if not, determine the cause.



Problem-solving skills are essential to your success in Physics. (credit: scui3asteveo, Flickr)

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in this text and for applying physics in everyday and professional life.

# **Problem-Solving Steps**

While there is no simple step-by-step method that works for every problem, the following general procedures facilitate problem solving and make it more meaningful. A certain amount of creativity and insight is required as well.

### Step 1

Examine the situation to determine which physical principles are involved. It often helps to *draw a simple sketch* at the outset. You will also need to decide which direction is positive and note that on your sketch. Once you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.

### Step 2

Make a list of what is given or can be inferred from the problem as stated (identify the knowns). Many problems are stated very succinctly and require some inspection to determine what is known. A sketch can also be very useful at this point. Formally identifying the knowns is of particular importance in applying physics to real-world situations. Remember, "stopped" means velocity is zero, and we often can take initial time and position as zero.

### Step 3

*Identify exactly what needs to be determined in the problem (identify the unknowns).* In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help.

### Step 4

Find an equation or set of equations that can help you solve the problem. Your list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all of the other variables are known, so you can easily solve for the unknown. If the equation contains more than one unknown, then an additional equation is needed to solve the problem. In some problems, several unknowns must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

### Step 5

Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units. This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made. However, be warned that correct units do not guarantee that the numerical part of the answer is also correct.

### Step 6

Check the answer to see if it is reasonable: Does it make sense? This final step is extremely important—the goal of physics is to accurately describe nature. To see if the answer is reasonable, check both its magnitude and its sign, in addition to its units. Your judgment will improve as you solve more and more physics problems, and it will become possible for you to make finer and finer judgments regarding whether nature is adequately described by the answer to a problem. This step brings the problem back to its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than just being able to mechanically solve a problem.

When solving problems, we often perform these steps in different order, and we also tend to do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience, and the basics of problem solving become almost automatic. One way to get practice is to work out the text's examples for yourself as you read. Another is to work as many end-of-section problems as possible, starting with the easiest to build confidence and progressing to the more difficult. Once you become involved in physics, you will see it all around you, and you can begin to apply it to situations you encounter outside the classroom, just as is done in many of the applications in this text.

#### **Unreasonable Results**

Physics must describe nature accurately. Some problems have results that are unreasonable because one premise is unreasonable or because certain premises are inconsistent with one another. The physical principle applied correctly then produces an unreasonable result. For example, if a person starting a foot race accelerates at  $0.40~\mathrm{m/s^2}$  for  $100~\mathrm{s}$ , his final speed will be  $40~\mathrm{m/s}$  (about  $150~\mathrm{km/h}$ )—clearly unreasonable because the time of  $100~\mathrm{s}$  is an unreasonable premise. The physics is correct in a sense, but there is more to describing nature than just manipulating equations correctly. Checking the result of a problem to see if it is reasonable does more than help uncover errors in problem solving—it also builds intuition in judging whether nature is being accurately described.

Use the following strategies to determine whether an answer is reasonable and, if it is not, to determine what is the cause.

### Step 1

Solve the problem using strategies as outlined and in the format followed in the worked examples in the text. In the example given in the preceding paragraph, you would identify the givens as the acceleration and time and use the equation below to find the unknown final velocity. That is,

## **Equation:**

$$v = v_0 + {
m at} = 0 + \left(0.40\ {
m m/s}^2
ight) (100\ {
m s}) = 40\ {
m m/s}.$$

### Step 2

*Check to see if the answer is reasonable.* Is it too large or too small, or does it have the wrong sign, improper units, ...? In this case, you may need to convert meters per second into a more familiar unit, such as miles per hour.

### **Equation:**

$$\left(\frac{40~\text{m}}{\text{s}}\right)\!\left(\frac{3.28~\text{ft}}{\text{m}}\right)\!\left(\frac{1~\text{mi}}{5280~\text{ft}}\right)\!\left(\frac{60~\text{s}}{\text{min}}\right)\!\left(\frac{60~\text{min}}{1~\text{h}}\right) = 89~\text{mph}$$

This velocity is about four times greater than a person can run—so it is too large.

### Step 3

If the answer is unreasonable, look for what specifically could cause the identified difficulty. In the example of the runner, there are only two assumptions that are suspect. The acceleration could be too great or the time too long. First look at the acceleration and think about what the number means. If someone accelerates at  $0.40~\rm m/s^2$ , their velocity is increasing by  $0.4~\rm m/s$  each second. Does this seem reasonable? If so, the time must be too long. It is not possible for someone to accelerate at a constant rate of  $0.40~\rm m/s^2$  for  $100~\rm s$  (almost two minutes).

# **Section Summary**

• The six basic problem solving steps for physics are:

- *Step 1*. Examine the situation to determine which physical principles are involved.
- *Step 2*. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).
- *Step 3*. Identify exactly what needs to be determined in the problem (identify the unknowns).
- *Step 4*. Find an equation or set of equations that can help you solve the problem.
- *Step 5*. Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.
- *Step 6*. Check the answer to see if it is reasonable: Does it make sense?

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

What information do you need in order to choose which equation or equations to use to solve a problem? Explain.

#### **Exercise:**

#### **Problem:**

What is the last thing you should do when solving a problem? Explain.

### **Falling Objects**

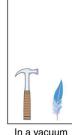
- Describe the effects of gravity on objects in motion.
- Describe the motion of objects that are in free fall.
- Calculate the position and velocity of objects in free fall.

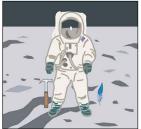
Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

# Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration*, *independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.







In a vacuum In a vacuum (the hard way)

A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the

# acceleration due to gravity is only $1.67 \text{ m/s}^2$ .

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in **free-fall**.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the **acceleration due to gravity**. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol, *g*. It is constant at any given location on Earth and has the average value

## **Equation:**

$$g = 9.80 \text{ m/s}^2$$
.

Although g varies from  $9.78 \text{ m/s}^2$  to  $9.83 \text{ m/s}^2$ , depending on latitude, altitude, underlying geological formations, and local topography, the average value of  $9.80 \text{ m/s}^2$  will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is *downward* (towards the center of *Earth*). In fact, its direction *defines* what we call vertical. Note that whether the acceleration a in the kinematic equations has the value +g or -g depends on how we define our coordinate system. If we define the upward direction as positive, then  $a = -g = -9.80 \text{ m/s}^2$ , and if we define the downward direction as positive, then  $a = g = 9.80 \text{ m/s}^2$ .

# **One-Dimensional Motion Involving Gravity**

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g. We will also represent vertical displacement with the symbol y and use x for horizontal displacement.

#### Note:

Kinematic Equations for Objects in Free-Fall where Acceleration = -g **Equation:** 

$$v = v_0 - \operatorname{gt}$$

**Equation:** 

$$y=y_0+v_0t-rac{1}{2}gt^2$$

**Equation:** 

$$v^2 = v_0^2 - 2g(y-y_0)$$

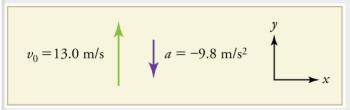
### **Example:**

# Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

### Strategy

Draw a sketch.



We are asked to determine the position y at various times. It is reasonable to take the initial position  $y_0$  to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so a is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs. Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as  $y_1$  and  $v_1$ ;  $y_2$  and  $v_2$ ; and  $v_3$  and  $v_3$ .

### **Solution for Position** $y_1$

- 1. Identify the knowns. We know that  $y_0 = 0$ ;  $v_0 = 13.0 \text{ m/s}$ ;  $a = -g = -9.80 \text{ m/s}^2$ ; and t = 1.00 s.
- 2. Identify the best equation to use. We will use  $y = y_0 + v_0 t + \frac{1}{2}at^2$  because it includes only one unknown, y (or  $y_1$ , here), which is the value we want to find.
- 3. Plug in the known values and solve for  $y_1$ .

## **Equation:**

$$y_1 = 0 + (13.0 \ \mathrm{m/s})(1.00 \ \mathrm{s}) + rac{1}{2} \Big( -9.80 \ \mathrm{m/s}^2 \Big) (1.00 \ \mathrm{s})^2 = 8.10 \ \mathrm{m}$$

#### Discussion

The rock is 8.10 m above its starting point at t = 1.00 s, since  $y_1 > y_0$ . It could be *moving* up or down; the only way to tell is to calculate  $v_1$  and find out if it is positive or negative.

# **Solution for Velocity** $v_1$

1. Identify the knowns. We know that  $y_0=0$ ;  $v_0=13.0~{\rm m/s}$ ;  $a=-g=-9.80~{\rm m/s}^2$ ; and  $t=1.00~{\rm s}$ . We also know from the solution above that  $y_1=8.10~{\rm m}$ .

2. Identify the best equation to use. The most straightforward is  $v = v_0 - \operatorname{gt}$  (from  $v = v_0 + \operatorname{at}$ , where  $a = \operatorname{gravitational} \operatorname{acceleration} = -g$ ).

3. Plug in the knowns and solve.

### **Equation:**

$$v_1 = v_0 - {
m gt} = 13.0 \ {
m m/s} - \Big( 9.80 \ {
m m/s}^2 \Big) (1.00 \ {
m s}) = 3.20 \ {
m m/s}$$

#### Discussion

The positive value for  $v_1$  means that the rock is still heading upward at  $t=1.00~\mathrm{s}$ . However, it has slowed from its original 13.0 m/s, as expected.

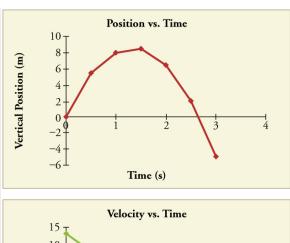
### **Solution for Remaining Times**

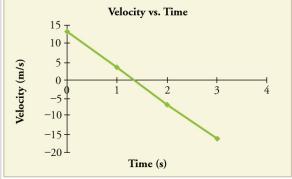
The procedures for calculating the position and velocity at t = 2.00 s and 3.00 s are the same as those above. The results are summarized in [link] and illustrated in [link].

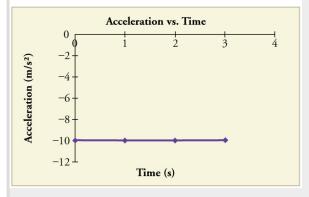
Time, t	Position, y	Velocity, v	Acceleration, a
1.00 s	8.10 m	$3.20~\mathrm{m/s}$	$-9.80~\mathrm{m/s}^2$
$2.00~\mathrm{s}$	$6.40~\mathrm{m}$	$-6.60~\mathrm{m/s}$	$-9.80~\mathrm{m/s}^2$
$3.00~\mathrm{s}$	$-5.10~\mathrm{m}$	$-16.4~\mathrm{m/s}$	$-9.80~\mathrm{m/s}^2$

#### Results

Graphing the data helps us understand it more clearly.







Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant. *Misconception Alert!* Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some

horizontal motion—the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is *time*, not space. The actual path of the rock in space is straight up, and straight down.

#### Discussion

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since  $y_1$  and  $v_1$  are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both  $y_3$  and  $v_3$  are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still  $-9.80 \, \mathrm{m/s^2}$ . Its acceleration is  $-9.80 \, \mathrm{m/s^2}$  for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.

#### Note:

# Making Connections: Take-Home Experiment—Reaction Time

A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

# Example:

# Calculating Velocity of a Falling Object: A Rock Thrown Down

What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when it is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.

## **Strategy**

Draw a sketch.

$$v_0 = -13.0 \text{ m/s}$$
 $a = -9.8 \text{ m/s}^2$ 

Since up is positive, the final position of the rock will be negative because it finishes below the starting point at  $y_0 = 0$ . Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.

#### Solution

- 1. Identify the knowns.  $y_0 = 0$ ;  $y_1 = -5.10 \text{ m}$ ;  $v_0 = -13.0 \text{ m/s}$ ;  $a = -g = -9.80 \text{ m/s}^2$ .
- 2. Choose the kinematic equation that makes it easiest to solve the problem. The equation  $v^2 = v_0^2 + 2a(y y_0)$  works well because the only unknown in it is v. (We will plug  $y_1$  in for y.)
- 3. Enter the known values

# **Equation:**

$$v^2 = (-13.0 \ {
m m/s})^2 + 2 \Big(-9.80 \ {
m m/s}^2\Big) (-5.10 \ {
m m} - 0 \ {
m m}) = 268.96 \ {
m m}^2/{
m s}^2,$$

where we have retained extra significant figures because this is an intermediate result.

Taking the square root, and noting that a square root can be positive or negative, gives

# **Equation:**

$$v=\pm 16.4~\mathrm{m/s}.$$

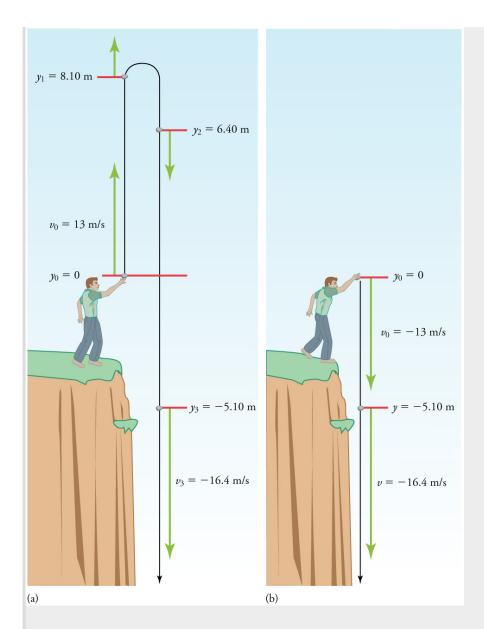
The negative root is chosen to indicate that the rock is still heading down. Thus,

# **Equation:**

$$v = -16.4 \text{ m/s}.$$

#### Discussion

Note that this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed. (See [link] and [link](a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the speed of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from [link]) when the initial velocity is 13.0 m/s straight up, a result of  $\pm 3.20$  m/s is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same speed but the opposite direction.



(a) A person throws a rock straight up, as explored in [link]. The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in [link]. Note that at the same distance below the point of release, the rock has the same velocity in both cases.

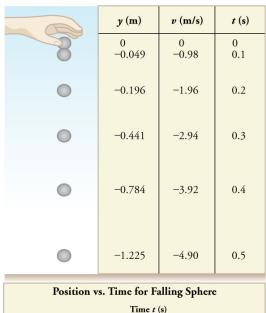
Another way to look at it is this: In  $[\underline{link}]$ , the rock is thrown up with an initial velocity of 13.0 m/s. It rises and then falls back down. When its

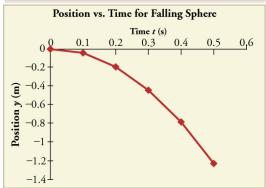
position is y=0 on its way back down, its velocity is  $-13.0~\mathrm{m/s}$ . That is, it has the same speed on its way down as on its way up. We would then expect its velocity at a position of  $y=-5.10~\mathrm{m}$  to be the same whether we have thrown it upwards at  $+13.0~\mathrm{m/s}$  or thrown it downwards at  $-13.0~\mathrm{m/s}$ . The velocity of the rock on its way down from y=0 is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.

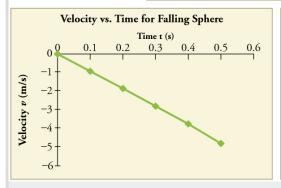
# **Example:**

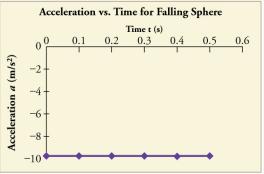
# Find *q* from Data on a Falling Object

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course. An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See, for example, [link]. Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.









Positions and velocities of a metal ball released from rest when air resistance is negligible. Velocity is seen to increase linearly with time while displacement increases with time squared.

Acceleration is a constant and is equal to gravitational acceleration.

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

### Strategy

Draw a sketch.

$$v_0 = 0 \text{ m/s}$$
  $a = ?$ 

We need to solve for acceleration a. Note that in this case, displacement is downward and therefore negative, as is acceleration.

#### **Solution**

- 1. Identify the knowns.  $y_0 = 0$ ; y = -1.0000 m; t = 0.45173;  $v_0 = 0$ .
- 2. Choose the equation that allows you to solve for a using the known values.

## **Equation:**

$$y=y_0+v_0t+rac{1}{2}at^2$$

3. Substitute 0 for  $v_0$  and rearrange the equation to solve for a. Substituting 0 for  $v_0$  yields

# **Equation:**

$$y=y_0+rac{1}{2}at^2.$$

Solving for *a* gives

# **Equation:**

$$a=rac{2(y-y_0)}{t^2}.$$

4. Substitute known values yields

# **Equation:**

$$a = \frac{2(-1.0000 \text{ m} - 0)}{(0.45173 \text{ s})^2} = -9.8010 \text{ m/s}^2,$$

so, because a = -g with the directions we have chosen,

# **Equation:**

$$g = 9.8010 \text{ m/s}^2.$$

#### Discussion

The negative value for a indicates that the gravitational acceleration is downward, as expected. We expect the value to be somewhere around the average value of  $9.80~\mathrm{m/s}^2$ , so  $9.8010~\mathrm{m/s}^2$  makes sense. Since the data going into the calculation are relatively precise, this value for g is more precise than the average value of  $9.80~\mathrm{m/s}^2$ ; it represents the local value for the acceleration due to gravity.

#### **Exercise:**

## **Check Your Understanding**

#### **Problem:**

A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

#### **Solution:**

We know that initial position  $y_0=0$ , final position y=-30.0 m, and a=-g=-9.80 m/s $^2$ . We can then use the equation  $y=y_0+v_0t+\frac{1}{2}at^2$  to solve for t. Inserting a=-g, we obtain **Equation:** 

$$egin{array}{lll} y &=& 0+0-rac{1}{2}gt^2 \ t^2 &=& rac{2y}{-g} \ && t &=& \pm\sqrt{rac{2y}{-g}} = \pm\sqrt{rac{2(-30.0\ ext{m})}{-9.80\ ext{m/s}^2}} = \pm\sqrt{6.12\ ext{s}^2} = 2.47\ ext{s} pprox 2.5\ ext{s} \end{array}$$

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

#### Note:

# PhET Explorations: Equation Grapher

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. y = bx) to see how they add to generate the polynomial curve.

https://phet.colorado.edu/sims/equation-grapher/equation-grapher en.html

# **Section Summary**

- An object in free-fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration due to gravity *g*, which averages

# **Equation:**

$$g = 9.80 \text{ m/s}^2.$$

- Whether the acceleration a should be taken as +g or -g is determined by your choice of coordinate system. If you choose the upward direction as positive,  $a = -g = -9.80 \text{ m/s}^2$  is negative. In the opposite case,  $a = +g = 9.80 \text{ m/s}^2$  is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate +g or -g substituted for a.
- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?

#### **Exercise:**

#### **Problem:**

An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the acceleration due to gravity have the same sign on the way up as on the way down?

#### **Exercise:**

#### **Problem:**

Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.

#### **Exercise:**

#### **Problem:**

If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?

#### **Exercise:**

#### **Problem:**

The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about 1/6 that of the Earth)?

#### **Exercise:**

#### **Problem:**

How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about 1/6 of g on Earth)?

#### **Problems & Exercises**

Assume air resistance is negligible unless otherwise stated.

#### **Exercise:**

#### **Problem:**

Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be  $y_0 = 0$ .

### **Solution:**

(a) 
$$y_1 = 6.28 \text{ m}$$
;  $v_1 = 10.1 \text{ m/s}$ 

(b) 
$$y_2 = 10.1 \text{ m}$$
;  $v_2 = 5.20 \text{ m/s}$ 

(c) 
$$y_3 = 11.5 \text{ m}$$
;  $v_3 = 0.300 \text{ m/s}$ 

(d) 
$$y_4 = 10.4 \text{ m}$$
;  $v_4 = -4.60 \text{ m/s}$ 

#### **Exercise:**

#### **Problem:**

Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, (d) 2.00, and (e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.

#### **Exercise:**

#### **Problem:**

A basketball referee tosses the ball straight up for the starting tip-off. At what velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?

#### **Solution:**

$$v_0 = 4.95 \; \mathrm{m/s}$$

#### **Exercise:**

#### **Problem:**

A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.40 m/s and observes that it takes 1.8 s to reach the water. (a) List the knowns in this problem. (b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.

#### **Exercise:**

#### **Problem:**

A dolphin in an aquatic show jumps straight up out of the water at a velocity of 13.0 m/s. (a) List the knowns in this problem. (b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known and identify its value. Then identify the unknown, and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the answer is reasonable. (c) How long is the dolphin in the air? Neglect any effects due to his size or orientation.

#### **Solution:**

(a) 
$$a = -9.80 \text{ m/s}^2$$
;  $v_0 = 13.0 \text{ m/s}$ ;  $y_0 = 0 \text{ m}$ 

(b)  $v=0\mathrm{m/s}$ . Unknown is distance y to top of trajectory, where velocity is zero. Use equation  $v^2=v_0^2+2a(y-y_0)$  because it contains all known values except for y, so we can solve for y. Solving for y gives

## **Equation:**

$$egin{array}{lcl} v^2-v_0^2&=&2a(y-y_0)\ rac{v^2-v_0^2}{2a}&=&y-y_0\ y&=&y_0+rac{v^2-v_0^2}{2a}=0\ \mathrm{m}+rac{(0\ \mathrm{m/s})^2-(13.0\ \mathrm{m/s})^2}{2\left(-9.80\ \mathrm{m/s}^2
ight)}=8.62\ \mathrm{m} \end{array}$$

Dolphins measure about 2 meters long and can jump several times their length out of the water, so this is a reasonable result.

(c) 2.65 s

#### **Exercise:**

#### **Problem:**

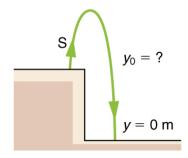
A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s, and her takeoff point is 1.80 m above the pool. (a) How long are her feet in the air? (b) What is her highest point above the board? (c) What is her velocity when her feet hit the water?

#### **Exercise:**

#### **Problem:**

(a) Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s. (b) How long would it take to reach the ground if it is thrown straight down with the same speed?

#### **Solution:**



- (a) 8.26 m
- (b) 0.717 s

#### **Exercise:**

#### **Problem:**

A very strong, but inept, shot putter puts the shot straight up vertically with an initial velocity of 11.0 m/s. How long does he have to get out of the way if the shot was released at a height of 2.20 m, and he is 1.80 m tall?

#### **Exercise:**

#### **Problem:**

You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?

#### **Solution:**

1.91 s

#### **Exercise:**

#### **Problem:**

A kangaroo can jump over an object 2.50 m high. (a) Calculate its vertical speed when it leaves the ground. (b) How long is it in the air?

#### **Exercise:**

#### **Problem:**

Standing at the base of one of the cliffs of Mt. Arapiles in Victoria, Australia, a hiker hears a rock break loose from a height of 105 m. He can't see the rock right away but then does, 1.50 s later. (a) How far above the hiker is the rock when he can see it? (b) How much time does he have to move before the rock hits his head?

#### **Solution:**

- (a) 94.0 m
- (b) 3.13 s

#### **Exercise:**

### **Problem:**

An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance traveled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.

#### **Exercise:**

#### **Problem:**

There is a 250-m-high cliff at Half Dome in Yosemite National Park in California. Suppose a boulder breaks loose from the top of this cliff. (a) How fast will it be going when it strikes the ground? (b) Assuming a reaction time of 0.300 s, how long will a tourist at the bottom have to get out of the way after hearing the sound of the rock breaking loose (neglecting the height of the tourist, which would become negligible anyway if hit)? The speed of sound is 335 m/s on this day.

#### **Solution:**

- (a) -70.0 m/s (downward)
- (b) 6.10 s

#### **Exercise:**

#### **Problem:**

A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes 0.312 s to go past the window. What was the ball's initial velocity? Hint: First consider only the distance along the window, and solve for the ball's velocity at the bottom of the window. Next, consider only the distance from the ground to the bottom of the window, and solve for the initial velocity using the velocity at the bottom of the window as the final velocity.

#### **Exercise:**

#### **Problem:**

Suppose you drop a rock into a dark well and, using precision equipment, you measure the time for the sound of a splash to return. (a) Neglecting the time required for sound to travel up the well, calculate the distance to the water if the sound returns in 2.0000 s. (b) Now calculate the distance taking into account the time for sound to travel up the well. The speed of sound is 332.00 m/s in this well.

#### **Solution:**

- (a) 19.6 m
- (b) 18.5 m

#### **Exercise:**

#### **Problem:**

A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms  $(8.00 \times 10^{-5} \text{ s})$ . (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

#### **Exercise:**

#### **Problem:**

A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.

#### **Solution:**

- (a) 305 m
- (b) 262 m, -29.2 m/s
- (c) 8.91 s

#### **Exercise:**

#### **Problem:**

A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms  $(3.50 \times 10^{-3} \text{ s})$ . (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

# Glossary

free-fall

the state of movement that results from gravitational force only

acceleration due to gravity acceleration of an object as a result of gravity

# Graphical Analysis of One-Dimensional Motion

- Describe a straight-line graph in terms of its slope and y-intercept.
- Determine average velocity or instantaneous velocity from a graph of position vs. time.
- Determine average or instantaneous acceleration from a graph of velocity vs. time.
- Derive a graph of velocity vs. time from a graph of position vs. time.
- Derive a graph of acceleration vs. time from a graph of velocity vs. time.

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities. This section uses graphs of position, velocity, and acceleration versus time to illustrate one-dimensional kinematics.

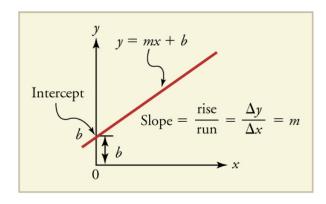
# **Slopes and General Relationships**

First note that graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against one another in such a graph, the horizontal axis is usually considered to be an **independent variable** and the vertical axis a **dependent variable**. If we call the horizontal axis the x-axis and the vertical axis the y-axis, as in  $[\underline{link}]$ , a straight-line graph has the general form

# **Equation:**

$$y = mx + b$$
.

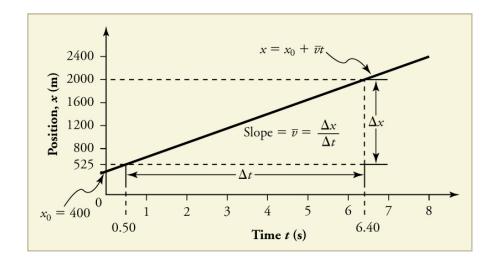
Here m is the **slope**, defined to be the rise divided by the run (as seen in the figure) of the straight line. The letter b is used for the y-intercept, which is the point at which the line crosses the vertical axis.



A straight-line graph. The equation for a straight line is y = mx + b.

# Graph of Position vs. Time (a = 0, so v is constant)

Time is usually an independent variable that other quantities, such as position, depend upon. A graph of position versus time would, thus, have x on the vertical axis and t on the horizontal axis. [link] is just such a straight-line graph. It shows a graph of position versus time for a jet-powered car on a very flat dry lake bed in Nevada.



# Graph of position versus time for a jet-powered car on the Bonneville Salt Flats.

Using the relationship between dependent and independent variables, we see that the slope in the graph above is average velocity v and the intercept is position at time zero—that is,  $x_0$ . Substituting these symbols into  $y = \max + b$  gives

# **Equation:**

$$x = \overline{v}t + x_0$$

or

## **Equation:**

$$x = x_0 + \overline{v}t.$$

Thus a graph of position versus time gives a general relationship among displacement(change in position), velocity, and time, as well as giving detailed numerical information about a specific situation.

#### Note:

The Slope of *x* vs. *t* 

The slope of the graph of position x vs. time t is velocity v.

# **Equation:**

slope = 
$$\frac{\Delta x}{\Delta t} = v$$

Notice that this equation is the same as that derived algebraically from other motion equations in <u>Motion Equations for Constant Acceleration in One Dimension</u>.

From the figure we can see that the car has a position of 25 m at 0.50 s and 2000 m at 6.40 s. Its position at other times can be read from the graph; furthermore, information about its velocity and acceleration can also be obtained from the graph.

## **Example:**

# Determining Average Velocity from a Graph of Position versus Time: Jet Car

Find the average velocity of the car whose position is graphed in [link]. **Strategy** 

The slope of a graph of x vs. t is average velocity, since slope equals rise over run. In this case, rise = change in position and run = change in time, so that

# **Equation:**

slope = 
$$\frac{\Delta x}{\Delta t} = \bar{v}$$
.

Since the slope is constant here, any two points on the graph can be used to find the slope. (Generally speaking, it is most accurate to use two widely separated points on the straight line. This is because any error in reading data from the graph is proportionally smaller if the interval is larger.)

Solution

- 1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)
- 2. Substitute the x and t values of the chosen points into the equation. Remember in calculating change  $(\Delta)$  we always use final value minus initial value.

# **Equation:**

$$ar{v} = rac{\Delta x}{\Delta t} = rac{2000 ext{ m} - 525 ext{ m}}{6.4 ext{ s} - 0.50 ext{ s}},$$

yielding

# **Equation:**

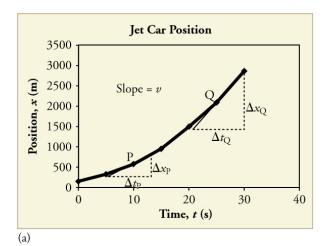
$$\overline{v}=250~\mathrm{m/s}.$$

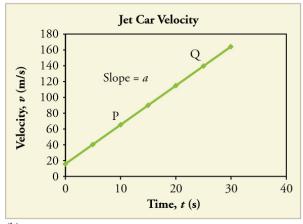
#### Discussion

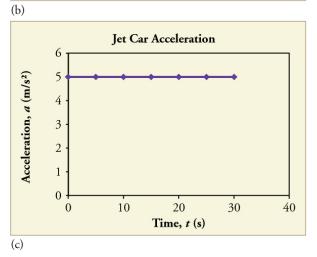
This is an impressively large land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 60 mi/h (27 m/s or 96 km/h), but considerably shy of the record of 343 m/s (1234 km/h or 766 mi/h) set in 1997.

# Graphs of Motion when a is constant but $a \neq 0$

The graphs in [link] below represent the motion of the jet-powered car as it accelerates toward its top speed, but only during the time when its acceleration is constant. Time starts at zero for this motion (as if measured with a stopwatch), and the position and velocity are initially 200 m and 15 m/s, respectively.







Graphs of motion of a jetpowered car during the time span when its acceleration is constant. (a) The slope of an xvs. t graph is velocity. This is shown at two points, and the instantaneous velocities obtained are plotted in the next graph. Instantaneous velocity at any point is the slope of the tangent at that point. (b) The slope of the v vs. t graph is constant for this part of the motion, indicating constant acceleration. (c) Acceleration has the constant value of  $5.0 \text{ m/s}^2$  over the time interval plotted.



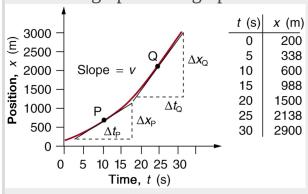
A U.S. Air Force jet car speeds down a track. (credit: Matt Trostle, Flickr)

The graph of position versus time in [link](a) is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses,

showing that the velocity is increasing over time. The slope at any point on a position-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line tangent to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in [link](a). If this is done at every point on the curve and the values are plotted against time, then the graph of velocity versus time shown in [link](b) is obtained. Furthermore, the slope of the graph of velocity versus time is acceleration, which is shown in [link](c).

## **Example:**

**Determining Instantaneous Velocity from the Slope at a Point: Jet Car** Calculate the velocity of the jet car at a time of 25 s by finding the slope of the x vs. t graph in the graph below.



The slope of an x vs. t graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

# **Strategy**

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point. This principle is illustrated in [link], where Q is the point at t = 25 s.

#### Solution

1. Find the tangent line to the curve at t = 25 s.

- 2. Determine the endpoints of the tangent. These correspond to a position of 1300 m at time 19 s and a position of 3120 m at time 32 s.
- 3. Plug these endpoints into the equation to solve for the slope, v.

## **Equation:**

$$ext{slope} = v_{ ext{Q}} = rac{\Delta x_{ ext{Q}}}{\Delta t_{ ext{Q}}} = rac{(3120 ext{ m} - 1300 ext{ m})}{(32 ext{ s} - 19 ext{ s})}$$

Thus,

## **Equation:**

$$v_{
m Q} = rac{1820 \ {
m m}}{13 \ {
m s}} = 140 \ {
m m/s}.$$

#### Discussion

This is the value given in this figure's table for v at t=25 s. The value of 140 m/s for  $v_Q$  is plotted in [link]. The entire graph of v vs. t can be obtained in this fashion.

Carrying this one step further, we note that the slope of a velocity versus time graph is acceleration. Slope is rise divided by run; on a v vs. t graph, rise = change in velocity  $\Delta v$  and run = change in time  $\Delta t$ .

## Note:

The Slope of *v* vs. *t* 

The slope of a graph of velocity v vs. time t is acceleration a.

# **Equation:**

slope = 
$$\frac{\Delta v}{\Delta t} = a$$

Since the velocity versus time graph in [link](b) is a straight line, its slope is the same everywhere, implying that acceleration is constant. Acceleration versus time is graphed in [link](c).

Additional general information can be obtained from [link] and the expression for a straight line, y = mx + b.

In this case, the vertical axis y is V, the intercept b is  $v_0$ , the slope m is a, and the horizontal axis x is t. Substituting these symbols yields **Equation:** 

$$v = v_0 + {
m at.}$$

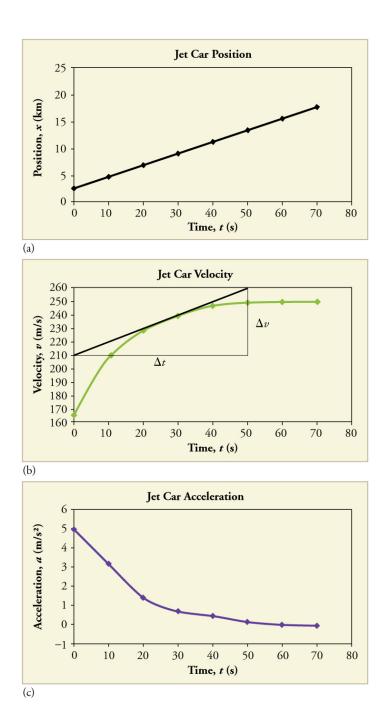
A general relationship for velocity, acceleration, and time has again been obtained from a graph. Notice that this equation was also derived algebraically from other motion equations in <a href="Motion Equations for Constant Acceleration">Motion Equations for Constant Acceleration</a> in One Dimension.

It is not accidental that the same equations are obtained by graphical analysis as by algebraic techniques. In fact, an important way to *discover* physical relationships is to measure various physical quantities and then make graphs of one quantity against another to see if they are correlated in any way. Correlations imply physical relationships and might be shown by smooth graphs such as those above. From such graphs, mathematical relationships can sometimes be postulated. Further experiments are then performed to determine the validity of the hypothesized relationships.

# **Graphs of Motion Where Acceleration is Not Constant**

Now consider the motion of the jet car as it goes from 165 m/s to its top velocity of 250 m/s, graphed in [link]. Time again starts at zero, and the initial position and velocity are 2900 m and 165 m/s, respectively. (These were the final position and velocity of the car in the motion graphed in [link].) Acceleration gradually decreases from  $5.0 \, \mathrm{m/s}^2$  to zero when the car hits 250 m/s. The slope of the x vs. t graph increases until  $t = 55 \, \mathrm{s}$ , after which time the slope is constant. Similarly, velocity increases until 55

s and then becomes constant, since acceleration decreases to zero at 55 s and remains zero afterward.



Graphs of motion of a jet-powered car as it reaches its top velocity. This motion begins where the motion in

[link] ends. (a) The slope of this graph is velocity; it is plotted in the next graph. (b) The velocity gradually approaches its top value. The slope of this graph is acceleration; it is plotted in the final graph. (c) Acceleration gradually declines to zero when velocity becomes constant.

## **Example:**

# Calculating Acceleration from a Graph of Velocity versus Time

Calculate the acceleration of the jet car at a time of 25 s by finding the slope of the v vs. t graph in [link](b).

## Strategy

The slope of the curve at t = 25 s is equal to the slope of the line tangent at that point, as illustrated in [link](b).

#### Solution

Determine endpoints of the tangent line from the figure, and then plug them into the equation to solve for slope, a.

# **Equation:**

$$ext{slope} = rac{\Delta v}{\Delta t} = rac{(260 ext{ m/s} - 210 ext{ m/s})}{(51 ext{ s} - 1.0 ext{ s})}$$

# **Equation:**

$$a = \frac{50 \text{ m/s}}{50 \text{ s}} = 1.0 \text{ m/s}^2.$$

#### Discussion

Note that this value for a is consistent with the value plotted in  $[\underline{link}](c)$  at t=25 s.

A graph of position versus time can be used to generate a graph of velocity versus time, and a graph of velocity versus time can be used to generate a graph of acceleration versus time. We do this by finding the slope of the graphs at every point. If the graph is linear (i.e., a line with a constant slope), it is easy to find the slope at any point and you have the slope for every point. Graphical analysis of motion can be used to describe both specific and general characteristics of kinematics. Graphs can also be used for other topics in physics. An important aspect of exploring physical relationships is to graph them and look for underlying relationships.

#### **Exercise:**

# **Check Your Understanding**

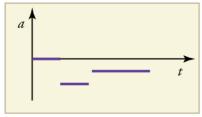
#### **Problem:**

A graph of velocity vs. time of a ship coming into a harbor is shown below. (a) Describe the motion of the ship based on the graph. (b)What would a graph of the ship's acceleration look like?



### **Solution:**

- (a) The ship moves at constant velocity and then begins to decelerate at a constant rate. At some point, its deceleration rate decreases. It maintains this lower deceleration rate until it stops moving.
- (b) A graph of acceleration vs. time would show zero acceleration in the first leg, large and constant negative acceleration in the second leg, and constant negative acceleration.



# **Section Summary**

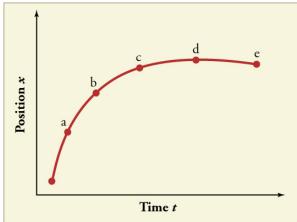
- Graphs of motion can be used to analyze motion.
- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of displacement x vs. time t is velocity v.
- The slope of a graph of velocity v vs. time t graph is acceleration a.
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing graphs.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

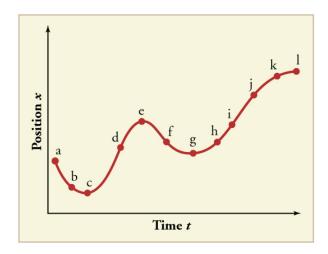
(a) Explain how you can use the graph of position versus time in [link] to describe the change in velocity over time. Identify (b) the time ( $t_a$ ,  $t_b$ ,  $t_c$ ,  $t_d$ , or  $t_e$ ) at which the instantaneous velocity is greatest, (c) the time at which it is zero, and (d) the time at which it is negative.



#### **Exercise:**

#### **Problem:**

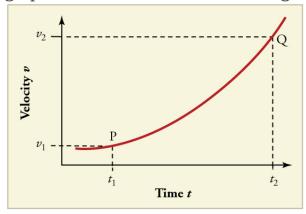
(a) Sketch a graph of velocity versus time corresponding to the graph of position versus time given in [link]. (b) Identify the time or times ( $t_a$ ,  $t_b$ ,  $t_c$ , etc.) at which the instantaneous velocity is greatest. (c) At which times is it zero? (d) At which times is it negative?



## **Exercise:**

## **Problem:**

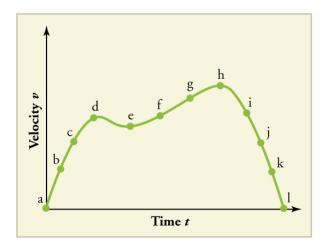
(a) Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in [link]. (b) Based on the graph, how does acceleration change over time?



## **Exercise:**

## **Problem:**

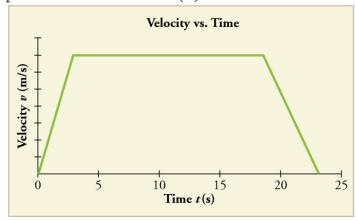
(a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in [link]. (b) Identify the time or times ( $t_a$ ,  $t_b$ ,  $t_c$ , etc.) at which the acceleration is greatest. (c) At which times is it zero? (d) At which times is it negative?



#### **Exercise:**

## **Problem:**

Consider the velocity vs. time graph of a person in an elevator shown in [link]. Suppose the elevator is initially at rest. It then accelerates for 3 seconds, maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion from Motion Equations for Constant Acceleration in One Dimension for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of (a) position vs. time and (b) acceleration vs. time for this trip.



## **Exercise:**

#### **Problem:**

A cylinder is given a push and then rolls up an inclined plane. If the origin is the starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

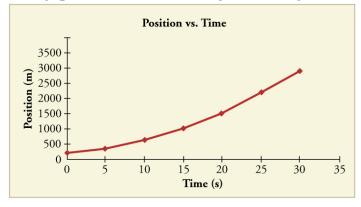
## **Problems & Exercises**

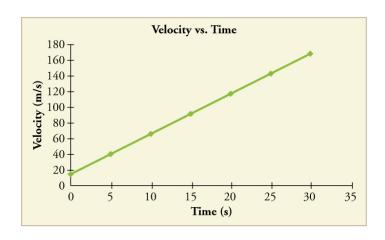
Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

#### **Exercise:**

### **Problem:**

(a) By taking the slope of the curve in [link], verify that the velocity of the jet car is 115 m/s at t=20 s. (b) By taking the slope of the curve at any point in [link], verify that the jet car's acceleration is  $5.0 \text{ m/s}^2$ .





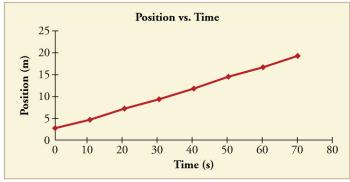
# **Solution:**

- (a) 115 m/s
- (b)  $5.0 \text{ m/s}^2$

## **Exercise:**

## **Problem:**

Using approximate values, calculate the slope of the curve in [link] to verify that the velocity at  $t=10.0~\rm s$  is 0.208 m/s. Assume all values are known to 3 significant figures.



## **Exercise:**

## **Problem:**

Using approximate values, calculate the slope of the curve in [ $\underline{\text{link}}$ ] to verify that the velocity at t=30.0~s is approximately 0.24 m/s.

**Solution:** 

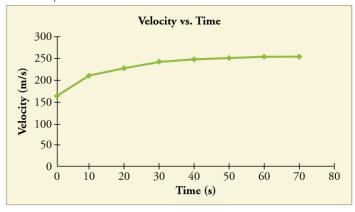
**Equation:** 

$$v = rac{(11.7 - 6.95) imes 10^3 ext{ m}}{(40.0 - 20.0) ext{ s}} = 238 ext{ m/s}$$

#### **Exercise:**

### **Problem:**

By taking the slope of the curve in [link], verify that the acceleration is  $3.2~{\rm m/s}^2$  at  $t=10~{\rm s}$ .

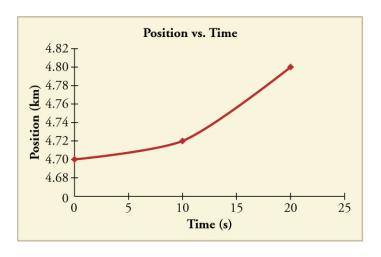


#### **Exercise:**

#### **Problem:**

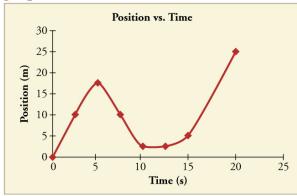
Construct the position graph for the subway shuttle train as shown in  $[\underline{link}](a)$ . Your graph should show the position of the train, in kilometers, from t = 0 to 20 s. You will need to use the information on acceleration and velocity given in the examples for this figure.

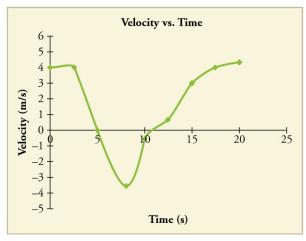
#### **Solution:**

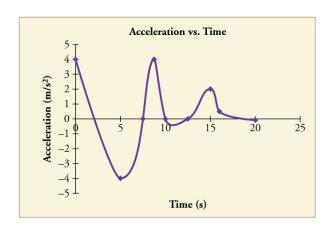


### **Problem:**

(a) Take the slope of the curve in [<u>link</u>] to find the jogger's velocity at  $t=2.5~\rm s$ . (b) Repeat at 7.5 s. These values must be consistent with the graph in [<u>link</u>].

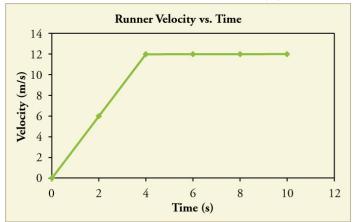






### **Problem:**

A graph of v(t) is shown for a world-class track sprinter in a 100-m race. (See [link]). (a) What is his average velocity for the first 4 s? (b) What is his instantaneous velocity at t=5 s? (c) What is his average acceleration between 0 and 4 s? (d) What is his time for the race?

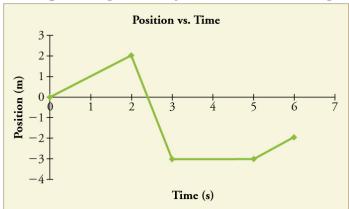


### **Solution:**

- (a) 6 m/s
- (b) 12 m/s
- (c)  $3 \text{ m/s}^2$
- (d) 10 s

### **Problem:**

[link] shows the position graph for a particle for 5 s. Draw the corresponding velocity and acceleration graphs.



### Glossary

### independent variable

the variable that the dependent variable is measured with respect to; usually plotted along the x-axis

### dependent variable

the variable that is being measured; usually plotted along the *y*-axis

### slope

the difference in y-value (the rise) divided by the difference in x-value (the run) of two points on a straight line

### y-intercept

the *y*-value when x=0, or when the graph crosses the *y*-axis

## Introduction to Two-Dimensional Kinematics class="introduction"

Everyday motion that we experience is, thankfully, rarely as tortuous as a rollercoaster ride like this—the Dragon Khan in Spain's Universal Port Aventura Amusement Park. However, most motion is in curved, rather than straight-line, paths. Motion along a curved path is twoor threedimensional motion, and can be described in a similar fashion to one-dimensional motion. (credit: Boris23/Wikimedi a Commons)



The arc of a basketball, the orbit of a satellite, a bicycle rounding a curve, a swimmer diving into a pool, blood gushing out of a wound, and a puppy chasing its tail are but a few examples of motions along curved paths. In fact, most motions in nature follow curved paths rather than straight lines. Motion along a curved path on a flat surface or a plane (such as that of a ball on a pool table or a skater on an ice rink) is two-dimensional, and thus described by two-dimensional kinematics. Motion not confined to a plane, such as a car following a winding mountain road, is described by three-dimensional kinematics. Both two- and three-dimensional kinematics are simple extensions of the one-dimensional kinematics developed for straight-line motion in the previous chapter. This simple extension will allow us to apply physics to many more situations, and it will also yield unexpected insights about nature.

#### Kinematics in Two Dimensions: An Introduction

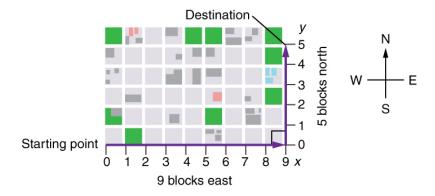
- Observe that motion in two dimensions consists of horizontal and vertical components.
- Understand the independence of horizontal and vertical vectors in twodimensional motion.



Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths. (credit: Margaret W. Carruthers)

### **Two-Dimensional Motion: Walking in a City**

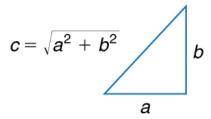
Suppose you want to walk from one point to another in a city with uniform square blocks, as pictured in [link].



A pedestrian walks a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size.

The straight-line path that a helicopter might fly is blocked to you as a pedestrian, and so you are forced to take a two-dimensional path, such as the one shown. You walk 14 blocks in all, 9 east followed by 5 north. What is the straight-line distance?

An old adage states that the shortest distance between two points is a straight line. The two legs of the trip and the straight-line path form a right triangle, and so the Pythagorean theorem,  $a^2 + b^2 = c^2$ , can be used to find the straight-line distance.

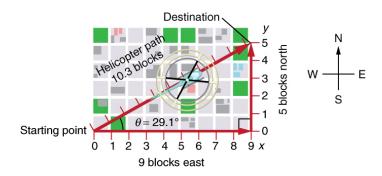


The Pythagorean theorem relates the length of the legs of a right triangle,

labeled a and b, with the hypotenuse, labeled c. The relationship is given by:  $a^2+b^2=c^2$ . This can be rewritten, solving for c:  $c=\sqrt{a^2+b^2}$ .

The hypotenuse of the triangle is the straight-line path, and so in this case its length in units of city blocks is

 $\sqrt{(9 \text{ blocks})^2 + (5 \text{ blocks})^2} = 10.3 \text{ blocks}$ , considerably shorter than the 14 blocks you walked. (Note that we are using three significant figures in the answer. Although it appears that "9" and "5" have only one significant digit, they are discrete numbers. In this case "9 blocks" is the same as "9.0 or 9.00 blocks." We have decided to use three significant figures in the answer in order to show the result more precisely.)



The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian. All blocks are square and the same size.

The fact that the straight-line distance (10.3 blocks) in [link] is less than the total distance walked (14 blocks) is one example of a general characteristic of vectors. (Recall that **vectors** are quantities that have both magnitude and direction.)

As for one-dimensional kinematics, we use arrows to represent vectors. The length of the arrow is proportional to the vector's magnitude. The arrow's length is indicated by hash marks in [link] and [link]. The arrow points in the same direction as the vector. For two-dimensional motion, the path of an object can be represented with three vectors: one vector shows the straightline path between the initial and final points of the motion, one vector shows the horizontal component of the motion, and one vector shows the vertical component of the motion. The horizontal and vertical components of the motion add together to give the straight-line path. For example, observe the three vectors in [link]. The first represents a 9-block displacement east. The second represents a 5-block displacement north. These vectors are added to give the third vector, with a 10.3-block total displacement. The third vector is the straight-line path between the two points. Note that in this example, the vectors that we are adding are perpendicular to each other and thus form a right triangle. This means that we can use the Pythagorean theorem to calculate the magnitude of the total displacement. (Note that we cannot use the Pythagorean theorem to add vectors that are not perpendicular. We will develop techniques for adding vectors having any direction, not just those perpendicular to one another, in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods.)

### The Independence of Perpendicular Motions

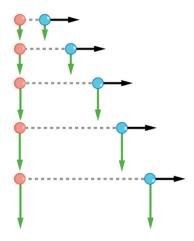
The person taking the path shown in [link] walks east and then north (two perpendicular directions). How far he or she walks east is only affected by his or her motion eastward. Similarly, how far he or she walks north is only affected by his or her motion northward.

#### Note:

### Independence of Motion

The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

This is true in a simple scenario like that of walking in one direction first, followed by another. It is also true of more complicated motion involving movement in two directions at once. For example, let's compare the motions of two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.



This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent

position is an equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, and not by any horizontal forces.) Careful examination of the

ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). Note that this case is true only for ideal conditions. In the real world, air resistance will affect the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called *projectile motion*, is to *resolve* (break) it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent. We shall see how to resolve vectors in <u>Vector Addition and Subtraction: Graphical Methods</u> and <u>Vector Addition and Subtraction: Analytical Methods</u>. We will find such techniques to be useful in many areas of physics.

### Note:

### PhET Explorations: Ladybug Motion 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

https://archive.cnx.org/specials/317a2b1e-2fbd-11e5-99b5-e38ffb545fe6/ladybug-motion/#sim-ladybug-motion

### **Summary**

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion

in the vertical direction, and vice versa.

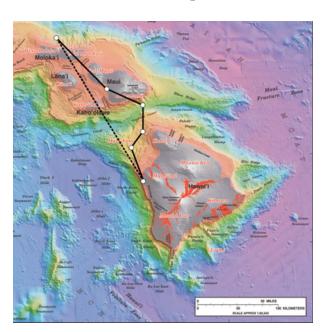
## Glossary

### vector

a quantity that has both magnitude and direction; an arrow used to represent quantities with both magnitude and direction

### Vector Addition and Subtraction: Graphical Methods

- Understand the rules of vector addition, subtraction, and multiplication.
- Apply graphical methods of vector addition and subtraction to determine the displacement of moving objects.



Displacement can be determined graphically using a scale map, such as this one of the Hawaiian Islands. A journey from Hawai'i to Moloka'i has a number of legs, or journey segments. These segments can be added graphically with a ruler to determine the total two-dimensional displacement of the journey. (credit: US Geological Survey)

### **Vectors in Two Dimensions**

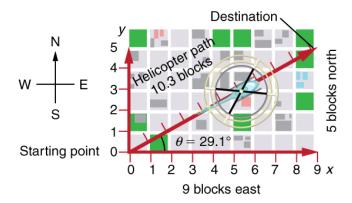
A **vector** is a quantity that has magnitude and direction. Displacement, velocity, acceleration, and force, for example, are all vectors. In one-dimensional, or straight-line, motion, the direction of a vector can be given simply by a plus or minus sign. In two dimensions (2-d), however, we specify the direction of a vector relative to some reference frame (i.e., coordinate system), using an arrow having length proportional to the vector's magnitude and pointing in the direction of the vector.

[link] shows such a *graphical representation of a vector*, using as an example the total displacement for the person walking in a city considered in Kinematics in Two Dimensions: An Introduction. We shall use the notation that a boldface symbol, such as D, stands for a vector. Its magnitude is represented by the symbol in italics, D, and its direction by  $\theta$ .

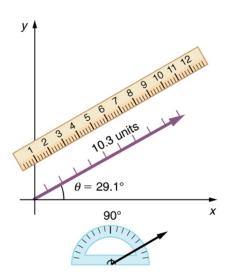
#### Note:

#### Vectors in this Text

In this text, we will represent a vector with a boldface variable. For example, we will represent the quantity force with the vector F, which has both magnitude and direction. The magnitude of the vector will be represented by a variable in italics, such as F, and the direction of the variable will be given by an angle  $\theta$ .



A person walks 9 blocks east and 5 blocks north. The displacement is 10.3 blocks at an angle 29.1° north of east.

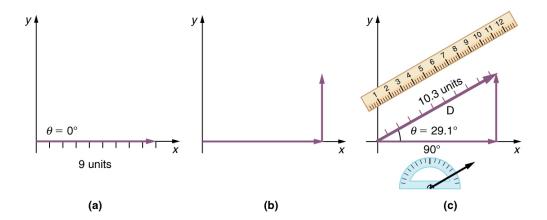


To describe the resultant vector for the person walking in a city considered in [link] graphically, draw an arrow to represent the total displacement vector D. Using a protractor, draw a line at an angle  $\theta$ relative to the eastwest axis. The length D of the arrow is proportional to the vector's

magnitude and is measured along the line with a ruler. In this example, the magnitude D of the vector is 10.3 units, and the direction  $\theta$  is  $29.1^{\circ}$  north of east.

### **Vector Addition: Head-to-Tail Method**

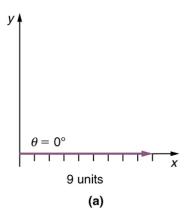
The **head-to-tail method** is a graphical way to add vectors, described in [link] below and in the steps following. The **tail** of the vector is the starting point of the vector, and the **head** (or tip) of a vector is the final, pointed end of the arrow.



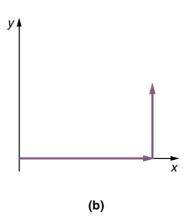
**Head-to-Tail Method:** The head-to-tail method of graphically adding vectors is illustrated for the two displacements of the person walking in a city considered in [link]. (a) Draw a vector representing the displacement to the east. (b) Draw a vector representing the displacement to the north. The tail of this vector should originate from the head of the first, east-pointing vector.

(c) Draw a line from the tail of the east-pointing vector to the head of the north-pointing vector to form the sum or **resultant vector** D. The length of the arrow D is proportional to the vector's magnitude and is measured to be 10.3 units . Its direction, described as the angle with respect to the east (or horizontal axis)  $\theta$  is measured with a protractor to be 29.1°.

**Step 1.** Draw an arrow to represent the first vector (9 blocks to the east) using a ruler and protractor.

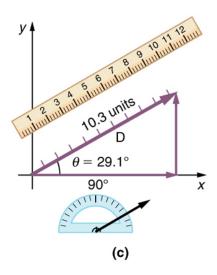


**Step 2.** Now draw an arrow to represent the second vector (5 blocks to the north). *Place the tail of the second vector at the head of the first vector*.



**Step 3.** If there are more than two vectors, continue this process for each vector to be added. Note that in our example, we have only two vectors, so we have finished placing arrows tip to tail.

**Step 4.** Draw an arrow from the tail of the first vector to the head of the last vector. This is the **resultant**, or the sum, of the other vectors.



**Step 5.** To get the **magnitude** of the resultant, *measure its length with a ruler.* (Note that in most calculations, we will use the Pythagorean theorem to determine this length.)

**Step 6.** To get the **direction** of the resultant, measure the angle it makes with the reference frame using a protractor. (Note that in most calculations, we will use trigonometric relationships to determine this angle.)

The graphical addition of vectors is limited in accuracy only by the precision with which the drawings can be made and the precision of the measuring tools. It is valid for any number of vectors.

### **Example:**

# Adding Vectors Graphically Using the Head-to-Tail Method: A Woman Takes a Walk

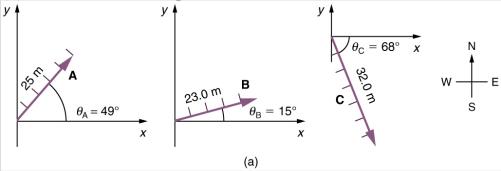
Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, she walks 25.0 m in a direction 49.0° north of east. Then, she walks 23.0 m heading 15.0° north of east. Finally, she turns and walks 32.0 m in a direction 68.0° south of east.

### Strategy

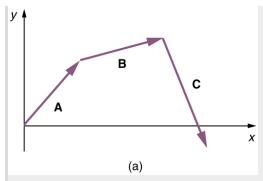
Represent each displacement vector graphically with an arrow, labeling the first A, the second B, and the third C, making the lengths proportional to the distance and the directions as specified relative to an east-west line. The head-to-tail method outlined above will give a way to determine the magnitude and direction of the resultant displacement, denoted  $\mathbf{R}$ .

#### Solution

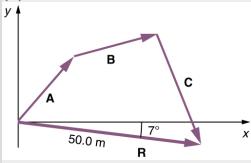
(1) Draw the three displacement vectors.



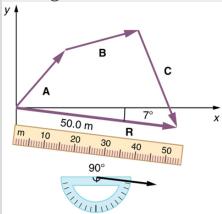
(2) Place the vectors head to tail retaining both their initial magnitude and direction.



(3) Draw the resultant vector, R.



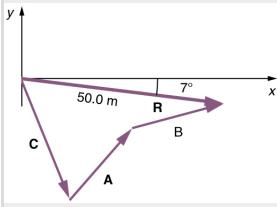
(4) Use a ruler to measure the magnitude of  $\mathbf{R}$ , and a protractor to measure the direction of  $\mathbf{R}$ . While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since the resultant vector is south of the eastward pointing axis, we flip the protractor upside down and measure the angle between the eastward axis and the vector.



In this case, the total displacement  ${\bf R}$  is seen to have a magnitude of 50.0 m and to lie in a direction 7.0° south of east. By using its magnitude and direction, this vector can be expressed as R=50.0 m and  $\theta=7.0$ ° south of east.

### Discussion

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that the resultant is independent of the order in which the vectors are added. Therefore, we could add the vectors in any order as illustrated in [link] and we will still get the same solution.



Here, we see that when the same vectors are added in a different order, the result is the same. This characteristic is true in every case and is an important characteristic of vectors. Vector addition is **commutative**. Vectors can be added in any order.

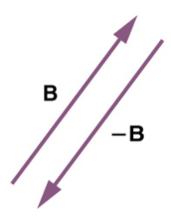
### **Equation:**

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}.$$

(This is true for the addition of ordinary numbers as well—you get the same result whether you add  $\mathbf{2} + \mathbf{3}$  or  $\mathbf{3} + \mathbf{2}$ , for example).

### **Vector Subtraction**

Vector subtraction is a straightforward extension of vector addition. To define subtraction (say we want to subtract  $\mathbf{B}$  from  $\mathbf{A}$ , written  $\mathbf{A} - \mathbf{B}$ , we must first define what we mean by subtraction. The *negative* of a vector  $\mathbf{B}$  is defined to be  $-\mathbf{B}$ ; that is, graphically *the negative of any vector has the same magnitude but the opposite direction*, as shown in [link]. In other words,  $\mathbf{B}$  has the same length as  $-\mathbf{B}$ , but points in the opposite direction. Essentially, we just flip the vector so it points in the opposite direction.



The negative of a vector is just another vector of the same magnitude but pointing in the opposite direction. So **B** is the negative of **-B**; it has the same length but opposite direction.

The *subtraction* of vector  $\mathbf{B}$  from vector  $\mathbf{A}$  is then simply defined to be the addition of  $-\mathbf{B}$  to  $\mathbf{A}$ . Note that vector subtraction is the addition of a negative vector. The order of subtraction does not affect the results.

### **Equation:**

$$A - B = A + (-\mathbf{B}).$$

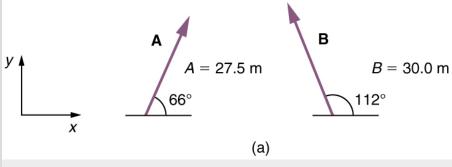
This is analogous to the subtraction of scalars (where, for example, 5-2=5+(-2)). Again, the result is independent of the order in which

the subtraction is made. When vectors are subtracted graphically, the techniques outlined above are used, as the following example illustrates.

### **Example:**

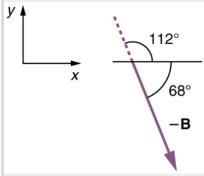
### Subtracting Vectors Graphically: A Woman Sailing a Boat

A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction 66.0° north of east from her current location, and then travel 30.0 m in a direction 112° north of east (or 22.0° west of north). If the woman makes a mistake and travels in the *opposite* direction for the second leg of the trip, where will she end up? Compare this location with the location of the dock.



### Strategy

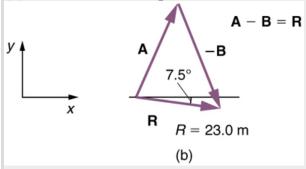
We can represent the first leg of the trip with a vector  $\mathbf{A}$ , and the second leg of the trip with a vector  $\mathbf{B}$ . The dock is located at a location  $\mathbf{A} + \mathbf{B}$ . If the woman mistakenly travels in the *opposite* direction for the second leg of the journey, she will travel a distance B (30.0 m) in the direction  $180^{\circ}-112^{\circ}=68^{\circ}$  south of east. We represent this as  $-\mathbf{B}$ , as shown below. The vector  $-\mathbf{B}$  has the same magnitude as  $\mathbf{B}$  but is in the opposite direction. Thus, she will end up at a location  $\mathbf{A}+(-\mathbf{B})$ , or  $\mathbf{A}-\mathbf{B}$ .



We will perform vector addition to compare the location of the dock, A + B, with the location at which the woman mistakenly arrives, A + (-B).

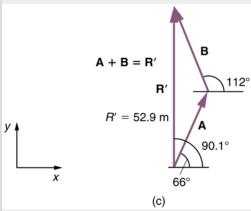
#### Solution

- (1) To determine the location at which the woman arrives by accident, draw vectors  $\mathbf{A}$  and  $-\mathbf{B}$ .
- (2) Place the vectors head to tail.
- (3) Draw the resultant vector  $\mathbf{R}$ .
- (4) Use a ruler and protractor to measure the magnitude and direction of  $\mathbf{R}$ .



In this case,  $R=23.0~\mathrm{m}$  and  $\theta=7.5^{\circ}$  south of east.

(5) To determine the location of the dock, we repeat this method to add vectors  $\mathbf{A}$  and  $\mathbf{B}$ . We obtain the resultant vector  $\mathbf{R}'$ :



In this case  $R=52.9~\mathrm{m}$  and  $\theta=90.1^{\circ}$  north of east.

We can see that the woman will end up a significant distance from the dock if she travels in the opposite direction for the second leg of the trip.

#### Discussion

Because subtraction of a vector is the same as addition of a vector with the opposite direction, the graphical method of subtracting vectors works the same as for addition.

### **Multiplication of Vectors and Scalars**

If we decided to walk three times as far on the first leg of the trip considered in the preceding example, then we would walk  $3\times 27.5$  m, or 82.5 m, in a direction  $66.0^{\circ}$  north of east. This is an example of multiplying a vector by a positive **scalar**. Notice that the magnitude changes, but the direction stays the same.

If the scalar is negative, then multiplying a vector by it changes the vector's magnitude and gives the new vector the *opposite* direction. For example, if you multiply by -2, the magnitude doubles but the direction changes. We can summarize these rules in the following way: When vector  $\mathbf{A}$  is multiplied by a scalar c,

- the magnitude of the vector becomes the absolute value of cA,
- if *c* is positive, the direction of the vector does not change,
- if *c* is negative, the direction is reversed.

In our case, c=3 and A=27.5 m. Vectors are multiplied by scalars in many situations. Note that division is the inverse of multiplication. For example, dividing by 2 is the same as multiplying by the value (1/2). The rules for multiplication of vectors by scalars are the same for division; simply treat the divisor as a scalar between 0 and 1.

### **Resolving a Vector into Components**

In the examples above, we have been adding vectors to determine the resultant vector. In many cases, however, we will need to do the opposite. We will need to take a single vector and find what other vectors added together produce it. In most cases, this involves determining the perpendicular **components** of a single vector, for example the *x- and y-* components, or the north-south and east-west components.

For example, we may know that the total displacement of a person walking in a city is 10.3 blocks in a direction 29.0° north of east and want to find out how many blocks east and north had to be walked. This method is called *finding the components (or parts)* of the displacement in the east and north

directions, and it is the inverse of the process followed to find the total displacement. It is one example of finding the components of a vector. There are many applications in physics where this is a useful thing to do. We will see this soon in <a href="Projectile Motion">Projectile Motion</a>, and much more when we cover forces in <a href="Dynamics: Newton's Laws of Motion">Dynamics: Newton's Laws of Motion</a>. Most of these involve finding components along perpendicular axes (such as north and east), so that right triangles are involved. The analytical techniques presented in <a href="Vector Addition and Subtraction: Analytical Methods">Vector Addition and Subtraction: Analytical Methods</a> are ideal for finding vector components.

#### Note:

PhET Explorations: Maze Game

Learn about position, velocity, and acceleration in the "Arena of Pain". Use the green arrow to move the ball. Add more walls to the arena to make the game more difficult. Try to make a goal as fast as you can.

https://archive.cnx.org/specials/30e37034-2fbd-11e5-83a2-

03be60006ece/maze-game/#sim-maze-game

### Summary

- The **graphical method of adding vectors A** and **B** involves drawing vectors on a graph and adding them using the head-to-tail method. The resultant vector **R** is defined such that  $\mathbf{A} + \mathbf{B} = \mathbf{R}$ . The magnitude and direction of **R** are then determined with a ruler and protractor, respectively.
- The **graphical method of subtracting vector B** from **A** involves adding the opposite of vector **B**, which is defined as  $-\mathbf{B}$ . In this case,  $\mathbf{A}-\mathbf{B}=\mathbf{A}+(-\mathbf{B})=\mathbf{R}$ . Then, the head-to-tail method of addition is followed in the usual way to obtain the resultant vector **R**.
- Addition of vectors is **commutative** such that  $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$ .
- The **head-to-tail method** of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at

- the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- If a vector **A** is multiplied by a scalar quantity *c*, the magnitude of the product is given by cA. If *c* is positive, the direction of the product points in the same direction as **A**; if *c* is negative, the direction of the product points in the opposite direction as **A**.

### **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

Which of the following is a vector: a person's height, the altitude on Mt. Everest, the age of the Earth, the boiling point of water, the cost of this book, the Earth's population, the acceleration of gravity?

#### **Exercise:**

#### **Problem:**

Give a specific example of a vector, stating its magnitude, units, and direction.

#### **Exercise:**

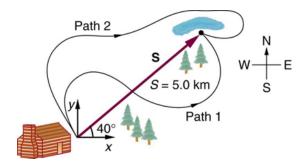
#### **Problem:**

What do vectors and scalars have in common? How do they differ?

#### **Exercise:**

#### **Problem:**

Two campers in a national park hike from their cabin to the same spot on a lake, each taking a different path, as illustrated below. The total distance traveled along Path 1 is 7.5 km, and that along Path 2 is 8.2 km. What is the final displacement of each camper?



### **Problem:**

If an airplane pilot is told to fly 123 km in a straight line to get from San Francisco to Sacramento, explain why he could end up anywhere on the circle shown in [link]. What other information would he need to get to Sacramento?



#### **Exercise:**

#### **Problem:**

Suppose you take two steps  $\bf A$  and  $\bf B$  (that is, two nonzero displacements). Under what circumstances can you end up at your starting point? More generally, under what circumstances can two nonzero vectors add to give zero? Is the maximum distance you can end up from the starting point  $\bf A + \bf B$  the sum of the lengths of the two steps?

**Problem:** Explain why it is not possible to add a scalar to a vector.

#### **Exercise:**

#### **Problem:**

If you take two steps of different sizes, can you end up at your starting point? More generally, can two vectors with different magnitudes ever add to zero? Can three or more?

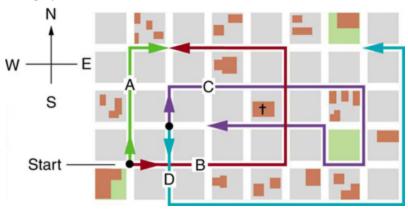
### **Problems & Exercises**

Use graphical methods to solve these problems. You may assume data taken from graphs is accurate to three digits.

#### **Exercise:**

#### **Problem:**

Find the following for path A in [link]: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.



The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

### **Solution:**

- (a) 480 m
- (b) 379 m, 18.4° east of north

#### **Exercise:**

#### **Problem:**

Find the following for path B in [link]: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

#### **Exercise:**

#### **Problem:**

Find the north and east components of the displacement for the hikers shown in [link].

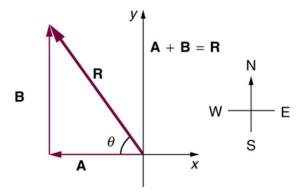
#### **Solution:**

north component 3.21 km, east component 3.83 km

#### **Exercise:**

#### **Problem:**

Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements  $\bf A$  and  $\bf B$ , as in [link], then this problem asks you to find their sum  $\bf R = \bf A + \bf B$ .)

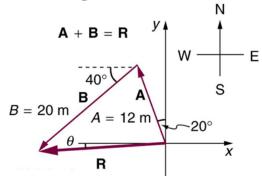


The two displacements  $\mathbf{A}$  and  $\mathbf{B}$  add to give a total displacement  $\mathbf{R}$  having magnitude R and direction  $\theta$ .

### **Exercise:**

### **Problem:**

Suppose you first walk 12.0 m in a direction  $20^{\circ}$  west of north and then 20.0 m in a direction  $40.0^{\circ}$  south of west. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements  $\bf A$  and  $\bf B$ , as in [link], then this problem finds their sum  $\bf R=\bf A+\bf B$ .)



#### **Solution:**

 $19.5 \text{ m}, 4.65^{\circ} \text{ south of west}$ 

#### **Problem:**

Repeat the problem above, but reverse the order of the two legs of the walk; show that you get the same final result. That is, you first walk leg  $\bf B$ , which is 20.0 m in a direction exactly  $40^{\circ}$  south of west, and then leg  $\bf A$ , which is 12.0 m in a direction exactly  $20^{\circ}$  west of north. (This problem shows that  $\bf A + \bf B = \bf B + \bf A$ .)

#### **Exercise:**

#### **Problem:**

(a) Repeat the problem two problems prior, but for the second leg you walk 20.0 m in a direction  $40.0^{\circ}$  north of east (which is equivalent to subtracting  $\mathbf{B}$  from  $\mathbf{A}$  —that is, to finding  $\mathbf{R}/=\mathbf{A}-\mathbf{B}$ ). (b) Repeat the problem two problems prior, but now you first walk 20.0 m in a direction  $40.0^{\circ}$  south of west and then 12.0 m in a direction  $20.0^{\circ}$  east of south (which is equivalent to subtracting  $\mathbf{A}$  from  $\mathbf{B}$  —that is, to finding  $\mathbf{R}//=\mathbf{B}-\mathbf{A}=-\mathbf{R}/$ ). Show that this is the case.

#### **Solution:**

- (a) 26.6 m,  $65.1^{\circ}$  north of east
- (b) 26.6 m,  $65.1^{\circ}$  south of west

#### **Exercise:**

#### **Problem:**

Show that the *order* of addition of three vectors does not affect their sum. Show this property by choosing any three vectors  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$ , all having different lengths and directions. Find the sum  $\mathbf{A} + \mathbf{B} + \mathbf{C}$  then find their sum when added in a different order and show the result is the same. (There are five other orders in which  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$  can be added; choose only one.)

#### **Exercise:**

### **Problem:**

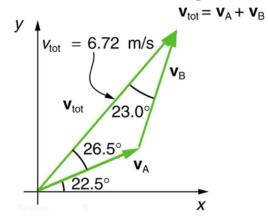
Show that the sum of the vectors discussed in [link] gives the result shown in [link].

#### **Solution:**

52.9 m,  $90.1^{\circ}$  with respect to the *x*-axis.

### **Exercise:**

**Problem:** Find the magnitudes of velocities  $v_{\rm A}$  and  $v_{\rm B}$  in [link]



The two velocities  $\mathbf{v}_{\mathrm{A}}$  and  $\mathbf{v}_{\mathrm{B}}$  add to give a total  $\mathbf{v}_{\mathrm{tot}}.$ 

### **Exercise:**

### **Problem:**

Find the components of  $v_{\text{tot}}$  along the x- and y-axes in [link].

### **Solution:**

*x*-component 4.41 m/s

### y-component 5.07 m/s

#### **Exercise:**

#### **Problem:**

Find the components of  $v_{\text{tot}}$  along a set of perpendicular axes rotated  $30^{\circ}$  counterclockwise relative to those in [link].

### **Glossary**

### component (of a 2-d vector)

a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components

#### commutative

refers to the interchangeability of order in a function; vector addition is commutative because the order in which vectors are added together does not affect the final sum

### direction (of a vector)

the orientation of a vector in space

### head (of a vector)

the end point of a vector; the location of the tip of the vector's arrowhead; also referred to as the "tip"

#### head-to-tail method

a method of adding vectors in which the tail of each vector is placed at the head of the previous vector

### magnitude (of a vector)

the length or size of a vector; magnitude is a scalar quantity

#### resultant

the sum of two or more vectors

# resultant vector

the vector sum of two or more vectors

# scalar

a quantity with magnitude but no direction

# tail

the start point of a vector; opposite to the head or tip of the arrow

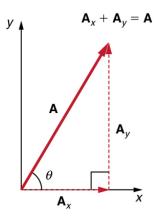
# Vector Addition and Subtraction: Analytical Methods

- Understand the rules of vector addition and subtraction using analytical methods.
- Apply analytical methods to determine vertical and horizontal component vectors.
- Apply analytical methods to determine the magnitude and direction of a resultant vector.

Analytical methods of vector addition and subtraction employ geometry and simple trigonometry rather than the ruler and protractor of graphical methods. Part of the graphical technique is retained, because vectors are still represented by arrows for easy visualization. However, analytical methods are more concise, accurate, and precise than graphical methods, which are limited by the accuracy with which a drawing can be made. Analytical methods are limited only by the accuracy and precision with which physical quantities are known.

# **Resolving a Vector into Perpendicular Components**

Analytical techniques and right triangles go hand-in-hand in physics because (among other things) motions along perpendicular directions are independent. We very often need to separate a vector into perpendicular components. For example, given a vector like  $\mathbf{A}$  in [link], we may wish to find which two perpendicular vectors,  $\mathbf{A}_x$  and  $\mathbf{A}_y$ , add to produce it.



The vector  $\mathbf{A}$ , with its tail at the origin of an x, ycoordinate system, is shown together with its *x*- and *y*components,  $\mathbf{A}_x$  and  $\mathbf{A}_y$ . These vectors form a right triangle. The analytical relationships among these vectors are summarized below.

 $\mathbf{A}_x$  and  $\mathbf{A}_y$  are defined to be the components of  $\mathbf{A}$  along the x- and y-axes. The three vectors  $\mathbf{A}$ ,  $\mathbf{A}_x$ , and  $\mathbf{A}_y$  form a right triangle:

# **Equation:**

$$\mathbf{A}_x + \mathbf{A}_y = \mathbf{A}.$$

Note that this relationship between vector components and the resultant vector holds only for vector quantities (which include both magnitude and direction). The relationship does not apply for the magnitudes alone. For example, if  $\mathbf{A}_x = 3$  m east,  $\mathbf{A}_y = 4$  m north, and  $\mathbf{A} = 5$  m north-east, then it is true that the vectors  $\mathbf{A}_x + \mathbf{A}_y = \mathbf{A}$ . However, it is *not* true that the sum of the magnitudes of the vectors is also equal. That is,

$$3~m+4~m~\neq~5~m$$

Thus,

# **Equation:**

$$A_x + A_y 
eq A$$

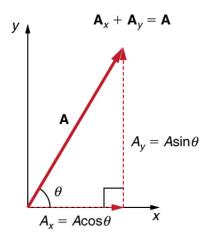
If the vector  $\mathbf{A}$  is known, then its magnitude A (its length) and its angle  $\theta$  (its direction) are known. To find  $A_x$  and  $A_y$ , its x- and y-components, we use the following relationships for a right triangle.

# **Equation:**

$$A_x = A \cos \theta$$

and

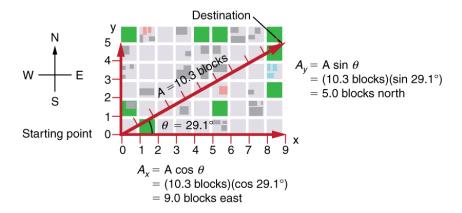
$$A_y = A \sin \theta$$
.



The magnitudes of the vector

components  $\mathbf{A}_x$  and  $\mathbf{A}_y$  can be related to the resultant vector  $\mathbf{A}$  and the angle  $\theta$  with trigonometric identities. Here we see that  $A_x = A \cos \theta$  and  $A_y = A \sin \theta$ .

Suppose, for example, that **A** is the vector representing the total displacement of the person walking in a city considered in <u>Kinematics in Two Dimensions: An Introduction</u> and <u>Vector Addition and Subtraction:</u> <u>Graphical Methods</u>.



We can use the relationships  $A_x = A \cos \theta$  and  $A_y = A \sin \theta$  to determine the magnitude of the horizontal and vertical component vectors in this example.

Then A=10.3 blocks and  $\theta=29.1^{\rm o}$  , so that

# **Equation:**

$$A_x = A\cos heta = (10.3 ext{ blocks})(\cos29.1^\circ) = 9.0 ext{ blocks}$$

# **Equation:**

$$A_y = A \sin \theta = (10.3 ext{ blocks})(\sin 29.1^{\circ}) = 5.0 ext{ blocks}.$$

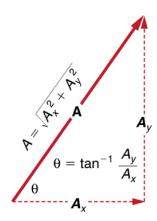
# **Calculating a Resultant Vector**

If the perpendicular components  $\mathbf{A}_x$  and  $\mathbf{A}_y$  of a vector  $\mathbf{A}$  are known, then  $\mathbf{A}$  can also be found analytically. To find the magnitude A and direction  $\theta$  of a vector from its perpendicular components  $\mathbf{A}_x$  and  $\mathbf{A}_y$ , we use the following relationships:

# **Equation:**

$$A=\sqrt{A_{x^2}+A_{y^2}}$$

$$heta= an^{-1}(A_y/A_x).$$



The magnitude and direction of the resultant vector can be determined once the horizontal and vertical components  $A_x$  and  $A_y$  have been determined.

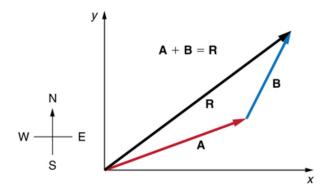
Note that the equation  $A=\sqrt{A_x^2+A_y^2}$  is just the Pythagorean theorem relating the legs of a right triangle to the length of the hypotenuse. For example, if  $A_x$  and  $A_y$  are 9 and 5 blocks, respectively, then  $A=\sqrt{9^2+5^2}{=}10.3$  blocks, again consistent with the example of the person walking in a city. Finally, the direction is  $\theta=\tan^{-1}(5/9){=}29.1^\circ$ , as before.

## **Note:**

Determining Vectors and Vector Components with Analytical Methods Equations  $A_x = A\cos\theta$  and  $A_y = A\sin\theta$  are used to find the perpendicular components of a vector—that is, to go from A and  $\theta$  to  $A_x$  and  $A_y$ . Equations  $A = \sqrt{A_x^2 + A_y^2}$  and  $\theta = \tan^{-1}(A_y/A_x)$  are used to find a vector from its perpendicular components—that is, to go from  $A_x$  and  $A_y$  to A and  $\theta$ . Both processes are crucial to analytical methods of vector addition and subtraction.

# **Adding Vectors Using Analytical Methods**

To see how to add vectors using perpendicular components, consider  $[\underline{link}]$ , in which the vectors  $\mathbf{A}$  and  $\mathbf{B}$  are added to produce the resultant  $\mathbf{R}$ .



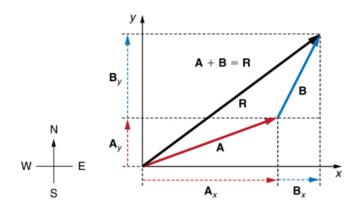
Vectors **A** and **B** are two legs of a walk, and **R** is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of **R**.

If **A** and **B** represent two legs of a walk (two displacements), then **R** is the total displacement. The person taking the walk ends up at the tip of **R**. There are many ways to arrive at the same point. In particular, the person could have walked first in the x-direction and then in the y-direction. Those paths are the x- and y-components of the resultant,  $\mathbf{R}_x$  and  $\mathbf{R}_y$ . If we know

 $\mathbf{R}_x$  and  $\mathbf{R}_y$ , we can find R and  $\theta$  using the equations  $A = \sqrt{{A_x}^2 + {A_y}^2}$  and  $\theta = \tan^{-1}(A_y/A_x)$ . When you use the analytical method of vector addition, you can determine the components or the magnitude and direction of a vector.

**Step 1.** Identify the x- and y-axes that will be used in the problem. Then, find the components of each vector to be added along the chosen

perpendicular axes. Use the equations  $A_x = A \cos \theta$  and  $A_y = A \sin \theta$  to find the components. In [link], these components are  $A_x$ ,  $A_y$ ,  $B_x$ , and  $B_y$ . The angles that vectors  $\mathbf{A}$  and  $\mathbf{B}$  make with the x-axis are  $\theta_A$  and  $\theta_B$ , respectively.



To add vectors  $\mathbf{A}$  and  $\mathbf{B}$ , first determine the horizontal and vertical components of each vector. These are the dotted vectors  $\mathbf{A}_x$ ,  $\mathbf{A}_y$ ,  $\mathbf{B}_x$  and  $\mathbf{B}_y$  shown in the image.

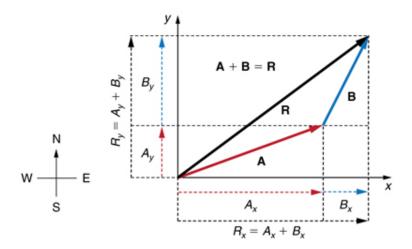
**Step 2.** Find the components of the resultant along each axis by adding the components of the individual vectors along that axis. That is, as shown in  $[\underline{link}]$ ,

# **Equation:**

$$R_x = A_x + B_x$$

and

$$R_y = A_y + B_y.$$



The magnitude of the vectors  $\mathbf{A}_x$  and  $\mathbf{B}_x$  add to give the magnitude  $R_x$  of the resultant vector in the horizontal direction. Similarly, the magnitudes of the vectors  $\mathbf{A}_y$  and  $\mathbf{B}_y$  add to give the magnitude  $R_y$  of the resultant vector in the vertical direction.

Components along the same axis, say the x-axis, are vectors along the same line and, thus, can be added to one another like ordinary numbers. The same is true for components along the y-axis. (For example, a 9-block eastward walk could be taken in two legs, the first 3 blocks east and the second 6 blocks east, for a total of 9, because they are along the same direction.) So resolving vectors into components along common axes makes it easier to add them. Now that the components of  $\mathbf{R}$  are known, its magnitude and direction can be found.

**Step 3.** To get the magnitude R of the resultant, use the Pythagorean theorem:

$$R = \sqrt{R_x^2 + R_y^2}.$$

**Step 4.** To get the direction of the resultant: **Equation:** 

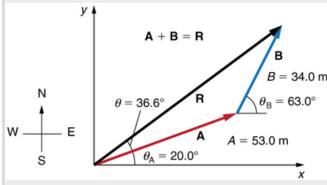
$$heta= an^{-1}(R_y/R_x).$$

The following example illustrates this technique for adding vectors using perpendicular components.

# **Example:**

# **Adding Vectors Using Analytical Methods**

Add the vector  $\mathbf{A}$  to the vector  $\mathbf{B}$  shown in [link], using perpendicular components along the x- and y-axes. The x- and y-axes are along the eastwest and north—south directions, respectively. Vector  $\mathbf{A}$  represents the first leg of a walk in which a person walks 53.0 m in a direction 20.0° north of east. Vector  $\mathbf{B}$  represents the second leg, a displacement of 34.0 m in a direction 63.0° north of east.



Vector **A** has magnitude 53.0 m and direction 20.0° north of the *x*-axis. Vector **B** has magnitude 34.0 m and direction 63.0° north of the *x*-axis. You can use analytical methods to determine the magnitude and direction of **R**.

# **Strategy**

The components of A and B along the x- and y-axes represent walking due east and due north to get to the same ending point. Once found, they are combined to produce the resultant.

## Solution

Following the method outlined above, we first find the components of  $\bf A$  and  $\bf B$  along the x- and y-axes. Note that A=53.0 m,  $\theta_{\rm A}=20.0^{\circ}$ , B=34.0 m, and  $\theta_{\rm B}=63.0^{\circ}$ . We find the x-components by using  $A_x=A\cos\theta$ , which gives

# **Equation:**

$$A_x = A \cos heta_{
m A} = (53.0 \ {
m m})(\cos 20.0^{
m o}) = (53.0 \ {
m m})(0.940) = 49.8 \ {
m m}$$

and

# **Equation:**

$$B_x = B \cos \theta_{\rm B} = (34.0 \text{ m})(\cos 63.0^{\circ})$$
  
=  $(34.0 \text{ m})(0.454) = 15.4 \text{ m}.$ 

Similarly, the *y*-components are found using  $A_y = A \sin \theta_A$ :

# **Equation:**

$$A_y = A \sin heta_{
m A} = (53.0 \ {
m m})(\sin 20.0^{
m o}) \ = (53.0 \ {
m m})(0.342) = 18.1 \ {
m m}$$

and

# **Equation:**

$$B_y = B \sin \theta_{\rm B} = (34.0 \text{ m})(\sin 63.0^{\circ})$$
  
=  $(34.0 \text{ m})(0.891) = 30.3 \text{ m}.$ 

The *x*- and *y*-components of the resultant are thus

# **Equation:**

$$R_x = A_x + B_x = 49.8 \text{ m} + 15.4 \text{ m} = 65.2 \text{ m}$$

and

# **Equation:**

$$R_y = A_y + B_y = 18.1 \text{ m} + 30.3 \text{ m} = 48.4 \text{ m}.$$

Now we can find the magnitude of the resultant by using the Pythagorean theorem:

# **Equation:**

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(65.2)^2 + (48.4)^2 ext{ m}}$$

so that

# **Equation:**

$$R = 81.2 \text{ m}.$$

Finally, we find the direction of the resultant:

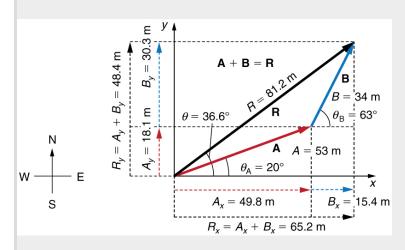
# **Equation:**

$$\theta = \tan^{-1}(R_y/R_x) = +\tan^{-1}(48.4/65.2).$$

Thus,

# **Equation:**

$$\theta = \tan^{-1}(0.742) = 36.6^{\circ}.$$



Using analytical methods, we see that the magnitude of  ${f R}$  is  $81.2~{f m}$  and its

direction is 36.6° north of east.

## Discussion

This example illustrates the addition of vectors using perpendicular components. Vector subtraction using perpendicular components is very similar—it is just the addition of a negative vector.

Subtraction of vectors is accomplished by the addition of a negative vector. That is,  $\mathbf{A} - \mathbf{B} \equiv \mathbf{A} + (-\mathbf{B})$ . Thus, the method for the subtraction of vectors using perpendicular components is identical to that for addition. The components of  $-\mathbf{B}$  are the negatives of the components of  $\mathbf{B}$ . The *x*-and *y*-components of the resultant  $\mathbf{A} - \mathbf{B} = \mathbf{R}$  are thus

# **Equation:**

$$R_x = A_x + (-B_x)$$

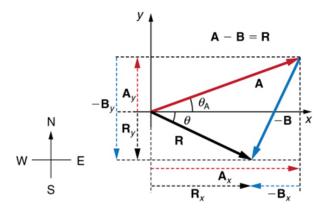
and

# **Equation:**

$$R_y = A_y + (-B_y)$$

and the rest of the method outlined above is identical to that for addition. (See [link].)

Analyzing vectors using perpendicular components is very useful in many areas of physics, because perpendicular quantities are often independent of one another. The next module, <u>Projectile Motion</u>, is one of many in which using perpendicular components helps make the picture clear and simplifies the physics.



The subtraction of the two vectors shown in [link]. The components of  $-\mathbf{B}$  are the negatives of the components of  $\mathbf{B}$ . The method of subtraction is the same as that for addition.

## Note:

PhET Explorations: Vector Addition

Learn how to add vectors. Drag vectors onto a graph, change their length and angle, and sum them together. The magnitude, angle, and components of each vector can be displayed in several formats.

https://phet.colorado.edu/sims/vector-addition/vector-addition en.html

# **Summary**

- The analytical method of vector addition and subtraction involves using the Pythagorean theorem and trigonometric identities to determine the magnitude and direction of a resultant vector.
- The steps to add vectors **A** and **B** using the analytical method are as follows:

Step 1: Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations

# **Equation:**

$$A_x = A \cos \theta$$
  
$$B_x = B \cos \theta$$

and

# **Equation:**

$$A_y = A \sin \theta$$
  
$$B_y = B \sin \theta.$$

Step 2: Add the horizontal and vertical components of each vector to determine the components  $R_x$  and  $R_y$  of the resultant vector,  $\mathbf{R}$ :

# **Equation:**

$$R_x = A_x + B_x$$

and

# **Equation:**

$$R_y = A_y + B_{y.}$$

Step 3: Use the Pythagorean theorem to determine the magnitude, R, of the resultant vector  $\mathbf{R}$ :

# **Equation:**

$$R = \sqrt{R_x^2 + R_y^2}.$$

Step 4: Use a trigonometric identity to determine the direction,  $\theta$ , of  ${\bf R}$  .

# **Equation:**

$$\theta = \tan^{-1}(R_y/R_x).$$

# **Conceptual Questions**

## **Exercise:**

## **Problem:**

Suppose you add two vectors **A** and **B**. What relative direction between them produces the resultant with the greatest magnitude? What is the maximum magnitude? What relative direction between them produces the resultant with the smallest magnitude? What is the minimum magnitude?

## **Exercise:**

## **Problem:**

Give an example of a nonzero vector that has a component of zero.

## **Exercise:**

## **Problem:**

Explain why a vector cannot have a component greater than its own magnitude.

## **Exercise:**

## **Problem:**

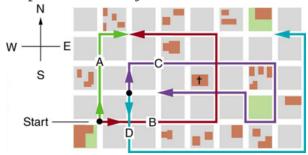
If the vectors **A** and **B** are perpendicular, what is the component of **A** along the direction of **B**? What is the component of **B** along the direction of **A**?

# **Problems & Exercises**

## **Exercise:**

## **Problem:**

Find the following for path C in [link]: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.



The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

## **Solution:**

- (a) 1.56 km
- (b) 120 m east

## **Exercise:**

## **Problem:**

Find the following for path D in [link]: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

## **Exercise:**

## **Problem:**

Find the north and east components of the displacement from San Francisco to Sacramento shown in [link].



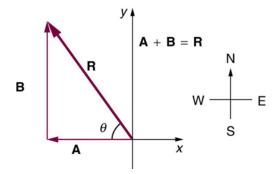
# **Solution:**

North-component 87.0 km, east-component 87.0 km

## **Exercise:**

## **Problem:**

Solve the following problem using analytical techniques: Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements  $\bf A$  and  $\bf B$ , as in [link], then this problem asks you to find their sum  $\bf R = \bf A + \bf B$ .)



The two displacements  $\mathbf{A}$  and  $\mathbf{B}$  add to give a total displacement  $\mathbf{R}$  having magnitude R and direction  $\theta$ .

Note that you can also solve this graphically. Discuss why the analytical technique for solving this problem is potentially more accurate than the graphical technique.

## **Exercise:**

### **Problem:**

Repeat [link] using analytical techniques, but reverse the order of the two legs of the walk and show that you get the same final result. (This problem shows that adding them in reverse order gives the same result —that is,  $\mathbf{B} + \mathbf{A} = \mathbf{A} + \mathbf{B}$ .) Discuss how taking another path to reach the same point might help to overcome an obstacle blocking you other path.

## **Solution:**

30.8 m, 35.8 west of north

## **Exercise:**

## **Problem:**

You drive 7.50 km in a straight line in a direction 15° east of north. (a) Find the distances you would have to drive straight east and then straight north to arrive at the same point. (This determination is equivalent to find the components of the displacement along the east and north directions.) (b) Show that you still arrive at the same point if the east and north legs are reversed in order.

## **Exercise:**

## **Problem:**

Do [link] again using analytical techniques and change the second leg of the walk to 25.0 m straight south. (This is equivalent to subtracting  $\mathbf{B}$  from  $\mathbf{A}$  —that is, finding  $\mathbf{R}\prime = \mathbf{A} - \mathbf{B}$ ) (b) Repeat again, but now you first walk 25.0 m north and then 18.0 m east. (This is equivalent to subtract  $\mathbf{A}$  from  $\mathbf{B}$  —that is, to find  $\mathbf{A} = \mathbf{B} + \mathbf{C}$ . Is that consistent with your result?)

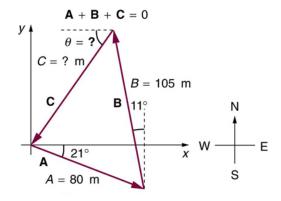
## **Solution:**

- (a) 30.8 m,  $54.2^{\circ}$  south of west
- (b) 30.8 m, 54.2° north of east

## **Exercise:**

## **Problem:**

A new landowner has a triangular piece of flat land she wishes to fence. Starting at the west corner, she measures the first side to be 80.0 m long and the next to be 105 m. These sides are represented as displacement vectors **A** from **B** in [link]. She then correctly calculates the length and orientation of the third side C. What is her result?



## **Exercise:**

## **Problem:**

You fly 32.0 km in a straight line in still air in the direction  $35.0^{\circ}$  south of west. (a) Find the distances you would have to fly straight south and then straight west to arrive at the same point. (This determination is equivalent to finding the components of the displacement along the south and west directions.) (b) Find the distances you would have to fly first in a direction  $45.0^{\circ}$  south of west and then in a direction  $45.0^{\circ}$  west of north. These are the components of the displacement along a different set of axes—one rotated  $45^{\circ}$ .

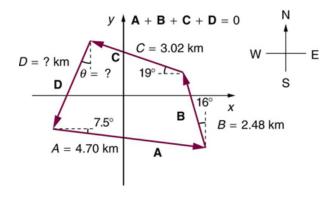
## **Solution:**

18.4 km south, then 26.2 km west(b) 31.5 km at  $45.0^{\circ}$  south of west, then 5.56 km at  $45.0^{\circ}$  west of north

## **Exercise:**

## **Problem:**

A farmer wants to fence off his four-sided plot of flat land. He measures the first three sides, shown as  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$  in [link], and then correctly calculates the length and orientation of the fourth side  $\mathbf{D}$ . What is his result?



## **Exercise:**

## **Problem:**

In an attempt to escape his island, Gilligan builds a raft and sets to sea. The wind shifts a great deal during the day, and he is blown along the following straight lines:  $2.50 \text{ km } 45.0^{\circ}$  north of west; then  $4.70 \text{ km } 60.0^{\circ}$  south of east; then  $1.30 \text{ km } 25.0^{\circ}$  south of west; then  $5.10 \text{ km } 5.00^{\circ}$  east of north; then  $7.20 \text{ km } 55.0^{\circ}$  south of west; and finally  $2.80 \text{ km } 10.0^{\circ}$  north of east. What is his final position relative to the island?

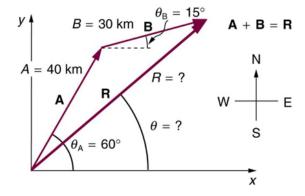
## **Solution:**

7.34 km,  $63.5^{\circ}$  south of east

## **Exercise:**

## **Problem:**

Suppose a pilot flies 40.0 km in a direction  $60^{\circ}$  north of east and then flies 30.0 km in a direction  $15^{\circ}$  north of east as shown in [link]. Find her total distance R from the starting point and the direction  $\theta$  of the straight-line path to the final position. Discuss qualitatively how this flight would be altered by a wind from the north and how the effect of the wind would depend on both wind speed and the speed of the plane relative to the air mass.



# Glossary

# analytical method

the method of determining the magnitude and direction of a resultant vector using the Pythagorean theorem and trigonometric identities

### Projectile Motion

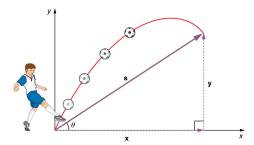
- Identify and explain the properties of a projectile, such as acceleration due to gravity, range, maximum height, and trajectory.
- Determine the location and velocity of a projectile at different points in its trajectory.
- Apply the principle of independence of motion to solve projectile motion problems.

**Projectile motion** is the **motion** of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a **projectile**, and its path is called its **trajectory**. The motion of falling objects, as covered in <u>Problem-Solving Basics for One-Dimensional Kinematics</u>, is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football or other object for which **air resistance** *is negligible*.

The most important fact to remember here is that *motions along perpendicular axes are independent* and thus can be analyzed separately. This fact was discussed in Kinematics in Two Dimensions: An Introduction, where vertical and horizontal motions were seen to be independent. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible, because acceleration due to gravity is vertical—thus, there will be no acceleration along the horizontal axis when air resistance is negligible.) As is customary, we call the horizontal axis the *x*-axis and the vertical axis the *y*-axis. [link] illustrates the notation for displacement, where **s** is defined to be the total displacement and **x** and **y** are its components along the horizontal and vertical axes, respectively. The magnitudes of these vectors are  $\mathbf{s}$ ,  $\mathbf{x}$ , and  $\mathbf{y}$ . (Note that in the last section we used the notation **A** to represent a vector with components  $\mathbf{A}_x$  and  $\mathbf{A}_y$ . If we continued this format, we would call displacement **s** with components  $\mathbf{s}_x$  and  $\mathbf{s}_y$ . However, to simplify the notation, we will simply represent the component vectors as  $\mathbf{x}$  and  $\mathbf{y}$ .)

Of course, to describe motion we must deal with velocity and acceleration, as well as with displacement. We must find their components along the x- and y-axes, too. We will assume all forces except gravity (such as air resistance and friction, for example) are negligible. The components of acceleration are then very simple:  $a_y = -g = -9.80 \,\mathrm{m/s^2}$ . (Note that this definition assumes that the upwards direction is defined as the positive direction. If you arrange the coordinate system instead such that the downwards direction is positive, then acceleration due to gravity takes a positive value.) Because gravity is vertical,  $a_x = 0$ . Both accelerations are constant, so the kinematic equations can be used.

Note:	
Review of Kinematic Equations (constant <i>a</i> )	
Equation:	
	$x=x_0+ar{v}t$
Equation:	
	a. La
	$\overline{v} = rac{v_0 + v}{2}$
	2
Equation:	
_4	
	$v = v_0 + at$
Equation:	
	1 .
x =	$x_0 + v_0 t + rac{1}{2} a t^2$
	L
<b>Equation:</b>	
$v^2 =$	$=v_0^2+2a(x-x_0).$



The total displacement  $\mathbf{s}$  of a soccer ball at a point along its path. The vector  $\mathbf{s}$  has components  $\mathbf{x}$  and  $\mathbf{y}$  along the horizontal and vertical axes. Its magnitude is s, and it makes an angle  $\theta$  with the horizontal.

Given these assumptions, the following steps are then used to analyze projectile motion:

Step 1. Resolve or break the motion into horizontal and vertical components along the x- and y-axes. These axes are perpendicular, so  $A_x=A\cos\theta$  and  $A_y=A\sin\theta$  are used. The magnitude of the components of displacement s along these axes are x and y. The magnitudes of the components of the velocity  $\mathbf{v}$  are  $v_x=v\cos\theta$  and  $v_y=v\sin\theta$ , where v is the magnitude of the velocity and  $\theta$  is its direction, as shown in [link]. Initial values are denoted with a subscript 0, as usual.

*Step 2. Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical.* The kinematic equations for horizontal and vertical motion take the following forms:

#### **Equation:**

Horizontal Motion $(a_x = 0)$ 

**Equation:** 

$$x = x_0 + v_x t$$

**Equation:** 

$$v_x = v_{0x} = v_x = \text{velocity is a constant.}$$

**Equation:** 

Vertical Motion  
(assuming positive is up 
$$a_y = -g = -9.80 \mathrm{m/s}^2$$
)

**Equation:** 

$$y=y_0+rac{1}{2}(v_{0y}+v_y)t$$

**Equation:** 

$$v_y = v_{0y} - \operatorname{gt}$$

$$y=y_0+v_{0y}t-rac{1}{2}gt^2$$

**Equation:** 

$$v_y^2 = v_{0y}^2 - 2g(y-y_0).$$

**Step 3.** Solve for the unknowns in the two separate motions—one horizontal and one vertical. Note that the only common variable between the motions is time t. The problem solving procedures here are the same as for one-dimensional **kinematics** and are illustrated in the solved examples below.

**Step 4.** Recombine the two motions to find the total displacement  ${\bf s}$  and velocity  ${\bf v}$ . Because the x - and y -motions are perpendicular, we determine these vectors by using the techniques outlined in the <u>Vector Addition and Subtraction: Analytical Methods</u> and employing  $A=\sqrt{A_x^2+A_y^2}$  and  $\theta=\tan^{-1}(A_y/A_x)$  in the following form, where  $\theta$  is the direction of the displacement  ${\bf s}$  and  $\theta_v$  is the direction of the velocity  ${\bf v}$ :

# Total displacement and velocity **Equation:**

 $s=\sqrt{x^2+y^2}$ 

**Equation:** 

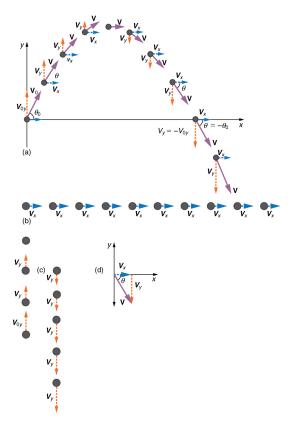
 $heta = an^{-1}(y/x)$ 

**Equation:** 

 $v=\sqrt{v_x^2+v_y^2}$ 

**Equation:** 

 $heta_v = an^{-1}(v_y/v_x).$ 



(a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because  $a_x=0$  and  $v_x$  is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The x - and y -motions are recombined to give the total velocity at any given point on the trajectory.

#### **Example:**

#### A Fireworks Projectile Explodes High and Away

During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of 75.0° above the horizontal, as illustrated in [link]. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passed between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes?

#### Strategy

Because air resistance is negligible for the unexploded shell, the analysis method outlined above can be used. The motion can be broken into horizontal and vertical motions in which  $a_x = 0$  and  $a_y = -g$ . We can then define  $x_0$ 

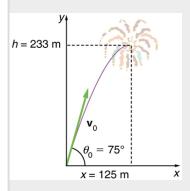
and  $y_0$  to be zero and solve for the desired quantities.

#### Solution for (a)

By "height" we mean the altitude or vertical position y above the starting point. The highest point in any trajectory, called the apex, is reached when  $v_y = 0$ . Since we know the initial and final velocities as well as the initial position, we use the following equation to find y:

### **Equation:**

$$v_y^2 = v_{0y}^2 - 2g(y-y_0).$$



The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.

Because  $y_0$  and  $v_y$  are both zero, the equation simplifies to

#### **Equation:**

$$0 = v_{0y}^2 - 2gy.$$

Solving for y gives

### **Equation:**

$$y = \frac{v_{0y}^2}{2g}.$$

Now we must find  $v_{0y}$ , the component of the initial velocity in the *y*-direction. It is given by  $v_{0y} = v_0 \sin \theta$ , where  $v_{0y}$  is the initial velocity of 70.0 m/s, and  $\theta_0 = 75.0^{\circ}$  is the initial angle. Thus,

#### **Equation:**

$$v_{0y} = v_0 \sin \theta_0 = (70.0 \text{ m/s})(\sin 75^\circ) = 67.6 \text{ m/s}.$$

and y is

#### **Equation:**

$$y = rac{(67.6 ext{ m/s})^2}{2(9.80 ext{ m/s}^2)},$$

so that

## **Equation:**

$$y = 233$$
m.

#### Discussion for (a)

Note that because up is positive, the initial velocity is positive, as is the maximum height, but the acceleration due to gravity is negative. Note also that the maximum height depends only on the vertical component of the initial velocity, so that any projectile with a 67.6 m/s initial vertical component of velocity will reach a maximum height of 233 m (neglecting air resistance). The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding. In practice, air resistance is not completely negligible, and so the initial velocity would have to be somewhat larger than that given to reach the same height.

#### Solution for (b)

As in many physics problems, there is more than one way to solve for the time to the highest point. In this case, the easiest method is to use  $y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$ . Because  $y_0$  is zero, this equation reduces to simply

## **Equation:**

$$y=rac{1}{2}(v_{0y}+v_y)t.$$

Note that the final vertical velocity,  $v_y$ , at the highest point is zero. Thus,

#### **Equation:**

$$t = \frac{2y}{(v_{0y} + v_y)} = \frac{2(233 \text{ m})}{(67.6 \text{ m/s})}$$
  
= 6.90 s

#### Discussion for (b)

This time is also reasonable for large fireworks. When you are able to see the launch of fireworks, you will notice several seconds pass before the shell explodes. (Another way of finding the time is by using  $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ , and solving the quadratic equation for t.)

#### Solution for (c)

Because air resistance is negligible,  $a_x=0$  and the horizontal velocity is constant, as discussed above. The horizontal displacement is horizontal velocity multiplied by time as given by  $x=x_0+v_xt$ , where  $x_0$  is equal to zero:

### **Equation:**

$$x = v_x t$$
,

where  $v_x$  is the x-component of the velocity, which is given by  $v_x = v_0 \cos \theta_0$ . Now,

#### **Equation:**

$$v_x = v_0 \cos \theta_0 = (70.0 \text{ m/s})(\cos 75.0^{\circ}) = 18.1 \text{ m/s}.$$

The time t for both motions is the same, and so x is

#### **Equation:**

$$x = (18.1 \text{ m/s})(6.90 \text{ s}) = 125 \text{ m}.$$

#### Discussion for (c)

The horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. Once the shell explodes, air resistance has a major effect, and many fragments will land directly below.

In solving part (a) of the preceding example, the expression we found for y is valid for any projectile motion where air resistance is negligible. Call the maximum height y = h; then,

$$h=rac{v_{0y}^2}{2g}.$$

This equation defines the *maximum height of a projectile* and depends only on the vertical component of the initial velocity.

#### Note:

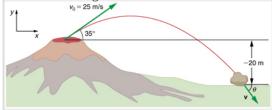
#### Defining a Coordinate System

It is important to set up a coordinate system when analyzing projectile motion. One part of defining the coordinate system is to define an origin for the x and y positions. Often, it is convenient to choose the initial position of the object as the origin such that  $x_0=0$  and  $y_0=0$ . It is also important to define the positive and negative directions in the x and y directions. Typically, we define the positive vertical direction as upwards, and the positive horizontal direction is usually the direction of the object's motion. When this is the case, the vertical acceleration, y0, takes a negative value (since it is directed downwards towards the Earth). However, it is occasionally useful to define the coordinates differently. For example, if you are analyzing the motion of a ball thrown downwards from the top of a cliff, it may make sense to define the positive direction downwards since the motion of the ball is solely in the downwards direction. If this is the case, y1 takes a positive value.

#### **Example:**

#### **Calculating Projectile Motion: Hot Rock Projectile**

Kilauea in Hawaii is the world's most continuously active volcano. Very active volcanoes characteristically eject red-hot rocks and lava rather than smoke and ash. Suppose a large rock is ejected from the volcano with a speed of 25.0 m/s and at an angle 35.0° above the horizontal, as shown in [link]. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path. (b) What are the magnitude and direction of the rock's velocity at impact?



The trajectory of a rock ejected from the Kilauea volcano.

#### Strategy

Again, resolving this two-dimensional motion into two independent one-dimensional motions will allow us to solve for the desired quantities. The time a projectile is in the air is governed by its vertical motion alone. We will solve for t first. While the rock is rising and falling vertically, the horizontal motion continues at a constant velocity. This example asks for the final velocity. Thus, the vertical and horizontal results will be recombined to obtain v and  $\theta_v$  at the final time t determined in the first part of the example.

### Solution for (a)

While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by using

$$y=y_0+v_{0y}t-rac{1}{2}\mathrm{gt}^2.$$

If we take the initial position  $y_0$  to be zero, then the final position is y=-20.0 m. Now the initial vertical velocity is the vertical component of the initial velocity, found from  $v_{0y}=v_0\sin\theta_0$  =  $(25.0 \text{ m/s})(\sin 35.0^\circ)$  = 14.3 m/s. Substituting known values yields

#### **Equation:**

$$-20.0 \text{ m} = (14.3 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2.$$

Rearranging terms gives a quadratic equation in t:

#### **Equation:**

$$(4.90 \text{ m/s}^2)t^2 - (14.3 \text{ m/s})t - (20.0 \text{ m}) = 0.$$

This expression is a quadratic equation of the form at  $^2 + bt + c = 0$ , where the constants are a = 4.90, b = -14.3, and c = -20.0. Its solutions are given by the quadratic formula:

#### **Equation:**

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This equation yields two solutions: t = 3.96 and t = -1.03. (It is left as an exercise for the reader to verify these solutions.) The time is t = 3.96 s or -1.03 s. The negative value of time implies an event before the start of motion, and so we discard it. Thus,

#### **Equation:**

$$t = 3.96 \, s.$$

#### Discussion for (a)

The time for projectile motion is completely determined by the vertical motion. So any projectile that has an initial vertical velocity of 14.3 m/s and lands 20.0 m below its starting altitude will spend 3.96 s in the air.

#### Solution for (b)

From the information now in hand, we can find the final horizontal and vertical velocities  $v_x$  and  $v_y$  and combine them to find the total velocity v and the angle  $\theta_0$  it makes with the horizontal. Of course,  $v_x$  is constant so we can solve for it at any horizontal location. In this case, we chose the starting point since we know both the initial velocity and initial angle. Therefore:

## **Equation:**

$$v_x = v_0 \cos \theta_0 = (25.0 \text{ m/s})(\cos 35^\circ) = 20.5 \text{ m/s}.$$

The final vertical velocity is given by the following equation:

#### **Equation:**

$$v_y = v_{0y} - \operatorname{gt},$$

where  $v_{0y}$  was found in part (a) to be 14.3 m/s. Thus,

### **Equation:**

$$v_y = 14.3 \text{ m/s} - (9.80 \text{ m/s}^2)(3.96 \text{ s})$$

so that

## **Equation:**

$$v_y = -24.5 \text{ m/s}.$$

To find the magnitude of the final velocity v we combine its perpendicular components, using the following equation:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20.5 \ ext{m/s})^2 + (-24.5 \ ext{m/s})^2},$$

which gives

**Equation:** 

$$v = 31.9 \text{ m/s}.$$

The direction  $\theta_v$  is found from the equation:

**Equation:** 

$$heta_v = an^{-1}(v_y/v_x)$$

so that

**Equation:** 

$$\theta_v = \tan^{-1}(-24.5/20.5) = \tan^{-1}(-1.19).$$

Thus,

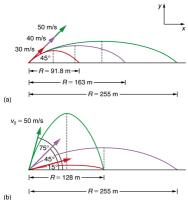
**Equation:** 

$$\theta_v = -50.1^{\circ}$$
.

#### Discussion for (b)

The negative angle means that the velocity is  $50.1^{\circ}$  below the horizontal. This result is consistent with the fact that the final vertical velocity is negative and hence downward—as you would expect because the final altitude is 20.0 m lower than the initial altitude. (See [link].)

One of the most important things illustrated by projectile motion is that vertical and horizontal motions are independent of each other. Galileo was the first person to fully comprehend this characteristic. He used it to predict the range of a projectile. On level ground, we define  ${\bf range}$  to be the horizontal distance  ${\cal R}$  traveled by a projectile. Galileo and many others were interested in the range of projectiles primarily for military purposes—such as aiming cannons. However, investigating the range of projectiles can shed light on other interesting phenomena, such as the orbits of satellites around the Earth. Let us consider projectile range further.



Trajectories of projectiles on level ground. (a) The greater the initial speed  $v_0$ , the greater the range for a given initial angle. (b) The effect of initial angle  $\theta_0$  on the range of a projectile with a

given initial speed. Note that the range is the same for 15° and 75°, although the maximum heights of those paths are different.

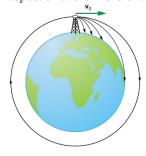
How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed  $v_0$ , the greater the range, as shown in [link](a). The initial angle  $\theta_0$  also has a dramatic effect on the range, as illustrated in [link] (b). For a fixed initial speed, such as might be produced by a cannon, the maximum range is obtained with  $\theta_0=45^\circ$ . This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is approximately 38°. Interestingly, for every initial angle except 45°, there are two angles that give the same range—the sum of those angles is 90°. The range also depends on the value of the acceleration of gravity g. The lunar astronaut Alan Shepherd was able to drive a golf ball a great distance on the Moon because gravity is weaker there. The range R of a projectile on level ground for which air resistance is negligible is given by **Equation:** 

$$R=rac{v_0^2\sin2 heta_0}{g},$$

where  $v_0$  is the initial speed and  $\theta_0$  is the initial angle relative to the horizontal. The proof of this equation is left as an end-of-chapter problem (hints are given), but it does fit the major features of projectile range as described.

When we speak of the range of a projectile on level ground, we assume that R is very small compared with the circumference of the Earth. If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path. The range is larger than predicted by the range equation given above because the projectile has farther to fall than it would on level ground. (See [link].) If the initial speed is great enough, the projectile goes into orbit. This possibility was recognized centuries before it could be accomplished. When an object is in orbit, the Earth curves away from underneath the object at the same rate as it falls. The object thus falls continuously but never hits the surface. These and other aspects of orbital motion, such as the rotation of the Earth, will be covered analytically and in greater depth later in this text.

Once again we see that thinking about one topic, such as the range of a projectile, can lead us to others, such as the Earth orbits. In <u>Addition of Velocities</u>, we will examine the addition of velocities, which is another important aspect of two-dimensional kinematics and will also yield insights beyond the immediate topic.



Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path.

With a large enough initial speed, orbit is achieved.

#### Note:

#### PhET Explorations: Projectile Motion

Blast a Buick out of a cannon! Learn about projectile motion by firing various objects. Set the angle, initial speed, and mass. Add air resistance. Make a game out of this simulation by trying to hit a target. <a href="https://phet.colorado.edu/sims/projectile-motion/projectile-motion">https://phet.colorado.edu/sims/projectile-motion/projectile-motion en.html</a>

#### **Summary**

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- To solve projectile motion problems, perform the following steps:

```
Sare given x and y, and the vare v_x = v \cos \theta and v_y = v \sin \theta,
                                                                                             v_{\rm is} the
                                                                                                         \thetais its
Determine a
                                   components given
                                                                                      where magnitude direction.
coordinate
                  by the
                                                 by
                                                                                              of the
system. Then,
                 quantities
                                   of the
resolve the
                                   velocity
                                                                                              velocity
position and/or
                                                                                              and
velocity of the
object in the
horizontal and
vertical
components.
The
components of
position
Analyze the
                  Equation:
                                                 Equation:
                                                                      Equation:
motion of the
projectile in the Horizontal motion (a_x = 0) x = x_0 + v_x t v_x = v_{0x} = \mathbf{v}_x = \text{velocity is a constant.}
horizontal
direction using
the following
equations:
                                                                                                   Equation:
Analyze Equation:
the motion of Vertical motion (Assuming positive direction is up; a_y = -g = -9.80 \text{ m/s}^2) y = y_0 + \frac{1}{2}(v_{0y})
the
projectile
in the
vertical
```

direction using the following equations:

Recombine the horizontal and vertical components of location and/or velocity using the following equations: **Equation:** 

**Equation:** 

**Equation:** 

 $s=\sqrt{x^2+y^2} \qquad heta= an^{-1}(y/x) \qquad v=\sqrt{v_x^2+v_y^2} \quad heta_{
m v}= an^{-1}(v_y/v_x).$ 

The maximum height h of a projectile launched with initial vertical velocity  $v_{0y}$  is given by **Equation:** 

$$h=rac{v_{0y}^2}{2g}.$$

• The maximum horizontal distance traveled by a projectile is called the **range**. The range R of a projectile on level ground launched at an angle  $\theta_0$  above the horizontal with initial speed  $v_0$  is given by **Equation:** 

$$R = \frac{v_0^2 \sin 2\theta_0}{g}.$$

### **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither  $0^{\circ}$  nor  $90^{\circ}$ ): (a) Is the velocity ever zero? (b) When is the velocity a minimum? A maximum? (c) Can the velocity ever be the same as the initial velocity at a time other than at t = 0? (d) Can the speed ever be the same as the initial speed at a time other than at t = 0?

#### **Exercise:**

#### **Problem:**

Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither  $0^{\circ}$  nor  $90^{\circ}$ ): (a) Is the acceleration ever zero? (b) Is the acceleration ever in the same direction as a component of velocity? (c) Is the acceleration ever opposite in direction to a component of velocity?

#### **Exercise:**

#### **Problem:**

For a fixed initial speed, the range of a projectile is determined by the angle at which it is fired. For all but the maximum, there are two angles that give the same range. Considering factors that might affect the ability of an archer to hit a target, such as wind, explain why the smaller angle (closer to the horizontal) is preferable. When would it be necessary for the archer to use the larger angle? Why does the punter in a football game use the higher trajectory?

#### **Exercise:**

During a lecture demonstration, a professor places two coins on the edge of a table. She then flicks one of the coins horizontally off the table, simultaneously nudging the other over the edge. Describe the subsequent motion of the two coins, in particular discussing whether they hit the floor at the same time.

#### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

A projectile is launched at ground level with an initial speed of 50.0 m/s at an angle of  $30.0^{\circ}$  above the horizontal. It strikes a target above the ground 3.00 seconds later. What are the x and y distances from where the projectile was launched to where it lands?

#### **Solution:**

```
x = 1.30 \text{ m} \times 10^2

y = 30.9 \text{ m}.
```

#### **Exercise:**

#### **Problem:**

A ball is kicked with an initial velocity of 16 m/s in the horizontal direction and 12 m/s in the vertical direction. (a) At what speed does the ball hit the ground? (b) For how long does the ball remain in the air? (c) What maximum height is attained by the ball?

#### **Exercise:**

#### **Problem:**

A ball is thrown horizontally from the top of a 60.0-m building and lands 100.0 m from the base of the building. Ignore air resistance. (a) How long is the ball in the air? (b) What must have been the initial horizontal component of the velocity? (c) What is the vertical component of the velocity just before the ball hits the ground? (d) What is the velocity (including both the horizontal and vertical components) of the ball just before it hits the ground?

#### **Solution:**

- (a) 3.50 s
- (b) 28.6 m/s (c) 34.3 m/s
- (d) 44.7 m/s, 50.2° below horizontal

#### Exercise:

### **Problem:**

(a) A daredevil is attempting to jump his motorcycle over a line of buses parked end to end by driving up a  $32^{\circ}$  ramp at a speed of 40.0~m/s (144~km/h). How many buses can he clear if the top of the takeoff ramp is at the same height as the bus tops and the buses are 20.0~m long? (b) Discuss what your answer implies about the margin of error in this act—that is, consider how much greater the range is than the horizontal distance he must travel to miss the end of the last bus. (Neglect air resistance.)

An archer shoots an arrow at a 75.0 m distant target; the bull's-eye of the target is at same height as the release height of the arrow. (a) At what angle must the arrow be released to hit the bull's-eye if its initial speed is 35.0 m/s? In this part of the problem, explicitly show how you follow the steps involved in solving projectile motion problems. (b) There is a large tree halfway between the archer and the target with an overhanging horizontal branch 3.50 m above the release height of the arrow. Will the arrow go over or under the branch?

#### **Solution:**

- (a)  $18.4^{\circ}$
- (b) The arrow will go over the branch.

#### **Exercise:**

#### **Problem:**

A rugby player passes the ball 7.00 m across the field, where it is caught at the same height as it left his hand. (a) At what angle was the ball thrown if its initial speed was 12.0 m/s, assuming that the smaller of the two possible angles was used? (b) What other angle gives the same range, and why would it not be used? (c) How long did this pass take?

#### **Exercise:**

**Problem:** Verify the ranges for the projectiles in [link](a) for  $\theta = 45^{\circ}$  and the given initial velocities.

#### **Solution:**

$$R=rac{v_0^2}{\sin 2 heta_0 g}$$
 For  $heta=45^{
m o},~~R=rac{v_0^2}{q}$ 

$$R = 91.8 \text{ m}$$
 for  $v_0 = 30 \text{ m/s}$ ;  $R = 163 \text{ m}$  for  $v_0 = 40 \text{ m/s}$ ;  $R = 255 \text{ m}$  for  $v_0 = 50 \text{ m/s}$ .

#### **Exercise:**

#### **Problem:**

Verify the ranges shown for the projectiles in [link](b) for an initial velocity of 50 m/s at the given initial angles.

#### **Exercise:**

#### **Problem:**

The cannon on a battleship can fire a shell a maximum distance of 32.0 km. (a) Calculate the initial velocity of the shell. (b) What maximum height does it reach? (At its highest, the shell is above 60% of the atmosphere—but air resistance is not really negligible as assumed to make this problem easier.) (c) The ocean is not flat, because the Earth is curved. Assume that the radius of the Earth is  $6.37 \times 10^3$  km. How many meters lower will its surface be 32.0 km from the ship along a horizontal line parallel to the surface at the ship? Does your answer imply that error introduced by the assumption of a flat Earth in projectile motion is significant here?

#### **Solution:**

- (a) 560 m/s
- (b)  $8.00 \times 10^3 \text{ m}$

(c) 80.0 m. This error is not significant because it is only 1% of the answer in part (b).

#### **Exercise:**

#### **Problem:**

An arrow is shot from a height of 1.5 m toward a cliff of height H. It is shot with a velocity of 30 m/s at an angle of  $60^{\circ}$  above the horizontal. It lands on the top edge of the cliff 4.0 s later. (a) What is the height of the cliff? (b) What is the maximum height reached by the arrow along its trajectory? (c) What is the arrow's impact speed just before hitting the cliff?

#### **Exercise:**

#### **Problem:**

In the standing broad jump, one squats and then pushes off with the legs to see how far one can jump. Suppose the extension of the legs from the crouch position is 0.600 m and the acceleration achieved from this position is 1.25 times the acceleration due to gravity, *g*. How far can they jump? State your assumptions. (Increased range can be achieved by swinging the arms in the direction of the jump.)

#### **Solution:**

1.50 m, assuming launch angle of  $45^{\circ}$ 

#### **Exercise:**

#### **Problem:**

The world long jump record is 8.95 m (Mike Powell, USA, 1991). Treated as a projectile, what is the maximum range obtainable by a person if he has a take-off speed of 9.5 m/s? State your assumptions.

#### **Exercise:**

#### **Problem:**

Serving at a speed of 170 km/h, a tennis player hits the ball at a height of 2.5 m and an angle  $\theta$  below the horizontal. The base line is 11.9 m from the net, which is 0.91 m high. What is the angle  $\theta$  such that the ball just crosses the net? Will the ball land in the service box, whose service line is 6.40 m from the net?

#### **Solution:**

 $\theta=6.1^{\rm o}$ 

yes, the ball lands at 5.3 m from the net

#### **Exercise:**

#### **Problem:**

A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. (a) If the ball is thrown at an angle of 25° relative to the ground and is caught at the same height as it is released, what is its initial speed relative to the ground? (b) How long does it take to get to the receiver? (c) What is its maximum height above its point of release?

#### **Exercise:**

#### **Problem:**

Gun sights are adjusted to aim high to compensate for the effect of gravity, effectively making the gun accurate only for a specific range. (a) If a gun is sighted to hit targets that are at the same height as the gun and 100.0 m away, how low will the bullet hit if aimed directly at a target 150.0 m away? The muzzle velocity of the bullet is 275 m/s. (b) Discuss qualitatively how a larger muzzle velocity would affect this problem and what would be the effect of air resistance.

#### **Solution:**

- (a) -0.486 m
- (b) The larger the muzzle velocity, the smaller the deviation in the vertical direction, because the time of flight would be smaller. Air resistance would have the effect of decreasing the time of flight, therefore increasing the vertical deviation.

#### **Exercise:**

#### **Problem:**

An eagle is flying horizontally at a speed of 3.00 m/s when the fish in her talons wiggles loose and falls into the lake 5.00 m below. Calculate the velocity of the fish relative to the water when it hits the water.

#### **Exercise:**

#### **Problem:**

An owl is carrying a mouse to the chicks in its nest. Its position at that time is 4.00 m west and 12.0 m above the center of the 30.0 cm diameter nest. The owl is flying east at 3.50 m/s at an angle 30.0 ° below the horizontal when it accidentally drops the mouse. Is the owl lucky enough to have the mouse hit the nest? To answer this question, calculate the horizontal position of the mouse when it has fallen 12.0 m.

#### **Solution:**

4.23 m. No, the owl is not lucky; he misses the nest.

#### **Exercise:**

#### **Problem:**

Suppose a soccer player kicks the ball from a distance 30 m toward the goal. Find the initial speed of the ball if it just passes over the goal, 2.4 m above the ground, given the initial direction to be  $40^{\circ}$  above the horizontal.

#### Exercise:

#### **Problem:**

Can a goalkeeper at her/ his goal kick a soccer ball into the opponent's goal without the ball touching the ground? The distance will be about 95 m. A goalkeeper can give the ball a speed of 30 m/s.

#### **Solution:**

No, the maximum range (neglecting air resistance) is about 92 m.

#### **Exercise:**

#### **Problem:**

The free throw line in basketball is 4.57 m (15 ft) from the basket, which is 3.05 m (10 ft) above the floor. A player standing on the free throw line throws the ball with an initial speed of 8.15 m/s, releasing it at a height of 2.44 m (8 ft) above the floor. At what angle above the horizontal must the ball be thrown to exactly hit the basket? Note that most players will use a large initial angle rather than a flat shot because it allows for a larger margin of error. Explicitly show how you follow the steps involved in solving projectile motion problems.

In 2007, Michael Carter (U.S.) set a world record in the shot put with a throw of 24.77 m. What was the initial speed of the shot if he released it at a height of 2.10 m and threw it at an angle of 38.0° above the horizontal? (Although the maximum distance for a projectile on level ground is achieved at 45° when air resistance is neglected, the actual angle to achieve maximum range is smaller; thus, 38° will give a longer range than 45° in the shot put.)

#### **Solution:**

15.0 m/s

#### Exercise:

#### **Problem:**

A basketball player is running at  $5.00~\mathrm{m/s}$  directly toward the basket when he jumps into the air to dunk the ball. He maintains his horizontal velocity. (a) What vertical velocity does he need to rise  $0.750~\mathrm{m}$  above the floor? (b) How far from the basket (measured in the horizontal direction) must he start his jump to reach his maximum height at the same time as he reaches the basket?

#### **Exercise:**

#### **Problem:**

A football player punts the ball at a 45.0° angle. Without an effect from the wind, the ball would travel 60.0 m horizontally. (a) What is the initial speed of the ball? (b) When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by 1.50 m/s. What distance does the ball travel horizontally?

#### **Solution:**

- (a) 24.2 m/s
- (b) The ball travels a total of 57.4 m with the brief gust of wind.

#### Exercise:

#### Problem:

Prove that the trajectory of a projectile is parabolic, having the form  $y = ax + bx^2$ . To obtain this expression, solve the equation  $x = v_{0x}t$  for t and substitute it into the expression for  $y = v_{0y}t - (1/2)gt^2$  (These equations describe the x and y positions of a projectile that starts at the origin.) You should obtain an equation of the form  $y = ax + bx^2$  where a and b are constants.

#### **Exercise:**

### **Problem:**

Derive  $R=\frac{v_0^2\sin 2\theta_0}{g}$  for the range of a projectile on level ground by finding the time t at which y becomes zero and substituting this value of t into the expression for  $x-x_0$ , noting that  $R=x-x_0$ 

### **Solution:**

$$y - y_0 = 0 = v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin \theta)t - \frac{1}{2}gt^2$$

so that 
$$t=rac{2(v_0\sin heta)}{g}$$

$$x - x_0 = v_{0x}t = (v_0 \cos \theta)t = R$$
, and substituting for t gives:

$$R = v_0 \cos heta \left( rac{2v_0 \sin heta}{g} 
ight) = rac{2v_0^2 \sin heta \cos heta}{g}$$

since  $2 \sin \theta \cos \theta = \sin 2\theta$ , the range is:

$$R = \frac{v_0^2 \sin 2\theta}{g}.$$

#### **Exercise:**

#### **Problem:**

**Unreasonable Results** (a) Find the maximum range of a super cannon that has a muzzle velocity of 4.0 km/s. (b) What is unreasonable about the range you found? (c) Is the premise unreasonable or is the available equation inapplicable? Explain your answer. (d) If such a muzzle velocity could be obtained, discuss the effects of air resistance, thinning air with altitude, and the curvature of the Earth on the range of the super cannon.

#### **Exercise:**

#### **Problem:**

Construct Your Own Problem Consider a ball tossed over a fence. Construct a problem in which you calculate the ball's needed initial velocity to just clear the fence. Among the things to determine are; the height of the fence, the distance to the fence from the point of release of the ball, and the height at which the ball is released. You should also consider whether it is possible to choose the initial speed for the ball and just calculate the angle at which it is thrown. Also examine the possibility of multiple solutions given the distances and heights you have chosen.

### Glossary

air resistance

a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

#### kinematics

the study of motion without regard to mass or force

motion

displacement of an object as a function of time

projectile

an object that travels through the air and experiences only acceleration due to gravity

projectile motion

the motion of an object that is subject only to the acceleration of gravity

range

the maximum horizontal distance that a projectile travels

trajectory

the path of a projectile through the air

# Introduction to Dynamics: Newton's Laws of Motion class="introduction"

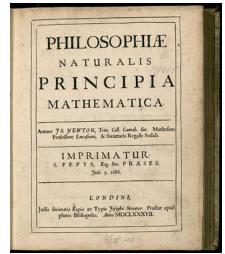
Newton's laws of motion describe the motion of the dolphin's path. (credit: Jin Jang)



Motion draws our attention. Motion itself can be beautiful, causing us to marvel at the forces needed to achieve spectacular motion, such as that of a

dolphin jumping out of the water, or a pole vaulter, or the flight of a bird, or the orbit of a satellite. The study of motion is kinematics, but kinematics only *describes* the way objects move—their velocity and their acceleration. **Dynamics** considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to similar situations on Earth as well as in space.

Isaac Newton's (1642–1727) laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance into the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. For many centuries natural philosophers had debated the nature of the universe based largely on certain rules of logic with great weight given to the thoughts of earlier classical philosophers such as Aristotle (384–322 BC). Among the many great thinkers who contributed to this change were Newton and Galileo.



Isaac Newton's monumental work, *Philosophiae Naturalis Principia Mathematica*, was published in 1687. It proposed scientific

laws that are still
used today to
describe the motion
of objects. (credit:
Service commun de
la documentation de
l'Université de
Strasbourg)

Galileo was instrumental in establishing *observation* as the absolute determinant of truth, rather than "logical" argument. Galileo's use of the telescope was his most notable achievement in demonstrating the importance of observation. He discovered moons orbiting Jupiter and made other observations that were inconsistent with certain ancient ideas and religious dogma. For this reason, and because of the manner in which he dealt with those in authority, Galileo was tried by the Inquisition and punished. He spent the final years of his life under a form of house arrest. Because others before Galileo had also made discoveries by *observing* the nature of the universe, and because repeated observations verified those of Galileo, his work could not be suppressed or denied. After his death, his work was verified by others, and his ideas were eventually accepted by the church and scientific communities.

Galileo also contributed to the formation of what is now called Newton's first law of motion. Newton made use of the work of his predecessors, which enabled him to develop laws of motion, discover the law of gravity, invent calculus, and make great contributions to the theories of light and color. It is amazing that many of these developments were made with Newton working alone, without the benefit of the usual interactions that take place among scientists today.

It was not until the advent of modern physics early in the 20th century that it was discovered that Newton's laws of motion produce a good approximation to motion only when the objects are moving at speeds much, much less than the speed of light and when those objects are larger than the

size of most molecules (about  $10^{-9}$  m in diameter). These constraints define the realm of classical mechanics, as discussed in <u>Introduction to the Nature of Science and Physics</u>. At the beginning of the  $20^{th}$  century, Albert Einstein (1879–1955) developed the theory of relativity and, along with many other scientists, developed quantum theory. This theory does not have the constraints present in classical physics. All of the situations we consider in this chapter, and all those preceding the introduction of relativity in <u>Special Relativity</u>, are in the realm of classical physics.

## Note:

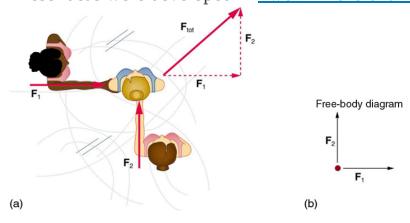
# Making Connections: Past and Present Philosophy

The importance of observation and the concept of cause and effect were not always so entrenched in human thinking. This realization was a part of the evolution of modern physics from natural philosophy. The achievements of Galileo, Newton, Einstein, and others were key milestones in the history of scientific thought. Most of the scientific theories that are described in this book descended from the work of these scientists.

# Development of Force Concept

• Understand the definition of force.

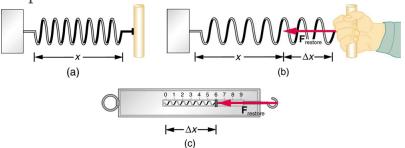
**Dynamics** is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of **force**—that is, a push or a pull—is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard. For example, a cannon exerts a strong force on a cannonball that is launched into the air. In contrast, Earth exerts only a tiny downward pull on a flea. Our everyday experiences also give us a good idea of how multiple forces add. If two people push in different directions on a third person, as illustrated in [link], we might expect the total force to be in the direction shown. Since force is a vector, it adds just like other vectors, as illustrated in [link](a) for two ice skaters. Forces, like other vectors, are represented by arrows and can be added using the familiar head-to-tail method or by trigonometric methods. These ideas were developed in Two-Dimensional Kinematics.



Part (a) shows an overhead view of two ice skaters pushing on a third. Forces are vectors and add like other vectors, so the total force on the third skater is in the direction shown. In part (b), we see a free-body diagram representing the forces acting on the third skater.

[link](b) is our first example of a **free-body diagram**, which is a technique used to illustrate all the **external forces** acting on a body. The body is represented by a single isolated point (or free body), and only those forces acting *on* the body from the outside (external forces) are shown. (These forces are the only ones shown, because only external forces acting on the body affect its motion. We can ignore any internal forces within the body.) Free-body diagrams are very useful in analyzing forces acting on a system and are employed extensively in the study and application of Newton's laws of motion.

A more quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard distance. One possibility is to stretch a spring a certain fixed distance, as illustrated in [link], and use the force it exerts to pull itself back to its relaxed shape—called a *restoring force*—as a standard. The magnitude of all other forces can be stated as multiples of this standard unit of force. Many other possibilities exist for standard forces. (One that we will encounter in Magnetism is the magnetic force between two wires carrying electric current.) Some alternative definitions of force will be given later in this chapter.



The force exerted by a stretched spring can be used as a standard unit of force. (a) This spring has a length x when undistorted. (b) When stretched a distance  $\Delta x$ , the spring exerts a restoring force,  $\mathbf{F}_{\text{restore}}$ , which is reproducible. (c) A spring scale is one device that uses a spring to measure force. The force  $\mathbf{F}_{\text{restore}}$  is exerted on whatever is attached to the hook. Here  $\mathbf{F}_{\text{restore}}$  has a

# magnitude of 6 units in the force standard being employed.

## Note:

# Take-Home Experiment: Force Standards

To investigate force standards and cause and effect, get two identical rubber bands. Hang one rubber band vertically on a hook. Find a small household item that could be attached to the rubber band using a paper clip, and use this item as a weight to investigate the stretch of the rubber band. Measure the amount of stretch produced in the rubber band with one, two, and four of these (identical) items suspended from the rubber band. What is the relationship between the number of items and the amount of stretch? How large a stretch would you expect for the same number of items suspended from two rubber bands? What happens to the amount of stretch of the rubber band (with the weights attached) if the weights are also pushed to the side with a pencil?

# **Section Summary**

- **Dynamics** is the study of how forces affect the motion of objects.
- **Force** is a push or pull that can be defined in terms of various standards, and it is a vector having both magnitude and direction.
- External forces are any outside forces that act on a body. A free-body diagram is a drawing of all external forces acting on a body.

# **Conceptual Questions**

Propose a force standard different from the example of a stretched spring discussed in the text. Your standard must be capable of producing the same force repeatedly.

## **Exercise:**

## **Problem:**

What properties do forces have that allow us to classify them as vectors?

# **Glossary**

# dynamics

the study of how forces affect the motion of objects and systems

## external force

a force acting on an object or system that originates outside of the object or system

# free-body diagram

a sketch showing all of the external forces acting on an object or system; the system is represented by a dot, and the forces are represented by vectors extending outward from the dot

## force

a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

## Newton's First Law of Motion: Inertia

- Define mass and inertia.
- Understand Newton's first law of motion.

Experience suggests that an object at rest will remain at rest if left alone, and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. What **Newton's first law of motion** states, however, is the following:

## Note:

## Newton's First Law of Motion

A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force.

Note the repeated use of the verb "remains." We can think of this law as preserving the status quo of motion.

Rather than contradicting our experience, **Newton's first law of motion** states that there must be a *cause* (which is a net external force) *for there to be any change in velocity (either a change in magnitude or direction)*. We will define *net external force* in the next section. An object sliding across a table or floor slows down due to the net force of friction acting on the object. If friction disappeared, would the object still slow down?

The idea of cause and effect is crucial in accurately describing what happens in various situations. For example, consider what happens to an object sliding along a rough horizontal surface. The object quickly grinds to a halt. If we spray the surface with talcum powder to make the surface smoother, the object slides farther. If we make the surface even smoother by rubbing lubricating oil on it, the object slides farther yet. Extrapolating to a frictionless surface, we can imagine the object sliding in a straight line indefinitely. Friction is thus the *cause* of the slowing (consistent with Newton's first law). The object would not slow down at all if friction were

completely eliminated. Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it creates a nearly frictionless surface, and the puck glides long distances without slowing down. Additionally, if we know enough about the friction, we can accurately predict how quickly the object will slow down. Friction is an external force.

Newton's first law is completely general and can be applied to anything from an object sliding on a table to a satellite in orbit to blood pumped from the heart. Experiments have thoroughly verified that any change in velocity (speed or direction) must be caused by an external force. The idea of *generally applicable or universal laws* is important not only here—it is a basic feature of all laws of physics. Identifying these laws is like recognizing patterns in nature from which further patterns can be discovered. The genius of Galileo, who first developed the idea for the first law, and Newton, who clarified it, was to ask the fundamental question, "What is the cause?" Thinking in terms of cause and effect is a worldview fundamentally different from the typical ancient Greek approach when questions such as "Why does a tiger have stripes?" would have been answered in Aristotelian fashion, "That is the nature of the beast." True perhaps, but not a useful insight.

## Mass

The property of a body to remain at rest or to remain in motion with constant velocity is called **inertia**. Newton's first law is often called the **law of inertia**. As we know from experience, some objects have more inertia than others. It is obviously more difficult to change the motion of a large boulder than that of a basketball, for example. The inertia of an object is measured by its **mass**. Roughly speaking, mass is a measure of the amount of "stuff" (or matter) in something. The quantity or amount of matter in an object is determined by the numbers of atoms and molecules of various types it contains. Unlike weight, mass does not vary with location. The mass of an object is the same on Earth, in orbit, or on the surface of the Moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so masses are not often determined in this

manner. Operationally, the masses of objects are determined by comparison with the standard kilogram.

## **Exercise:**

# **Check Your Understanding**

## **Problem:**

Which has more mass: a kilogram of cotton balls or a kilogram of gold?

## **Solution:**

## Answer

They are equal. A kilogram of one substance is equal in mass to a kilogram of another substance. The quantities that might differ between them are volume and density.

# **Section Summary**

- **Newton's first law of motion** states that a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force. This is also known as the **law of inertia**.
- **Inertia** is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object's mass.
- **Mass** is the quantity of matter in a substance.

# **Conceptual Questions**

## **Exercise:**

**Problem:** How are inertia and mass related?

## **Exercise:**

## **Problem:**

What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?

# Glossary

inertia

the tendency of an object to remain at rest or remain in motion

law of inertia

see Newton's first law of motion

mass

the quantity of matter in a substance; measured in kilograms

Newton's first law of motion

a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force; also known as the law of inertia

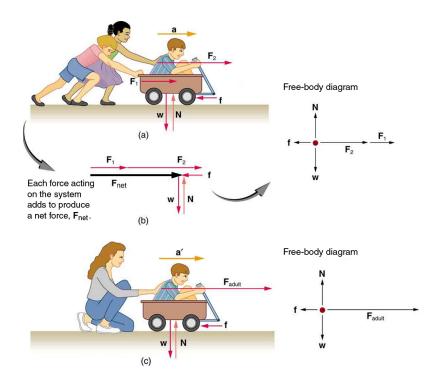
# Newton's Second Law of Motion: Concept of a System

- Define net force, external force, and system.
- Understand Newton's second law of motion.
- Apply Newton's second law to determine the weight of an object.

**Newton's second law of motion** is closely related to Newton's first law of motion. It mathematically states the cause and effect relationship between force and changes in motion. Newton's second law of motion is more quantitative and is used extensively to calculate what happens in situations involving a force. Before we can write down Newton's second law as a simple equation giving the exact relationship of force, mass, and acceleration, we need to sharpen some ideas that have already been mentioned.

First, what do we mean by a change in motion? The answer is that a change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is an **acceleration**. Newton's first law says that a net external force causes a change in motion; thus, we see that a *net* external force causes acceleration.

Another question immediately arises. What do we mean by an external force? An intuitive notion of external is correct—an **external force** acts from outside the **system** of interest. For example, in [link](a) the system of interest is the wagon plus the child in it. The two forces exerted by the other children are external forces. An internal force acts between elements of the system. Again looking at [link](a), the force the child in the wagon exerts to hang onto the wagon is an internal force between elements of the system of interest. Only external forces affect the motion of a system, according to Newton's first law. (The internal forces actually cancel, as we shall see in the next section.) You must define the boundaries of the system before you can determine which forces are external. Sometimes the system is obvious, whereas other times identifying the boundaries of a system is more subtle. The concept of a system is fundamental to many areas of physics, as is the correct application of Newton's laws. This concept will be revisited many times on our journey through physics.



Different forces exerted on the same mass produce different accelerations. (a) Two children push a wagon with a child in it. Arrows representing all external forces are shown. The system of interest is the wagon and its rider. The weight **w** of the system and the support of the ground N are also shown for completeness and are assumed to cancel. The vector  $\mathbf{f}$  represents the friction acting on the wagon, and it acts to the left, opposing the motion of the wagon. (b) All of the external forces acting on the system add together to produce a net force,  $\mathbf{F}_{\text{net}}$ . The free-body diagram shows all of the forces acting on the system of interest. The dot represents the center of mass of the system. Each force vector extends from this dot. Because there are two forces acting to the right, we draw the vectors collinearly. (c) A larger net external force produces a larger

# acceleration $(\mathbf{a}\prime > \mathbf{a})$ when an adult pushes the child.

Now, it seems reasonable that acceleration should be directly proportional to and in the same direction as the net (total) external force acting on a system. This assumption has been verified experimentally and is illustrated in [link]. In part (a), a smaller force causes a smaller acceleration than the larger force illustrated in part (c). For completeness, the vertical forces are also shown; they are assumed to cancel since there is no acceleration in the vertical direction. The vertical forces are the weight  ${\bf w}$  and the support of the ground  ${\bf N}$ , and the horizontal force  ${\bf f}$  represents the force of friction. These will be discussed in more detail in later sections. For now, we will define **friction** as a force that opposes the motion past each other of objects that are touching. [link](b) shows how vectors representing the external forces add together to produce a net force,  ${\bf F}_{\rm net}$ .

To obtain an equation for Newton's second law, we first write the relationship of acceleration and net external force as the proportionality **Equation:** 

$$\mathbf{a} \propto \mathbf{F}_{\mathrm{net}},$$

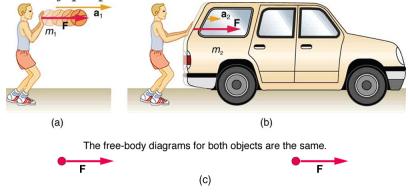
where the symbol  $\propto$  means "proportional to," and  $\mathbf{F}_{\mathrm{net}}$  is the **net external force**. (The net external force is the vector sum of all external forces and can be determined graphically, using the head-to-tail method, or analytically, using components. The techniques are the same as for the addition of other vectors, and are covered in <u>Two-Dimensional Kinematics</u>.) This proportionality states what we have said in words—*acceleration is directly proportional to the net external force*. Once the system of interest is chosen, it is important to identify the external forces and ignore the internal ones. It is a tremendous simplification not to have to consider the numerous internal forces acting between objects within the system, such as muscular forces within the child's body, let alone the myriad of forces between atoms in the objects, but by doing so, we can easily solve some very complex problems with only minimal error due to our simplification

Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. And indeed, as illustrated in [link], the same net external force applied to a car produces a much smaller acceleration than when applied to a basketball. The proportionality is written as

# **Equation:**

$$\mathbf{a} \propto rac{1}{m}$$

where m is the mass of the system. Experiments have shown that acceleration is exactly inversely proportional to mass, just as it is exactly linearly proportional to the net external force.



The same force exerted on systems of different masses produces different accelerations. (a) A basketball player pushes on a basketball to make a pass. (The effect of gravity on the ball is ignored.) (b) The same player exerts an identical force on a stalled SUV and produces a far smaller acceleration (even if friction is negligible). (c) The free-body diagrams are identical, permitting direct comparison of the two situations. A series of patterns for the free-body diagram will emerge as you do more problems.

It has been found that the acceleration of an object depends *only* on the net external force and the mass of the object. Combining the two proportionalities just given yields Newton's second law of motion.

## Note:

## Newton's Second Law of Motion

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.

In equation form, Newton's second law of motion is

## **Equation:**

$$\mathbf{a} = rac{\mathbf{F}_{ ext{net}}}{m}.$$

This is often written in the more familiar form

# **Equation:**

$$\mathbf{F}_{\mathrm{net}} = m\mathbf{a}$$
.

When only the magnitude of force and acceleration are considered, this equation is simply

# **Equation:**

$$F_{
m net}={
m ma.}$$

Although these last two equations are really the same, the first gives more insight into what Newton's second law means. The law is a *cause and effect relationship* among three quantities that is not simply based on their definitions. The validity of the second law is completely based on experimental verification.

## **Units of Force**

 ${f F}_{
m net}=m{f a}$  is used to define the units of force in terms of the three basic units for mass, length, and time. The SI unit of force is called the **newton** (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of  $1{
m m/s}^2$ . That is, since  ${f F}_{
m net}=m{f a}$ ,

# **Equation:**

$$1 N = 1 kg \cdot m/s^2.$$

While almost the entire world uses the newton for the unit of force, in the United States the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb.

# Weight and the Gravitational Force

When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that a net force on an object is responsible for its acceleration. If air resistance is negligible, the net force on a falling object is the gravitational force, commonly called its **weight w**. Weight can be denoted as a vector  $\mathbf{w}$  because it has a direction; *down* is, by definition, the direction of gravity, and hence weight is a downward force. The magnitude of weight is denoted as w. Galileo was instrumental in showing that, in the absence of air resistance, all objects fall with the same acceleration g. Using Galileo's result and Newton's second law, we can derive an equation for weight.

Consider an object with mass m falling downward toward Earth. It experiences only the downward force of gravity, which has magnitude w. Newton's second law states that the magnitude of the net external force on an object is  $F_{\rm net} = {\rm ma}$ .

Since the object experiences only the downward force of gravity,  $F_{\text{net}} = w$ . We know that the acceleration of an object due to gravity is g, or a = g. Substituting these into Newton's second law gives

## Note:

Weight

This is the equation for *weight*—the gravitational force on a mass m:

# **Equation:**

$$w = mg$$
.

Since  $g = 9.80 \text{ m/s}^2$  on Earth, the weight of a 1.0 kg object on Earth is 9.8 N, as we see:

# **Equation:**

$$w = \text{mg} = (1.0 \text{ kg})(9.80 \text{ m/s}^2) = 9.8 \text{ N}.$$

Recall that g can take a positive or negative value, depending on the positive direction in the coordinate system. Be sure to take this into consideration when solving problems with weight.

When the net external force on an object is its weight, we say that it is in **free-fall**. That is, the only force acting on the object is the force of gravity. In the real world, when objects fall downward toward Earth, they are never truly in free-fall because there is always some upward force from the air acting on the object.

The acceleration due to gravity g varies slightly over the surface of Earth, so that the weight of an object depends on location and is not an intrinsic property of the object. Weight varies dramatically if one leaves Earth's surface. On the Moon, for example, the acceleration due to gravity is only  $1.67~\mathrm{m/s}^2$ . A 1.0-kg mass thus has a weight of  $9.8~\mathrm{N}$  on Earth and only about  $1.7~\mathrm{N}$  on the Moon.

The broadest definition of weight in this sense is that the weight of an object is the gravitational force on it from the nearest large body, such as Earth, the Moon, the Sun, and so on. This is the most common and useful definition of weight in physics. It differs dramatically, however, from the definition of weight used by NASA and the popular media in relation to space travel and exploration. When they speak of "weightlessness" and

"microgravity," they are really referring to the phenomenon we call "free-fall" in physics. We shall use the above definition of weight, and we will make careful distinctions between free-fall and actual weightlessness.

It is important to be aware that weight and mass are very different physical quantities, although they are closely related. Mass is the quantity of matter (how much "stuff") and does not vary in classical physics, whereas weight is the gravitational force and does vary depending on gravity. It is tempting to equate the two, since most of our examples take place on Earth, where the weight of an object only varies a little with the location of the object. Furthermore, the terms *mass* and *weight* are used interchangeably in everyday language; for example, our medical records often show our "weight" in kilograms, but never in the correct units of newtons.

## Note:

# Common Misconceptions: Mass vs. Weight

Mass and weight are often used interchangeably in everyday language. However, in science, these terms are distinctly different from one another. Mass is a measure of how much matter is in an object. The typical measure of mass is the kilogram (or the "slug" in English units). Weight, on the other hand, is a measure of the force of gravity acting on an object. Weight is equal to the mass of an object (m) multiplied by the acceleration due to gravity (g). Like any other force, weight is measured in terms of newtons (or pounds in English units).

Assuming the mass of an object is kept intact, it will remain the same, regardless of its location. However, because weight depends on the acceleration due to gravity, the weight of an object *can change* when the object enters into a region with stronger or weaker gravity. For example, the acceleration due to gravity on the Moon is  $1.67~\mathrm{m/s^2}$  (which is much less than the acceleration due to gravity on Earth,  $9.80~\mathrm{m/s^2}$ ). If you measured your weight on Earth and then measured your weight on the Moon, you would find that you "weigh" much less, even though you do not look any skinnier. This is because the force of gravity is weaker on the Moon. In fact, when people say that they are "losing weight," they really

mean that they are losing "mass" (which in turn causes them to weigh less).

## Note:

# Take-Home Experiment: Mass and Weight

What do bathroom scales measure? When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled. The springs provide a measure of your weight (for an object which is not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is divided by 9.80 to give a reading in mass units of kilograms. The scale measures weight but is calibrated to provide information about mass. While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why? Would your scale measure the same "mass" on Earth as on the Moon?

# **Example:**

# What Acceleration Can a Person Produce when Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N (about 11 lb) parallel to the ground. The mass of the mower is 24 kg. What is its acceleration?



The net force on a lawn mower is 51

N to the right. At what rate does the lawn mower accelerate to the right?

# **Strategy**

Since  $\mathbf{F}_{\mathrm{net}}$  and m are given, the acceleration can be calculated directly from Newton's second law as stated in  $\mathbf{F}_{\mathrm{net}} = m\mathbf{a}$ .

## Solution

The magnitude of the acceleration a is  $a = \frac{F_{\text{net}}}{m}$ . Entering known values gives

# **Equation:**

$$a = \frac{51 \text{ N}}{24 \text{ kg}}$$

Substituting the units  $kg \cdot m/s^2$  for N yields

# **Equation:**

$$a = rac{51 \; ext{kg} \cdot ext{m/s}^2}{24 \; ext{kg}} = 2.1 \; ext{m/s}^2.$$

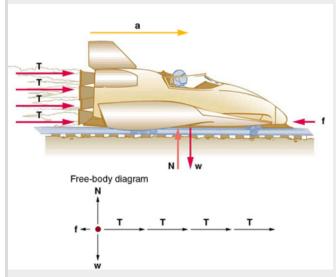
## Discussion

The direction of the acceleration is the same direction as that of the net force, which is parallel to the ground. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion (since we know the mower moves forward), and the vertical forces must cancel if there is to be no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is small enough to be reasonable for a person pushing a mower. Such an effort would not last too long because the person's top speed would soon be reached.

# **Example:**

## What Rocket Thrust Accelerates This Sled?

Prior to manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on human subjects at high speeds. They consisted of a platform that was mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its thrust  $\mathbf{T}$ , for the four-rocket propulsion system shown in [link]. The sled's initial acceleration is  $49 \text{ m/s}^2$ , the mass of the system is 2100 kg, and the force of friction opposing the motion is known to be 650 N.



A sled experiences a rocket thrust that accelerates it to the right. Each rocket creates an identical thrust **T**. As in other situations where there is only horizontal acceleration, the vertical forces cancel. The ground exerts an upward force **N** on the system that is equal in magnitude and opposite in direction to its weight, **w**. The system here is the sled, its rockets, and rider, so none of the forces *between* these objects are considered. The arrow representing friction (**f**) is drawn larger than scale.

# **Strategy**

Although there are forces acting vertically and horizontally, we assume the vertical forces cancel since there is no vertical acceleration. This leaves us with only horizontal forces and a simpler one-dimensional problem. Directions are indicated with plus or minus signs, with right taken as the positive direction. See the free-body diagram in the figure.

## Solution

Since acceleration, mass, and the force of friction are given, we start with Newton's second law and look for ways to find the thrust of the engines. Since we have defined the direction of the force and acceleration as acting "to the right," we need to consider only the magnitudes of these quantities in the calculations. Hence we begin with

# **Equation:**

$$F_{\rm net} = {
m ma}$$
,

where  $F_{\text{net}}$  is the net force along the horizontal direction. We can see from [link] that the engine thrusts add, while friction opposes the thrust. In equation form, the net external force is

# **Equation:**

$$F_{\rm net} = 4T - f$$
.

Substituting this into Newton's second law gives

# **Equation:**

$$F_{
m net} = {
m ma} = 4T - f.$$

Using a little algebra, we solve for the total thrust 4T:

# **Equation:**

$$4T = \text{ma} + f$$
.

Substituting known values yields

# **Equation:**

$$4T = \text{ma} + f = (2100 \text{ kg})(49 \text{ m/s}^2) + 650 \text{ N}.$$

So the total thrust is

# **Equation:**

$$4T = 1.0 \times 10^5 \text{ N},$$

and the individual thrusts are

## **Equation:**

$$T = rac{1.0 imes 10^5 ext{ N}}{4} = 2.6 imes 10^4 ext{ N}.$$

## Discussion

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance and the setup designed to protect human subjects in jet fighter emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 g's. (Recall that g, the acceleration due to gravity, is  $9.80~\text{m/s}^2$ . When we say that an acceleration is 45~g's, it is  $45\times9.80~\text{m/s}^2$ , which is approximately  $440~\text{m/s}^2$ .) While living subjects are not used any more, land speeds of 10,000 km/h have been obtained with rocket sleds. In this example, as in the preceding one, the system of interest is obvious. We will see in later examples that choosing the system of interest is crucial—and the choice is not always obvious.

Newton's second law of motion is more than a definition; it is a relationship among acceleration, force, and mass. It can help us make predictions. Each of those physical quantities can be defined independently, so the second law tells us something basic and universal about nature. The next section introduces the third and final law of motion.

# **Section Summary**

- Acceleration, **a**, is defined as a change in velocity, meaning a change in its magnitude or direction, or both.
- An external force is one acting on a system from outside the system, as opposed to internal forces, which act between components within the

system.

- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.
- In equation form, Newton's second law of motion is  $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$ .
- This is often written in the more familiar form:  $\mathbf{F}_{\mathrm{net}} = m\mathbf{a}$ .
- The weight **w** of an object is defined as the force of gravity acting on an object of mass *m*. The object experiences an acceleration due to gravity **g**:

# **Equation:**

$$\mathbf{w} = m\mathbf{g}$$
.

- If the only force acting on an object is due to gravity, the object is in free fall.
- Friction is a force that opposes the motion past each other of objects that are touching.

# **Conceptual Questions**

### **Exercise:**

## **Problem:**

Which statement is correct? (a) Net force causes motion. (b) Net force causes change in motion. Explain your answer and give an example.

#### Exercise:

## Problem:

Why can we neglect forces such as those holding a body together when we apply Newton's second law of motion?

Explain how the choice of the "system of interest" affects which forces must be considered when applying Newton's second law of motion.

## **Exercise:**

## **Problem:**

Describe a situation in which the net external force on a system is not zero, yet its speed remains constant.

## Exercise:

## **Problem:**

A system can have a nonzero velocity while the net external force on it *is* zero. Describe such a situation.

## **Exercise:**

## **Problem:**

A rock is thrown straight up. What is the net external force acting on the rock when it is at the top of its trajectory?

#### **Exercise:**

## **Problem:**

(a) Give an example of different net external forces acting on the same system to produce different accelerations. (b) Give an example of the same net external force acting on systems of different masses, producing different accelerations. (c) What law accurately describes both effects? State it in words and as an equation.

#### Exercise:

## **Problem:**

If the acceleration of a system is zero, are no external forces acting on it? What about internal forces? Explain your answers.

If a constant, nonzero force is applied to an object, what can you say about the velocity and acceleration of the object?

## **Exercise:**

## **Problem:**

The gravitational force on the basketball in [link] is ignored. When gravity *is* taken into account, what is the direction of the net external force on the basketball—above horizontal, below horizontal, or still horizontal?

## **Problem Exercises**

You may assume data taken from illustrations is accurate to three digits.

## **Exercise:**

## **Problem:**

A 63.0-kg sprinter starts a race with an acceleration of  $4.20 \text{ m/s}^2$ . What is the net external force on him?

## **Solution:**

265 N

## **Exercise:**

## **Problem:**

If the sprinter from the previous problem accelerates at that rate for 20 m, and then maintains that velocity for the remainder of the 100-m dash, what will be his time for the race?

A cleaner pushes a 4.50-kg laundry cart in such a way that the net external force on it is 60.0 N. Calculate the magnitude of its acceleration.

## **Solution:**

 $13.3 \text{ m/s}^2$ 

## **Exercise:**

## **Problem:**

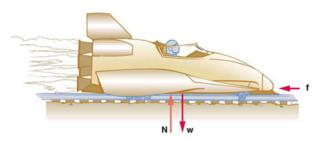
Since astronauts in orbit are apparently weightless, a clever method of measuring their masses is needed to monitor their mass gains or losses to adjust diets. One way to do this is to exert a known force on an astronaut and measure the acceleration produced. Suppose a net external force of 50.0 N is exerted and the astronaut's acceleration is measured to be  $0.893~\text{m/s}^2$ . (a) Calculate her mass. (b) By exerting a force on the astronaut, the vehicle in which they orbit experiences an equal and opposite force. Discuss how this would affect the measurement of the astronaut's acceleration. Propose a method in which recoil of the vehicle is avoided.

## **Exercise:**

## **Problem:**

In [link], the net external force on the 24-kg mower is stated to be 51 N. If the force of friction opposing the motion is 24 N, what force F (in newtons) is the person exerting on the mower? Suppose the mower is moving at 1.5 m/s when the force F is removed. How far will the mower go before stopping?

The same rocket sled drawn in [link] is decelerated at a rate of  $196 \text{ m/s}^2$ . What force is necessary to produce this deceleration? Assume that the rockets are off. The mass of the system is 2100 kg.



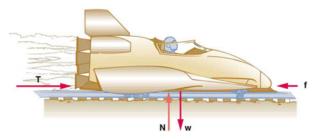
## **Exercise:**

## **Problem:**

(a) If the rocket sled shown in [link] starts with only one rocket burning, what is the magnitude of its acceleration? Assume that the mass of the system is 2100 kg, the thrust T is  $2.4 \times 10^4$  N, and the force of friction opposing the motion is known to be 650 N. (b) Why is the acceleration not one-fourth of what it is with all rockets burning?

## **Solution:**

- (a)  $12 \text{ m/s}^2$ .
- (b) The acceleration is not one-fourth of what it was with all rockets burning because the frictional force is still as large as it was with all rockets burning.



### **Exercise:**

### **Problem:**

What is the deceleration of the rocket sled if it comes to rest in 1.1 s from a speed of 1000 km/h? (Such deceleration caused one test subject to black out and have temporary blindness.)

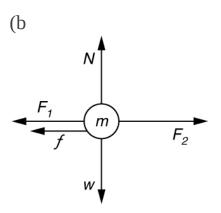
### **Exercise:**

### **Problem:**

Suppose two children push horizontally, but in exactly opposite directions, on a third child in a wagon. The first child exerts a force of 75.0 N, the second a force of 90.0 N, friction is 12.0 N, and the mass of the third child plus wagon is 23.0 kg. (a) What is the system of interest if the acceleration of the child in the wagon is to be calculated? (b) Draw a free-body diagram, including all forces acting on the system. (c) Calculate the acceleration. (d) What would the acceleration be if friction were 15.0 N?

### **Solution:**

(a) The system is the child in the wagon plus the wagon.



(c)  $a = 0.130 \text{ m/s}^2$  in the direction of the second child's push.

(d) 
$$a = 0.00 \text{ m/s}^2$$

### **Exercise:**

### **Problem:**

A powerful motorcycle can produce an acceleration of  $3.50~\mathrm{m/s}^2$  while traveling at 90.0 km/h. At that speed the forces resisting motion, including friction and air resistance, total 400 N. (Air resistance is analogous to air friction. It always opposes the motion of an object.) What is the magnitude of the force the motorcycle exerts backward on the ground to produce its acceleration if the mass of the motorcycle with rider is 245 kg?

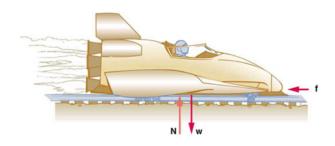
### **Exercise:**

### **Problem:**

The rocket sled shown in [link] accelerates at a rate of  $49.0 \text{ m/s}^2$ . Its passenger has a mass of 75.0 kg. (a) Calculate the horizontal component of the force the seat exerts against his body. Compare this with his weight by using a ratio. (b) Calculate the direction and magnitude of the total force the seat exerts against his body.

### **Solution:**

- (a)  $3.68\times 10^3\ N$  . This force is 5.00 times greater than his weight.
- (b) 3750 N; 11.3° above horizontal



### **Exercise:**

### **Problem:**

Repeat the previous problem for the situation in which the rocket sled decelerates at a rate of  $201 \text{ m/s}^2$ . In this problem, the forces are exerted by the seat and restraining belts.

#### **Exercise:**

### **Problem:**

The weight of an astronaut plus his space suit on the Moon is only 250 N. How much do they weigh on Earth? What is the mass on the Moon? On Earth?

### **Solution:**

 $1.5 \times 10^3 \; \mathrm{N}, 150 \; \mathrm{kg}, 150 \; \mathrm{kg}$ 

### **Exercise:**

### **Problem:**

Suppose the mass of a fully loaded module in which astronauts take off from the Moon is 10,000 kg. The thrust of its engines is 30,000 N. (a) Calculate its the magnitude of acceleration in a vertical takeoff from the Moon. (b) Could it lift off from Earth? If not, why not? If it could, calculate the magnitude of its acceleration.

# Glossary

#### acceleration

the rate at which an object's velocity changes over a period of time

#### free-fall

a situation in which the only force acting on an object is the force due to gravity

#### friction

a force past each other of objects that are touching; examples include rough surfaces and air resistance

### net external force

the vector sum of all external forces acting on an object or system; causes a mass to accelerate

### Newton's second law of motion

the net external force  $\mathbf{F}_{\mathrm{net}}$  on an object with mass m is proportional to and in the same direction as the acceleration of the object,  $\mathbf{a}$ , and inversely proportional to the mass; defined mathematically as  $\mathbf{F}_{\mathrm{net}}$ 

$$\mathbf{a} = \frac{\mathbf{F}_{ ext{net}}}{m}$$

### system

defined by the boundaries of an object or collection of objects being observed; all forces originating from outside of the system are considered external forces

## weight

the force **w**due to gravity acting on an object of mass m; defined mathematically as:  $\mathbf{w} = m\mathbf{g}$ , where  $\mathbf{g}$  is the magnitude and direction of the acceleration due to gravity

## Newton's Third Law of Motion: Symmetry in Forces

- Understand Newton's third law of motion.
- Apply Newton's third law to define systems and solve problems of motion.

There is a passage in the musical *Man of la Mancha* that relates to Newton's third law of motion. Sancho, in describing a fight with his wife to Don Quixote, says, "Of course I hit her back, Your Grace, but she's a lot harder than me and you know what they say, 'Whether the stone hits the pitcher or the pitcher hits the stone, it's going to be bad for the pitcher." This is exactly what happens whenever one body exerts a force on another—the first also experiences a force (equal in magnitude and opposite in direction). Numerous common experiences, such as stubbing a toe or throwing a ball, confirm this. It is precisely stated in **Newton's third law of motion**.

### Note:

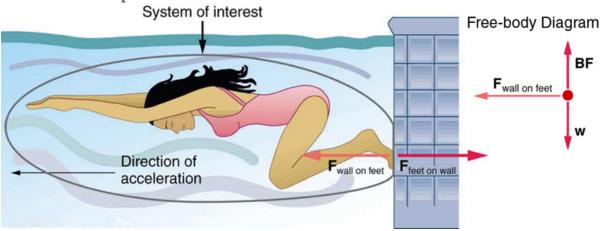
### Newton's Third Law of Motion

Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.

This law represents a certain *symmetry in nature*: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself. We sometimes refer to this law loosely as "action-reaction," where the force exerted is the action and the force experienced as a consequence is the reaction. Newton's third law has practical uses in analyzing the origin of forces and understanding which forces are external to a system.

We can readily see Newton's third law at work by taking a look at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in [link]. She pushes against the pool wall with her feet

and accelerates in the direction *opposite* to that of her push. The wall has exerted an equal and opposite force back on the swimmer. You might think that two equal and opposite forces would cancel, but they do not *because they act on different systems*. In this case, there are two systems that we could investigate: the swimmer or the wall. If we select the swimmer to be the system of interest, as in the figure, then  $\mathbf{F}_{\text{wall on feet}}$  is an external force on this system and affects its motion. The swimmer moves in the direction of  $\mathbf{F}_{\text{wall on feet}}$ . In contrast, the force  $\mathbf{F}_{\text{feet on wall}}$  acts on the wall and not on our system of interest. Thus  $\mathbf{F}_{\text{feet on wall}}$  does not directly affect the motion of the system and does not cancel  $\mathbf{F}_{\text{wall on feet}}$ . Note that the swimmer pushes in the direction opposite to that in which she wishes to move. The reaction to her push is thus in the desired direction.



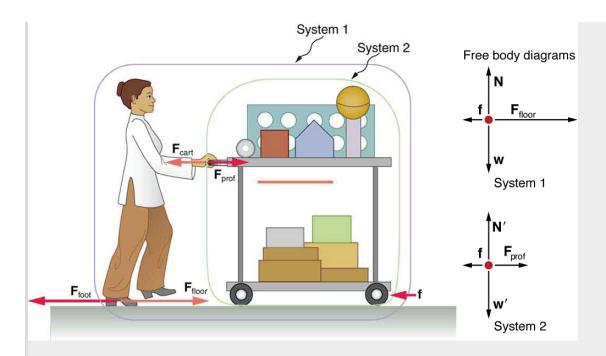
When the swimmer exerts a force  $\mathbf{F}_{\mathrm{feet\ on\ wall}}$  on the wall, she accelerates in the direction opposite to that of her push. This means the net external force on her is in the direction opposite to  $\mathbf{F}_{\mathrm{feet\ on\ wall}}$ . This opposition occurs because, in accordance with Newton's third law of motion, the wall exerts a force  $\mathbf{F}_{\mathrm{wall\ on\ feet}}$  on her, equal in magnitude but in the direction opposite to the one she exerts on it. The line around the swimmer indicates the system of interest. Note that  $\mathbf{F}_{\mathrm{feet\ on\ wall}}$  does not act on this system (the swimmer) and, thus, does not cancel  $\mathbf{F}_{\mathrm{wall\ on\ feet}}$ . Thus the free-body diagram shows only  $\mathbf{F}_{\mathrm{wall\ on\ feet}}$ ,  $\mathbf{w}$ , the gravitational force, and  $\mathbf{BF}$ , the buoyant force of the water supporting the swimmer's weight. The vertical forces  $\mathbf{w}$  and  $\mathbf{BF}$  cancel since there is no vertical motion.

Other examples of Newton's third law are easy to find. As a professor paces in front of a whiteboard, she exerts a force backward on the floor. The floor exerts a reaction force forward on the professor that causes her to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the drive wheels in reaction to the drive wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward. In another example, rockets move forward by expelling gas backward at high velocity. This means the rocket exerts a large backward force on the gas in the rocket combustion chamber, and the gas therefore exerts a large reaction force forward on the rocket. This reaction force is called **thrust**. It is a common misconception that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can more readily expel the exhaust gases. Helicopters similarly create lift by pushing air down, thereby experiencing an upward reaction force. Birds and airplanes also fly by exerting force on air in a direction opposite to that of whatever force they need. For example, the wings of a bird force air downward and backward in order to get lift and move forward. An octopus propels itself in the water by ejecting water through a funnel from its body, similar to a jet ski. In a situation similar to Sancho's, professional cage fighters experience reaction forces when they punch, sometimes breaking their hand by hitting an opponent's body.

## **Example:**

## **Getting Up To Speed: Choosing the Correct System**

A physics professor pushes a cart of demonstration equipment to a lecture hall, as seen in [link]. Her mass is 65.0 kg, the cart's is 12.0 kg, and the equipment's is 7.0 kg. Calculate the acceleration produced when the professor exerts a backward force of 150 N on the floor. All forces opposing the motion, such as friction on the cart's wheels and air resistance, total 24.0 N.



A professor pushes a cart of demonstration equipment. The lengths of the arrows are proportional to the magnitudes of the forces (except for  ${\bf f}$ , since it is too small to draw to scale). Different questions are asked in each example; thus, the system of interest must be defined differently for each. System 1 is appropriate for this example, since it asks for the acceleration of the entire group of objects. Only  ${\bf F}_{\rm floor}$  and  ${\bf f}$  are external forces acting on System 1 along the line of motion. All other forces either cancel or act on the outside world. System 2 is chosen for [link] so that  ${\bf F}_{\rm prof}$  will be an external force and enter into Newton's second law. Note that the free-body diagrams, which allow us to apply Newton's second law, vary with the system chosen.

# **Strategy**

Since they accelerate as a unit, we define the system to be the professor, cart, and equipment. This is System 1 in [link]. The professor pushes backward with a force  $\mathbf{F}_{\mathrm{foot}}$  of 150 N. According to Newton's third law, the floor exerts a forward reaction force  $\mathbf{F}_{\mathrm{floor}}$  of 150 N on System 1. Because all motion is horizontal, we can assume there is no net force in the vertical direction. The problem is therefore one-dimensional along the

horizontal direction. As noted,  $\mathbf{f}$  opposes the motion and is thus in the opposite direction of  $\mathbf{F}_{\mathrm{floor}}$ . Note that we do not include the forces  $\mathbf{F}_{\mathrm{prof}}$  or  $\mathbf{F}_{\mathrm{cart}}$  because these are internal forces, and we do not include  $\mathbf{F}_{\mathrm{foot}}$  because it acts on the floor, not on the system. There are no other significant forces acting on System 1. If the net external force can be found from all this information, we can use Newton's second law to find the acceleration as requested. See the free-body diagram in the figure.

### Solution

Newton's second law is given by

## **Equation:**

$$a=rac{F_{
m net}}{m}.$$

The net external force on System 1 is deduced from [link] and the discussion above to be

## **Equation:**

$$F_{\rm net} = F_{
m floor} - f = 150 \; {
m N} - 24.0 \; {
m N} = 126 \; {
m N}.$$

The mass of System 1 is

## **Equation:**

$$m = (65.0 + 12.0 + 7.0) \text{ kg} = 84 \text{ kg}.$$

These values of  $F_{
m net}$  and m produce an acceleration of

## **Equation:**

$$a = rac{F_{
m net}}{m}, \ a = rac{126 \ {
m N}}{84 \ {
m kg}} = 1.5 \ {
m m/s^2}.$$

#### Discussion

None of the forces between components of System 1, such as between the professor's hands and the cart, contribute to the net external force because they are internal to System 1. Another way to look at this is to note that forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the

professor on the cart results in an equal and opposite force back on her. In this case both forces act on the same system and, therefore, cancel. Thus internal forces (between components of a system) cancel. Choosing System 1 was crucial to solving this problem.

## **Example:**

## Force on the Cart—Choosing a New System

Calculate the force the professor exerts on the cart in [link] using data from the previous example if needed.

## Strategy

If we now define the system of interest to be the cart plus equipment (System 2 in [link]), then the net external force on System 2 is the force the professor exerts on the cart minus friction. The force she exerts on the cart,  $\mathbf{F}_{prof}$ , is an external force acting on System 2.  $\mathbf{F}_{prof}$  was internal to System 1, but it is external to System 2 and will enter Newton's second law for System 2.

### Solution

Newton's second law can be used to find  $\mathbf{F}_{\mathrm{prof}}$ . Starting with

## **Equation:**

$$a=rac{F_{
m net}}{m}$$

and noting that the magnitude of the net external force on System 2 is **Equation:** 

$$F_{
m net} = F_{
m prof} - f,$$

we solve for  $F_{\text{prof}}$ , the desired quantity:

## **Equation:**

$$F_{
m prof} = F_{
m net} + f$$
.

The value of f is given, so we must calculate net  $F_{\rm net}$ . That can be done since both the acceleration and mass of System 2 are known. Using Newton's second law we see that

## **Equation:**

$$F_{
m net}={
m ma},$$

where the mass of System 2 is 19.0 kg (m= 12.0 kg + 7.0 kg) and its acceleration was found to be  $a = 1.5 \text{ m/s}^2$  in the previous example. Thus,

## **Equation:**

$$F_{\rm net} = {
m ma}$$
,

## **Equation:**

$$F_{
m net} = (19.0 \ {
m kg})(1.5 \ {
m m/s^2}) = 29 \ {
m N}.$$

Now we can find the desired force:

## **Equation:**

$$F_{
m prof} = F_{
m net} + f,$$

## **Equation:**

$$F_{\text{prof}} = 29 \text{ N} + 24.0 \text{ N} = 53 \text{ N}.$$

### Discussion

It is interesting that this force is significantly less than the 150-N force the professor exerted backward on the floor. Not all of that 150-N force is transmitted to the cart; some of it accelerates the professor.

The choice of a system is an important analytical step both in solving problems and in thoroughly understanding the physics of the situation (which is not necessarily the same thing).

### Note:

## PhET Explorations: Gravity Force Lab

Visualize the gravitational force that two objects exert on each other. Change properties of the objects in order to see how it changes the gravity force. https://phet.colorado.edu/sims/html/gravity-force-lab/latest/gravity-force-lab en.html

# **Section Summary**

- **Newton's third law of motion** represents a basic symmetry in nature. It states: Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts.
- A **thrust** is a reaction force that pushes a body forward in response to a backward force. Rockets, airplanes, and cars are pushed forward by a thrust reaction force.

## **Conceptual Questions**

### **Exercise:**

#### **Problem:**

When you take off in a jet aircraft, there is a sensation of being pushed back into the seat. Explain why you move backward in the seat—is there really a force backward on you? (The same reasoning explains whiplash injuries, in which the head is apparently thrown backward.)

#### **Exercise:**

#### **Problem:**

A device used since the 1940s to measure the kick or recoil of the body due to heart beats is the "ballistocardiograph." What physics principle(s) are involved here to measure the force of cardiac contraction? How might we construct such a device?

#### **Exercise:**

### **Problem:**

Describe a situation in which one system exerts a force on another and, as a consequence, experiences a force that is equal in magnitude and opposite in direction. Which of Newton's laws of motion apply?

### **Exercise:**

#### **Problem:**

Why does an ordinary rifle recoil (kick backward) when fired? The barrel of a recoilless rifle is open at both ends. Describe how Newton's third law applies when one is fired. Can you safely stand close behind one when it is fired?

#### **Exercise:**

### **Problem:**

An American football lineman reasons that it is senseless to try to outpush the opposing player, since no matter how hard he pushes he will experience an equal and opposite force from the other player. Use Newton's laws and draw a free-body diagram of an appropriate system to explain how he can still out-push the opposition if he is strong enough.

#### **Exercise:**

#### **Problem:**

Newton's third law of motion tells us that forces always occur in pairs of equal and opposite magnitude. Explain how the choice of the "system of interest" affects whether one such pair of forces cancels.

### **Problem Exercises**

### **Exercise:**

### **Problem:**

What net external force is exerted on a 1100-kg artillery shell fired from a battleship if the shell is accelerated at  $2.40\times10^4~\mathrm{m/s}^2$ ? What is the magnitude of the force exerted on the ship by the artillery shell?

### **Solution:**

Force on shell:  $2.64 \times 10^7~\mathrm{N}$ 

Force exerted on ship =  $-2.64 \times 10^7$  N, by Newton's third law

### **Exercise:**

### **Problem:**

A brave but inadequate rugby player is being pushed backward by an opposing player who is exerting a force of 800 N on him. The mass of the losing player plus equipment is 90.0 kg, and he is accelerating at  $1.20~{\rm m/s}^2$  backward. (a) What is the force of friction between the losing player's feet and the grass? (b) What force does the winning player exert on the ground to move forward if his mass plus equipment is  $110~{\rm kg}$ ? (c) Draw a sketch of the situation showing the system of interest used to solve each part. For this situation, draw a free-body diagram and write the net force equation.

# Glossary

Newton's third law of motion

whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts

#### thrust

a reaction force that pushes a body forward in response to a backward force; rockets, airplanes, and cars are pushed forward by a thrust reaction force

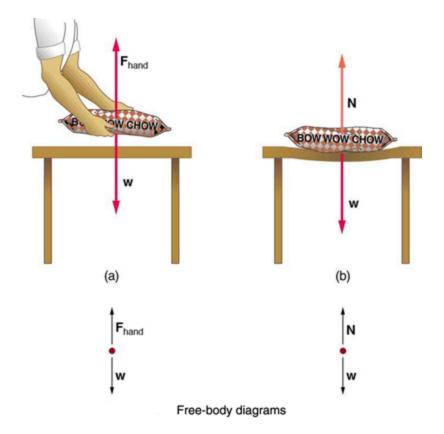
## Normal, Tension, and Other Examples of Forces

- Define normal and tension forces.
- Apply Newton's laws of motion to solve problems involving a variety of forces.
- Use trigonometric identities to resolve weight into components.

Forces are given many names, such as push, pull, thrust, lift, weight, friction, and tension. Traditionally, forces have been grouped into several categories and given names relating to their source, how they are transmitted, or their effects. The most important of these categories are discussed in this section, together with some interesting applications. Further examples of forces are discussed later in this text.

### **Normal Force**

**Weight** (also called force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You definitely notice that you must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in [link](a). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in [link](b)? When the bag of dog food is placed on the table, the table actually sags slightly under the load. This would be noticeable if the load were placed on a card table, but even rigid objects deform when a force is applied to them. Unless the object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or trampoline or diving board). The greater the deformation, the greater the restoring force. So when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly, and the sag is slight so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.



(a) The person holding the bag of dog food must supply an upward force F<sub>hand</sub> equal in magnitude and opposite in direction to the weight of the food w. (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force N equal in magnitude and opposite in direction to the weight of the load.

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. If the force supporting a load is perpendicular to the surface of contact between the load and its support, this force is defined to be a **normal force** and here is given the symbol **N**. (This is not the unit for force N.) The word *normal* means perpendicular to a

surface. The normal force can be less than the object's weight if the object is on an incline, as you will see in the next example.

### Note:

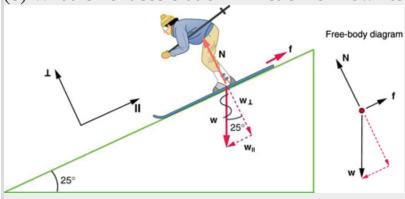
## Common Misconception: Normal Force (N) vs. Newton (N)

In this section we have introduced the quantity normal force, which is represented by the variable  $\mathbf{N}$ . This should not be confused with the symbol for the newton, which is also represented by the letter  $\mathbf{N}$ . These symbols are particularly important to distinguish because the units of a normal force ( $\mathbf{N}$ ) happen to be newtons ( $\mathbf{N}$ ). For example, the normal force  $\mathbf{N}$  that the floor exerts on a chair might be  $\mathbf{N}=100~\mathrm{N}$ . One important difference is that normal force is a vector, while the newton is simply a unit. Be careful not to confuse these letters in your calculations! You will encounter more similarities among variables and units as you proceed in physics. Another example of this is the quantity work (W) and the unit watts (W).

## **Example:**

## Weight on an Incline, a Two-Dimensional Problem

Consider the skier on a slope shown in [link]. Her mass including equipment is 60.0 kg. (a) What is her acceleration if friction is negligible? (b) What is her acceleration if friction is known to be 45.0 N?



Since motion and friction are parallel to the slope, it is most convenient to project all

forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier).

 ${f N}$  is perpendicular to the slope and  ${f f}$  is parallel to the slope, but  ${f w}$  has components along both axes, namely  ${f w}_{\perp}$  and  ${f w}_{\parallel}$ .  ${f N}$  is equal in magnitude to  ${f w}_{\perp}$ , so that there is no motion perpendicular to the slope, but f is less than  $w_{\parallel}$ , so that there is a downslope acceleration (along the parallel axis).

## Strategy

This is a two-dimensional problem, since the forces on the skier (the system of interest) are not parallel. The approach we have used in twodimensional kinematics also works very well here. Choose a convenient coordinate system and project the vectors onto its axes, creating two connected *one*-dimensional problems to solve. The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. (Remember that motions along mutually perpendicular axes are independent.) We use the symbols  $\perp$  and  $\parallel$  to represent perpendicular and parallel, respectively. This choice of axes simplifies this type of problem, because there is no motion perpendicular to the slope and because friction is always parallel to the surface between two objects. The only external forces acting on the system are the skier's weight, friction, and the support of the slope, respectively labeled  $\mathbf{w}$ ,  $\mathbf{f}$ , and  $\mathbf{N}$  in [link].  $\mathbf{N}$  is always perpendicular to the slope, and  $\mathbf{f}$  is parallel to it. But  $\mathbf{w}$  is not in the direction of either axis, and so the first step we take is to project it into components along the chosen axes, defining  $w_{\parallel}$  to be the component of weight parallel to the slope and  $w_{\perp}$  the component of weight perpendicular to the slope. Once this is done, we can consider the two separate problems of forces parallel to the slope and forces perpendicular to the slope.

### Solution

The magnitude of the component of the weight parallel to the slope is  $w_{\parallel}=w\sin{(25^{\circ})}=mg\sin{(25^{\circ})}$ , and the magnitude of the component of

the weight perpendicular to the slope is

$$w_{\perp}=w\cos{(25^{
m o})}=mg\cos{(25^{
m o})}.$$

(a) Neglecting friction. Since the acceleration is parallel to the slope, we need only consider forces parallel to the slope. (Forces perpendicular to the slope add to zero, since there is no acceleration in that direction.) The forces parallel to the slope are the amount of the skier's weight parallel to the slope  $w_{\parallel}$  and friction f. Using Newton's second law, with subscripts to denote quantities parallel to the slope,

## **Equation:**

$$a_\parallel = rac{F_{
m net\parallel}}{m}$$

where  $F_{
m net\parallel}=w_{\parallel}={
m mg~sin}~(25^{
m o})$ , assuming no friction for this part, so that

## **Equation:**

$$a_\parallel = rac{F_{
m net\parallel}}{m} = rac{{
m mg\,sin}\,(25^{
m o})}{m} = g\,{
m sin}\,(25^{
m o})$$

## **Equation:**

$$(9.80 \text{ m/s}^2)(0.4226) = 4.14 \text{ m/s}^2$$

is the acceleration.

(b) Including friction. We now have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is now

## **Equation:**

$$|F_{
m net\parallel}=w_\parallel-f,$$

and substituting this into Newton's second law,  $a_{\parallel}=rac{F_{
m net\parallel}}{m}$  , gives

## **Equation:**

$$a_\parallel = rac{F_{
m net}_\parallel}{m} = rac{w_\parallel - f}{m} = rac{{
m mg\,sin}\left(25^{
m o}
ight) - f}{m}.$$

We substitute known values to obtain

## **Equation:**

$$a_{\parallel} = rac{(60.0 \ ext{kg})(9.80 \ ext{m/s}^2)(0.4226) - 45.0 \ ext{N}}{60.0 \ ext{kg}},$$

which yields

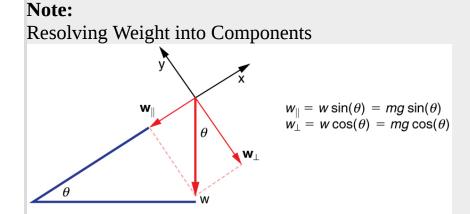
## **Equation:**

$$a_\parallel=3.39~\mathrm{m/s}^2,$$

which is the acceleration parallel to the incline when there is 45.0 N of opposing friction.

### Discussion

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is none. In fact, it is a general result that if friction on an incline is negligible, then the acceleration down the incline is  $a = g \sin \theta$ , regardless of mass. This is related to the previously discussed fact that all objects fall with the same acceleration in the absence of air resistance. Similarly, all objects, regardless of mass, slide down a frictionless incline with the same acceleration (if the angle is the same).



An object rests on an incline that makes an

angle  $\theta$  with the horizontal.

When an object rests on an incline that makes an angle  $\theta$  with the horizontal, the force of gravity acting on the object is divided into two components: a force acting perpendicular to the plane,  $\mathbf{w}_{\perp}$ , and a force acting parallel to the plane,  $\mathbf{w}_{\parallel}$ . The perpendicular force of weight,  $\mathbf{w}_{\perp}$ , is typically equal in magnitude and opposite in direction to the normal force,  $\mathbf{N}$ . The force acting parallel to the plane,  $\mathbf{w}_{\parallel}$ , causes the object to accelerate down the incline. The force of friction,  $\mathbf{f}$ , opposes the motion of the object, so it acts upward along the plane.

It is important to be careful when resolving the weight of the object into components. If the angle of the incline is at an angle  $\theta$  to the horizontal, then the magnitudes of the weight components are

## **Equation:**

$$w_\parallel = w \sin{( heta)} = \mathrm{mg} \sin{( heta)}$$

and

# **Equation:**

$$w_{\perp}=w\cos{( heta)}=\mathrm{mg}\cos{( heta)}.$$

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, draw the right triangle formed by the three weight vectors. Notice that the angle  $\theta$  of the incline is the same as the angle formed between  $\mathbf{w}$  and  $\mathbf{w}_{\perp}$ . Knowing this property, you can use trigonometry to determine the magnitude of the weight components:

## **Equation:**

$$egin{array}{lll} \cos \left( heta 
ight) &=& rac{w_{\perp}}{w} \ w_{\perp} &=& w \cos \left( heta 
ight) = \operatorname{mg} \cos \left( heta 
ight) \end{array}$$

## **Equation:**

$$egin{array}{lcl} \sin \left( heta 
ight) & = & rac{w_{\parallel}}{w} \ w_{\parallel} & = & w \sin \left( heta 
ight) = \mathrm{mg} \sin \left( heta 
ight) \end{array}$$

#### Note:

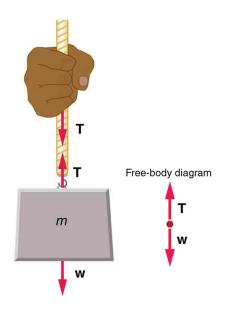
## Take-Home Experiment: Force Parallel

To investigate how a force parallel to an inclined plane changes, find a rubber band, some objects to hang from the end of the rubber band, and a board you can position at different angles. How much does the rubber band stretch when you hang the object from the end of the board? Now place the board at an angle so that the object slides off when placed on the board. How much does the rubber band extend if it is lined up parallel to the board and used to hold the object stationary on the board? Try two more angles. What does this show?

### **Tension**

A **tension** is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable. The word "tension" comes from a Latin word meaning "to stretch." Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*. Any flexible connector, such as a string, rope, chain, wire, or cable, can exert pulls only parallel to its length; thus, a force carried by a flexible connector is a tension with direction parallel to the connector. It is important to understand that tension is a pull in a connector. In contrast, consider the phrase: "You can't push a rope." The tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope as shown in [link].



When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force T, that force must be parallel to the length of the rope, as shown. The pull such a flexible connector exerts is a tension. Note that the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). This is an example of Newton's third law. The rope is the medium that carries

the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.

Tension in the rope must equal the weight of the supported mass, as we can prove using Newton's second law. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero, and thus  $\mathbf{F}_{\rm net}=0$ . The only external forces acting on the mass are its weight  $\mathbf{w}$  and the tension  $\mathbf{T}$  supplied by the rope. Thus,

## **Equation:**

$$F_{
m net} = T - w = 0,$$

where T and w are the magnitudes of the tension and weight and their signs indicate direction, with up being positive here. Thus, just as you would expect, the tension equals the weight of the supported mass:

## **Equation:**

$$T = w = mg$$
.

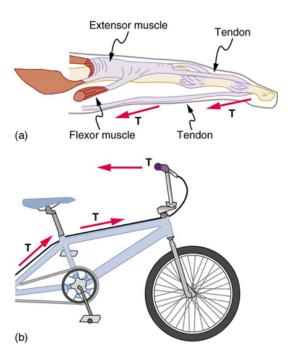
For a 5.00-kg mass, then (neglecting the mass of the rope) we see that

## **Equation:**

$$T = \text{mg} = (5.00 \text{ kg})(9.80 \text{ m/s}^2) = 49.0 \text{ N}.$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a finger joint, or a bicycle brake cable. If there is no friction, the tension is transmitted undiminished. Only its direction changes, and it is always parallel to the flexible connector. This is illustrated in [link] (a) and (b).



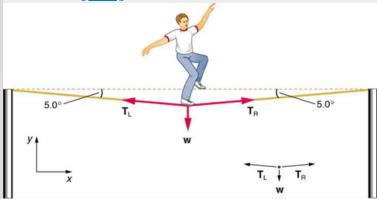
(a) Tendons in the finger carry force **T** from the muscles to other parts of the finger, usually changing the force's direction, but not its magnitude (the

tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension **T** from the handlebars to the brake mechanism. Again, the direction but not the magnitude of **T** is changed.

## **Example:**

## What Is the Tension in a Tightrope?

Calculate the tension in the wire supporting the 70.0-kg tightrope walker shown in [link].



The weight of a tightrope walker causes a wire to sag by 5.0 degrees. The system of interest here is the point in the wire at which the tightrope walker is standing.

## Strategy

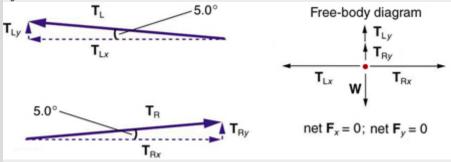
As you can see in the figure, the wire is not perfectly horizontal (it cannot be!), but is bent under the person's weight. Thus, the tension on either side of the person has an upward component that can support his weight. As

usual, forces are vectors represented pictorially by arrows having the same directions as the forces and lengths proportional to their magnitudes. The system is the tightrope walker, and the only external forces acting on him are his weight  $\mathbf{w}$  and the two tensions  $\mathbf{T}_{L}$  (left tension) and  $\mathbf{T}_{R}$  (right tension), as illustrated. It is reasonable to neglect the weight of the wire itself. The net external force is zero since the system is stationary. A little trigonometry can now be used to find the tensions. One conclusion is possible at the outset—we can see from part (b) of the figure that the magnitudes of the tensions  $T_{L}$  and  $T_{R}$  must be equal. This is because there is no horizontal acceleration in the rope, and the only forces acting to the left and right are  $T_{L}$  and  $T_{R}$ . Thus, the magnitude of those forces must be equal so that they cancel each other out.

Whenever we have two-dimensional vector problems in which no two vectors are parallel, the easiest method of solution is to pick a convenient coordinate system and project the vectors onto its axes. In this case the best coordinate system has one axis horizontal and the other vertical. We call the horizontal the x-axis and the vertical the y-axis.

#### Solution

First, we need to resolve the tension vectors into their horizontal and vertical components. It helps to draw a new free-body diagram showing all of the horizontal and vertical components of each force acting on the system.



When the vectors are projected onto vertical and horizontal axes, their components along those axes must add to zero, since the tightrope walker is stationary. The small angle results in T being much greater than w.

Consider the horizontal components of the forces (denoted with a subscript x):

## **Equation:**

$$F_{\text{net}x} = T_{\text{L}x} - T_{\text{R}x}$$
.

The net external horizontal force  $F_{\mathrm{net}x}=0$ , since the person is stationary. Thus,

## **Equation:**

$$egin{array}{lcl} F_{
m net}x = 0 &=& T_{
m L}x - T_{
m R}x \ T_{
m L}x &=& T_{
m R}x. \end{array}$$

Now, observe [link]. You can use trigonometry to determine the magnitude of  $T_L$  and  $T_R$ . Notice that:

## **Equation:**

$$egin{array}{lll} \cos{(5.0^{
m o})} &=& rac{T_{
m L}x}{T_{
m L}} \ T_{
m L}x &=& T_{
m L}\cos{(5.0^{
m o})} \ \cos{(5.0^{
m o})} &=& rac{T_{
m R}x}{T_{
m R}} \ T_{
m R}x &=& T_{
m R}\cos{(5.0^{
m o})}. \end{array}$$

Equating  $T_{Lx}$  and  $T_{Rx}$ :

## **Equation:**

$$T_{
m L} \cos{(5.0^{
m o})} = T_{
m R} \cos{(5.0^{
m o})}.$$

Thus,

## **Equation:**

$$T_{\mathrm{L}} = T_{\mathrm{R}} = T$$

as predicted. Now, considering the vertical components (denoted by a subscript y), we can solve for T. Again, since the person is stationary, Newton's second law implies that net  $F_y = 0$ . Thus, as illustrated in the free-body diagram in [link],

## **Equation:**

$$F_{\mathrm{net}y} = T_{\mathrm{L}y} + T_{\mathrm{R}y} - w = 0.$$

Observing [link], we can use trigonometry to determine the relationship between  $T_{Ly}$ ,  $T_{Ry}$ , and T. As we determined from the analysis in the horizontal direction,  $T_L = T_R = T$ :

## **Equation:**

$$egin{array}{lll} \sin{(5.0^{
m o})} &=& rac{T_{
m L}y}{T_{
m L}} \ T_{
m L}y = T_{
m L} \sin{(5.0^{
m o})} &=& T \sin{(5.0^{
m o})} \ \sin{(5.0^{
m o})} &=& rac{T_{
m R}y}{T_{
m R}} \ T_{
m R}y = T_{
m R} \sin{(5.0^{
m o})} &=& T \sin{(5.0^{
m o})}. \end{array}$$

Now, we can substitute the values for  $T_{Ly}$  and  $T_{Ry}$ , into the net force equation in the vertical direction:

## **Equation:**

$$egin{array}{lll} F_{
m nety} & = & T_{
m L}_y + T_{
m R}_y - w = 0 \ & = & T \sin{(5.0^{
m o})} + T \sin{(5.0^{
m o})} - w = 0 \ & 2 \, T \sin{(5.0^{
m o})} - w & = & 0 \ & 2 \, T \sin{(5.0^{
m o})} & = & w \end{array}$$

and

# **Equation:**

$$T = rac{w}{2 \sin{(5.0^{
m o})}} = rac{
m mg}{2 \sin{(5.0^{
m o})}},$$

so that

## **Equation:**

$$T = rac{(70.0 ext{ kg})(9.80 ext{ m/s}^2)}{2(0.0872)},$$

and the tension is

# **Equation:**

$$T = 3900 \text{ N}.$$

### **Discussion**

Note that the vertical tension in the wire acts as a normal force that supports the weight of the tightrope walker. The tension is almost six times the 686-N weight of the tightrope walker. Since the wire is nearly horizontal, the vertical component of its tension is only a small fraction of the tension in the wire. The large horizontal components are in opposite directions and cancel, and so most of the tension in the wire is not used to support the weight of the tightrope walker.

If we wish to *create* a very large tension, all we have to do is exert a force perpendicular to a flexible connector, as illustrated in [link]. As we saw in the last example, the weight of the tightrope walker acted as a force perpendicular to the rope. We saw that the tension in the roped related to the weight of the tightrope walker in the following way:

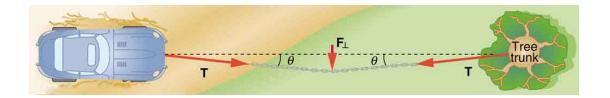
## **Equation:**

$$T=rac{w}{2\sin{( heta)}}.$$

We can extend this expression to describe the tension T created when a perpendicular force ( $\mathbf{F}_{\perp}$ ) is exerted at the middle of a flexible connector: **Equation:** 

$$T=rac{F_{\perp}}{2\sin{( heta)}}.$$

Note that  $\theta$  is the angle between the horizontal and the bent connector. In this case, T becomes very large as  $\theta$  approaches zero. Even the relatively small weight of any flexible connector will cause it to sag, since an infinite tension would result if it were horizontal (i.e.,  $\theta=0$  and  $\sin\theta=0$ ). (See  $\lceil \ln k \rceil$ .)



We can create a very large tension in the chain by pushing on it perpendicular to its length, as shown. Suppose we wish to pull a car out of the mud when no tow truck is available. Each time the car moves forward, the chain is tightened to keep it as nearly straight as possible. The tension in the chain is given by  $T = \frac{F_\perp}{2\sin{(\theta)}}$ ; since  $\theta$  is small, T is very large. This situation is analogous to the tightrope walker shown in [link], except that the tensions shown here are those transmitted to the car and the tree rather than those acting at the point where  $\mathbf{F}_\perp$  is applied.



Unless an infinite tension is exerted, any flexible connector—such as the chain at the bottom of the picture—will sag under its own weight, giving a characteristic curve when the weight is evenly

distributed along the length.
Suspension bridges—such as the
Golden Gate Bridge shown in this
image—are essentially very heavy
flexible connectors. The weight of
the bridge is evenly distributed
along the length of flexible
connectors, usually cables, which
take on the characteristic shape.
(credit: Leaflet, Wikimedia
Commons)

## **Extended Topic: Real Forces and Inertial Frames**

There is another distinction among forces in addition to the types already mentioned. Some forces are real, whereas others are not. *Real forces* are those that have some physical origin, such as the gravitational pull. Contrastingly, *fictitious forces* are those that arise simply because an observer is in an accelerating frame of reference, such as one that rotates (like a merry-go-round) or undergoes linear acceleration (like a car slowing down). For example, if a satellite is heading due north above Earth's northern hemisphere, then to an observer on Earth it will appear to experience a force to the west that has no physical origin. Of course, what is happening here is that Earth is rotating toward the east and moves east under the satellite. In Earth's frame this looks like a westward force on the satellite, or it can be interpreted as a violation of Newton's first law (the law of inertia). An **inertial frame of reference** is one in which all forces are real and, equivalently, one in which Newton's laws have the simple forms given in this chapter.

Earth's rotation is slow enough that Earth is nearly an inertial frame. You ordinarily must perform precise experiments to observe fictitious forces and the slight departures from Newton's laws, such as the effect just described. On the large scale, such as for the rotation of weather systems and ocean currents, the effects can be easily observed.

The crucial factor in determining whether a frame of reference is inertial is whether it accelerates or rotates relative to a known inertial frame. Unless stated otherwise, all phenomena discussed in this text are considered in inertial frames.

All the forces discussed in this section are real forces, but there are a number of other real forces, such as lift and thrust, that are not discussed in this section. They are more specialized, and it is not necessary to discuss every type of force. It is natural, however, to ask where the basic simplicity we seek to find in physics is in the long list of forces. Are some more basic than others? Are some different manifestations of the same underlying force? The answer to both questions is yes, as will be seen in the next (extended) section and in the treatment of modern physics later in the text.

#### Note:

## PhET Explorations: Forces in 1 Dimension

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. View a free-body diagram of all the forces (including gravitational and normal forces).

Forces in <u>1</u> Dimensio <u>n</u>

## **Section Summary**

• When objects rest on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts

perpendicular to and away from the surface. It is called a normal force, N.

• When objects rest on a non-accelerating horizontal surface, the magnitude of the normal force is equal to the weight of the object: **Equation:** 

$$N = mg.$$

• When objects rest on an inclined plane that makes an angle  $\theta$  with the horizontal surface, the weight of the object can be resolved into components that act perpendicular  $(\mathbf{w}_{\perp})$  and parallel  $(\mathbf{w}_{\parallel})$  to the surface of the plane. These components can be calculated using: **Equation:** 

$$w_{\parallel} = w \sin{(\theta)} = \operatorname{mg}{\sin{(\theta)}}$$

## **Equation:**

$$w_{\perp} = w \cos{(\theta)} = \operatorname{mg}{\cos{(\theta)}}.$$

• The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension, **T**. When a rope supports the weight of an object that is at rest, the tension in the rope is equal to the weight of the object:

## **Equation:**

$$T=\mathrm{mg}.$$

• In any inertial frame of reference (one that is not accelerated or rotated), Newton's laws have the simple forms given in this chapter and all forces are real forces having a physical origin.

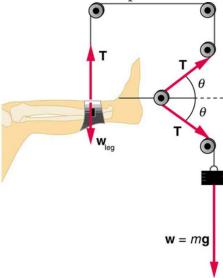
# **Conceptual Questions**

#### **Exercise:**

### **Problem:**

If a leg is suspended by a traction setup as shown in [link], what is the

tension in the rope?



A leg is suspended by a traction system in which wires are used to transmit forces. Frictionless pulleys change the direction of the force *T* without changing its magnitude.

### **Exercise:**

### **Problem:**

In a traction setup for a broken bone, with pulleys and rope available, how might we be able to increase the force along the tibia using the same weight? (See [link].) (Note that the tibia is the shin bone shown in this image.)

## **Problem Exercises**

### **Exercise:**

### **Problem:**

Two teams of nine members each engage in a tug of war. Each of the first team's members has an average mass of 68 kg and exerts an average force of 1350 N horizontally. Each of the second team's members has an average mass of 73 kg and exerts an average force of 1365 N horizontally. (a) What is magnitude of the acceleration of the two teams? (b) What is the tension in the section of rope between the teams?

### **Solution:**

a. 
$$0.11 \text{ m/s}^2$$
  
b.  $1.2 \times 10^4 \text{ N}$ 

### **Exercise:**

#### **Problem:**

What force does a trampoline have to apply to a 45.0-kg gymnast to accelerate her straight up at  $7.50~\mathrm{m/s}^2$ ? Note that the answer is independent of the velocity of the gymnast—she can be moving either up or down, or be stationary.

#### **Exercise:**

### **Problem:**

(a) Calculate the tension in a vertical strand of spider web if a spider of mass  $8.00 \times 10^{-5}$  kg hangs motionless on it. (b) Calculate the tension in a horizontal strand of spider web if the same spider sits motionless in the middle of it much like the tightrope walker in [link]. The strand sags at an angle of  $12^{\circ}$  below the horizontal. Compare this with the tension in the vertical strand (find their ratio).

## **Solution:**

- (a)  $7.84 \times 10^{-4} \text{ N}$
- (b)  $1.89\times 10^{-3}\ N$  . This is 2.41 times the tension in the vertical strand.

## **Exercise:**

## **Problem:**

Suppose a 60.0-kg gymnast climbs a rope. (a) What is the tension in the rope if he climbs at a constant speed? (b) What is the tension in the rope if he accelerates upward at a rate of  $1.50 \, \mathrm{m/s}^2$ ?

## **Exercise:**

## **Problem:**

Show that, as stated in the text, a force  $\mathbf{F}_{\perp}$  exerted on a flexible medium at its center and perpendicular to its length (such as on the tightrope wire in  $[\underline{\operatorname{link}}]$ ) gives rise to a tension of magnitude  $T = \frac{F_{\perp}}{2\sin{(\theta)}}$ .

## **Solution:**

Newton's second law applied in vertical direction gives

# **Equation:**

$$F_{y} = F - 2T \sin \theta = 0$$

# **Equation:**

$$F = 2T \sin \theta$$

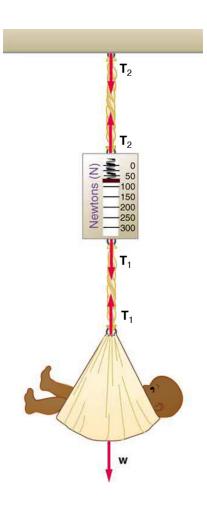
# **Equation:**

$$T = rac{F}{2\sin heta}.$$

## **Exercise:**

## **Problem:**

Consider the baby being weighed in [link]. (a) What is the mass of the child and basket if a scale reading of 55 N is observed? (b) What is the tension  $T_1$  in the cord attaching the baby to the scale? (c) What is the tension  $T_2$  in the cord attaching the scale to the ceiling, if the scale has a mass of 0.500 kg? (d) Draw a sketch of the situation indicating the system of interest used to solve each part. The masses of the cords are negligible.



A baby is weighed using a spring scale.

# Glossary

### inertial frame of reference

a coordinate system that is not accelerating; all forces acting in an inertial frame of reference are real forces, as opposed to fictitious forces that are observed due to an accelerating frame of reference

## normal force

the force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests

### tension

the pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a tension force

### Friction

- Discuss the general characteristics of friction.
- Describe the various types of friction.
- Calculate the magnitude of static and kinetic friction.

**Friction** is a force that is around us all the time that opposes relative motion between systems in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behavior of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

## Note:

### Friction

Friction is a force that opposes relative motion between systems in contact.

One of the simpler characteristics of friction is that it is parallel to the contact surface between systems and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called **kinetic friction**. For example, friction slows a hockey puck sliding on ice. But when objects are stationary, **static friction** can act between them; the static friction is usually greater than the kinetic friction between the objects.

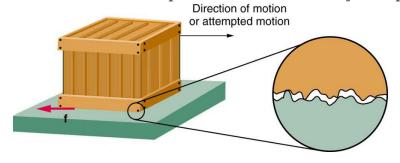
### Note:

## Kinetic Friction

If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

[link] is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explain the dependence of friction on the nature of the substances. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.



Frictional forces, such as f, always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of

the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the  $\boldsymbol{magnitude}$  of static  $\boldsymbol{friction}$   $\boldsymbol{f}_{\mathrm{s}}$  is

# **Equation:**

$$f_{
m s} \leq \mu_{
m s} N,$$

where  $\mu_s$  is the coefficient of static friction and N is the magnitude of the normal force (the force perpendicular to the surface).

#### Note:

Magnitude of Static Friction Magnitude of static friction  $f_{\rm s}$  is

# **Equation:**

$$f_{
m s} \leq \mu_{
m s} N,$$

where  $\mu_s$  is the coefficient of static friction and N is the magnitude of the normal force.

The symbol  $\leq$  means *less than or equal to*, implying that static friction can have a minimum and a maximum value of  $\mu_s N$ . Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds  $f_{s(max)}$ , the object will move. Thus

# **Equation:**

$$f_{
m s(max)} = \mu_{
m s} N.$$

Once an object is moving, the **magnitude of kinetic friction f\_k** is given by **Equation:** 

$$f_{
m k}=\mu_{
m k}N,$$

where  $\mu_k$  is the coefficient of kinetic friction. A system in which  $f_k = \mu_k N$  is described as a system in which *friction behaves simply*.

## Note:

Magnitude of Kinetic Friction

The magnitude of kinetic friction  $f_{
m k}$  is given by

# **Equation:**

$$f_{
m k}=\mu_{
m k}N,$$

where  $\mu_k$  is the coefficient of kinetic friction.

As seen in [link], the coefficients of kinetic friction are less than their static counterparts. That values of  $\mu$  in [link] are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

	Static friction	Kinetic friction
System	$\mu_{ m s}$	$\mu_{ m k}$
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7

	Static friction	Kinetic friction
System	$\mu_{ m s}$	$\mu_{ m k}$
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.04	0.02

## Coefficients of Static and Kinetic Friction

The equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight,  $W = mg = (100 \text{ kg})(9.80 \text{ m/s}^2) = 980 \text{ N}$ , perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than

 $f_{\rm s(max)} = \mu_{\rm s} N = (0.45)(980~{\rm N}) = 440~{\rm N}$  to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N ( $f_{\rm k} = \mu_{\rm k} N = (0.30)(980~{\rm N}) = 290~{\rm N}$ ) would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unit less quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

#### Note:

# Take-Home Experiment

Find a small plastic object (such as a food container) and slide it on a kitchen table by giving it a gentle tap. Now spray water on the table,

simulating a light shower of rain. What happens now when you give the object the same-sized tap? Now add a few drops of (vegetable or olive) oil on the surface of the water and give the same tap. What happens now? This latter situation is particularly important for drivers to note, especially after a light rain shower. Why?

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint ([link]). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.





Artificial knee
replacement is a
procedure that has been
performed for more than
20 years. In this figure,
we see the post-op x rays
of the right knee joint
replacement. (credit:
Mike Baird, Flickr)

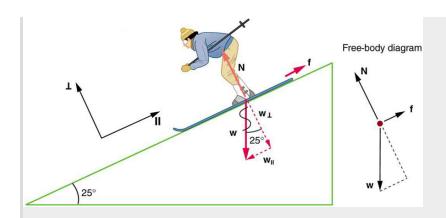
Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Artificial lubricants are also common in hospitals and doctor's clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to to lubricate the surface between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to move freely over the skin.

# **Example:**

# **Skiing Exercise**

A skier with a mass of 62 kg is sliding down a snowy slope. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N. **Strategy** 

The magnitude of kinetic friction was given in to be 45.0 N. Kinetic friction is related to the normal force N as  $f_k = \mu_k N$ ; thus, the coefficient of kinetic friction can be found if we can find the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope. (See the skier and free-body diagram in [link].)



The motion of the skier and friction are parallel to the slope and so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier).  $\mathbf{N}$  (the normal force) is perpendicular to the slope, and  $\mathbf{f}$  (the friction) is parallel to the slope, but  $\mathbf{w}$  (the skier's weight) has components along both axes, namely  $\mathbf{w}_{\perp}$  and  $\mathbf{W}_{//}$ .  $\mathbf{N}$  is equal in magnitude to  $\mathbf{w}_{\perp}$ , so there is no motion perpendicular to the slope. However,  $\mathbf{f}$  is less than  $\mathbf{W}_{//}$  in magnitude, so there is acceleration down the slope (along the x-axis).

That is,

# **Equation:**

$$N=w_{\perp}=w\cos25^{
m o}=mg\cos25^{
m o}.$$

Substituting this into our expression for kinetic friction, we get **Equation:** 

$$f_{
m k}=\mu_{
m k}{
m mg}\cos25^{
m o},$$

which can now be solved for the coefficient of kinetic friction  $\mu_k$ .

### Solution

Solving for  $\mu_k$  gives

## **Equation:**

$$\mu_{\mathrm{k}} = rac{f_{\mathrm{k}}}{N} = rac{f_{\mathrm{k}}}{w\cos25^{\mathrm{o}}} = rac{f_{\mathrm{k}}}{\mathrm{mg}\cos25^{\mathrm{o}}}.$$

Substituting known values on the right-hand side of the equation,

# **Equation:**

$$\mu_{
m k} = rac{45.0 \ {
m N}}{(62 \ {
m kg})(9.80 \ {
m m/s}^2)(0.906)} = 0.082.$$

## Discussion

This result is a little smaller than the coefficient listed in [link] for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle  $\theta$  with the horizontal, friction is given by  $f_k = \mu_k \operatorname{mg} \cos \theta$ . All objects will slide down a slope with constant acceleration under these circumstances. Proof of this is left for this chapter's Problems and Exercises.

### Note:

# Take-Home Experiment

An object will slide down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in [link], the kinetic friction on a slope  $f_k = \mu_k \text{mg cos } \theta$ . The component of the weight down the slope is equal to  $\text{mg sin } \theta$  (see the free-body diagram in [link]). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out:

# **Equation:**

$$f_{
m k}={
m Fg}_{
m x}$$

# **Equation:**

$$\mu_{\rm k}$$
mg cos  $\theta$  = mg sin  $\theta$ .

Solving for  $\mu_k$ , we find that

# **Equation:**

$$\mu_{
m k} = rac{{
m mg} \sin heta}{{
m mg} \cos heta} = an heta.$$

Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find  $\mu_k$ . Note that the coin will not start to slide at all until an angle greater than  $\theta$  is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Discuss how this may affect the value for  $\mu_k$  and its uncertainty.

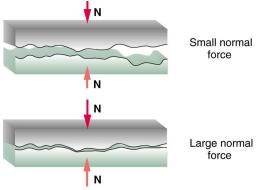
We have discussed that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force.

### Note:

# Making Connections: Submicroscopic Explanations of Friction

The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) to heat.

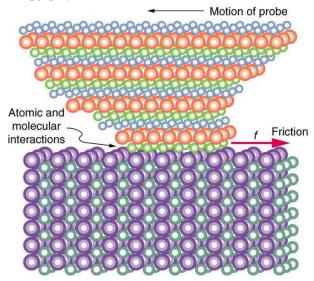
[link] illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area since only high spots touch. When a greater normal force is exerted, the actual contact area increases, and it is found that the friction is proportional to this area.



Two rough surfaces in contact have a much smaller area of actual contact than their total area. When there is a greater normal force as a result of a greater applied force, the area of actual contact increases as does friction.

But the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur

between atoms and molecules on the surfaces. [link] shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which will be discussed later in this chapter. The variation in shear stress is remarkable (more than a factor of  $10^{12}$ ) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.



The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.

## Note:

PhET Explorations: Forces and Motion

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. Draw a free-body diagram of all the forces (including gravitational and normal forces).

Forces
and
Motio
n

# **Section Summary**

• Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force N pushing the systems together. (A normal force is always perpendicular to the contact surface between systems.) Friction depends on both of the materials involved. The magnitude of static friction  $f_{\rm s}$  between systems stationary relative to one another is given by

# **Equation:**

$$f_{ ext{ iny S}} \leq \mu_{ ext{ iny S}} N,$$

where  $\mu_s$  is the coefficient of static friction, which depends on both of the materials.

• The kinetic friction force  $f_{\rm k}$  between systems moving relative to one another is given by

# **Equation:**

$$f_{
m k}=\mu_{
m k}N,$$

where  $\mu_k$  is the coefficient of kinetic friction, which also depends on both materials.

# **Conceptual Questions**

### **Exercise:**

### **Problem:**

Define normal force. What is its relationship to friction when friction behaves simply?

### **Exercise:**

#### **Problem:**

The glue on a piece of tape can exert forces. Can these forces be a type of simple friction? Explain, considering especially that tape can stick to vertical walls and even to ceilings.

#### Exercise:

### **Problem:**

When you learn to drive, you discover that you need to let up slightly on the brake pedal as you come to a stop or the car will stop with a jerk. Explain this in terms of the relationship between static and kinetic friction.

### **Exercise:**

### **Problem:**

When you push a piece of chalk across a chalkboard, it sometimes screeches because it rapidly alternates between slipping and sticking to the board. Describe this process in more detail, in particular explaining how it is related to the fact that kinetic friction is less than static friction. (The same slip-grab process occurs when tires screech on pavement.)

## **Problems & Exercises**

## **Exercise:**

## **Problem:**

A physics major is cooking breakfast when he notices that the frictional force between his steel spatula and his Teflon frying pan is only 0.200 N. Knowing the coefficient of kinetic friction between the two materials, he quickly calculates the normal force. What is it?

## **Solution:**

5.00 N

## **Exercise:**

### **Problem:**

(a) When rebuilding her car's engine, a physics major must exert 300 N of force to insert a dry steel piston into a steel cylinder. What is the magnitude of the normal force between the piston and cylinder? (b) What is the magnitude of the force would she have to exert if the steel parts were oiled?

### **Exercise:**

## **Problem:**

(a) What is the maximum frictional force in the knee joint of a person who supports 66.0 kg of her mass on that knee? (b) During strenuous exercise it is possible to exert forces to the joints that are easily ten times greater than the weight being supported. What is the maximum force of friction under such conditions? The frictional forces in joints are relatively small in all circumstances except when the joints deteriorate, such as from injury or arthritis. Increased frictional forces can cause further damage and pain.

#### **Exercise:**

## **Problem:**

Suppose you have a 120-kg wooden crate resting on a wood floor. (a) What maximum force can you exert horizontally on the crate without moving it? (b) If you continue to exert this force once the crate starts to slip, what will the magnitude of its acceleration then be?

## **Solution:**

- (a) 588 N
- (b)  $1.96 \text{ m/s}^2$

## **Exercise:**

### **Problem:**

(a) If half of the weight of a small  $1.00 \times 10^3$  kg utility truck is supported by its two drive wheels, what is the magnitude of the maximum acceleration it can achieve on dry concrete? (b) Will a metal cabinet lying on the wooden bed of the truck slip if it accelerates at this rate? (c) Solve both problems assuming the truck has four-wheel drive.

#### **Exercise:**

### **Problem:**

A team of eight dogs pulls a sled with waxed wood runners on wet snow (mush!). The dogs have average masses of 19.0 kg, and the loaded sled with its rider has a mass of 210 kg. (a) Calculate the magnitude of the acceleration starting from rest if each dog exerts an average force of 185 N backward on the snow. (b) What is the magnitude of the acceleration once the sled starts to move? (c) For both situations, calculate the magnitude of the force in the coupling between the dogs and the sled.

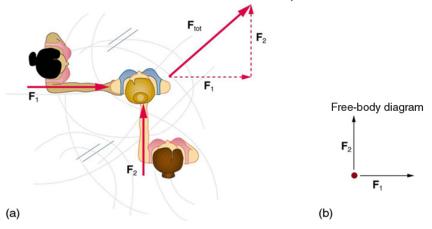
## **Solution:**

- (a)  $3.29 \text{ m/s}^2$
- (b)  $3.52 \text{ m/s}^2$
- (c) 980 N; 945 N

## **Exercise:**

## **Problem:**

Consider the 65.0-kg ice skater being pushed by two others shown in  $[\underline{link}]$ . (a) Find the direction and magnitude of  $\mathbf{F}_{tot}$ , the total force exerted on her by the others, given that the magnitudes  $F_1$  and  $F_2$  are 26.4 N and 18.6 N, respectively. (b) What is her initial acceleration if she is initially stationary and wearing steel-bladed skates that point in the direction of  $\mathbf{F}_{tot}$ ? (c) What is her acceleration assuming she is already moving in the direction of  $\mathbf{F}_{tot}$ ? (Remember that friction always acts in the direction opposite that of motion or attempted motion between surfaces in contact.)



### **Exercise:**

## **Problem:**

Show that the acceleration of any object down a frictionless incline that makes an angle  $\theta$  with the horizontal is  $a = g \sin \theta$ . (Note that this acceleration is independent of mass.)

### **Exercise:**

## **Problem:**

Show that the acceleration of any object down an incline where friction behaves simply (that is, where  $f_{\rm k}=\mu_{\rm k}N$ ) is  $a=g(\sin\theta-\mu_{\rm k}\cos\theta$ ). Note that the acceleration is independent of mass and reduces to the expression found in the previous problem when friction becomes negligibly small ( $\mu_{\rm k}=0$ ).

### **Exercise:**

## **Problem:**

Calculate the deceleration of a snow boarder going up a 5.0°, slope assuming the coefficient of friction for waxed wood on wet snow. The result of [link] may be useful, but be careful to consider the fact that the snow boarder is going uphill. Explicitly show how you follow the steps in Problem-Solving Strategies.

## **Solution:**

 $1.83 \mathrm{m/s}^2$ 

#### **Exercise:**

#### **Problem:**

(a) Calculate the acceleration of a skier heading down a 10.0° slope, assuming the coefficient of friction for waxed wood on wet snow. (b) Find the angle of the slope down which this skier could coast at a constant velocity. You can neglect air resistance in both parts, and you will find the result of [link] to be useful. Explicitly show how you follow the steps in the Problem-Solving Strategies.

### **Exercise:**

## **Problem:**

If an object is to rest on an incline without slipping, then friction must equal the component of the weight of the object parallel to the incline. This requires greater and greater friction for steeper slopes. Show that the maximum angle of an incline above the horizontal for which an object will not slide down is  $\theta = \tan^{-1} \mu_s$ . You may use the result of the previous problem. Assume that a=0 and that static friction has reached its maximum value.

### **Exercise:**

### **Problem:**

Calculate the maximum deceleration of a car that is heading down a  $6^{\circ}$  slope (one that makes an angle of  $6^{\circ}$  with the horizontal) under the following road conditions. You may assume that the weight of the car is evenly distributed on all four tires and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the deceleration. (Ignore rolling.) Calculate for a car: (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that  $\mu_{\rm s}=0.100$ , the same as for shoes on ice.

#### **Exercise:**

#### **Problem:**

Calculate the maximum acceleration of a car that is heading up a  $4^{\circ}$  slope (one that makes an angle of  $4^{\circ}$  with the horizontal) under the following road conditions. Assume that only half the weight of the car is supported by the two drive wheels and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the acceleration. (Ignore rolling.) (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that  $\mu_s=0.100$ , the same as for shoes on ice.

### **Solution:**

(a) 
$$4.20 \text{ m/s}^2$$

(b)  $2.74 \text{ m/s}^2$ 

(c)  $-0.195 \text{ m/s}^2$ 

## **Exercise:**

**Problem:** Repeat [link] for a car with four-wheel drive.

**Exercise:** 

### **Problem:**

A freight train consists of two  $8.00 \times 10^5$ -kg engines and 45 cars with average masses of  $5.50 \times 10^5$  kg. (a) What force must each engine exert backward on the track to accelerate the train at a rate of  $5.00 \times 10^{-2}$  m/s $^2$  if the force of friction is  $7.50 \times 10^5$  N, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the magnitude of the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

## **Solution:**

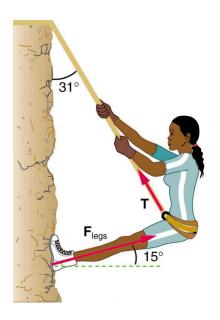
(a) 
$$1.03 \times 10^6 \text{ N}$$

(b) 
$$3.48 \times 10^5 \text{ N}$$

### **Exercise:**

## **Problem:**

Consider the 52.0-kg mountain climber in [link]. (a) Find the tension in the rope and the force that the mountain climber must exert with her feet on the vertical rock face to remain stationary. Assume that the force is exerted parallel to her legs. Also, assume negligible force exerted by her arms. (b) What is the minimum coefficient of friction between her shoes and the cliff?



Part of the climber's weight is supported by her rope and part by friction between her feet and the rock face.

## **Exercise:**

## **Problem:**

A contestant in a winter sporting event pushes a 45.0-kg block of ice across a frozen lake as shown in  $[\underline{link}]$ (a). (a) Calculate the minimum force F he must exert to get the block moving. (b) What is the magnitude of its acceleration once it starts to move, if that force is maintained?

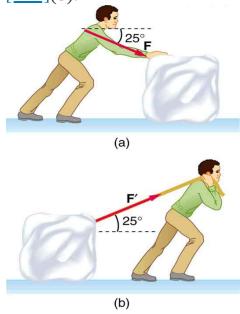
## **Solution:**

- (a) 51.0 N
- (b)  $0.720 \mathrm{\ m/s}^2$

## **Exercise:**

## **Problem:**

Repeat [link] with the contestant pulling the block of ice with a rope over his shoulder at the same angle above the horizontal as shown in [link](b).



Which method of sliding a block of ice requires less force—(a) pushing or (b) pulling at the same angle above the horizontal?

# Glossary

## friction

a force that opposes relative motion or attempts at motion between systems in contact

## kinetic friction

a force that opposes the motion of two systems that are in contact and moving relative to one another

## static friction

a force that opposes the motion of two systems that are in contact and are not moving relative to one another

# magnitude of static friction

 $f_{\rm s} \leq \mu_{\rm s} \ N$ , where  $\mu_{\rm s}$  is the coefficient of static friction and N is the magnitude of the normal force

## magnitude of kinetic friction

 $f_{
m k}=\mu_{
m k}N$  , where  $\mu_{
m k}$  is the coefficient of kinetic friction

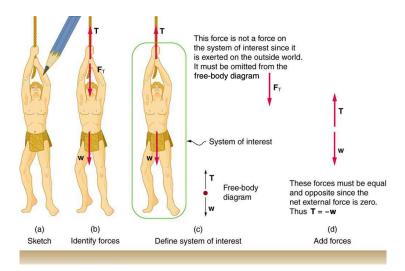
#### **Problem-Solving Strategies**

• Understand and apply a problem-solving procedure to solve problems using Newton's laws of motion.

Success in problem solving is obviously necessary to understand and apply physical principles, not to mention the more immediate need of passing exams. The basics of problem solving, presented earlier in this text, are followed here, but specific strategies useful in applying Newton's laws of motion are emphasized. These techniques also reinforce concepts that are useful in many other areas of physics. Many problem-solving strategies are stated outright in the worked examples, and so the following techniques should reinforce skills you have already begun to develop.

#### Problem-Solving Strategy for Newton's Laws of Motion

Step 1. As usual, it is first necessary to identify the physical principles involved. *Once it is determined that Newton's laws of motion are involved (if the problem involves forces), it is particularly important to draw a careful sketch of the situation*. Such a sketch is shown in [link](a). Then, as in [link](b), use arrows to represent all forces, label them carefully, and make their lengths and directions correspond to the forces they represent (whenever sufficient information exists).



(a) A sketch of Tarzan hanging from a vine. (b) Arrows are used to represent all forces.  ${\bf T}$  is the tension in the vine above Tarzan,  ${\bf F}_T$  is the force he exerts on the vine, and  ${\bf w}$  is his weight. All other forces, such as the nudge of a breeze, are assumed negligible. (c) Suppose we are given the ape man's mass and asked to find the tension in the vine. We then define the system of interest as shown and draw a free-body diagram.  ${\bf F}_T$  is no longer shown, because it is not a force acting on the system of interest; rather,  ${\bf F}_T$  acts on the outside world. (d) Showing only the arrows, the head-to-tail method of addition is used. It is apparent that  ${\bf T}=-{\bf w}$ , if Tarzan is stationary.

Step 2. Identify what needs to be determined and what is known or can be inferred from the problem as stated. That is, make a list of knowns and unknowns. *Then carefully determine the system of interest*. This decision is a crucial step, since Newton's second law involves only external forces. Once the system of interest has been identified, it becomes possible to determine which forces are external and which are internal, a necessary step to

employ Newton's second law. (See [link](c).) Newton's third law may be used to identify whether forces are exerted between components of a system (internal) or between the system and something outside (external). As illustrated earlier in this chapter, the system of interest depends on what question we need to answer. This choice becomes easier with practice, eventually developing into an almost unconscious process. Skill in clearly defining systems will be beneficial in later chapters as well.

A diagram showing the system of interest and all of the external forces is called a **free-body diagram**. Only forces are shown on free-body diagrams, not acceleration or velocity. We have drawn several of these in worked examples. [link](c) shows a free-body diagram for the system of interest. Note that no internal forces are shown in a free-body diagram.

Step 3. Once a free-body diagram is drawn, *Newton's second law can be applied to solve the problem*. This is done in [link](d) for a particular situation. In general, once external forces are clearly identified in free-body diagrams, it should be a straightforward task to put them into equation form and solve for the unknown, as done in all previous examples. If the problem is one-dimensional—that is, if all forces are parallel—then they add like scalars. If the problem is two-dimensional, then it must be broken down into a pair of one-dimensional problems. This is done by projecting the force vectors onto a set of axes chosen for convenience. As seen in previous examples, the choice of axes can simplify the problem. For example, when an incline is involved, a set of axes with one axis parallel to the incline and one perpendicular to it is most convenient. It is almost always convenient to make one axis parallel to the direction of motion, if this is known.

#### Note:

#### Applying Newton's Second Law

Before you write net force equations, it is critical to determine whether the system is accelerating in a particular direction. If the acceleration is zero in a particular direction, then the net force is zero in that direction. Similarly, if the acceleration is nonzero in a particular direction, then the net force is described by the equation:  $F_{\rm net} = {\rm ma.}$  For example, if the system is accelerating in the horizontal direction, but it is not accelerating in the vertical direction, then you will have the following conclusions:

#### **Equation:**

$$F_{\text{net }x} = \text{ma},$$

#### **Equation:**

$$F_{\text{net } y} = 0.$$

You will need this information in order to determine unknown forces acting in a system.

Step 4. As always, *check the solution to see whether it is reasonable*. In some cases, this is obvious. For example, it is reasonable to find that friction causes an object to slide down an incline more slowly than when no friction exists. In practice, intuition develops gradually through problem solving, and with experience it becomes progressively easier to judge whether an answer is reasonable. Another way to check your solution is to check the units. If you are solving for force and end up with units of m/s, then you have made a mistake.

#### **Section Summary**

- To solve problems involving Newton's laws of motion, follow the procedure described:
  - 1. Draw a sketch of the problem.
  - 2. Identify known and unknown quantities, and identify the system of interest. Draw a free-body diagram, which is a sketch showing all of the forces acting on an object. The object is represented by a dot, and the forces are represented by vectors extending in different directions from the dot. If vectors act in

directions that are not horizontal or vertical, resolve the vectors into horizontal and vertical components and draw them on the free-body diagram.

- 3. Write Newton's second law in the horizontal and vertical directions and add the forces acting on the object. If the object does not accelerate in a particular direction (for example, the x-direction) then  $F_{\text{net }x}=0$ . If the object does accelerate in that direction,  $F_{\text{net }x}=\text{ma}$ .
- 4. Check your answer. Is the answer reasonable? Are the units correct?

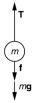
#### **Problem Exercises**

#### **Exercise:**

#### **Problem:**

 $A~5.00 \times 10^5$ -kg rocket is accelerating straight up. Its engines produce  $1.250 \times 10^7~N$  of thrust, and air resistance is  $4.50 \times 10^6~N$ . What is the rocket's acceleration? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

#### **Solution:**



Using the free-body diagram:

$$F_{\text{net}} = T - f - mg = \text{ma},$$

so that

$$a = \frac{{T - f - {\rm{mg}}}}{m} = \frac{{1.250 \times 10^7 \; {\rm{N}} - 4.50 \times 10^6 \; N - (5.00 \times 10^5 \; {\rm{kg}})(9.80 \; {\rm{m/s}^2})}}{{5.00 \times 10^5 \; {\rm{kg}}}} = 6.20 \; {\rm{m/s}^2}.$$

#### **Exercise:**

#### **Problem:**

The wheels of a midsize car exert a force of 2100 N backward on the road to accelerate the car in the forward direction. If the force of friction including air resistance is 250 N and the acceleration of the car is  $1.80~\mathrm{m/s}^2$ , what is the mass of the car plus its occupants? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. For this situation, draw a free-body diagram and write the net force equation.

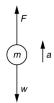
#### Exercise:

#### **Problem:**

Calculate the force a 70.0-kg high jumper must exert on the ground to produce an upward acceleration 4.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

#### **Solution:**

Use Newton's laws of motion.



Given : 
$$a=4.00g=(4.00)(9.80 \text{ m/s}^2)=39.2 \text{ m/s}^2; m=70.0 \text{ kg,}$$
 Find:  $F$ . 
$$\sum F=+F-w=\text{ma,so} \quad F=\text{ma}+w=\text{ma}+\text{mg}=m(a+g).$$
 that 
$$F=(70.0 \text{ kg})[(39.2 \text{ m/s}^2)+(9.80 \text{ m/s}^2)+($$

This result is reasonable, since it is quite possible for a person to exert a force of the magnitude of  $10^3$  N.

#### **Exercise:**

#### **Problem:**

When landing after a spectacular somersault, a 40.0-kg gymnast decelerates by pushing straight down on the mat. Calculate the force she must exert if her deceleration is 7.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

#### Exercise:

#### **Problem:**

A freight train consists of two  $8.00 \times 10^4$ -kg engines and 45 cars with average masses of  $5.50 \times 10^4$  kg . (a) What force must each engine exert backward on the track to accelerate the train at a rate of  $5.00 \times 10^{-2}$  m/s $^2$  if the force of friction is  $7.50 \times 10^5$  N, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

#### **Solution:**

- (a)  $4.41 \times 10^5 \text{ N}$
- (b)  $1.50 \times 10^5 \text{ N}$

#### **Exercise:**

#### **Problem:**

Commercial airplanes are sometimes pushed out of the passenger loading area by a tractor. (a) An 1800-kg tractor exerts a force of  $1.75 \times 10^4~\rm N$  backward on the pavement, and the system experiences forces resisting motion that total 2400 N. If the acceleration is  $0.150~\rm m/s^2$ , what is the mass of the airplane? (b) Calculate the force exerted by the tractor on the airplane, assuming 2200 N of the friction is experienced by the airplane. (c) Draw two sketches showing the systems of interest used to solve each part, including the free-body diagrams for each.

#### **Exercise:**

#### **Problem:**

A 1100-kg car pulls a boat on a trailer. (a) What total force resists the motion of the car, boat, and trailer, if the car exerts a 1900-N force on the road and produces an acceleration of  $0.550~\rm m/s^2$ ? The mass of the boat plus trailer is 700 kg. (b) What is the force in the hitch between the car and the trailer if 80% of the resisting forces are experienced by the boat and trailer?

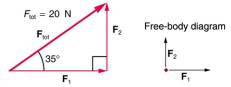
#### **Solution:**

- (a) 910 N
- (b)  $1.11 \times 10^3 \text{ N}$

#### **Exercise:**

#### **Problem:**

(a) Find the magnitudes of the forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  that add to give the total force  $\mathbf{F}_{tot}$  shown in [link]. This may be done either graphically or by using trigonometry. (b) Show graphically that the same total force is obtained independent of the order of addition of  $\mathbf{F}_1$  and  $\mathbf{F}_2$ . (c) Find the direction and magnitude of some other pair of vectors that add to give  $\mathbf{F}_{tot}$ . Draw these to scale on the same drawing used in part (b) or a similar picture.



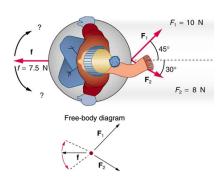
#### **Exercise:**

#### **Problem:**

Two children pull a third child on a snow saucer sled exerting forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  as shown from above in [link]. Find the acceleration of the 49.00-kg sled and child system. Note that the direction of the frictional force is unspecified; it will be in the opposite direction of the sum of  $\mathbf{F}_1$  and  $\mathbf{F}_2$ .

#### **Solution:**

 $a=0.139 \mathrm{\ m/s}, \, \theta=12.4^{\circ}$  north of east



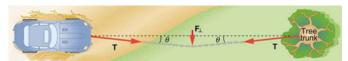
An overhead view of the horizontal forces acting on a

child's snow saucer sled.

#### **Exercise:**

#### **Problem:**

Suppose your car was mired deeply in the mud and you wanted to use the method illustrated in [link] to pull it out. (a) What force would you have to exert perpendicular to the center of the rope to produce a force of 12,000 N on the car if the angle is 2.00°? In this part, explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. (b) Real ropes stretch under such forces. What force would be exerted on the car if the angle increases to 7.00° and you still apply the force found in part (a) to its center?



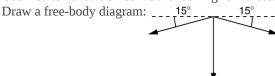
#### **Exercise:**

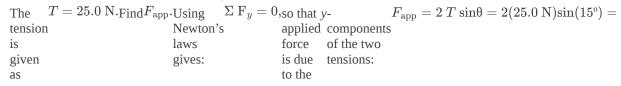
#### **Problem:**

What force is exerted on the tooth in [link] if the tension in the wire is 25.0 N? Note that the force applied to the tooth is smaller than the tension in the wire, but this is necessitated by practical considerations of how force can be applied in the mouth. Explicitly show how you follow steps in the Problem-Solving Strategy for Newton's laws of motion.

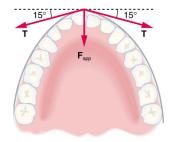
#### **Solution:**

Use Newton's laws since we are looking for forces.





This seems reasonable, since the applied tensions should be greater than the force applied to the tooth.



Braces are used to apply forces to teeth to realign them. Shown in this figure are the tensions applied by the wire to the protruding tooth. The total force applied to the tooth by the wire,  $\mathbf{F}_{app}$ , points straight toward the back of the mouth.

#### **Exercise:**

#### **Problem:**

[link] shows Superhero and Trusty Sidekick hanging motionless from a rope. Superhero's mass is 90.0 kg, while Trusty Sidekick's is 55.0 kg, and the mass of the rope is negligible. (a) Draw a free-body diagram of the situation showing all forces acting on Superhero, Trusty Sidekick, and the rope. (b) Find the tension in the rope above Superhero. (c) Find the tension in the rope between Superhero and Trusty Sidekick. Indicate on your free-body diagram the system of interest used to solve each part.



Superhero and Trusty Sidekick hang motionless on a rope as they try to figure out what to do next. Will the tension be the same everywher e in the rope?

#### **Exercise:**

#### **Problem:**

A nurse pushes a cart by exerting a force on the handle at a downward angle 35.0° below the horizontal. The loaded cart has a mass of 28.0 kg, and the force of friction is 60.0 N. (a) Draw a free-body diagram for the system of interest. (b) What force must the nurse exert to move at a constant velocity?

#### Exercise:

#### **Problem:**

**Construct Your Own Problem** Consider the tension in an elevator cable during the time the elevator starts from rest and accelerates its load upward to some cruising velocity. Taking the elevator and its load to be the system of interest, draw a free-body diagram. Then calculate the tension in the cable. Among the things to consider are the mass of the elevator and its load, the final velocity, and the time taken to reach that velocity.

#### **Exercise:**

#### **Problem:**

**Construct Your Own Problem** Consider two people pushing a toboggan with four children on it up a snow-covered slope. Construct a problem in which you calculate the acceleration of the toboggan and its load. Include a free-body diagram of the appropriate system of interest as the basis for your analysis. Show vector forces and their components and explain the choice of coordinates. Among the things to be considered are the forces exerted by those pushing, the angle of the slope, and the masses of the toboggan and children.

#### **Exercise:**

#### **Problem:**

**Unreasonable Results** (a) Repeat [link], but assume an acceleration of  $1.20 \text{ m/s}^2$  is produced. (b) What is unreasonable about the result? (c) Which premise is unreasonable, and why is it unreasonable?

#### **Exercise:**

#### **Problem:**

**Unreasonable Results** (a) What is the initial acceleration of a rocket that has a mass of  $1.50 \times 10^6$  kg at takeoff, the engines of which produce a thrust of  $2.00 \times 10^6$  N? Do not neglect gravity. (b) What is unreasonable about the result? (This result has been unintentionally achieved by several real rockets.) (c) Which premise is unreasonable, or which premises are inconsistent? (You may find it useful to compare this problem to the rocket problem earlier in this section.)

## Further Applications of Newton's Laws of Motion

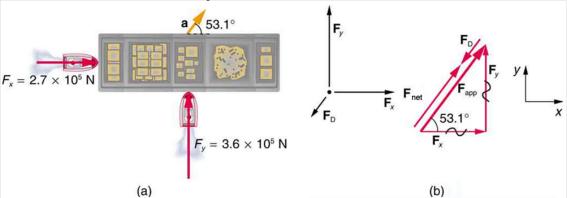
- Apply problem-solving techniques to solve for quantities in more complex systems of forces.
- Integrate concepts from kinematics to solve problems using Newton's laws of motion.

There are many interesting applications of Newton's laws of motion, a few more of which are presented in this section. These serve also to illustrate some further subtleties of physics and to help build problem-solving skills.

## **Example:**

## **Drag Force on a Barge**

Suppose two tugboats push on a barge at different angles, as shown in [link]. The first tugboat exerts a force of  $2.7 \times 10^5$  N in the *x*-direction, and the second tugboat exerts a force of  $3.6 \times 10^5$  N in the *y*-direction.



(a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Since the applied forces are perpendicular, the x- and y-axes are in the same direction as  $\mathbf{F}_x$  and  $\mathbf{F}_y$ . The problem quickly becomes a one-dimensional problem along the direction of  $\mathbf{F}_{app}$ , since friction is in the direction opposite to  $\mathbf{F}_{app}$ .

If the mass of the barge is  $5.0 \times 10^6$  kg and its acceleration is observed to be  $7.5 \times 10^{-2}~{\rm m/s}^2$  in the direction shown, what is the drag force of the water on the

barge resisting the motion? (Note: drag force is a frictional force exerted by fluids, such as air or water. The drag force opposes the motion of the object.)

#### **Strategy**

The directions and magnitudes of acceleration and the applied forces are given in  $[\underline{link}](a)$ . We will define the total force of the tugboats on the barge as  $\mathbf{F}_{app}$  so that:

# **Equation:**

$$\mathbf{F}_{\mathrm{app}} = \mathbf{F}_x + \mathbf{F}_y$$

Since the barge is flat bottomed, the drag of the water  $\mathbf{F}_D$  will be in the direction opposite to  $\mathbf{F}_{app}$ , as shown in the free-body diagram in [link](b). The system of interest here is the barge, since the forces on it are given as well as its acceleration. Our strategy is to find the magnitude and direction of the net applied force  $\mathbf{F}_{app}$ , and then apply Newton's second law to solve for the drag force  $\mathbf{F}_D$ .

#### **Solution**

Since  $\mathbf{F}_x$  and  $\mathbf{F}_y$  are perpendicular, the magnitude and direction of  $\mathbf{F}_{app}$  are easily found. First, the resultant magnitude is given by the Pythagorean theorem:

#### **Equation:**

The angle is given by

## **Equation:**

$$egin{array}{lcl} heta &=& an^{-1}\Big(rac{F_y}{F_x}\Big) \ heta &=& an^{-1}\Big(rac{3.6 imes10^5~ ext{N}}{2.7 imes10^5~ ext{N}}\Big) = 53^{ ext{o}}, \end{array}$$

which we know, because of Newton's first law, is the same direction as the acceleration.  $\mathbf{F}_D$  is in the opposite direction of  $\mathbf{F}_{app}$ , since it acts to slow down the acceleration. Therefore, the net external force is in the same direction as  $\mathbf{F}_{app}$ , but its magnitude is slightly less than  $\mathbf{F}_{app}$ . The problem is now one-dimensional. From  $[\underline{link}](\mathbf{b})$ , we can see that

# **Equation:**

$$F_{
m net} = F_{
m app} - F_{
m D}$$
.

But Newton's second law states that

## **Equation:**

$$F_{
m net}={
m ma.}$$

Thus,

## **Equation:**

$$F_{\rm app} - F_{\rm D} = {
m ma.}$$

This can be solved for the magnitude of the drag force of the water  $F_{\rm D}$  in terms of known quantities:

# **Equation:**

$$F_{
m D} = F_{
m app} - {
m ma.}$$

Substituting known values gives

# **Equation:**

$${
m F_D} = (4.5 imes 10^5 \ {
m N}) - (5.0 imes 10^6 \ {
m kg}) (7.5 imes 10^{-2} \ {
m m/s}^2) = 7.5 imes 10^4 \ {
m N}.$$

The direction of  $\mathbf{F}_D$  has already been determined to be in the direction opposite to  $\mathbf{F}_{app}$ , or at an angle of 53° south of west.

#### Discussion

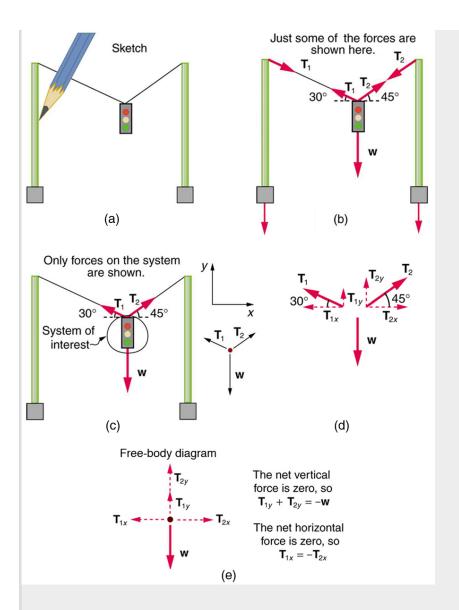
The numbers used in this example are reasonable for a moderately large barge. It is certainly difficult to obtain larger accelerations with tugboats, and small speeds are desirable to avoid running the barge into the docks. Drag is relatively small for a well-designed hull at low speeds, consistent with the answer to this example, where  $F_{\rm D}$  is less than 1/600th of the weight of the ship.

In the earlier example of a tightrope walker we noted that the tensions in wires supporting a mass were equal only because the angles on either side were equal. Consider the following example, where the angles are not equal; slightly more trigonometry is involved.

# **Example:**

# **Different Tensions at Different Angles**

Consider the traffic light (mass 15.0 kg) suspended from two wires as shown in [link]. Find the tension in each wire, neglecting the masses of the wires.



A traffic light is suspended from two wires. (b) Some of the forces involved. (c) Only forces acting on the system are shown here. The free-body diagram for the traffic light is also shown. (d) The forces projected onto vertical (*y*) and horizontal (*x*) axes. The horizontal components of the tensions must cancel, and the sum of the vertical components of the tensions must equal the weight of the traffic light. (e) The free-body diagram shows the vertical and horizontal forces acting on the traffic light.

# Strategy

The system of interest is the traffic light, and its free-body diagram is shown in [link] (c). The three forces involved are not parallel, and so they must be projected onto a coordinate system. The most convenient coordinate system has one axis vertical and one horizontal, and the vector projections on it are shown in part (d) of the figure. There are two unknowns in this problem ( $T_1$  and  $T_2$ ), so two equations are needed to find them. These two equations come from applying Newton's second law along the vertical and horizontal axes, noting that the net external force is zero along each axis because acceleration is zero.

#### Solution

First consider the horizontal or *x*-axis:

## **Equation:**

$$F_{
m net} x = T_{2x} - T_{1x} = 0.$$

Thus, as you might expect,

#### **Equation:**

$$T_{1x}=T_{2x}$$
.

This gives us the following relationship between  $T_1$  and  $T_2$ :

#### **Equation:**

$$T_1 \cos (30^{\circ}) = T_2 \cos (45^{\circ}).$$

Thus,

# **Equation:**

$$T_2 = (1.225)T_1.$$

Note that  $T_1$  and  $T_2$  are not equal in this case, because the angles on either side are not equal. It is reasonable that  $T_2$  ends up being greater than  $T_1$ , because it is exerted more vertically than  $T_1$ .

Now consider the force components along the vertical or *y*-axis:

## **Equation:**

$$F_{ ext{net }y} = T_{1y} + T_{2y} - w = 0.$$

This implies

## **Equation:**

$$T_{1y} + T_{2y} = w.$$

Substituting the expressions for the vertical components gives

# **Equation:**

$$T_1 \sin{(30^\circ)} + T_2 \sin{(45^\circ)} = w.$$

There are two unknowns in this equation, but substituting the expression for  $T_2$  in terms of  $T_1$  reduces this to one equation with one unknown:

## **Equation:**

$$T_1(0.500) + (1.225T_1)(0.707) = w = mg,$$

which yields

## **Equation:**

$$(1.366)T_1 = (15.0 \text{ kg})(9.80 \text{ m/s}^2).$$

Solving this last equation gives the magnitude of  $T_1$  to be

## **Equation:**

$$T_1 = 108 \text{ N}.$$

Finally, the magnitude of  $T_2$  is determined using the relationship between them,  $T_2$  = 1.225  $T_1$ , found above. Thus we obtain

## **Equation:**

$$T_2 = 132 \text{ N}.$$

### Discussion

Both tensions would be larger if both wires were more horizontal, and they will be equal if and only if the angles on either side are the same (as they were in the earlier example of a tightrope walker).

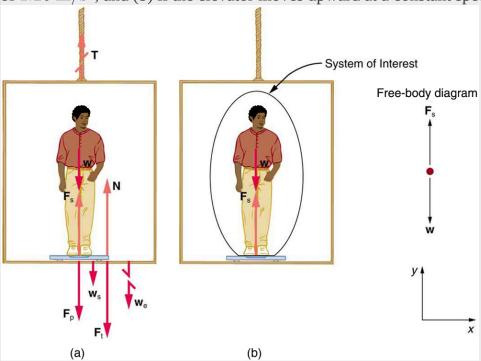
The bathroom scale is an excellent example of a normal force acting on a body. It provides a quantitative reading of how much it must push upward to support the weight of an object. But can you predict what you would see on the dial of a bathroom scale if you stood on it during an elevator ride? Will you see a value greater than your weight when the elevator starts up? What about when the elevator moves upward at a constant speed: will the scale still read more than your weight at rest? Consider the following example.

# **Example:**

#### What Does the Bathroom Scale Read in an Elevator?

[link] shows a 75.0-kg man (weight of about 165 lb) standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate

of  $1.20 \text{ m/s}^2$ , and (b) if the elevator moves upward at a constant speed of 1 m/s.



(a) The various forces acting when a person stands on a bathroom scale in an elevator. The arrows are approximately correct for when the elevator is accelerating upward—broken arrows represent forces too large to be drawn to scale.  $\mathbf{T}$  is the tension in the supporting cable,  $\mathbf{w}$  is the weight of the person,  $\mathbf{w}_s$  is the weight of the scale,  $\mathbf{w}_e$  is the weight of the elevator,  $\mathbf{F}_s$  is the force of the scale on the person,  $\mathbf{F}_p$  is the force of the person on the scale,  $\mathbf{F}_t$  is the force of the scale on the floor of the elevator, and  $\mathbf{N}$  is the force of the floor upward on the scale. (b) The free-body diagram shows only the external forces acting on the designated system of interest—the person.

# Strategy

If the scale is accurate, its reading will equal  $F_{\rm p}$ , the magnitude of the force the person exerts downward on it. [link](a) shows the numerous forces acting on the elevator, scale, and person. It makes this one-dimensional problem look much more formidable than if the person is chosen to be the system of interest and a free-body diagram is drawn as in [link](b). Analysis of the free-body diagram using Newton's laws can produce answers to both parts (a) and (b) of this example, as well as some other questions that might arise. The only forces acting on the person are his weight  $\mathbf{w}$  and the upward force of the scale  $\mathbf{F}_{\rm s}$ . According to Newton's third law  $\mathbf{F}_{\rm p}$  and  $\mathbf{F}_{\rm s}$  are

equal in magnitude and opposite in direction, so that we need to find  $F_{\rm s}$  in order to find what the scale reads. We can do this, as usual, by applying Newton's second law,

## **Equation:**

$$F_{
m net}={
m ma.}$$

From the free-body diagram we see that  $F_{
m net} = F_{
m s} - w$ , so that

# **Equation:**

$$F_{\rm s}-w={
m ma}.$$

Solving for  $F_s$  gives an equation with only one unknown:

## **Equation:**

$$F_{\rm s}={
m ma}+w,$$

or, because w = mg, simply

## **Equation:**

$$F_{\rm s}={
m ma+mg.}$$

No assumptions were made about the acceleration, and so this solution should be valid for a variety of accelerations in addition to the ones in this exercise.

# Solution for (a)

In this part of the problem,  $a = 1.20 \text{ m/s}^2$ , so that

# **Equation:**

$$F_{
m s} = (75.0~{
m kg})(1.20~{
m m/s^2}) + (75.0~{
m kg})(9.80~{
m m/s^2}),$$

yielding

# **Equation:**

$$F_{\rm s}=825~{
m N}.$$

# Discussion for (a)

This is about 185 lb. What would the scale have read if he were stationary? Since his acceleration would be zero, the force of the scale would be equal to his weight:

# **Equation:**

$$egin{array}{lcl} F_{
m net} &=& {
m ma} = 0 = F_{
m s} - w \ F_{
m s} &=& w = {
m mg} \ F_{
m s} &=& (75.0\ {
m kg})(9.80\ {
m m/s}^2) \ F_{
m s} &=& 735\ {
m N}. \end{array}$$

So, the scale reading in the elevator is greater than his 735-N (165 lb) weight. This means that the scale is pushing up on the person with a force greater than his weight, as it must in order to accelerate him upward. Clearly, the greater the acceleration of the elevator, the greater the scale reading, consistent with what you feel in rapidly accelerating versus slowly accelerating elevators.

## Solution for (b)

Now, what happens when the elevator reaches a constant upward velocity? Will the scale still read more than his weight? For any constant velocity—up, down, or stationary—acceleration is zero because  $a=\frac{\Delta v}{\Delta t}$ , and  $\Delta v=0$ .

Thus,

# **Equation:**

$$F_{\mathrm{s}} = \mathrm{ma} + \mathrm{mg} = 0 + \mathrm{mg}.$$

Now

## **Equation:**

$$F_{
m s} = (75.0~{
m kg})(9.80~{
m m/s}^2),$$

which gives

# **Equation:**

$$F_{\rm s} = 735 \; {
m N}.$$

# Discussion for (b)

The scale reading is 735 N, which equals the person's weight. This will be the case whenever the elevator has a constant velocity—moving up, moving down, or stationary.

The solution to the previous example also applies to an elevator accelerating downward, as mentioned. When an elevator accelerates downward, a is negative, and the scale reading is *less* than the weight of the person, until a constant downward velocity is reached, at which time the scale reading again becomes equal to the person's weight. If the elevator is in free-fall and accelerating downward at g, then the scale reading will be zero and the person will *appear* to be weightless.

# **Integrating Concepts: Newton's Laws of Motion and Kinematics**

Physics is most interesting and most powerful when applied to general situations that involve more than a narrow set of physical principles. Newton's laws of motion can also be integrated with other concepts that have been discussed previously in this text to

solve problems of motion. For example, forces produce accelerations, a topic of kinematics, and hence the relevance of earlier chapters. When approaching problems that involve various types of forces, acceleration, velocity, and/or position, use the following steps to approach the problem:

## **Problem-Solving Strategy**

Step 1. *Identify which physical principles are involved*. Listing the givens and the quantities to be calculated will allow you to identify the principles involved. Step 2. *Solve the problem using strategies outlined in the text*. If these are available for the specific topic, you should refer to them. You should also refer to the sections of the text that deal with a particular topic. The following worked example illustrates how these strategies are applied to an integrated concept problem.

## **Example:**

## What Force Must a Soccer Player Exert to Reach Top Speed?

A soccer player starts from rest and accelerates forward, reaching a velocity of 8.00 m/s in 2.50 s. (a) What was his average acceleration? (b) What average force did he exert backward on the ground to achieve this acceleration? The player's mass is 70.0 kg, and air resistance is negligible.

# Strategy

To	integrate	d, we must <i>accelera</i>	tionalong a <i>kinen</i>	natics. fo	orce, a	<i>dynamics</i> found
solv	econcept	first	straight	Part	topi	c in this
an	problem	identify	line.	(b)	of	chapter.
	_	the	This is	deals		
		physical	a topic	with		
		principles	of			
		involved				
		and				
		identify				
		the				
		chapters				
		in which				
		they are				
		found.				
		Part (a)				
		of this				
		example				
		considers				
The following solutions to each part of the example illustrate how the specific						

problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so forth.

# Solution for (a)

We are given the initial and final velocities (zero and 8.00 m/s forward); thus, the change in velocity is  $\Delta v = 8.00$  m/s. We are given the elapsed time, and so  $\Delta t = 2.50$  s. The unknown is acceleration, which can be found from its definition:

## **Equation:**

$$a = rac{\Delta v}{\Delta t}.$$

Substituting the known values yields

## **Equation:**

$$a = \frac{8.00 \text{ m/s}}{2.50 \text{ s}}$$
  
=  $3.20 \text{ m/s}^2$ .

#### Discussion for (a)

This is an attainable acceleration for an athlete in good condition.

## Solution for (b)

Here we are asked to find the average force the player exerts backward to achieve this forward acceleration. Neglecting air resistance, this would be equal in magnitude to the net external force on the player, since this force causes his acceleration. Since we now know the player's acceleration and are given his mass, we can use Newton's second law to find the force exerted. That is,

## **Equation:**

$$F_{
m net}={
m ma.}$$

Substituting the known values of m and a gives

# **Equation:**

$$F_{\text{net}} = (70.0 \text{ kg})(3.20 \text{ m/s}^2)$$
  
= 224 N.

# Discussion for (b)

This is about 50 pounds, a reasonable average force.

This worked example illustrates how to apply problem-solving strategies to situations that include topics from different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These strategies are found throughout the text, and many worked examples show how to use them for single topics. You will find these

techniques for integrated concept problems useful in applications of physics outside of a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

# **Summary**

- Newton's laws of motion can be applied in numerous situations to solve problems of motion
- Some problems will contain multiple force vectors acting in different directions on an object. Be sure to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram. Always analyze the direction in which an object accelerates so that you can determine whether  $F_{\rm net} = {\rm ma}$  or  $F_{\rm net} = 0$ .
- The normal force on an object is not always equal in magnitude to the weight of the object. If an object is accelerating, the normal force will be less than or greater than the weight of the object. Also, if the object is on an inclined plane, the normal force will always be less than the full weight of the object.
- Some problems will contain various physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics in order to solve these problems of motion.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

To simulate the apparent weightlessness of space orbit, astronauts are trained in the hold of a cargo aircraft that is accelerating downward at g. Why will they appear to be weightless, as measured by standing on a bathroom scale, in this accelerated frame of reference? Is there any difference between their apparent weightlessness in orbit and in the aircraft?

#### **Exercise:**

#### **Problem:**

A cartoon shows the toupee coming off the head of an elevator passenger when the elevator rapidly stops during an upward ride. Can this really happen without the person being tied to the floor of the elevator? Explain your answer.

## **Problem Exercises**

#### **Exercise:**

#### **Problem:**

A flea jumps by exerting a force of  $1.20\times10^{-5}~N$  straight down on the ground. A breeze blowing on the flea parallel to the ground exerts a force of  $0.500\times10^{-6}~N$  on the flea. Find the direction and magnitude of the acceleration of the flea if its mass is  $6.00\times10^{-7}~kg$ . Do not neglect the gravitational force.

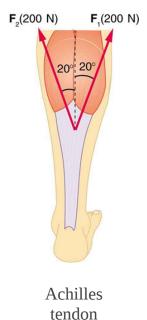
#### **Solution:**

 $10.2 \text{ m/s}^2$ ,  $4.67^{\circ}$  from vertical

#### **Exercise:**

#### **Problem:**

Two muscles in the back of the leg pull upward on the Achilles tendon, as shown in [link]. (These muscles are called the medial and lateral heads of the gastrocnemius muscle.) Find the magnitude and direction of the total force on the Achilles tendon. What type of movement could be caused by this force?

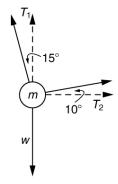


#### **Exercise:**

# **Problem:**

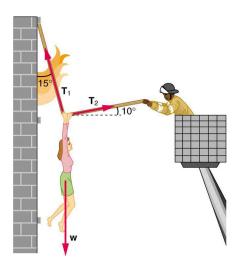
A 76.0-kg person is being pulled away from a burning building as shown in [link]. Calculate the tension in the two ropes if the person is momentarily motionless. Include a free-body diagram in your solution.

# **Solution:**



$$T_1=736\;\mathrm{N}$$

$$T_2 = 194 \mathrm{\ N}$$



The force  $\mathbf{T}_2$  needed to hold steady the person being rescued from the fire is less than her weight and less than the force  $\mathbf{T}_1$  in the other rope, since the more

vertical rope supports a greater part of her weight (a vertical force).

#### **Exercise:**

#### **Problem:**

**Integrated Concepts** A 35.0-kg dolphin decelerates from 12.0 to 7.50 m/s in 2.30 s to join another dolphin in play. What average force was exerted to slow him if he was moving horizontally? (The gravitational force is balanced by the buoyant force of the water.)

#### **Exercise:**

#### **Problem:**

**Integrated Concepts** When starting a foot race, a 70.0-kg sprinter exerts an average force of 650 N backward on the ground for 0.800 s. (a) What is his final speed? (b) How far does he travel?

#### **Solution:**

- (a) 7.43 m/s
- (b) 2.97 m

#### **Exercise:**

#### **Problem:**

**Integrated Concepts** A large rocket has a mass of  $2.00 \times 10^6$  kg at takeoff, and its engines produce a thrust of  $3.50 \times 10^7$  N. (a) Find its initial acceleration if it takes off vertically. (b) How long does it take to reach a velocity of 120 km/h straight up, assuming constant mass and thrust? (c) In reality, the mass of a rocket decreases significantly as its fuel is consumed. Describe qualitatively how this affects the acceleration and time for this motion.

#### **Exercise:**

#### **Problem:**

**Integrated Concepts** A basketball player jumps straight up for a ball. To do this, he lowers his body 0.300 m and then accelerates through this distance by forcefully straightening his legs. This player leaves the floor with a vertical velocity sufficient to carry him 0.900 m above the floor. (a) Calculate his velocity when he leaves the floor. (b) Calculate his acceleration while he is straightening his legs. He goes from zero to the velocity found in part (a) in a distance of 0.300 m. (c) Calculate the force he exerts on the floor to do this, given that his mass is 110 kg.

#### **Solution:**

- (a) 4.20 m/s
- (b)  $29.4 \text{ m/s}^2$
- (c)  $4.31 \times 10^3 \text{ N}$

#### **Exercise:**

#### **Problem:**

**Integrated Concepts** A 2.50-kg fireworks shell is fired straight up from a mortar and reaches a height of 110 m. (a) Neglecting air resistance (a poor assumption, but we will make it for this example), calculate the shell's velocity when it leaves the mortar. (b) The mortar itself is a tube 0.450 m long. Calculate the average acceleration of the shell in the tube as it goes from zero to the velocity found in (a). (c) What is the average force on the shell in the mortar? Express your answer in newtons and as a ratio to the weight of the shell.

#### **Exercise:**

#### **Problem:**

**Integrated Concepts** Repeat [link] for a shell fired at an angle 10.0° from the vertical.

## **Solution:**

- (a) 47.1 m/s
- (b)  $2.47 \times 10^3 \text{ m/s}^2$
- (c)  $6.18\times 10^3\ N$  . The average force is 252 times the shell's weight.

#### **Exercise:**

#### **Problem:**

**Integrated Concepts** An elevator filled with passengers has a mass of 1700 kg. (a) The elevator accelerates upward from rest at a rate of  $1.20 \, \mathrm{m/s^2}$  for  $1.50 \, \mathrm{s}$ . Calculate the tension in the cable supporting the elevator. (b) The elevator continues upward at constant velocity for  $8.50 \, \mathrm{s}$ . What is the tension in the cable during this time? (c) The elevator decelerates at a rate of  $0.600 \, \mathrm{m/s^2}$  for  $3.00 \, \mathrm{s}$ . What is the tension in the cable during deceleration? (d) How high has the elevator moved above its original starting point, and what is its final velocity?

#### **Exercise:**

#### **Problem:**

**Unreasonable Results** (a) What is the final velocity of a car originally traveling at 50.0 km/h that decelerates at a rate of  $0.400~\mathrm{m/s}^2$  for 50.0 s? (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

#### **Exercise:**

#### **Problem:**

**Unreasonable Results** A 75.0-kg man stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s. (a) Calculate the scale reading in newtons and compare it with his weight. (The scale exerts an upward force on him equal to its reading.) (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

# The First Condition for Equilibrium

- State the first condition of equilibrium.
- Explain static equilibrium.
- Explain dynamic equilibrium.

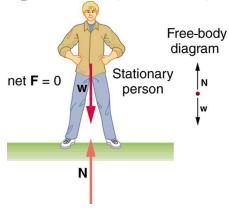
The first condition necessary to achieve equilibrium is the one already mentioned: the net external force on the system must be zero. Expressed as an equation, this is simply

# **Equation:**

Note that if net is zero, then the net external force in *any* direction is zero. For example, the net external forces along the typical *x*- and *y*-axes are zero. This is written as

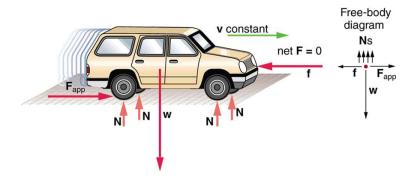
# **Equation:**

[link] and [link] illustrate situations where for both **static equilibrium** (motionless), and **dynamic equilibrium** (constant velocity).



This motionless person is in static equilibrium. The forces acting on him add up to zero. Both

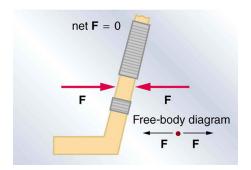
# forces are vertical in this case.



This car is in dynamic equilibrium because it is moving at constant velocity. There are horizontal and vertical forces, but the net external force in any direction is zero. The applied force between the tires and the road is balanced by air friction, and the weight of the car is supported by the normal forces, here shown to be equal for all four tires.

However, it is not sufficient for the net external force of a system to be zero for a system to be in equilibrium. Consider the two situations illustrated in [link] and [link] where forces are applied to an ice hockey stick lying flat on ice. The net external force is zero in both situations shown in the figure; but in one case, equilibrium is achieved, whereas in the other, it is not. In [link], the ice hockey stick remains motionless. But in [link], with the same forces applied in different places, the stick experiences accelerated rotation. Therefore, we know that the point at which a force is applied is another factor in determining whether or not equilibrium is achieved. This will be explored further in the next section.

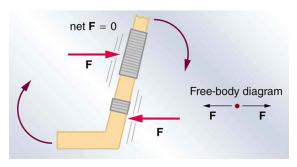
#### Equilibrium: remains stationary



An ice hockey stick lying flat on ice with two equal and opposite horizontal forces applied to it. Friction is negligible, and the gravitational force is balanced by the support of the ice (a normal force). Thus,

Equilibrium is achieved, which is static equilibrium in this case.

Nonequilibrium: rotation accelerates



The same forces are applied at other points and the stick

rotates—in fact, it
experiences an accelerated
rotation. Here but
the system is *not* at
equilibrium. Hence, the
is a necessary—
but not sufficient—condition
for achieving equilibrium.

# Note:

# PhET Explorations: Torque

Investigate how torque causes an object to rotate. Discover the relationships between angular acceleration, moment of inertia, angular momentum and torque.

<u>Torqu</u>

# **Section Summary**

- Statics is the study of forces in equilibrium.
- Two conditions must be met to achieve equilibrium, which is defined to be motion without linear or rotational acceleration.
- The first condition necessary to achieve equilibrium is that the net external force on the system must be zero, so that .

# **Conceptual Questions**

## **Exercise:**

# **Problem:**

What can you say about the velocity of a moving body that is in dynamic equilibrium? Draw a sketch of such a body using clearly labeled arrows to represent all external forces on the body.

## **Exercise:**

# **Problem:**

Under what conditions can a rotating body be in equilibrium? Give an example.

# **Glossary**

# static equilibrium

a state of equilibrium in which the net external force and torque acting on a system is zero

# dynamic equilibrium

a state of equilibrium in which the net external force and torque on a system moving with constant velocity are zero

# Introduction to Work, Energy, and Energy Resources class="introduction"

How many forms of energy can you identify in this photograph of a wind farm in Iowa? (credit: Jürgen from Sandesneben , Germany, Wikimedia Commons)



*Energy* plays an essential role both in everyday events and in scientific phenomena. You can no doubt name many forms of energy, from that provided by our foods, to the energy we use to run our cars, to the sunlight that warms us on the beach. You can also cite examples of what people call energy that may not be scientific, such as someone having an energetic personality. Not only does energy have many interesting forms, it is

involved in almost all phenomena, and is one of the most important concepts of physics. What makes it even more important is that the total amount of energy in the universe is constant. Energy can change forms, but it cannot appear from nothing or disappear without a trace. Energy is thus one of a handful of physical quantities that we say is *conserved*.

**Conservation of energy** (as physicists like to call the principle that energy can neither be created nor destroyed) is based on experiment. Even as scientists discovered new forms of energy, conservation of energy has always been found to apply. Perhaps the most dramatic example of this was supplied by Einstein when he suggested that mass is equivalent to energy (his famous equation  $E = \mathrm{mc}^2$ ).

From a societal viewpoint, energy is one of the major building blocks of modern civilization. Energy resources are key limiting factors to economic growth. The world use of energy resources, especially oil, continues to grow, with ominous consequences economically, socially, politically, and environmentally. We will briefly examine the world's energy use patterns at the end of this chapter.

There is no simple, yet accurate, scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define **energy** as the ability to do work, admitting that in some circumstances not all energy is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form.

#### Work: The Scientific Definition

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.

# What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy —whenever work is done, energy is transferred.

For work, in the scientific sense, to be done, a force must be exerted and there must be displacement in the direction of the force.

Formally, the **work** done on a system by a constant force is defined to be the product of the component of the force in the direction of motion times the distance through which the force acts. For one-way motion in one dimension, this is expressed in equation form as

# **Equation:**

$$W = |\mathbf{F}| (\cos \theta) |\mathbf{d}|,$$

where W is work,  $\mathbf{d}$  is the displacement of the system, and  $\theta$  is the angle between the force vector  $\mathbf{F}$  and the displacement vector  $\mathbf{d}$ , as in [link]. We can also write this as

# **Equation:**

$$W = \operatorname{Fd} \cos \theta$$
.

To find the work done on a system that undergoes motion that is not oneway or that is in two or three dimensions, we divide the motion into oneway one-dimensional segments and add up the work done over each segment.

# Note:

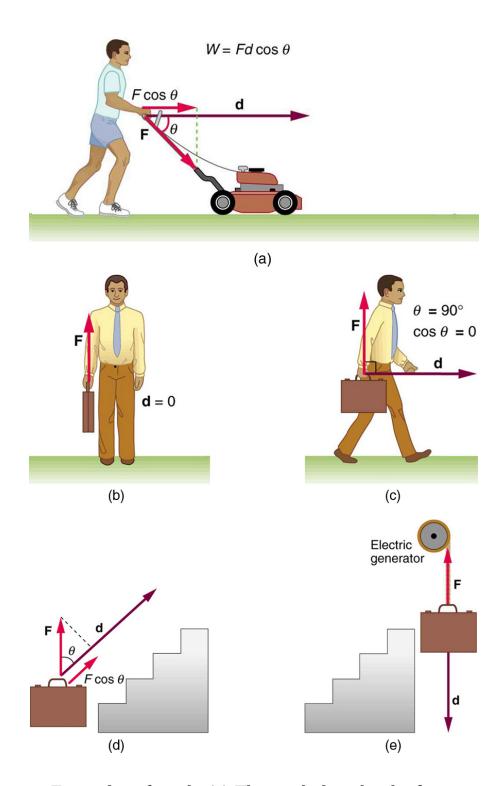
# What is Work?

The work done on a system by a constant force is the product of the component of the force in the direction of motion times the distance through which the force acts. For one-way motion in one dimension, this is expressed in equation form as

# **Equation:**

$$W = \operatorname{Fd} \cos \theta$$
,

where W is work, F is the magnitude of the force on the system, d is the magnitude of the displacement of the system, and  $\theta$  is the angle between the force vector  $\mathbf{F}$  and the displacement vector  $\mathbf{d}$ .



Examples of work. (a) The work done by the force  ${\bf F}$  on this lawn mower is Fd  $\cos\theta$ . Note that  $F\cos\theta$  is the component of the force in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no

displacement. No energy is transferred to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (d) Work *is* done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force **F** in the direction of the motion. Energy is transferred to the briefcase and could in turn be used to do work. (e) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because **F** and **d** are in opposite directions.

To examine what the definition of work means, let us consider the other situations shown in [link]. The person holding the briefcase in [link](b) does no work, for example. Here d=0, so W=0. Why is it you get tired just holding a load? The answer is that your muscles are doing work against one another, but they are doing no work on the system of interest (the "briefcase-Earth system"—see Gravitational Potential Energy for more details). There must be displacement for work to be done, and there must be a component of the force in the direction of the motion. For example, the person carrying the briefcase on level ground in [link](c) does no work on it, because the force is perpendicular to the motion. That is,  $\cos 90^\circ = 0$ , and so W=0.

In contrast, when a force exerted on the system has a component in the direction of motion, such as in [link](d), work *is* done—energy is transferred to the briefcase. Finally, in [link](e), energy is transferred from the briefcase to a generator. There are two good ways to interpret this energy transfer. One interpretation is that the briefcase's weight does work on the generator, giving it energy. The other interpretation is that the generator does negative work on the briefcase, thus removing energy from it. The drawing shows the latter, with the force from the generator upward

on the briefcase, and the displacement downward. This makes  $\theta=180^{\circ}$ , and  $\cos 180^{\circ}=-1$ ; therefore, W is negative.

# **Calculating Work**

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in **newton-meters**. A newton-meter is given the special name **joule** (J), and  $1 J = 1 N \cdot m = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$ . One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.

# **Example:**

# Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn

How much work is done on the lawn mower by the person in [link](a) if he exerts a constant force of 75.0 N at an angle 35° below the horizontal and pushes the mower 25.0 m on level ground? Convert the amount of work from joules to kilocalories and compare it with this person's average daily intake of 10,000 kJ (about 2400 kcal) of food energy. One *calorie* (1 cal) of heat is the amount required to warm 1 g of water by 1°C, and is equivalent to 4.184 J, while one *food calorie* (1 kcal) is equivalent to 4184 J.

# Strategy

We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation  $W = \operatorname{Fd} \cos \theta$ . The force, angle, and displacement are given, so that only the work W is unknown.

## Solution

The equation for the work is

# **Equation:**

$$W = \operatorname{Fd} \cos \theta$$
.

Substituting the known values gives

# **Equation:**

$$W = (75.0 \text{ N})(25.0 \text{ m}) \cos (35.0^{\circ})$$
  
=  $1536 \text{ J} = 1.54 \times 10^{3} \text{ J}.$ 

Converting the work in joules to kilocalories yields  $W=(1536~{
m J})(1~{
m kcal}/4184~{
m J})=0.367~{
m kcal}.$  The ratio of the work done to the daily consumption is

# **Equation:**

$$rac{W}{2400 ext{ kcal}} = 1.53 imes 10^{-4}.$$

## Discussion

This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we "work" all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.

# **Section Summary**

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work W that a force  ${\bf F}$  does on an object is the product of the magnitude F of the force, times the magnitude d of the displacement, times the cosine of the angle  $\theta$  between them. In symbols, **Equation:**

$$W = \operatorname{Fd} \cos \theta$$
.

- The SI unit for work and energy is the joule (J), where  $1~J=1~N\cdot m=1~kg\cdot m^2/s^2.$
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.

• The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

# **Conceptual Questions**

## **Exercise:**

#### **Problem:**

Give an example of something we think of as work in everyday circumstances that is not work in the scientific sense. Is energy transferred or changed in form in your example? If so, explain how this is accomplished without doing work.

## **Exercise:**

# **Problem:**

Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.

#### **Exercise:**

#### **Problem:**

Describe a situation in which a force is exerted for a long time but does no work. Explain.

## **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.

#### **Solution:**

# **Equation:**

$$3.00~{
m J} = 7.17 imes 10^{-4}~{
m kcal}$$

#### **Exercise:**

## **Problem:**

A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task.

#### **Exercise:**

## **Problem:**

(a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N. (b) What is the work done on the lift by the gravitational force in this process? (c) What is the total work done on the lift?

# **Solution:**

(a) 
$$5.92 \times 10^5 \text{ J}$$

(b) 
$$-5.88 \times 10^5 \ J$$

(c) The net force is zero.

#### **Exercise:**

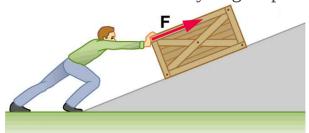
#### **Problem:**

Suppose a car travels 108 km at a speed of 30.0 m/s, and uses 2.0 gal of gasoline. Only 30% of the gasoline goes into useful work by the force that keeps the car moving at constant speed despite friction. (See [link] for the energy content of gasoline.) (a) What is the magnitude of the force exerted to keep the car moving at constant speed? (b) If the required force is directly proportional to speed, how many gallons will be used to drive 108 km at a speed of 28.0 m/s?

## **Exercise:**

# **Problem:**

Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of 20.0° with the horizontal. (See [link].) He exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed. Be certain to include the work he does on the crate *and* on his body to get up the ramp.



A man pushes a crate up a ramp.

**Solution:** 

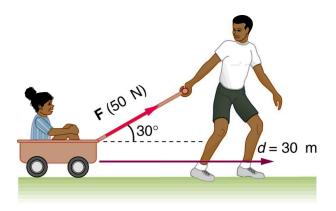
**Equation:** 

$$3.14 \times 10^3 \ \mathrm{J}$$

#### **Exercise:**

## **Problem:**

How much work is done by the boy pulling his sister 30.0 m in a wagon as shown in [link]? Assume no friction acts on the wagon.



The boy does work on the system of the wagon and the child when he pulls them as shown.

# **Exercise:**

# **Problem:**

A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction 25.0° below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?

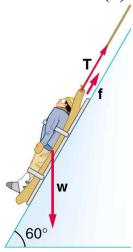
# **Solution:**

- (a) -700 J
- (b) 0
- (c) 700 J
- (d) 38.6 N

### **Exercise:**

## **Problem:**

Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60.0° slope at constant speed, as shown in [link]. The coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?



A rescue sled and victim are lowered down a steep slope.

# Glossary

energy

the ability to do work

# work

the transfer of energy by a force that causes an object to be displaced; the product of the component of the force in the direction of the displacement and the magnitude of the displacement

# joule

SI unit of work and energy, equal to one newton-meter

# Kinetic Energy and the Work-Energy Theorem

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

# **Work Transfers Energy**

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, if the lawn mower in [link] (a) is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on the briefcase by the person carrying it up stairs in [link](d) is stored in the briefcase-Earth system and can be recovered at any time, as shown in [link](e). In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system. Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

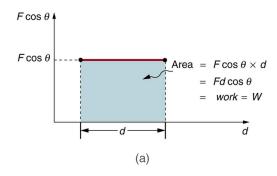
In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

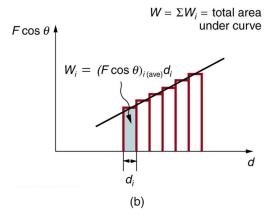
# **Net Work and the Work-Energy Theorem**

We know from the study of Newton's laws in <u>Dynamics: Force and Newton's Laws of Motion</u> that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

Let us start by considering the total, or net, work done on a system. Net work is defined to be the sum of work done by all external forces—that is, **net work** is the work done by the net external force  $\mathbf{F}_{\rm net}$ . In equation form, this is  $W_{\rm net} = F_{\rm net} d \cos \theta$  where  $\theta$  is the angle between the force vector and the displacement vector.

[link](a) shows a graph of force versus displacement for the component of the force in the direction of the displacement—that is, an  $F \cos \theta$  vs. d graph. In this case,  $F \cos \theta$  is constant. You can see that the area under the graph is  $Fd \cos \theta$ , or the work done. [link](b) shows a more general process where the force varies. The area under the curve is divided into strips, each having an average force  $(F \cos \theta)_{i(ave)}$ . The work done is  $(F \cos \theta)_{i(ave)}d_i$  for each strip, and the total work done is the sum of the  $W_i$ . Thus the total work done is the total area under the curve, a useful property to which we shall refer later.

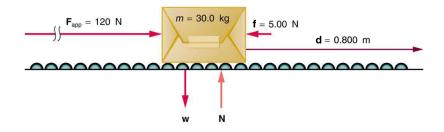




(a) A graph of  $F \cos \theta$  vs. d, when  $F \cos \theta$  is

constant. The area under the curve represents the work done by the force. (b) A graph of  $F \cos \theta$  vs. d in which the force varies. The work done for each interval is the area of each strip; thus, the total area under the curve equals the total work done.

Net work will be simpler to examine if we consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in [link].



A package on a roller belt is pushed horizontally through a distance **d**.

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force  $\mathbf{F}_{\mathrm{app}}$  and the horizontal friction force  $\mathbf{f}$ . Thus, as expected, the net force is

parallel to the displacement, so that  $\theta=0^\circ$  and  $\cos\theta=1$ , and the net work is given by

## **Equation:**

$$W_{
m net} = F_{
m net} d.$$

The effect of the net force  $\mathbf{F}_{\mathrm{net}}$  is to accelerate the package from  $v_0$  to v. The kinetic energy of the package increases, indicating that the net work done on the system is positive. (See [link].) By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting  $F_{\mathrm{net}} = \mathrm{ma}$  from Newton's second law gives

## **Equation:**

$$W_{\rm net} = {
m mad.}$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take  $d=x-x_0$  and use the equation studied in Motion Equations for Constant Acceleration in One Dimension for the change in speed over a distance d if the acceleration has the constant value a; namely,  $v^2=v_0^2+2{\rm ad}$  (note that a appears in the expression for the net work). Solving for acceleration gives  $a=\frac{v^2-v_0^2}{2d}$ . When a is substituted into the preceding expression for  $W_{\rm net}$ , we obtain

# **Equation:**

$$W_{
m net} = migg(rac{v^2-{v_0}^2}{2d}igg)d.$$

The d cancels, and we rearrange this to obtain

# **Equation:**

$${W}_{
m net} = rac{1}{2} m v^2 - rac{1}{2} m v_0^2.$$

This expression is called the **work-energy theorem**, and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity  $\frac{1}{2}mv^2$ . This quantity is our first example of a form of energy.

#### Note:

The Work-Energy Theorem

The net work on a system equals the change in the quantity  $\frac{1}{2}mv^2$ .

**Equation:** 

$$W_{
m net} = rac{1}{2} m v^2 - rac{1}{2} {
m mv}_0^2$$

The quantity  $\frac{1}{2}mv^2$  in the work-energy theorem is defined to be the translational **kinetic energy** (KE) of a mass m moving at a speed v. (*Translational* kinetic energy is distinct from *rotational* kinetic energy, which is considered later.) In equation form, the translational kinetic energy, **Equation:** 

$$ext{KE} = rac{1}{2}mv^2,$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

We are aware that it takes energy to get an object, like a car or the package in [link], up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 100 km/h has four times the kinetic energy it has at 50

km/h, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

# **Example:**

# Calculating the Kinetic Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in [link] is moving at 0.500 m/s. What is its kinetic energy?

# Strategy

Because the mass m and speed v are given, the kinetic energy can be calculated from its definition as given in the equation  $KE = \frac{1}{2}mv^2$ .

#### Solution

The kinetic energy is given by

## **Equation:**

$$ext{KE} = rac{1}{2}mv^2.$$

Entering known values gives

# **Equation:**

$$KE = 0.5(30.0 \text{ kg})(0.500 \text{ m/s})^2$$

which yields

# **Equation:**

$$KE = 3.75 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 3.75 \text{ J}.$$

#### Discussion

Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

# **Example:**

# **Determining the Work to Accelerate a Package**

Suppose that you push on the 30.0-kg package in [link] with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N.

(a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.

# Strategy and Concept for (a)

This is a motion in one dimension problem, because the downward force (from the weight of the package) and the normal force have equal magnitude and opposite direction, so that they cancel in calculating the net force, while the applied force, friction, and the displacement are all horizontal. (See [link].) As expected, the net work is the net force times distance.

# Solution for (a)

The net force is the push force minus friction, or  $F_{\rm net} = 120~{
m N} - 5.00~{
m N} = 115~{
m N}$ . Thus the net work is

# **Equation:**

$$W_{\text{net}} = F_{\text{net}}d = (115 \text{ N})(0.800 \text{ m})$$
  
= 92.0 N · m = 92.0 J.

# Discussion for (a)

This value is the net work done on the package. The person actually does more work than this, because friction opposes the motion. Friction does negative work and removes some of the energy the person expends and converts it to thermal energy. The net work equals the sum of the work done by each individual force.

# **Strategy and Concept for (b)**

The forces acting on the package are gravity, the normal force, the force of friction, and the applied force. The normal force and force of gravity are each perpendicular to the displacement, and therefore do no work.

# Solution for (b)

The applied force does work.

# **Equation:**

$$egin{array}{lll} W_{
m app} &=& F_{
m app} d \cos(0^{
m o}) = F_{
m app} d \ &=& (120\ {
m N})(0.800\ {
m m}) \ &=& 96.0\ {
m J} \end{array}$$

The friction force and displacement are in opposite directions, so that  $\theta=180^{\circ}$ , and the work done by friction is

## **Equation:**

$$egin{array}{lll} W_{
m fr} &=& F_{
m fr} d \cos(180^{
m o}) = - F_{
m fr} d \ &=& - (5.00 \ 
m N) (0.800 \ 
m m) \ &=& - 4.00 \ 
m J. \end{array}$$

So the amounts of work done by gravity, by the normal force, by the applied force, and by friction are, respectively,

## **Equation:**

$$egin{array}{lll} W_{
m gr} &=& 0, \ W_{
m N} &=& 0, \ W_{
m app} &=& 96.0 \ 
m J, \ W_{
m fr} &=& -4.00 \ 
m J. \end{array}$$

The total work done as the sum of the work done by each force is then seen to be

# **Equation:**

$$W_{
m total} = W_{
m gr} + W_{
m N} + W_{
m app} + W_{
m fr} = 92.0~
m J.$$

# Discussion for (b)

The calculated total work  $W_{\rm total}$  as the sum of the work by each force agrees, as expected, with the work  $W_{\rm net}$  done by the net force. The work done by a collection of forces acting on an object can be calculated by either approach.

# **Example:**

# **Determining Speed from Work and Energy**

Find the speed of the package in [link] at the end of the push, using work and energy concepts.

# **Strategy**

Here the work-energy theorem can be used, because we have just calculated the net work,  $W_{\rm net}$ , and the initial kinetic energy,  $\frac{1}{2}mv_0^2$ . These calculations allow us to find the final kinetic energy,  $\frac{1}{2}mv^2$ , and thus the final speed v.

## Solution

The work-energy theorem in equation form is

## **Equation:**

$$W_{
m net} = rac{1}{2} m v^2 - rac{1}{2} m {v_0}^2.$$

Solving for  $\frac{1}{2}mv^2$  gives

## **Equation:**

$$rac{1}{2} {
m mv}^2 = W_{
m net} + rac{1}{2} m {v_0}^2.$$

Thus,

# **Equation:**

$$rac{1}{2}mv^2 = 92.0 \ \mathrm{J} + 3.75 \ \mathrm{J} = 95.75 \ \mathrm{J}.$$

Solving for the final speed as requested and entering known values gives **Equation:** 

$$egin{array}{lcl} v & = & \sqrt{rac{2(95.75 \, {
m J})}{m}} = \sqrt{rac{191.5 \, {
m kg \cdot m^2/s^2}}{30.0 \, {
m kg}}} \ & = & 2.53 \, {
m m/s}. \end{array}$$

## Discussion

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work

done on the package. This means that the work indeed adds to the energy of the package.

## **Example:**

## Work and Energy Can Reveal Distance, Too

How far does the package in [link] coast after the push, assuming friction remains constant? Use work and energy considerations.

## Strategy

We know that once the person stops pushing, friction will bring the package to rest. In terms of energy, friction does negative work until it has removed all of the package's kinetic energy. The work done by friction is the force of friction times the distance traveled times the cosine of the angle between the friction force and displacement; hence, this gives us a way of finding the distance traveled after the person stops pushing.

## Solution

The normal force and force of gravity cancel in calculating the net force. The horizontal friction force is then the net force, and it acts opposite to the displacement, so  $\theta=180^{\circ}$ . To reduce the kinetic energy of the package to zero, the work  $W_{\rm fr}$  by friction must be minus the kinetic energy that the package started with plus what the package accumulated due to the pushing. Thus  $W_{\rm fr}=-95.75$  J. Furthermore,  $W_{\rm fr}=fdt\cos\theta=-fdt$ , where dt is the distance it takes to stop. Thus,

# **Equation:**

$$d\prime = -rac{W_{
m fr}}{f} = -rac{-95.75 \ 
m J}{5.00 \ 
m N},$$

and so

# **Equation:**

$$d\prime = 19.2 \text{ m}.$$

#### Discussion

This is a reasonable distance for a package to coast on a relatively frictionfree conveyor system. Note that the work done by friction is negative (the force is in the opposite direction of motion), so it removes the kinetic energy.

Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

# **Section Summary**

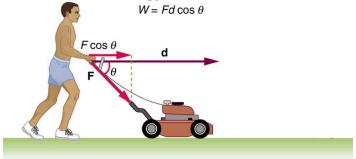
- The net work  $W_{\mathrm{net}}$  is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass m moving at speed v is  $KE = \frac{1}{2}mv^2$ .
- The work-energy theorem states that the net work  $W_{\rm net}$  on a system changes its kinetic energy,  $W_{\rm net}=\frac{1}{2}mv^2-\frac{1}{2}m{v_0}^2.$

# **Conceptual Questions**

## **Exercise:**

## **Problem:**

The person in [link] does work on the lawn mower. Under what conditions would the mower gain energy? Under what conditions would it lose energy?



# **Exercise:**

## **Problem:**

Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.

## **Exercise:**

#### **Problem:**

When solving for speed in [link], we kept only the positive root. Why?

## **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

Compare the kinetic energy of a 20,000-kg truck moving at 110 km/h with that of an 80.0-kg astronaut in orbit moving at 27,500 km/h.

#### **Solution:**

1/250

#### **Exercise:**

#### **Problem:**

(a) How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s? (b) Discuss how the larger energies needed for the movement of larger animals would relate to metabolic rates.

#### **Exercise:**

#### **Problem:**

Confirm the value given for the kinetic energy of an aircraft carrier in [link]. You will need to look up the definition of a nautical mile (1 knot = 1 nautical mile/h).

## **Solution:**

 $1.1 \times 10^{10} \, \mathrm{J}$ 

## **Exercise:**

## **Problem:**

(a) Calculate the force needed to bring a 950-kg car to rest from a speed of 90.0 km/h in a distance of 120 m (a fairly typical distance for a non-panic stop). (b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).

#### **Exercise:**

## **Problem:**

A car's bumper is designed to withstand a 4.0-km/h (1.1-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.

## **Solution:**

 $2.8 \times 10^3 \text{ N}$ 

#### **Exercise:**

#### **Problem:**

Boxing gloves are padded to lessen the force of a blow. (a) Calculate the force exerted by a boxing glove on an opponent's face, if the glove and face compress 7.50 cm during a blow in which the 7.00-kg arm and glove are brought to rest from an initial speed of 10.0 m/s. (b) Calculate the force exerted by an identical blow in the gory old days when no gloves were used and the knuckles and face would compress only 2.00 cm. (c) Discuss the magnitude of the force with glove on. Does it seem high enough to cause damage even though it is lower than the force with no glove?

## **Exercise:**

#### **Problem:**

Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

## **Solution:**

102 N

# **Glossary**

net work

work done by the net force, or vector sum of all the forces, acting on an object

# work-energy theorem

the result, based on Newton's laws, that the net work done on an object is equal to its change in kinetic energy

# kinetic energy

the energy an object has by reason of its motion, equal to  $\frac{1}{2}mv^2$  for the translational (i.e., non-rotational) motion of an object of mass m moving at speed v

## Gravitational Potential Energy

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass m at height h on Earth is given by  $PE_g = mgh$ .
- Show how knowledge of the potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

# **Work Done Against Gravity**

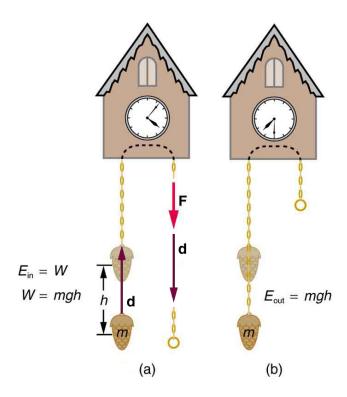
Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass m through a height h, such as in [link]. If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight mg. The work done on the mass is then W = Fd = mgh. We define this to be the **gravitational potential energy** (PE<sub> $\sigma$ </sub>) put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the PE<sub>g</sub> gained by the object, recognizing that this is energy stored in the gravitational field of Earth. Why do we use the word "system"? Potential energy is a property of a system rather than of a single object—due to its physical position. An object's gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth's surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in the Earthobject system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

# **Converting Between Potential Energy and Kinetic Energy**

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work

equal to mgh on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of  $PE_g$  to KE without explicitly considering the intermediate step of work. (See [link].) This shortcut makes it is easier to solve problems using energy (if possible) rather than explicitly using forces.



(a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.

More precisely, we define the *change* in gravitational potential energy  $\Delta PE_g$  to be **Equation:** 

$$\Delta \mathrm{PE}_{\mathrm{g}} = \mathrm{mgh},$$

where, for simplicity, we denote the change in height by h rather than the usual  $\Delta h$ . Note that h is positive when the final height is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

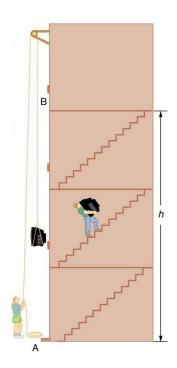
## **Equation:**

$$mgh = (0.500 \text{ kg}) (9.80 \text{ m/s}^2) (1.00 \text{ m})$$
  
=  $4.90 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 4.90 \text{ J}.$ 

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, without directly considering the force of gravity that does the work.

# **Using Potential Energy to Simplify Calculations**

The equation  $\Delta PE_g = mgh$  applies for any path that has a change in height of h, not just when the mass is lifted straight up. (See [link].) It is much easier to calculate mgh (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position h of a mass m is accompanied by a change in gravitational potential energy mgh, and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.



The change in gravitational potential energy  $(\Delta \mathrm{PE}_\mathrm{g})$ between points A and B is independent of the path.  $\Delta PE_g = mgh$ for any path between the two points. Gravity is one of a small class of forces where the work done by or against the force depends only on the starting and ending points, not on the path between them.

## **Example:**

## The Force to Stop Falling

A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.

## Strategy

This person's energy is brought to zero in this situation by the work done on him by the floor as he stops. The initial  $PE_g$  is transformed into KE as he falls. The work done by the floor reduces this kinetic energy to zero.

#### Solution

The work done on the person by the floor as he stops is given by

## **Equation:**

$$W = \mathrm{Fd} \cos \theta = -\mathrm{Fd},$$

with a minus sign because the displacement while stopping and the force from floor are in opposite directions ( $\cos \theta = \cos 180^{\circ} = -1$ ). The floor removes energy from the system, so it does negative work.

The kinetic energy the person has upon reaching the floor is the amount of potential energy lost by falling through height h:

## **Equation:**

$$ext{KE} = -\Delta ext{PE}_{ ext{g}} = - ext{mgh},$$

The distance d that the person's knees bend is much smaller than the height h of the fall, so the additional change in gravitational potential energy during the knee bend is ignored.

The work W done by the floor on the person stops the person and brings the person's kinetic energy to zero:

## **Equation:**

$$W = -KE = mgh.$$

Combining this equation with the expression for W gives

# **Equation:**

$$-Fd = mgh.$$

Recalling that h is negative because the person fell down, the force on the knee joints is given by

# **Equation:**

$$F = -rac{ ext{mgh}}{d} = -rac{(60.0 ext{ kg}) \Big( 9.80 ext{ m/s}^2 \Big) (-3.00 ext{ m})}{5.00 imes 10^{-3} ext{ m}} = 3.53 imes 10^5 ext{ N}.$$

#### Discussion

Such a large force (500 times more than the person's weight) over the short impact time is enough to break bones. A much better way to cushion the shock is by bending the legs or rolling on the ground, increasing the time over which the force acts. A bending motion of 0.5 m this way yields a force 100 times smaller than in the example. A kangaroo's hopping shows this method in action. The kangaroo is the only large animal to use hopping for locomotion, but the shock in hopping is cushioned by the bending of its hind legs in each jump. (See [link].)

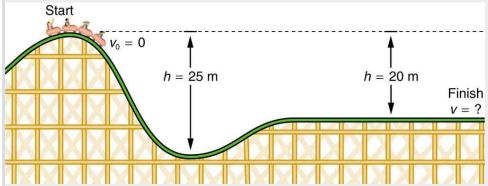


The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced. (credit: Chris Samuel, Flickr)

## **Example:**

## Finding the Speed of a Roller Coaster from its Height

(a) What is the final speed of the roller coaster shown in [link] if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible? (b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?



The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all  $\Delta PE_{\rm g}$  is converted to KE.

# Strategy

The roller coaster loses potential energy as it goes downhill. We neglect friction, so that the remaining force exerted by the track is the normal force, which is perpendicular to the direction of motion and does no work. The net work on the roller coaster is then done by gravity alone. The *loss* of gravitational potential energy from moving *downward* through a distance h equals the *gain* in kinetic energy. This can be written in equation form as  $-\Delta PE_g = \Delta KE$ . Using the equations for  $PE_g$  and KE, we can solve for the final speed v, which is the desired quantity.

# Solution for (a)

Here the initial kinetic energy is zero, so that  $\Delta KE = \frac{1}{2}mv^2$ . The equation for change in potential energy states that  $\Delta PE_{\rm g} = mgh$ . Since h is negative in this case, we will rewrite this as  $\Delta PE_{\rm g} = -mg \mid h \mid$  to show the minus sign clearly. Thus,

## **Equation:**

$$-\Delta PE_g = \Delta KE$$

becomes

## **Equation:**

$$egin{aligned} \operatorname{mg}\mid h\mid =rac{1}{2}mv^2. \end{aligned}$$

Solving for v, we find that mass cancels and that

## **Equation:**

$$v = \sqrt{2g\mid h\mid}.$$

Substituting known values,

## **Equation:**

$$v = \sqrt{2(9.80 \text{ m/s}^2)(20.0 \text{ m})}$$
  
= 19.8 m/s.

## Solution for (b)

Again  $-\Delta PE_g=\Delta KE$ . In this case there is initial kinetic energy, so  $\Delta KE=\frac{1}{2}mv^2-\frac{1}{2}mv_0^2$ . Thus,

# **Equation:**

$$|mg| \ h \mid = rac{1}{2} m v^2 - rac{1}{2} m {v_0}^2.$$

Rearranging gives

# **Equation:**

$$rac{1}{2}mv^2 = \mathrm{mg}\mid h\mid +rac{1}{2}m{v_0}^2.$$

This means that the final kinetic energy is the sum of the initial kinetic energy and the gravitational potential energy. Mass again cancels, and

# **Equation:**

$$v = \sqrt{2g\mid h\mid + {v_0}^2}.$$

This equation is very similar to the kinematics equation  $v = \sqrt{v_0^2 + 2 \mathrm{ad}}$ , but it is more general—the kinematics equation is valid only for constant acceleration, whereas our equation above is valid for any path regardless of whether the object moves with a constant acceleration. Now, substituting known values gives

## **Equation:**

$$v = \sqrt{2(9.80 \text{ m/s}^2)(20.0 \text{ m}) + (5.00 \text{ m/s})^2}$$
  
= 20.4 m/s.

## **Discussion and Implications**

First, note that mass cancels. This is quite consistent with observations made in Falling Objects that all objects fall at the same rate if friction is negligible. Second, only the speed of the roller coaster is considered; there is no information about its direction at any point. This reveals another general truth. When friction is negligible, the speed of a falling body depends only on its initial speed and height, and not on its mass or the path taken. For example, the roller coaster will have the same final speed whether it falls 20.0 m straight down or takes a more complicated path like the one in the figure. Third, and perhaps unexpectedly, the final speed in part (b) is greater than in part (a), but by far less than 5.00 m/s. Finally, note that speed can be found at *any* height along the way by simply using the appropriate value of h at the point of interest.

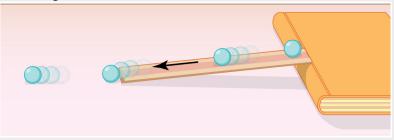
We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

#### Note:

Making Connections: Take-Home Investigation—Converting Potential to Kinetic Energy

One can study the conversion of gravitational potential energy into kinetic energy in this experiment. On a smooth, level surface, use a ruler of the kind that has a groove running along its length and a book to make an incline (see [link]). Place a marble at the 10-cm position on the ruler and let it roll down the ruler. When it hits the level surface, measure the time it takes to roll one meter. Now place the marble

at the 20-cm and the 30-cm positions and again measure the times it takes to roll 1 m on the level surface. Find the velocity of the marble on the level surface for all three positions. Plot velocity squared versus the distance traveled by the marble. What is the shape of each plot? If the shape is a straight line, the plot shows that the marble's kinetic energy at the bottom is proportional to its potential energy at the release point.



A marble rolls down a ruler, and its speed on the level surface is measured.

# **Section Summary**

- Work done against gravity in lifting an object becomes potential energy of the object-Earth system.
- The change in gravitational potential energy,  $\Delta PE_g$ , is  $\Delta PE_g = mgh$ , with h being the increase in height and g the acceleration due to gravity.
- The gravitational potential energy of an object near Earth's surface is due to its position in the mass-Earth system. Only differences in gravitational potential energy,  $\Delta PE_{\rm g}$ , have physical significance.
- As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that  $\Delta KE = -\Delta PE_g$ .

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

In [link], we calculated the final speed of a roller coaster that descended 20 m in height and had an initial speed of 5 m/s downhill. Suppose the roller coaster had had an initial speed of 5 m/s *uphill* instead, and it coasted uphill, stopped, and then rolled back down to a final point 20 m below the start. We would find in that case that its final speed is the same as its initial speed. Explain in terms of conservation of energy.

#### **Exercise:**

#### **Problem:**

Does the work you do on a book when you lift it onto a shelf depend on the path taken? On the time taken? On the height of the shelf? On the mass of the book?

## **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

A hydroelectric power facility (see [link]) converts the gravitational potential energy of water behind a dam to electric energy. (a) What is the gravitational potential energy relative to the generators of a lake of volume  $50.0~\rm km^3$  (  $\rm mass = 5.00 \times 10^{13}~\rm kg)$ , given that the lake has an average height of 40.0 m above the generators? (b) Compare this with the energy stored in a 9-megaton fusion bomb.



Hydroelectric facility (credit: Denis

## Belevich, Wikimedia Commons)

#### **Solution:**

- (a)  $1.96 \times 10^{16} \text{ J}$
- (b) The ratio of gravitational potential energy in the lake to the energy stored in the bomb is 0.52. That is, the energy stored in the lake is approximately half that in a 9-megaton fusion bomb.

#### **Exercise:**

## **Problem:**

(a) How much gravitational potential energy (relative to the ground on which it is built) is stored in the Great Pyramid of Cheops, given that its mass is about  $7 \times 10^9$  kg and its center of mass is 36.5 m above the surrounding ground? (b) How does this energy compare with the daily food intake of a person?

#### **Exercise:**

## **Problem:**

Suppose a 350-g kookaburra (a large kingfisher bird) picks up a 75-g snake and raises it 2.5 m from the ground to a branch. (a) How much work did the bird do on the snake? (b) How much work did it do to raise its own center of mass to the branch?

#### **Solution:**

- (a) 1.8 J
- (b) 8.6 J

#### **Exercise:**

#### **Problem:**

In [link], we found that the speed of a roller coaster that had descended 20.0 m was only slightly greater when it had an initial speed of 5.00 m/s than when it started from rest. This implies that  $\Delta PE >> KE_i$ . Confirm this statement by taking the ratio of  $\Delta PE$  to  $KE_i$ . (Note that mass cancels.)

#### **Exercise:**

#### **Problem:**

A 100-g toy car is propelled by a compressed spring that starts it moving. The car follows the curved track in [link]. Show that the final speed of the toy car is 0.687 m/s if its initial speed is 2.00 m/s and it coasts up the frictionless slope,

gaining 0.180 m in altitude.



A toy car moves up a sloped track. (credit: Leszek Leszczynski, Flickr)

## **Solution:**

# **Equation:**

$$v_f = \sqrt{2 {
m gh} + {v_0}^2} = \sqrt{2 (9.80 \ {
m m/s}^2) (-0.180 \ {
m m}) + (2.00 \ {
m m/s})^2} = 0.687 \ {
m m/s}$$

#### **Exercise:**

#### **Problem:**

In a downhill ski race, surprisingly, little advantage is gained by getting a running start. (This is because the initial kinetic energy is small compared with the gain in gravitational potential energy on even small hills.) To demonstrate this, find the final speed and the time taken for a skier who skies 70.0 m along a 30° slope neglecting friction: (a) Starting from rest. (b) Starting with an initial speed of 2.50 m/s. (c) Does the answer surprise you? Discuss why it is still advantageous to get a running start in very competitive events.

# Glossary

gravitational potential energy the energy an object has due to its position in a gravitational field

# Conservative Forces and Potential Energy

- Define conservative force, potential energy, and mechanical energy.
- Explain the potential energy of a spring in terms of its compression when Hooke's law applies.
- Use the work-energy theorem to show how having only conservative forces implies conservation of mechanical energy.

# **Potential Energy and Conservative Forces**

Work is done by a force, and some forces, such as weight, have special characteristics. A **conservative force** is one, like the gravitational force, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken. We can define a **potential energy** (PE) for any conservative force, just as we did for the gravitational force. For example, when you wind up a toy, an egg timer, or an old-fashioned watch, you do work against its spring and store energy in it. (We treat these springs as ideal, in that we assume there is no friction and no production of thermal energy.) This stored energy is recoverable as work, and it is useful to think of it as potential energy contained in the spring. Indeed, the reason that the spring has this characteristic is that its force is *conservative*. That is, a conservative force results in stored or potential energy. Gravitational potential energy is one example, as is the energy stored in a spring. We will also see how conservative forces are related to the conservation of energy.

#### Note:

# Potential Energy and Conservative Forces

Potential energy is the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable. A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.

We can define a potential energy (PE) for any conservative force. The work done against a conservative force to reach a final configuration

depends on the configuration, not the path followed, and is the potential energy added.

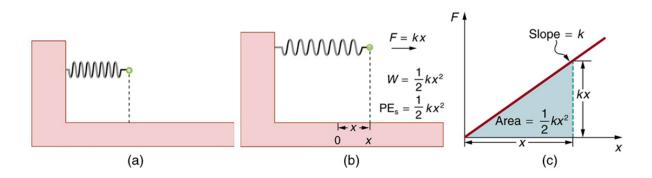
# **Potential Energy of a Spring**

First, let us obtain an expression for the potential energy stored in a spring ( $PE_s$ ). We calculate the work done to stretch or compress a spring that obeys Hooke's law. (Hooke's law was examined in Elasticity: Stress and Strain, and states that the magnitude of force F on the spring and the resulting deformation  $\Delta L$  are proportional,  $F = k\Delta L$ .) (See [link].) For our spring, we will replace  $\Delta L$  (the amount of deformation produced by a force F) by the distance x that the spring is stretched or compressed along its length. So the force needed to stretch the spring has magnitude F = kx, where k is the spring's force constant. The force increases linearly from 0 at the start to kx in the fully stretched position. The average force is kx/2. Thus the work done in stretching or compressing the spring is

 $W_{\rm s}={
m Fd}=\left(\frac{kx}{2}\right)x=\frac{1}{2}kx^2$ . Alternatively, we noted in <u>Kinetic Energy</u> and the Work-Energy Theorem that the area under a graph of F vs. x is the work done by the force. In  $[\underline{{
m link}}](c)$  we see that this area is also  $\frac{1}{2}kx^2$ . We therefore define the **potential energy of a spring**,  ${
m PE}_{\rm s}$ , to be **Equation:** 

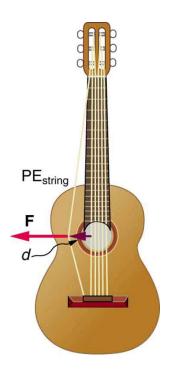
$$ext{PE}_{ ext{s}} = rac{1}{2} ext{kx}^2,$$

where k is the spring's force constant and x is the displacement from its undeformed position. The potential energy represents the work done *on* the spring and the energy stored in it as a result of stretching or compressing it a distance x. The potential energy of the spring  $PE_s$  does not depend on the path taken; it depends only on the stretch or squeeze x in the final configuration.



(a) An undeformed spring has no PE<sub>s</sub> stored in it. (b) The force needed to stretch (or compress) the spring a distance x has a magnitude F = kx, and the work done to stretch (or compress) it is \(\frac{1}{2}kx^2\). Because the force is conservative, this work is stored as potential energy (PE<sub>s</sub>) in the spring, and it can be fully recovered.
(c) A graph of F vs. x has a slope of k, and the area under the graph is \(\frac{1}{2}kx^2\). Thus the work done or potential energy stored is \(\frac{1}{2}kx^2\).

The equation  $PE_s = \frac{1}{2}kx^2$  has general validity beyond the special case for which it was derived. Potential energy can be stored in any elastic medium by deforming it. Indeed, the general definition of **potential energy** is energy due to position, shape, or configuration. For shape or position deformations, stored energy is  $PE_s = \frac{1}{2}kx^2$ , where k is the force constant of the particular system and x is its deformation. Another example is seen in [link] for a guitar string.



Work is done to deform the guitar string, giving it potential energy. When released, the potential energy is converted to kinetic energy and back to potential as the string oscillates back and forth. A very small fraction is dissipated as

sound
energy,
slowly
removing
energy from
the string.

# **Conservation of Mechanical Energy**

Let us now consider what form the work-energy theorem takes when only conservative forces are involved. This will lead us to the conservation of energy principle. The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy. In equation form, this is

## **Equation:**

$$W_{
m net} = rac{1}{2} m v^2 - rac{1}{2} m {v_0}^2 = \Delta {
m KE}.$$

If only conservative forces act, then

# **Equation:**

$$W_{
m net} = W_{
m c},$$

where  $W_{\rm c}$  is the total work done by all conservative forces. Thus, **Equation:** 

$$W_{\mathrm{c}} = \Delta \mathrm{KE}.$$

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy. That is,  $W_{\rm c}=-\Delta {\rm PE}$ . Therefore,

# **Equation:**

$$-\Delta PE = \Delta KE$$

or

## **Equation:**

$$\Delta \text{KE} + \Delta \text{PE} = 0.$$

This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

# **Equation:**

$$KE + PE = constant \label{eq:KE}$$
 or 
$$(conservative \ forces \ only), \ KE_i + PE_i = KE_f + PE_f$$

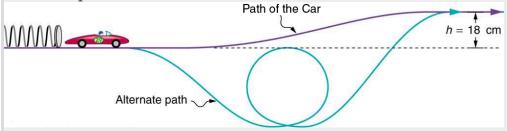
where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the **conservation of mechanical energy** principle. Remember that this applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its **mechanical energy**, (KE + PE). In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and the various types of PE , with the total energy remaining constant.

# **Example:**

# Using Conservation of Mechanical Energy to Calculate the Speed of a Toy Car

A 0.100-kg toy car is propelled by a compressed spring, as shown in [link]. The car follows a track that rises 0.180 m above the starting point. The spring is compressed 4.00 cm and has a force constant of 250.0 N/m. Assuming work done by friction to be negligible, find (a) how fast the car

is going before it starts up the slope and (b) how fast it is going at the top of the slope.



A toy car is pushed by a compressed spring and coasts up a slope. Assuming negligible friction, the potential energy in the spring is first completely converted to kinetic energy, and then to a combination of kinetic and gravitational potential energy as the car rises. The details of the path are unimportant because all forces are conservative—the car would have the same final speed if it took the alternate path shown.

# Strategy

The spring force and the gravitational force are conservative forces, so conservation of mechanical energy can be used. Thus,

# **Equation:**

$$KE_i + PE_i = KE_f + PE_f$$

or

# **Equation:**

$$rac{1}{2}m{v_{
m i}}^2 + mg{h_{
m i}} + rac{1}{2}k{x_{
m i}}^2 = rac{1}{2}m{v_{
m f}}^2 + mg{h_{
m f}} + rac{1}{2}k{x_{
m f}}^2,$$

where h is the height (vertical position) and x is the compression of the spring. This general statement looks complex but becomes much simpler when we start considering specific situations. First, we must identify the initial and final conditions in a problem; then, we enter them into the last equation to solve for an unknown.

# Solution for (a)

This part of the problem is limited to conditions just before the car is released and just after it leaves the spring. Take the initial height to be zero, so that both  $h_{\rm i}$  and  $h_{\rm f}$  are zero. Furthermore, the initial speed  $v_{\rm i}$  is zero and the final compression of the spring  $x_{\rm f}$  is zero, and so several terms in the conservation of mechanical energy equation are zero and it simplifies to

## **Equation:**

$$rac{1}{2}k{x_{
m i}}^2 = rac{1}{2}m{v_{
m f}}^2.$$

In other words, the initial potential energy in the spring is converted completely to kinetic energy in the absence of friction. Solving for the final speed and entering known values yields

## **Equation:**

$$egin{array}{lll} v_{
m f} &=& \sqrt{rac{k}{m}} x_{
m i} \ &=& \sqrt{rac{250.0\ {
m N/m}}{0.100\ {
m kg}}} (0.0400\ {
m m}) \ &=& 2.00\ {
m m/s}. \end{array}$$

# Solution for (b)

One method of finding the speed at the top of the slope is to consider conditions just before the car is released and just after it reaches the top of the slope, completely ignoring everything in between. Doing the same type of analysis to find which terms are zero, the conservation of mechanical energy becomes

# **Equation:**

$$rac{1}{2}k{x_i}^2 = rac{1}{2}m{v_f}^2 + mgh_f.$$

This form of the equation means that the spring's initial potential energy is converted partly to gravitational potential energy and partly to kinetic energy. The final speed at the top of the slope will be less than at the bottom. Solving for  $v_{\rm f}$  and substituting known values gives

# **Equation:**

$$egin{array}{lll} v_{
m f} &=& \sqrt{rac{kx_{
m i}^{2}}{m}-2gh_{
m f}} \ &=& \sqrt{\left(rac{250.0~{
m N/m}}{0.100~{
m kg}}
ight)(0.0400~{
m m})^{2}-2(9.80~{
m m/s}^{2})(0.180~{
m m})} \ &=& 0.687~{
m m/s}. \end{array}$$

#### Discussion

Another way to solve this problem is to realize that the car's kinetic energy before it goes up the slope is converted partly to potential energy—that is, to take the final conditions in part (a) to be the initial conditions in part (b).

Note that, for conservative forces, we do not directly calculate the work they do; rather, we consider their effects through their corresponding potential energies, just as we did in [link]. Note also that we do not consider details of the path taken—only the starting and ending points are important (as long as the path is not impossible). This assumption is usually a tremendous simplification, because the path may be complicated and forces may vary along the way.

### Note:

## PhET Explorations: Energy Skate Park

Learn about conservation of energy with a skater dude! Build tracks, ramps and jumps for the skater and view the kinetic energy, potential energy and friction as he moves. You can also take the skater to different planets or even space!

https://phet.colorado.edu/sims/html/energy-skate-park-basics/latest/energy-skate-park-basics en.html

# **Section Summary**

- A conservative force is one for which work depends only on the starting and ending points of a motion, not on the path taken.
- We can define potential energy (PE) for any conservative force, just as we defined  $PE_g$  for the gravitational force.
- The potential energy of a spring is  $PE_s = \frac{1}{2}kx^2$ , where k is the spring's force constant and x is the displacement from its undeformed position.
- Mechanical energy is defined to be KE + PE for a conservative force.
- When only conservative forces act on and within a system, the total mechanical energy is constant. In equation form,

### **Equation:**

$$KE + PE = constant \label{eq:KE}$$
 or 
$$KE_i + PE_i = KE_f + PE_f \label{eq:KE}$$

where i and f denote initial and final values. This is known as the conservation of mechanical energy.

# **Conceptual Questions**

#### **Exercise:**

**Problem:** What is a conservative force?

#### **Exercise:**

#### **Problem:**

The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential energy of a diving board as a swimmer dives from it, starting just before the swimmer steps on the board until just after his feet leave it.

#### **Exercise:**

### **Problem:**

Define mechanical energy. What is the relationship of mechanical energy to nonconservative forces? What happens to mechanical energy if only conservative forces act?

### **Exercise:**

#### **Problem:**

What is the relationship of potential energy to conservative force?

### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

A  $5.00 \times 10^5$ -kg subway train is brought to a stop from a speed of 0.500 m/s in 0.400 m by a large spring bumper at the end of its track. What is the force constant k of the spring?

### **Solution:**

**Equation:** 

$$7.81 \times 10^5 \, \mathrm{N/m}$$

#### **Exercise:**

#### **Problem:**

A pogo stick has a spring with a force constant of  $2.50 \times 10^4$  N/m, which can be compressed 12.0 cm. To what maximum height can a child jump on the stick using only the energy in the spring, if the child and stick have a total mass of 40.0 kg? Explicitly show how you follow the steps in the <u>Problem-Solving Strategies for Energy</u>.

# **Glossary**

### conservative force

a force that does the same work for any given initial and final configuration, regardless of the path followed

### potential energy

energy due to position, shape, or configuration

# potential energy of a spring

the stored energy of a spring as a function of its displacement; when Hooke's law applies, it is given by the expression  $\frac{1}{2}kx^2$  where x is the distance the spring is compressed or extended and k is the spring constant

# conservation of mechanical energy

the rule that the sum of the kinetic energies and potential energies remains constant if only conservative forces act on and within a system

# mechanical energy

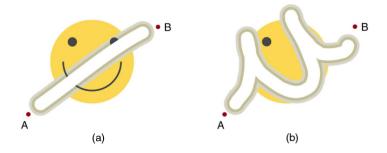
the sum of kinetic energy and potential energy

#### Nonconservative Forces

- Define nonconservative forces and explain how they affect mechanical energy.
- Show how the principle of conservation of energy can be applied by treating the conservative forces in terms of their potential energies and any nonconservative forces in terms of the work they do.

### **Nonconservative Forces and Friction**

Forces are either conservative or nonconservative. Conservative forces were discussed in <u>Conservative Forces and Potential Energy</u>. A **nonconservative force** is one for which work depends on the path taken. Friction is a good example of a nonconservative force. As illustrated in [link], work done against friction depends on the length of the path between the starting and ending points. Because of this dependence on path, there is no potential energy associated with nonconservative forces. An important characteristic is that the work done by a nonconservative force *adds or removes mechanical energy from a system*. **Friction**, for example, creates **thermal energy** that dissipates, removing energy from the system. Furthermore, even if the thermal energy is retained or captured, it cannot be fully converted back to work, so it is lost or not recoverable in that sense as well.

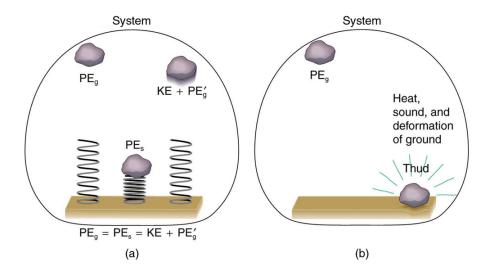


The amount of the happy face erased depends on the path taken by the eraser between points A and B, as does the work done against friction. Less work is done and less of the face

is erased for the path in (a) than for the path in (b). The force here is friction, and most of the work goes into thermal energy that subsequently leaves the system (the happy face plus the eraser). The energy expended cannot be fully recovered.

# **How Nonconservative Forces Affect Mechanical Energy**

*Mechanical* energy *may* not be conserved when nonconservative forces act. For example, when a car is brought to a stop by friction on level ground, it loses kinetic energy, which is dissipated as thermal energy, reducing its mechanical energy. [link] compares the effects of conservative and nonconservative forces. We often choose to understand simpler systems such as that described in [link](a) first before studying more complicated systems as in [link](b).



Comparison of the effects of conservative and nonconservative forces on the mechanical energy of a system. (a) A system with only conservative forces. When a rock is dropped onto a spring, its mechanical energy remains constant (neglecting air resistance) because the force in the spring is conservative. The spring can propel the rock back to its original height, where it once again has only potential energy due to gravity. (b) A system with nonconservative forces. When the same rock is dropped onto the ground, it is stopped by nonconservative forces that dissipate its mechanical energy as thermal energy, sound, and surface distortion. The rock has lost mechanical energy.

# **How the Work-Energy Theorem Applies**

Now let us consider what form the work-energy theorem takes when both conservative and nonconservative forces act. We will see that the work done by nonconservative forces equals the change in the mechanical energy of a system. As noted in <u>Kinetic Energy and the Work-Energy Theorem</u>, the work-energy theorem states that the net work on a system equals the change in its kinetic energy, or  $W_{\rm net} = \Delta KE$ . The net work is the sum of the work by nonconservative forces plus the work by conservative forces. That is, **Equation:** 

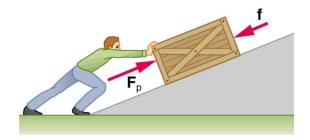
$$W_{\rm net} = W_{\rm nc} + W_{\rm c}$$

so that

# **Equation:**

$$W_{\rm nc} + W_{\rm c} = \Delta {\rm KE}$$

where  $W_{\rm nc}$  is the total work done by all nonconservative forces and  $W_{\rm c}$  is the total work done by all conservative forces.



A person pushes a crate up a ramp, doing work on the crate. Friction and gravitational force (not shown) also do work on the crate; both forces oppose the person's push. As the crate is pushed up the ramp, it gains mechanical energy, implying that the work done by the person is greater than the work done by friction.

Consider [link], in which a person pushes a crate up a ramp and is opposed by friction. As in the previous section, we note that work done by a conservative force comes from a loss of gravitational potential energy, so that  $W_{\rm c} = -\Delta {\rm PE}$ . Substituting this equation into the previous one and solving for  $W_{\rm nc}$  gives

## **Equation:**

$$W_{\rm nc} = \Delta {
m KE} + \Delta {
m PE}.$$

This equation means that the total mechanical energy (KE + PE) changes by exactly the amount of work done by nonconservative forces. In [link], this is the work done by the person minus the work done by friction. So even if energy is not conserved for the system of interest (such as the crate), we know that an equal amount of work was done to cause the change in total mechanical energy.

We rearrange  $W_{\rm nc} = \Delta {\rm KE} + \Delta {\rm PE}$  to obtain **Equation:** 

$$KE_i + PE_i + W_{nc} = KE_f + PE_f.$$

This means that the amount of work done by nonconservative forces adds to the mechanical energy of a system. If  $W_{\rm nc}$  is positive, then mechanical energy is increased, such as when the person pushes the crate up the ramp in [link]. If  $W_{\rm nc}$  is negative, then mechanical energy is decreased, such as when the rock hits the ground in [link](b). If  $W_{\rm nc}$  is zero, then mechanical energy is conserved, and nonconservative forces are balanced. For example, when you push a lawn mower at constant speed on level ground, your work done is removed by the work of friction, and the mower has a constant energy.

# **Applying Energy Conservation with Nonconservative Forces**

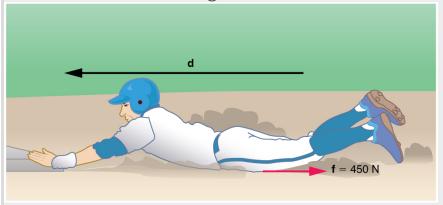
When no change in potential energy occurs, applying  $KE_i + PE_i + W_{nc} = KE_f + PE_f$  amounts to applying the work-energy theorem by setting the change in kinetic energy to be equal to the net work done on the system, which in the most general case includes both conservative and nonconservative forces. But when seeking instead to find a change in total mechanical energy in situations that involve changes in both potential and kinetic energy, the previous equation  $KE_i + PE_i + W_{nc} = KE_f + PE_f$  says that you can start by finding the change in mechanical energy that would have resulted from just the conservative forces, including the potential energy changes, and add to it the work done, with the proper sign, by any nonconservative forces involved.

## **Example:**

Calculating Distance Traveled: How Far a Baseball Player Slides

Consider the situation shown in [link], where a baseball player slides to a stop on level ground. Using energy considerations, calculate the distance

the 65.0-kg baseball player slides, given that his initial speed is 6.00 m/s and the force of friction against him is a constant 450 N.



The baseball player slides to a stop in a distance *d*. In the process, friction removes the player's kinetic energy by doing an amount of work fd equal to the initial kinetic energy.

## Strategy

Friction stops the player by converting his kinetic energy into other forms, including thermal energy. In terms of the work-energy theorem, the work done by friction, which is negative, is added to the initial kinetic energy to reduce it to zero. The work done by friction is negative, because  $\mathbf{f}$  is in the opposite direction of the motion (that is,  $\theta = 180^{\circ}$ , and so  $\cos \theta = -1$ ). Thus  $W_{\rm nc} = -\mathrm{fd}$ . The equation simplifies to

# **Equation:**

$$rac{1}{2}m{v_{\mathrm{i}}}^2-\mathrm{fd}=0$$

or

# **Equation:**

$$\mathrm{fd}=rac{1}{2}m{v_{\mathrm{i}}}^{2}.$$

This equation can now be solved for the distance d.

#### Solution

Solving the previous equation for d and substituting known values yields **Equation:** 

$$egin{array}{lcl} d & = & rac{m{v_{
m i}}^2}{2f} \ & = & rac{(65.0\ {
m kg})(6.00\ {
m m/s})^2}{(2)(450\ {
m N})} \ & = & 2.60\ {
m m.} \end{array}$$

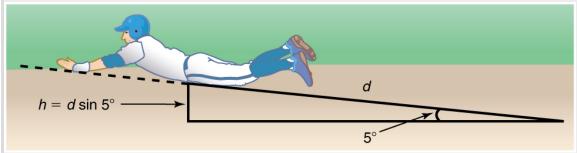
#### Discussion

The most important point of this example is that the amount of nonconservative work equals the change in mechanical energy. For example, you must work harder to stop a truck, with its large mechanical energy, than to stop a mosquito.

## **Example:**

## **Calculating Distance Traveled: Sliding Up an Incline**

Suppose that the player from [link] is running up a hill having a 5.00° incline upward with a surface similar to that in the baseball stadium. The player slides with the same initial speed, and the frictional force is still 450 N. Determine how far he slides.



The same baseball player slides to a stop on a  $5.00^{\circ}$  slope.

## Strategy

In this case, the work done by the nonconservative friction force on the player reduces the mechanical energy he has from his kinetic energy at zero height, to the final mechanical energy he has by moving through

distance d to reach height h along the hill, with  $h = d \sin 5.00^\circ$ . This is expressed by the equation

### **Equation:**

$$KE_i + PE_i + W_{nc} = KE_f + PE_f.$$

#### Solution

The work done by friction is again  $W_{\rm nc}=-{\rm fd}$ ; initially the potential energy is  ${\rm PE_i}={\rm mg}\cdot 0=0$  and the kinetic energy is  ${\rm KE_i}=\frac{1}{2}m{v_i}^2$ ; the final energy contributions are  ${\rm KE_f}=0$  for the kinetic energy and  ${\rm PE_f}={\rm mgh}={\rm mgd}\sin\theta$  for the potential energy. Substituting these values gives

### **Equation:**

$$rac{1}{2}m{v_{\mathrm{i}}}^2+0+\left(-fd
ight)=0+mgd\sin heta.$$

Solve this for d to obtain

### **Equation:**

$$egin{array}{lcl} d & = & rac{\left(rac{1}{2}
ight)m{v_{
m i}}^2}{f+mg\sin heta} \ & = & rac{(0.5)(65.0\,{
m kg})(6.00\,{
m m/s})^2}{450\,{
m N}+(65.0\,{
m kg})(9.80\,{
m m/s}^2)\sin{(5.00^{
m o})}} \ & = & 2.31\,{
m m}. \end{array}$$

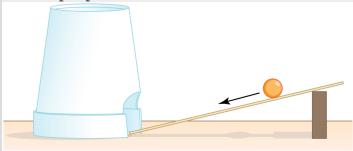
#### Discussion

As might have been expected, the player slides a shorter distance by sliding uphill. Note that the problem could also have been solved in terms of the forces directly and the work energy theorem, instead of using the potential energy. This method would have required combining the normal force and force of gravity vectors, which no longer cancel each other because they point in different directions, and friction, to find the net force. You could then use the net force and the net work to find the distance d that reduces the kinetic energy to zero. By applying conservation of energy and using the potential energy instead, we need only consider the gravitational potential energy mgh, without combining and resolving force vectors. This simplifies the solution considerably.

#### Note:

Making Connections: Take-Home Investigation—Determining Friction from the Stopping Distance

This experiment involves the conversion of gravitational potential energy into thermal energy. Use the ruler, book, and marble from <u>Take-Home</u> <u>Investigation—Converting Potential to Kinetic Energy</u>. In addition, you will need a foam cup with a small hole in the side, as shown in [link]. From the 10-cm position on the ruler, let the marble roll into the cup positioned at the bottom of the ruler. Measure the distance d the cup moves before stopping. What forces caused it to stop? What happened to the kinetic energy of the marble at the bottom of the ruler? Next, place the marble at the 20-cm and the 30-cm positions and again measure the distance the cup moves after the marble enters it. Plot the distance the cup moves versus the initial marble position on the ruler. Is this relationship linear? With some simple assumptions, you can use these data to find the coefficient of kinetic friction  $\mu_k$  of the cup on the table. The force of friction f on the cup is  $\mu_k N$ , where the normal force N is just the weight of the cup plus the marble. The normal force and force of gravity do no work because they are perpendicular to the displacement of the cup, which moves horizontally. The work done by friction is fd. You will need the mass of the marble as well to calculate its initial kinetic energy. It is interesting to do the above experiment also with a steel marble (or ball bearing). Releasing it from the same positions on the ruler as you did with the glass marble, is the velocity of this steel marble the same as the velocity of the marble at the bottom of the ruler? Is the distance the cup moves proportional to the mass of the steel and glass marbles?



Rolling a marble down a ruler into a foam cup.

#### Note:

## PhET Explorations: The Ramp

Explore forces, energy and work as you push household objects up and down a ramp. Lower and raise the ramp to see how the angle of inclination affects the parallel forces acting on the file cabinet. Graphs show forces, energy and work.

The Ram

# **Section Summary**

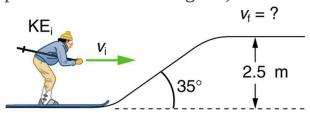
- A nonconservative force is one for which work depends on the path.
- Friction is an example of a nonconservative force that changes mechanical energy into thermal energy.
- Work  $W_{\rm nc}$  done by a nonconservative force changes the mechanical energy of a system. In equation form,  $W_{\rm nc} = \Delta {\rm KE} + \Delta {\rm PE}$  or, equivalently,  ${\rm KE_i} + {\rm PE_i} + W_{\rm nc} = {\rm KE_f} + {\rm PE_f}$ .
- When both conservative and nonconservative forces act, energy conservation can be applied and used to calculate motion in terms of the known potential energies of the conservative forces and the work done by nonconservative forces, instead of finding the net work from the net force, or having to directly apply Newton's laws.

### **Problems & Exercises**

#### **Exercise:**

### **Problem:**

A 60.0-kg skier with an initial speed of 12.0 m/s coasts up a 2.50-m-high rise as shown in [link]. Find her final speed at the top, given that the coefficient of friction between her skis and the snow is 0.0800. (Hint: Find the distance traveled up the incline assuming a straight-line path as shown in the figure.)



The skier's initial kinetic energy is partially used in coasting to the top of a rise.

### **Solution:**

 $9.46 \, \text{m/s}$ 

#### **Exercise:**

#### **Problem:**

(a) How high a hill can a car coast up (engine disengaged) if work done by friction is negligible and its initial speed is 110 km/h? (b) If, in actuality, a 750-kg car with an initial speed of 110 km/h is observed to coast up a hill to a height 22.0 m above its starting point, how much thermal energy was generated by friction? (c) What is the average force of friction if the hill has a slope 2.5° above the horizontal?

# Glossary

nonconservative force

a force whose work depends on the path followed between the given initial and final configurations

### friction

the force between surfaces that opposes one sliding on the other; friction changes mechanical energy into thermal energy

## Conservation of Energy

- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.
- Define efficiency of an energy conversion process as the fraction left as useful energy or work, rather than being transformed, for example, into thermal energy.

# **Law of Conservation of Energy**

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The **law of conservation of energy** can be stated as follows:

Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.

We have explored some forms of energy and some ways it can be transferred from one system to another. This exploration led to the definition of two major types of energy—mechanical energy (KE + PE) and energy transferred via work done by nonconservative forces ( $W_{\rm nc}$ ). But energy takes *many* other forms, manifesting itself in *many* different ways, and we need to be able to deal with all of these before we can write an equation for the above general statement of the conservation of energy.

# Other Forms of Energy than Mechanical Energy

At this point, we deal with all other forms of energy by lumping them into a single group called other energy (OE). Then we can state the conservation of energy in equation form as

# **Equation:**

$$KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f.$$

All types of energy and work can be included in this very general statement of conservation of energy. Kinetic energy is KE, work done by a conservative force is represented by PE, work done by nonconservative forces is  $W_{\rm nc}$ , and

all other energies are included as OE. This equation applies to all previous examples; in those situations OE was constant, and so it subtracted out and was not directly considered.

#### Note:

Making Connections: Usefulness of the Energy Conservation Principle
The fact that energy is conserved and has many forms makes it very
important. You will find that energy is discussed in many contexts, because it
is involved in all processes. It will also become apparent that many situations
are best understood in terms of energy and that problems are often most
easily conceptualized and solved by considering energy.

When does OE play a role? One example occurs when a person eats. Food is oxidized with the release of carbon dioxide, water, and energy. Some of this chemical energy is converted to kinetic energy when the person moves, to potential energy when the person changes altitude, and to thermal energy (another form of OE).

# Some of the Many Forms of Energy

What are some other forms of energy? You can probably name a number of forms of energy not yet discussed. Many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast in weapons.

Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

[link] gives the amount of energy stored, used, or released from various objects and in various phenomena. The range of energies and the variety of types and situations is impressive.

#### Note:

### Problem-Solving Strategies for Energy

You will find the following problem-solving strategies useful whenever you deal with energy. The strategies help in organizing and reinforcing energy concepts. In fact, they are used in the examples presented in this chapter. The familiar general problem-solving strategies presented earlier—involving identifying physical principles, knowns, and unknowns, checking units, and so on—continue to be relevant here.

**Step 1.** Determine the system of interest and identify what information is given and what quantity is to be calculated. A sketch will help.

**Step 2.** Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step 3 or step 4.

**Step 3.** If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is

# **Equation:**

$$KE_i + PE_i = KE_f + PE_f.$$

**Step 4.** If you know the potential energy for only some of the forces, possibly because some of them are nonconservative and do not have a potential energy, or if there are other energies that are not easily treated in terms of force and work, then the conservation of energy law in its most general form must be used.

## **Equation:**

$$KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f.$$

In most problems, one or more of the terms is zero, simplifying its solution. Do not calculate  $W_c$ , the work done by conservative forces; it is already incorporated in the PE terms.

**Step 5.** You have already identified the types of work and energy involved (in step 2). Before solving for the unknown, *eliminate terms wherever possible* to simplify the algebra. For example, choose h=0 at either the initial or final point, so that  $PE_{\rm g}$  is zero there. Then solve for the unknown in the customary manner.

**Step 6.** *Check the answer to see if it is reasonable.* Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on. Also check to see that the numerical value obtained is reasonable. For example, the final speed of a skateboarder who coasts down a 3-m-high ramp could reasonably be 20 km/h, but *not* 80 km/h.

# **Transformation of Energy**

The transformation of energy from one form into others is happening all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this section.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (see [link]) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.



Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. (credit: NASA)

Object/phenomenon	Energy in joules
Big Bang	$10^{68}$
Energy released in a supernova	$10^{44}$
Fusion of all the hydrogen in Earth's oceans	$10^{34}$
Annual world energy use	$4{ imes}10^{20}$

Object/phenomenon	Energy in joules
Large fusion bomb (9 megaton)	$3.8{ imes}10^{16}$
1 kg hydrogen (fusion to helium)	$6.4{\times}10^{14}$
1 kg uranium (nuclear fission)	$8.0{\times}10^{13}$
Hiroshima-size fission bomb (10 kiloton)	$4.2{\times}10^{13}$
90,000-ton aircraft carrier at 30 knots	$1.1{\times}10^{10}$
1 barrel crude oil	$5.9{ imes}10^9$
1 ton TNT	$4.2{ imes}10^9$
1 gallon of gasoline	$1.2{ imes}10^8$
Daily home electricity use (developed countries)	$7{ imes}10^7$
Daily adult food intake (recommended)	$1.2{\times}10^7$

Object/phenomenon	Energy in joules
1000-kg car at 90 km/h	$3.1{ imes}10^5$
1 g fat (9.3 kcal)	$3.9{\times}10^4$
ATP hydrolysis reaction	$3.2{ imes}10^4$
1 g carbohydrate (4.1 kcal)	$1.7{\times}10^4$
1 g protein (4.1 kcal)	$1.7{\times}10^4$
Tennis ball at 100 km/h	22
Mosquito $\left(10^{-2}~\mathrm{g~at~0.5~m/s}\right)$	$1.3{ imes}10^{-6}$
Single electron in a TV tube beam	$4.0{ imes}10^{-15}$
Energy to break one DNA strand	$10^{-19}$

Energy of Various Objects and Phenomena

# **Efficiency**

Even though energy is conserved in an energy conversion process, the output of *useful energy* or work will be less than the energy input. The **efficiency** Eff of an energy conversion process is defined as

## **Equation:**

$$\text{Efficiency(Eff)} = \frac{\text{useful energy or work output}}{\text{total energy input}} = \frac{W_{\text{out}}}{E_{\text{in}}}.$$

[link] lists some efficiencies of mechanical devices and human activities. In a coal-fired power plant, for example, about 40% of the chemical energy in the coal becomes useful electrical energy. The other 60% transforms into other (perhaps less useful) energy forms, such as thermal energy, which is then released to the environment through combustion gases and cooling towers.

Activity/device	Efficiency (%)[ <u>footnote</u> ] Representative values
Cycling and climbing	20
Swimming, surface	2
Swimming, submerged	4
Shoveling	3
Weightlifting	9
Steam engine	17
Gasoline engine	30

Activity/device	Efficiency (%)[footnote] Representative values
Diesel engine	35
Nuclear power plant	35
Coal power plant	42
Electric motor	98
Compact fluorescent light	20
Gas heater (residential)	90
Solar cell	10

Efficiency of the Human Body and Mechanical Devices

#### Note:

PhET Explorations: Masses and Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energies for each spring.

https://phet.colorado.edu/sims/mass-spring-lab/mass-spring-lab en.html

# **Section Summary**

- The law of conservation of energy states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.
- When all forms of energy are considered, conservation of energy is written in equation form as

 $KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f$ , where OE is all **other forms of energy** besides mechanical energy.

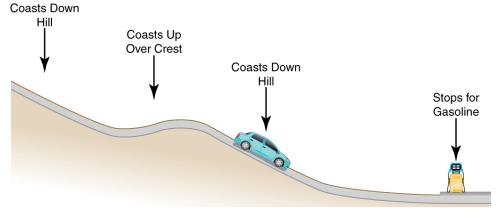
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.
- Energy is often utilized to do work, but it is not possible to convert all the energy of a system to work.
- The efficiency Eff of a machine or human is defined to be  $\mathrm{Eff} = \frac{W_{\mathrm{out}}}{E_{\mathrm{in}}}$ , where  $W_{\mathrm{out}}$  is useful work output and  $E_{\mathrm{in}}$  is the energy consumed.

# **Conceptual Questions**

#### **Exercise:**

### **Problem:**

Consider the following scenario. A car for which friction is *not* negligible accelerates from rest down a hill, running out of gasoline after a short distance. The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events. (See [link].)



A car experiencing non-negligible friction coasts down a hill, over a small crest, then downhill again, and comes to a stop at a gas station.

### **Exercise:**

#### **Problem:**

Describe the energy transfers and transformations for a javelin, starting from the point at which an athlete picks up the javelin and ending when the javelin is stuck into the ground after being thrown.

#### **Exercise:**

#### **Problem:**

Do devices with efficiencies of less than one violate the law of conservation of energy? Explain.

#### **Exercise:**

#### **Problem:**

List four different forms or types of energy. Give one example of a conversion from each of these forms to another form.

#### **Exercise:**

**Problem:** List the energy conversions that occur when riding a bicycle.

### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

Using values from [link], how many DNA molecules could be broken by the energy carried by a single electron in the beam of an old-fashioned TV tube? (These electrons were not dangerous in themselves, but they did create dangerous x rays. Later model tube TVs had shielding that absorbed x rays before they escaped and exposed viewers.)

#### **Solution:**

 $4 \times 10^4$  molecules

### **Exercise:**

#### **Problem:**

Using energy considerations and assuming negligible air resistance, show that a rock thrown from a bridge 20.0 m above water with an initial speed of 15.0 m/s strikes the water with a speed of 24.8 m/s independent of the direction thrown.

#### **Solution:**

Equating  $\Delta PE_g$  and  $\Delta KE$ , we obtain

$$v = \sqrt{2 ext{gh} + {v_0}^2} = \sqrt{2(9.80 ext{ m/s}^2)(20.0 ext{ m}) + (15.0 ext{ m/s})^2} = 24.8 ext{ m/s}$$

#### **Exercise:**

#### **Problem:**

If the energy in fusion bombs were used to supply the energy needs of the world, how many of the 9-megaton variety would be needed for a year's supply of energy (using data from [link])? This is not as far-fetched as it may sound—there are thousands of nuclear bombs, and their energy can be trapped in underground explosions and converted to electricity, as natural geothermal energy is.

#### Exercise:

#### **Problem:**

(a) Use of hydrogen fusion to supply energy is a dream that may be realized in the next century. Fusion would be a relatively clean and almost limitless supply of energy, as can be seen from [link]. To illustrate this, calculate how many years the present energy needs of the world could be supplied by one millionth of the oceans' hydrogen fusion energy. (b) How does this time compare with historically significant events, such as the duration of stable economic systems?

#### **Solution:**

(a) 
$$25 \times 10^6$$
 years

(b) This is much, much longer than human time scales.

# Glossary

### law of conservation of energy

the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same

### electrical energy

the energy carried by a flow of charge

## chemical energy

the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

### radiant energy

the energy carried by electromagnetic waves

### nuclear energy

energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

## thermal energy

the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

## efficiency

a measure of the effectiveness of the input of energy to do work; useful energy or work divided by the total input of energy

### Power

- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

### What is Power?

*Power*—the word conjures up many images: a professional football player muscling aside his opponent, a dragster roaring away from the starting line, a volcano blowing its lava into the atmosphere, or a rocket blasting off, as in [link].



This powerful rocket on the Space Shuttle *Endeavor* did work and consumed energy at a very high rate. (credit: NASA)

These images of power have in common the rapid performance of work, consistent with the scientific definition of **power** (P) as the rate at which work is done.

Note:

Power

Power is the rate at which work is done.

**Equation:** 

$$P=rac{W}{t}$$

The SI unit for power is the **watt** (W), where 1 watt equals 1 joule/second (1 W = 1 J/s).

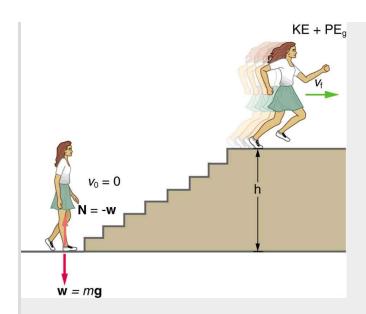
Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

# **Calculating Power from Energy**

# **Example:**

# **Calculating the Power to Climb Stairs**

What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See [link].)



When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

## **Strategy and Concept**

The work going into mechanical energy is W = KE + PE. At the bottom of the stairs, we take both KE and  $PE_g$  as initially zero; thus,

 $W = \mathrm{KE_f} + \mathrm{PE_g} = \frac{1}{2} m v_\mathrm{f}^2 + mgh$ , where h is the vertical height of the stairs. Because all terms are given, we can calculate W and then divide it by time to get power.

#### Solution

Substituting the expression for W into the definition of power given in the previous equation, P=W/t yields

# **Equation:**

$$P=rac{W}{t}=rac{rac{1}{2}m{v_{\mathrm{f}}}^2+mgh}{t}.$$

Entering known values yields

## **Equation:**

$$P = rac{0.5(60.0 \text{ kg})(2.00 \text{ m/s})^2 + (60.0 \text{ kg})(9.80 \text{ m/s}^2)(3.00 \text{ m})}{3.50 \text{ s}}$$
 $= rac{120 \text{ J} + 1764 \text{ J}}{3.50 \text{ s}}$ 
 $= 538 \text{ W}.$ 

#### Discussion

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 horsepower (1 hp=746~W)! People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

#### Note:

Making Connections: Take-Home Investigation—Measure Your Power Rating

Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Don't expect that your output will be more than about 0.5 hp.

# **Examples of Power**

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. (See [link] for some examples.) Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter  $(kW/m^2)$ . A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40% of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1000 megawatts; 1 megawatt (MW) is  $10^6$  W of electric power. But the power plant consumes chemical energy at a rate of about 2500 MW, creating heat transfer to the surroundings at a rate of 1500 MW. (See [link].)



Tremendous amounts of electric power are generated by coalfired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings.

The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like. (credit: Kleinolive, Wikimedia Commons)

Object or Phenomenon	Power in Watts
Supernova (at peak)	$5{ imes}10^{37}$
Milky Way galaxy	$10^{37}$
Crab Nebula pulsar	$10^{28}$
The Sun	$4{ imes}10^{26}$

Object or Phenomenon	Power in Watts
Volcanic eruption (maximum)	$4{ imes}10^{15}$
Lightning bolt	$2{\times}10^{12}$
Nuclear power plant (total electric and heat transfer)	$3{ imes}10^9$
Aircraft carrier (total useful and heat transfer)	$10^8$
Dragster (total useful and heat transfer)	$2{ imes}10^6$
Car (total useful and heat transfer)	$8{ imes}10^4$
Football player (total useful and heat transfer)	$5{ imes}10^3$
Clothes dryer	$4{ imes}10^3$
Person at rest (all heat transfer)	100

Object or Phenomenon	Power in Watts
Typical incandescent light bulb (total useful and heat transfer)	60
Heart, person at rest (total useful and heat transfer)	8
Electric clock	3
Pocket calculator	$10^{-3}$

Power Output or Consumption

# **Power and Energy Consumption**

We usually have to pay for the energy we use. It is interesting and easy to estimate the cost of energy for an electrical appliance if its power consumption rate and time used are known. The higher the power consumption rate and the longer the appliance is used, the greater the cost of that appliance. The power consumption rate is P = W/t = E/t, where E is the energy supplied by the electricity company. So the energy consumed over a time t is

# **Equation:**

$$E = Pt$$
.

Electricity bills state the energy used in units of **kilowatt-hours**  $(kW \cdot h)$ , which is the product of power in kilowatts and time in hours. This unit is convenient because electrical power consumption at the kilowatt level for hours at a time is typical.

# **Example:**

# **Calculating Energy Costs**

What is the cost of running a 0.200-kW computer 6.00 h per day for 30.0 d if the cost of electricity is 0.120 per kW  $\cdot$  h?

# Strategy

Cost is based on energy consumed; thus, we must find E from  $E = \operatorname{Pt}$  and then calculate the cost. Because electrical energy is expressed in  $kW \cdot h$ , at the start of a problem such as this it is convenient to convert the units into kW and hours.

## Solution

The energy consumed in  $kW \cdot h$  is

# **Equation:**

$$E = \text{Pt} = (0.200 \,\text{kW})(6.00 \,\text{h/d})(30.0 \,\text{d})$$
  
= 36.0 kW · h,

and the cost is simply given by

# **Equation:**

$$cost = (36.0 \text{ kW} \cdot \text{h})(\$0.120 \text{ per kW} \cdot \text{h}) = \$4.32 \text{ per month.}$$

#### Discussion

The cost of using the computer in this example is neither exorbitant nor negligible. It is clear that the cost is a combination of power and time. When both are high, such as for an air conditioner in the summer, the cost is high.

The motivation to save energy has become more compelling with its ever-increasing price. Armed with the knowledge that energy consumed is the product of power and time, you can estimate costs for yourself and make the necessary value judgments about where to save energy. Either power or time must be reduced. It is most cost-effective to limit the use of high-power devices that normally operate for long periods of time, such as water heaters and air conditioners. This would not include relatively high power devices like toasters, because they are on only a few minutes per day. It would also not include electric clocks, in spite of their 24-hour-per-day

usage, because they are very low power devices. It is sometimes possible to use devices that have greater efficiencies—that is, devices that consume less power to accomplish the same task. One example is the compact fluorescent light bulb, which produces over four times more light per watt of power consumed than its incandescent cousin.

Modern civilization depends on energy, but current levels of energy consumption and production are not sustainable. The likelihood of a link between global warming and fossil fuel use (with its concomitant production of carbon dioxide), has made reduction in energy use as well as a shift to non-fossil fuels of the utmost importance. Even though energy in an isolated system is a conserved quantity, the final result of most energy transformations is waste heat transfer to the environment, which is no longer useful for doing work. As we will discuss in more detail in <a href="https://december/>Thermodynamics">Thermodynamics</a>, the potential for energy to produce useful work has been "degraded" in the energy transformation.

# **Section Summary**

- Power is the rate at which work is done, or in equation form, for the average power P for work W done over a time t, P = W/t.
- The SI unit for power is the watt (W), where 1 W = 1 J/s.
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where  $1\ hp=746\ W$ .

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.

Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours rather than joules. What is the relationship between these two energy units?

#### **Exercise:**

#### **Problem:**

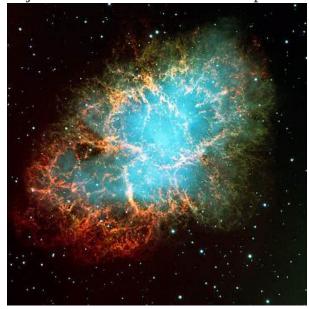
A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.

# **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

The Crab Nebula (see [link]) pulsar is the remnant of a supernova that occurred in A.D. 1054. Using data from [link], calculate the approximate factor by which the power output of this astronomical object has declined since its explosion.



Crab Nebula (credit: ESO, via Wikimedia Commons)

Solution: Equation:

 $2{\times}10^{-10}$ 

#### **Exercise:**

# **Problem:**

Suppose a star 1000 times brighter than our Sun (that is, emitting 1000 times the power) suddenly goes supernova. Using data from [link]: (a) By what factor does its power output increase? (b) How many times brighter than our entire Milky Way galaxy is the supernova? (c) Based on your answers, discuss whether it should be possible to observe supernovas in distant galaxies. Note that there are on the order of 10<sup>11</sup> observable galaxies, the average brightness of which is somewhat less than our own galaxy.

#### **Exercise:**

## **Problem:**

A person in good physical condition can put out 100 W of useful power for several hours at a stretch, perhaps by pedaling a mechanism that drives an electric generator. Neglecting any problems of generator efficiency and practical considerations such as resting time: (a) How many people would it take to run a 4.00-kW electric clothes dryer? (b) How many people would it take to replace a large electric power plant that generates 800 MW?

# **Solution:**

(a) 40

(b) 8 million

# **Exercise:**

## **Problem:**

What is the cost of operating a 3.00-W electric clock for a year if the cost of electricity is 0.0900 per kW  $\cdot$  h?

#### **Exercise:**

# **Problem:**

A large household air conditioner may consume 15.0 kW of power. What is the cost of operating this air conditioner 3.00 h per day for 30.0 d if the cost of electricity is \$0.110 per kW · h?

## **Solution:**

\$149

## **Exercise:**

#### **Problem:**

(a) What is the average power consumption in watts of an appliance that uses  $5.00~\mathrm{kW}\cdot\mathrm{h}$  of energy per day? (b) How many joules of energy does this appliance consume in a year?

#### **Exercise:**

# **Problem:**

(a) What is the average useful power output of a person who does  $6.00\times10^6~\rm J$  of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)

#### **Solution:**

(a) 208 W

(b) 141 s

# **Exercise:**

#### **Problem:**

A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m (about a quarter of a mile) and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?

## **Exercise:**

#### **Problem:**

(a) How long will it take an 850-kg car with a useful power output of 40.0 hp (1 hp = 746 W) to reach a speed of 15.0 m/s, neglecting friction? (b) How long will this acceleration take if the car also climbs a 3.00-m-high hill in the process?

# **Solution:**

- (a) 3.20 s
- (b) 4.04 s

#### **Exercise:**

#### **Problem:**

(a) Find the useful power output of an elevator motor that lifts a 2500-kg load a height of 35.0 m in 12.0 s, if it also increases the speed from rest to 4.00 m/s. Note that the total mass of the counterbalanced system is 10,000 kg—so that only 2500 kg is raised in height, but the full 10,000 kg is accelerated. (b) What does it cost, if electricity is \$0.0900 per  $kW \cdot h$ ?

(a) What is the available energy content, in joules, of a battery that operates a 2.00-W electric clock for 18 months? (b) How long can a battery that can supply  $8.00\times10^4~\mathrm{J}$  run a pocket calculator that consumes energy at the rate of  $1.00\times10^{-3}~\mathrm{W}$ ?

#### **Solution:**

- (a)  $9.46 \times 10^7 \text{ J}$
- (b) 2.54 y

#### **Exercise:**

## **Problem:**

(a) How long would it take a  $1.50 \times 10^5$ -kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible? (b) If it actually takes 900 s, what is the power? (c) Given this power, what is the average force of air resistance if the airplane takes 1200 s? (Hint: You must find the distance the plane travels in 1200 s assuming constant acceleration.)

#### **Exercise:**

#### **Problem:**

Calculate the power output needed for a 950-kg car to climb a 2.00° slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N. Explicitly show how you follow the steps in the <u>Problem-Solving Strategies for Energy</u>.

#### **Solution:**

Identify knowns: m=950 kg, slope angle  $\theta=2.00^{\circ},\,v=3.00$  m/s, f=600 N

Identify unknowns: power P of the car, force F that car applies to road

Solve for unknown:

$$P = \frac{W}{t} = \frac{\mathrm{Fd}}{t} = F(\frac{d}{t}) = \mathrm{Fv},$$

where F is parallel to the incline and must oppose the resistive forces and the force of gravity:

$$F = f + w = 600 \text{ N} + \text{mg sin } \theta$$

Insert this into the expression for power and solve:

$$P = (f + \text{mg sin } \theta)v$$
  
=  $\left[600 \text{ N} + (950 \text{ kg}) \left(9.80 \text{ m/s}^2\right) \text{sin } 2^{\circ}\right] (30.0 \text{ m/s})$   
=  $2.77 \times 10^4 \text{ W}$ 

About 28 kW (or about 37 hp) is reasonable for a car to climb a gentle incline.

#### **Exercise:**

# **Problem:**

(a) Calculate the power per square meter reaching Earth's upper atmosphere from the Sun. (Take the power output of the Sun to be  $4.00\times10^{26}~\rm W.$ ) (b) Part of this is absorbed and reflected by the atmosphere, so that a maximum of  $1.30~\rm kW/m^2$  reaches Earth's surface. Calculate the area in km² of solar energy collectors needed to replace an electric power plant that generates 750 MW if the collectors convert an average of 2.00% of the maximum power into electricity. (This small conversion efficiency is due to the devices themselves, and the fact that the sun is directly overhead only briefly.) With the same assumptions, what area would be needed to meet the United States' energy needs  $(1.05\times10^{20}~\rm J)$ ? Australia's energy needs  $(5.4\times10^{18}~\rm J)$ ? China's energy needs  $(6.3\times10^{19}~\rm J)$ ? (These energy consumption values are from 2006.)

# Glossary

power

the rate at which work is done

watt

(W) SI unit of power, with 1  $W=1~\mathrm{J/s}$ 

horsepower

an older non-SI unit of power, with 1  $\mathrm{hp} = 746~\mathrm{W}$ 

kilowatt-hour

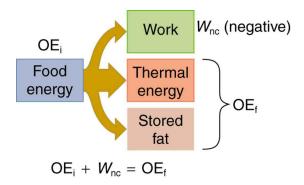
 $(\mathbf{k}\mathbf{W}\cdot\mathbf{h})$  unit used primarily for electrical energy provided by electric utility companies

# Work, Energy, and Power in Humans

- Explain the human body's consumption of energy when at rest vs. when engaged in activities that do useful work.
- Calculate the conversion of chemical energy in food into useful work.

# **Energy Conversion in Humans**

Our own bodies, like all living organisms, are energy conversion machines. Conservation of energy implies that the chemical energy stored in food is converted into work, thermal energy, and/or stored as chemical energy in fatty tissue. (See [link].) The fraction going into each form depends both on how much we eat and on our level of physical activity. If we eat more than is needed to do work and stay warm, the remainder goes into body fat.



Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity.

# **Power Consumed at Rest**

The *rate* at which the body uses food energy to sustain life and to do different activities is called the **metabolic rate**. The total energy conversion rate of a person *at rest* is called the **basal metabolic rate** (BMR) and is divided among various systems in the body, as shown in [link]. The largest fraction goes to the liver and spleen, with the brain coming next. Of course, during vigorous exercise, the energy consumption of the skeletal muscles and heart increase markedly. About 75% of the calories burned in a day go into these basic functions. The BMR is a function of age, gender, total body weight, and amount of muscle mass (which burns more calories than body fat). Athletes have a greater BMR due to this last factor.

Organ	Power consumed at rest (W)	Oxygen consumption (mL/min)	Percent of BMR
Liver & spleen	23	67	27
Brain	16	47	19
Skeletal muscle	15	45	18
Kidney	9	26	10
Heart	6	17	7
Other	16	48	19
Totals	85 W	250 mL/min	100%

# Basal Metabolic Rates (BMR)

Energy consumption is directly proportional to oxygen consumption because the digestive process is basically one of oxidizing food. We can measure the energy people use during various activities by measuring their oxygen use. (See [link].) Approximately 20 kJ of energy are produced for each liter of oxygen consumed, independent of the type of food. [link] shows energy and oxygen consumption rates (power expended) for a variety of activities.

# **Power of Doing Useful Work**

Work done by a person is sometimes called **useful work**, which is *work done on the outside world*, such as lifting weights. Useful work requires a force exerted through a distance on the outside world, and so it excludes internal work, such as that done by the heart when pumping blood. Useful work does include that done in climbing stairs or accelerating to a full run, because these are accomplished by exerting forces on the outside world. Forces exerted by the body are nonconservative, so that they can change the mechanical energy (KE + PE) of the system worked upon, and this is often the goal. A baseball player throwing a ball, for example, increases both the ball's kinetic and potential energy.

If a person needs more energy than they consume, such as when doing vigorous work, the body must draw upon the chemical energy stored in fat. So exercise can be helpful in losing fat. However, the amount of exercise needed to produce a loss in fat, or to burn off extra calories consumed that day, can be large, as [link] illustrates.

# **Example:**

# **Calculating Weight Loss from Exercising**

If a person who normally requires an average of 12,000 kJ (3000 kcal) of food energy per day consumes 13,000 kJ per day, he will steadily gain weight. How much bicycling per day is required to work off this extra 1000 kJ?

## Solution

[link] states that 400 W are used when cycling at a moderate speed. The time required to work off 1000 kJ at this rate is then

# **Equation:**

$$ext{Time} = rac{ ext{energy}}{\left(rac{ ext{energy}}{ ext{time}}
ight)} = rac{1000 ext{ kJ}}{400 ext{ W}} = 2500 ext{ s} = 42 ext{ min}.$$

#### Discussion

If this person uses more energy than he or she consumes, the person's body will obtain the needed energy by metabolizing body fat. If the person uses 13,000 kJ but consumes only 12,000 kJ, then the amount of fat loss will be

# **Equation:**

$${
m Fat \ loss} = (1000 \ {
m kJ}) igg( rac{1.0 \ {
m g \ fat}}{39 \ {
m kJ}} igg) = 26 \ {
m g},$$

assuming the energy content of fat to be 39 kJ/g.



A pulse oxymeter is an apparatus that measures the amount of oxygen in blood.
Oxymeters can be used to determine a person's metabolic rate, which is the rate at which food energy is converted to another form. Such

measurements can indicate the level of athletic conditioning as well as certain medical problems. (credit: UusiAjaja, Wikimedia Commons)

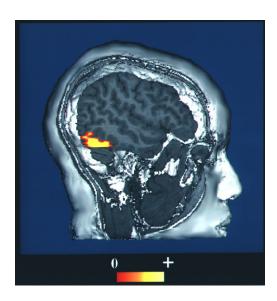
Activity	Energy consumption in watts	Oxygen consumption in liters O <sub>2</sub> /min
Sleeping	83	0.24
Sitting at rest	120	0.34
Standing relaxed	U 1/5	
Sitting in class	210	0.60
Walking (5 km/h)	280	0.80
Cycling (13–18 km/h)	400	1.14
Shivering	425	1.21
Playing tennis	440	1.26

Activity	Energy consumption in watts	Oxygen consumption in liters O <sub>2</sub> /min		
Swimming breaststroke	475	1.36		
Ice skating (14.5 km/h)	545	1.56		
Climbing stairs (116/min)	685	1.96		
Cycling (21 km/h)	700	2.00		
Running cross- country	740	2.12		
Playing basketball	800	2.28		
Cycling, professional racer	1855	5.30		
Sprinting	2415	6.90		

Energy and Oxygen Consumption Rates[<u>footnote</u>] (Power) for an average 76-kg male

All bodily functions, from thinking to lifting weights, require energy. (See [link].) The many small muscle actions accompanying all quiet activity, from sleeping to head scratching, ultimately become thermal energy, as do less visible muscle actions by the heart, lungs, and digestive tract. Shivering, in fact, is an involuntary response to low body temperature that pits muscles against one another to produce thermal energy in the body (and

do no work). The kidneys and liver consume a surprising amount of energy, but the biggest surprise of all is that a full 25% of all energy consumed by the body is used to maintain electrical potentials in all living cells. (Nerve cells use this electrical potential in nerve impulses.) This bioelectrical energy ultimately becomes mostly thermal energy, but some is utilized to power chemical processes such as in the kidneys and liver, and in fat production.



This fMRI scan shows an increased level of energy consumption in the vision center of the brain. Here, the patient was being asked to recognize faces. (credit: NIH via Wikimedia Commons)

# **Section Summary**

- The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue.
- The *rate* at which the body uses food energy to sustain life and to do different activities is called the metabolic rate, and the corresponding rate when at rest is called the basal metabolic rate (BMR)
- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next.
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate.
- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food.

# **Conceptual Questions**

#### **Exercise:**

## **Problem:**

Explain why it is easier to climb a mountain on a zigzag path rather than one straight up the side. Is your increase in gravitational potential energy the same in both cases? Is your energy consumption the same in both?

#### **Exercise:**

#### **Problem:**

Do you do work on the outside world when you rub your hands together to warm them? What is the efficiency of this activity?

#### **Exercise:**

#### **Problem:**

Shivering is an involuntary response to lowered body temperature. What is the efficiency of the body when shivering, and is this a desirable value?

Discuss the relative effectiveness of dieting and exercise in losing weight, noting that most athletic activities consume food energy at a rate of 400 to 500 W, while a single cup of yogurt can contain 1360 kJ (325 kcal). Specifically, is it likely that exercise alone will be sufficient to lose weight? You may wish to consider that regular exercise may increase the metabolic rate, whereas protracted dieting may reduce it.

# **Problems & Exercises**

## **Exercise:**

# **Problem:**

(a) How long can you rapidly climb stairs (116/min) on the 93.0 kcal of energy in a 10.0-g pat of butter? (b) How many flights is this if each flight has 16 stairs?

# **Solution:**

- (a) 9.5 min
- (b) 69 flights of stairs

#### **Exercise:**

#### **Problem:**

(a) What is the power output in watts and horsepower of a 70.0-kg sprinter who accelerates from rest to 10.0 m/s in 3.00 s? (b) Considering the amount of power generated, do you think a well-trained athlete could do this repetitively for long periods of time?

Calculate the power output in watts and horsepower of a shot-putter who takes 1.20 s to accelerate the 7.27-kg shot from rest to 14.0 m/s, while raising it 0.800 m. (Do not include the power produced to accelerate his body.)



Shot putter at the Dornoch Highland Gathering in 2007. (credit: John Haslam, Flickr)

# **Solution:**

641 W, 0.860 hp

#### **Exercise:**

## **Problem:**

(a) What is the efficiency of an out-of-condition professor who does  $2.10\times10^5~\rm J$  of useful work while metabolizing 500 kcal of food energy? (b) How many food calories would a well-conditioned athlete metabolize in doing the same work with an efficiency of 20%?

Energy that is not utilized for work or heat transfer is converted to the chemical energy of body fat containing about 39 kJ/g. How many grams of fat will you gain if you eat 10,000 kJ (about 2500 kcal) one day and do nothing but sit relaxed for 16.0 h and sleep for the other 8.00 h? Use data from [link] for the energy consumption rates of these activities.

# **Solution:**

31 g

#### **Exercise:**

## **Problem:**

Using data from [link], calculate the daily energy needs of a person who sleeps for 7.00 h, walks for 2.00 h, attends classes for 4.00 h, cycles for 2.00 h, sits relaxed for 3.00 h, and studies for 6.00 h. (Studying consumes energy at the same rate as sitting in class.)

#### **Exercise:**

#### **Problem:**

What is the efficiency of a subject on a treadmill who puts out work at the rate of 100 W while consuming oxygen at the rate of 2.00 L/min? (Hint: See [link].)

#### **Solution:**

14.3%

Shoveling snow can be extremely taxing because the arms have such a low efficiency in this activity. Suppose a person shoveling a footpath metabolizes food at the rate of 800 W. (a) What is her useful power output? (b) How long will it take her to lift 3000 kg of snow 1.20 m? (This could be the amount of heavy snow on 20 m of footpath.) (c) How much waste heat transfer in kilojoules will she generate in the process?

# **Exercise:**

#### **Problem:**

Very large forces are produced in joints when a person jumps from some height to the ground. (a) Calculate the magnitude of the force produced if an 80.0-kg person jumps from a 0.600-m-high ledge and lands stiffly, compressing joint material 1.50 cm as a result. (Be certain to include the weight of the person.) (b) In practice the knees bend almost involuntarily to help extend the distance over which you stop. Calculate the magnitude of the force produced if the stopping distance is 0.300 m. (c) Compare both forces with the weight of the person.

## **Solution:**

- (a)  $3.21 \times 10^4 \text{ N}$
- (b)  $2.35 \times 10^3 \text{ N}$
- (c) Ratio of net force to weight of person is 41.0 in part (a); 3.00 in part (b)

Jogging on hard surfaces with insufficiently padded shoes produces large forces in the feet and legs. (a) Calculate the magnitude of the force needed to stop the downward motion of a jogger's leg, if his leg has a mass of 13.0 kg, a speed of 6.00 m/s, and stops in a distance of 1.50 cm. (Be certain to include the weight of the 75.0-kg jogger's body.) (b) Compare this force with the weight of the jogger.

#### **Exercise:**

#### **Problem:**

(a) Calculate the energy in kJ used by a 55.0-kg woman who does 50 deep knee bends in which her center of mass is lowered and raised 0.400 m. (She does work in both directions.) You may assume her efficiency is 20%. (b) What is the average power consumption rate in watts if she does this in 3.00 min?

# **Solution:**

- (a) 108 kJ
- (b) 599 W

#### **Exercise:**

#### **Problem:**

Kanellos Kanellopoulos flew 119 km from Crete to Santorini, Greece, on April 23, 1988, in the *Daedalus 88*, an aircraft powered by a bicycle-type drive mechanism (see [link]). His useful power output for the 234-min trip was about 350 W. Using the efficiency for cycling from [link], calculate the food energy in kilojoules he metabolized during the flight.

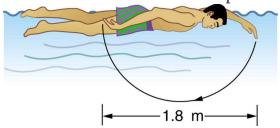


The Daedalus 88 in flight. (credit: NASA photo by Beasley)

# **Exercise:**

# **Problem:**

The swimmer shown in [link] exerts an average horizontal backward force of 80.0 N with his arm during each 1.80 m long stroke. (a) What is his work output in each stroke? (b) Calculate the power output of his arms if he does 120 strokes per minute.



# **Solution:**

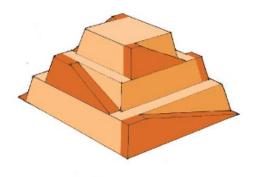
- (a) 144 J
- (b) 288 W

Mountain climbers carry bottled oxygen when at very high altitudes. (a) Assuming that a mountain climber uses oxygen at twice the rate for climbing 116 stairs per minute (because of low air temperature and winds), calculate how many liters of oxygen a climber would need for 10.0 h of climbing. (These are liters at sea level.) Note that only 40% of the inhaled oxygen is utilized; the rest is exhaled. (b) How much useful work does the climber do if he and his equipment have a mass of 90.0 kg and he gains 1000 m of altitude? (c) What is his efficiency for the 10.0-h climb?

#### **Exercise:**

## **Problem:**

The awe-inspiring Great Pyramid of Cheops was built more than 4500 years ago. Its square base, originally 230 m on a side, covered 13.1 acres, and it was 146 m high, with a mass of about  $7 \times 10^9$  kg. (The pyramid's dimensions are slightly different today due to quarrying and some sagging.) Historians estimate that 20,000 workers spent 20 years to construct it, working 12-hour days, 330 days per year. (a) Calculate the gravitational potential energy stored in the pyramid, given its center of mass is at one-fourth its height. (b) Only a fraction of the workers lifted blocks; most were involved in support services such as building ramps (see [link]), bringing food and water, and hauling blocks to the site. Calculate the efficiency of the workers who did the lifting, assuming there were 1000 of them and they consumed food energy at the rate of 300 kcal/h. What does your answer imply about how much of their work went into block-lifting, versus how much work went into friction and lifting and lowering their own bodies? (c) Calculate the mass of food that had to be supplied each day, assuming that the average worker required 3600 kcal per day and that their diet was 5% protein, 60% carbohydrate, and 35% fat. (These proportions neglect the mass of bulk and nondigestible materials consumed.)



Ancient pyramids were probably constructed using ramps as simple machines. (credit: Franck Monnier, Wikimedia Commons)

# **Solution:**

- (a)  $2.50 \times 10^{12} \, \mathrm{J}$
- (b) 2.52%
- (c)  $1.4 \times 10^4$  kg (14 metric tons)

## **Exercise:**

# **Problem:**

(a) How long can you play tennis on the 800 kJ (about 200 kcal) of energy in a candy bar? (b) Does this seem like a long time? Discuss why exercise is necessary but may not be sufficient to cause a person to lose weight.

# Glossary

# metabolic rate

the rate at which the body uses food energy to sustain life and to do different activities

# basal metabolic rate the total energy conversion rate of a person at rest

useful work work done on an external system

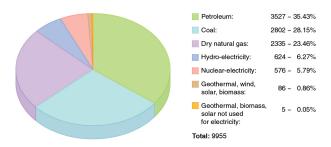
#### World Energy Use

- Describe the distinction between renewable and nonrenewable energy sources.
- Explain why the inevitable conversion of energy to less useful forms makes it necessary to conserve energy resources.

Energy is an important ingredient in all phases of society. We live in a very interdependent world, and access to adequate and reliable energy resources is crucial for economic growth and for maintaining the quality of our lives. But current levels of energy consumption and production are not sustainable. About 40% of the world's energy comes from oil, and much of that goes to transportation uses. Oil prices are dependent as much upon new (or foreseen) discoveries as they are upon political events and situations around the world. The U.S., with 4.5% of the world's population, consumes 24% of the world's oil production per year; 66% of that oil is imported!

#### **Renewable and Nonrenewable Energy Sources**

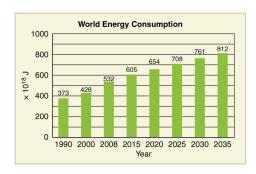
The principal energy resources used in the world are shown in [link]. The fuel mix has changed over the years but now is dominated by oil, although natural gas and solar contributions are increasing. **Renewable forms of energy** are those sources that cannot be used up, such as water, wind, solar, and biomass. About 85% of our energy comes from nonrenewable **fossil fuels**—oil, natural gas, coal. The likelihood of a link between global warming and fossil fuel use, with its production of carbon dioxide through combustion, has made, in the eyes of many scientists, a shift to non-fossil fuels of utmost importance—but it will not be easy.



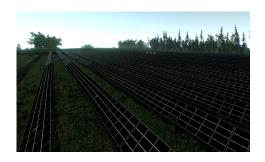
World energy consumption by source, in billions of kilowatt-hours: 2006. (credit: KVDP)

#### The World's Growing Energy Needs

World energy consumption continues to rise, especially in the developing countries. (See [link].) Global demand for energy has tripled in the past 50 years and might triple again in the next 30 years. While much of this growth will come from the rapidly booming economies of China and India, many of the developed countries, especially those in Europe, are hoping to meet their energy needs by expanding the use of renewable sources. Although presently only a small percentage, renewable energy is growing very fast, especially wind energy. For example, Germany plans to meet 20% of its electricity and 10% of its overall energy needs with renewable resources by the year 2020. (See [link].) Energy is a key constraint in the rapid economic growth of China and India. In 2003, China surpassed Japan as the world's second largest consumer of oil. However, over 1/3 of this is imported. Unlike most Western countries, coal dominates the commercial energy resources of China, accounting for 2/3 of its energy consumption. In 2009 China surpassed the United States as the largest generator of CO<sub>2</sub>. In India, the main energy resources are biomass (wood and dung) and coal. Half of India's oil is imported. About 70% of India's electricity is generated by highly polluting coal. Yet there are sizeable strides being made in renewable energy. India has a rapidly growing wind energy base, and it has the largest solar cooking program in the world.



Past and projected world energy use (source: Based on data from U.S. Energy Information Administration, 2011)



Solar cell arrays at a power plant in Steindorf, Germany (credit: Michael Betke, Flickr)

[link] displays the 2006 commercial energy mix by country for some of the prime energy users in the world. While non-renewable sources dominate, some countries get a sizeable percentage of their electricity from renewable resources. For example, about 67% of New Zealand's electricity demand is met by hydroelectric. Only 10% of the U.S. electricity is generated by renewable resources, primarily hydroelectric. It is difficult to determine total contributions of renewable energy in some countries with a large rural population, so these percentages in this table are left blank.

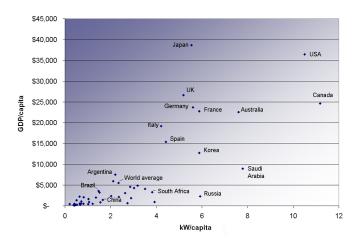
Country	Consumption, in EJ (10 <sup>18</sup> J)	Oil	Natural Gas	Coal	Nuclear	Hydro	Other Renewables
Australia	5.4	34%	17%	44%	0%	3%	1%

Country	Consumption, in EJ (10 <sup>18</sup> J)	Oil	Natural Gas	Coal	Nuclear	Hydro	Other Renewables
Brazil	9.6	48%	7%	5%	1%	35%	2%
China	63	22%	3%	69%	1%	6%	
Egypt	2.4	50%	41%	1%	0%	6%	
Germany	16	37%	24%	24%	11%	1%	3%
India	15	34%	7%	52%	1%	5%	
Indonesia	4.9	51%	26%	16%	0%	2%	3%
Japan	24	48%	14%	21%	12%	4%	1%
New Zealand	0.44	32%	26%	6%	0%	11%	19%
Russia	31	19%	53%	16%	5%	6%	
U.S.	105	40%	23%	22%	8%	3%	1%
World	432	39%	23%	24%	6%	6%	2%

Energy Consumption—Selected Countries (2006)

## **Energy and Economic Well-being**

The last two columns in this table examine the energy and electricity use per capita. Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (gross domestic product) per capita, are matched by higher levels of energy consumption per capita. This is borne out in [link]. Increased efficiency of energy use will change this dependency. A global problem is balancing energy resource development against the harmful effects upon the environment in its extraction and use.



Power consumption per capita versus GDP per capita for various countries. Note the increase in energy usage with increasing GDP. (2007, credit: Frank van Mierlo, Wikimedia Commons)

#### **Conserving Energy**

As we finish this chapter on energy and work, it is relevant to draw some distinctions between two sometimes misunderstood terms in the area of energy use. As has been mentioned elsewhere, the "law of the conservation of energy" is a very useful principle in analyzing physical processes. It is a statement that cannot be proven from basic principles, but is a very good bookkeeping device, and no exceptions have ever been found. It states that the total amount of energy in an isolated system will always remain constant. Related to this principle, but remarkably different from it, is the important philosophy of energy conservation. This concept has to do with seeking to decrease the amount of energy used by an individual or group through (1) reduced activities (e.g., turning down thermostats, driving fewer kilometers) and/or (2) increasing conversion efficiencies in the performance of a particular task—such as developing and using more efficient room heaters, cars that have greater miles-per-gallon ratings, energy-efficient compact fluorescent lights, etc.

Since energy in an isolated system is not destroyed or created or generated, one might wonder why we need to be concerned about our energy resources, since energy is a conserved quantity. The problem is that the final result of most energy transformations is waste heat transfer to the environment and conversion to energy forms no longer useful for doing work. To state it in another way, the potential for energy to produce useful work has been "degraded" in the energy transformation. (This will be discussed in more detail in <a href="https://decap.press.org/linearized-number-15">Thermodynamics</a>.)

#### **Section Summary**

- The relative use of different fuels to provide energy has changed over the years, but fuel use is currently dominated by oil, although natural gas and solar contributions are increasing.
- Although non-renewable sources dominate, some countries meet a sizeable percentage of their electricity needs from renewable resources.
- The United States obtains only about 10% of its energy from renewable sources, mostly hydroelectric power.
- Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (Gross Domestic Product) per capita, are matched by higher levels of energy consumption per capita.
- Even though, in accordance with the law of conservation of energy, energy can never be created or destroyed, energy that can be used to do work is always partly converted to less useful forms, such as waste heat to the environment, in all of our uses of energy for practical purposes.

#### **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

What is the difference between energy conservation and the law of conservation of energy? Give some examples of each.

#### **Exercise:**

#### **Problem:**

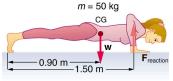
If the efficiency of a coal-fired electrical generating plant is 35%, then what do we mean when we say that energy is a conserved quantity?

#### **Problems & Exercises**

#### **Exercise:**

#### **Problem:** Integrated Concepts

(a) Calculate the force the woman in [link] exerts to do a push-up at constant speed, taking all data to be known to three digits. (b) How much work does she do if her center of mass rises 0.240 m? (c) What is her useful power output if she does 25 push-ups in 1 min? (Should work done lowering her body be included? See the discussion of useful work in Work, Energy, and Power in Humans.



Forces involved in doing push-ups. The woman's weight acts as a force exerted downward on her center of gravity (CG).

#### **Solution:**

- (a) 294 N
- (b) 118 J
- (c) 49.0 W

#### **Exercise:**

#### **Problem:** Integrated Concepts

A 75.0-kg cross-country skier is climbing a  $3.0^{\circ}$  slope at a constant speed of 2.00 m/s and encounters air resistance of 25.0 N. Find his power output for work done against the gravitational force and air resistance. (b) What average force does he exert backward on the snow to accomplish this? (c) If he continues to exert this force and to experience the same air resistance when he reaches a level area, how long will it take him to reach a velocity of 10.0 m/s?

#### **Exercise:**

#### **Problem:** Integrated Concepts

The 70.0-kg swimmer in [link] starts a race with an initial velocity of 1.25 m/s and exerts an average force of 80.0 N backward with his arms during each 1.80 m long stroke. (a) What is his initial acceleration if water resistance is 45.0 N? (b) What is the subsequent average resistance force from the water during the 5.00 s it takes him to reach his top velocity of 2.50 m/s? (c) Discuss whether water resistance seems to increase linearly with velocity.

#### **Solution:**

- (a)  $0.500 \text{ m/s}^2$
- (b) 62.5 N
- (c) Assuming the acceleration of the swimmer decreases linearly with time over the 5.00 s interval, the frictional force must therefore be increasing linearly with time, since f = F ma. If the acceleration decreases linearly with time, the velocity will contain a term dependent on time squared ( $t^2$ ). Therefore, the water resistance will not depend linearly on the velocity.

#### **Exercise:**

#### **Problem:** Integrated Concepts

A toy gun uses a spring with a force constant of 300 N/m to propel a 10.0-g steel ball. If the spring is compressed 7.00 cm and friction is negligible: (a) How much force is needed to compress the spring? (b) To what maximum height can the ball be shot? (c) At what angles above the horizontal may a child aim to hit a target 3.00 m away at the same height as the gun? (d) What is the gun's maximum range on level ground?

#### Exercise:

#### **Problem:** Integrated Concepts

(a) What force must be supplied by an elevator cable to produce an acceleration of  $0.800~\mathrm{m/s}^2$  against a 200-N frictional force, if the mass of the loaded elevator is 1500 kg? (b) How much work is done by the cable in lifting the elevator 20.0 m? (c) What is the final speed of the elevator if it starts from rest? (d) How much work went into thermal energy?

#### **Solution:**

- (a)  $16.1 \times 10^3 \text{ N}$
- (b)  $3.22 \times 10^5 \text{ J}$
- (c) 5.66 m/s
- (d) 4.00 kJ

#### **Exercise:**

#### **Problem:** Unreasonable Results

A car advertisement claims that its 900-kg car accelerated from rest to 30.0 m/s and drove 100 km, gaining 3.00 km in altitude, on 1.0 gal of gasoline. The average force of friction including air resistance was 700 N. Assume all values are known to three significant figures. (a) Calculate the car's efficiency. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

#### **Exercise:**

#### **Problem:** Unreasonable Results

Body fat is metabolized, supplying 9.30 kcal/g, when dietary intake is less than needed to fuel metabolism. The manufacturers of an exercise bicycle claim that you can lose 0.500 kg of fat per day by vigorously exercising for 2.00 h per day on their machine. (a) How many kcal are supplied by the metabolization of 0.500 kg of fat? (b) Calculate the kcal/min that you would have to utilize to metabolize fat at the rate of 0.500 kg in 2.00 h. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?

#### **Solution:**

- (a)  $4.65 \times 10^3 \text{ kcal}$
- (b) 38.8 kcal/min
- (c) This power output is higher than the highest value on [link], which is about 35 kcal/min (corresponding to 2415 watts) for sprinting.
- (d) It would be impossible to maintain this power output for 2 hours (imagine sprinting for 2 hours!).

#### **Exercise:**

#### **Problem:** Construct Your Own Problem

Consider a person climbing and descending stairs. Construct a problem in which you calculate the long-term rate at which stairs can be climbed considering the mass of the person, his ability to generate power with his legs, and the height of a single stair step. Also consider why the same person can descend stairs at a faster rate for a nearly unlimited time in spite of the fact that very similar forces are exerted going down as going up. (This points to a fundamentally different process for descending versus climbing stairs.)

#### Exercise:

#### **Problem:** Construct Your Own Problem

Consider humans generating electricity by pedaling a device similar to a stationary bicycle. Construct a problem in which you determine the number of people it would take to replace a large electrical generation facility. Among the things to consider are the power output that is reasonable using the legs, rest time, and the need for electricity 24 hours per day. Discuss the practical implications of your results.

#### Exercise:

#### **Problem:** Integrated Concepts

A 105-kg basketball player crouches down 0.400 m while waiting to jump. After exerting a force on the floor through this 0.400 m, his feet leave the floor and his center of gravity rises 0.950 m above its normal standing erect position. (a) Using energy considerations, calculate his velocity when he leaves the floor. (b) What average force did he exert on the floor? (Do not neglect the force to support his weight as well as that to accelerate him.) (c) What was his power output during the acceleration phase?

#### **Solution:**

- (a) 4.32 m/s
- (b)  $3.47 \times 10^3 \text{ N}$
- (c) 8.93 kW

# Glossary

 $\begin{array}{c} \text{renewable forms of energy} \\ \text{those sources that cannot be used up, such as water, wind, solar, and biomass} \end{array}$ 

fossil fuels oil, natural gas, and coal

# Introduction to Linear Momentum and Collisions class="introduction"

"Each rugby player has great momentum , which will affect the outcome of their collisions with each other and the ground. (credit: vjpaul, Flickr)"



We use the term momentum in various ways in everyday language, and most of these ways are consistent with its precise scientific definition. We speak of sports teams or politicians gaining and maintaining the momentum to win. We also recognize that momentum has something to do with collisions. For example, looking at the rugby players in the photograph colliding and falling to the ground, we expect their momenta to have great effects in the resulting collisions. Generally, momentum implies a tendency to continue on course—to move in the same direction—and is associated with great mass and speed.

Momentum, like energy, is important because it is conserved. Only a few physical quantities are conserved in nature, and studying them yields fundamental insight into how nature works, as we shall see in our study of momentum.

#### Linear Momentum and Force

- Define linear momentum.
- Explain the relationship between momentum and force.
- State Newton's second law of motion in terms of momentum.
- Calculate momentum given mass and velocity.

### **Linear Momentum**

The scientific definition of linear momentum is consistent with most people's intuitive understanding of momentum: a large, fast-moving object has greater momentum than a smaller, slower object. **Linear momentum** is defined as the product of a system's mass multiplied by its velocity. In symbols, linear momentum is expressed as

# **Equation:**

$$\mathbf{p}=m\mathbf{v}.$$

Momentum is directly proportional to the object's mass and also its velocity. Thus the greater an object's mass or the greater its velocity, the greater its momentum. Momentum  $\mathbf{p}$  is a vector having the same direction as the velocity  $\mathbf{v}$ . The SI unit for momentum is  $kg \cdot m/s$ .

### Note:

#### Linear Momentum

Linear momentum is defined as the product of a system's mass multiplied by its velocity:

# **Equation:**

$$\mathbf{p}=m\mathbf{v}.$$

# **Example:**

# Calculating Momentum: A Football Player and a Football

(a) Calculate the momentum of a 110-kg football player running at 8.00 m/s. (b) Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s.

### Strategy

No information is given regarding direction, and so we can calculate only the magnitude of the momentum, p. (As usual, a symbol that is in italics is a magnitude, whereas one that is italicized, boldfaced, and has an arrow is a vector.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum given in the equation, which becomes

# **Equation:**

$$p = mv$$

when only magnitudes are considered.

### Solution for (a)

To determine the momentum of the player, substitute the known values for the player's mass and speed into the equation.

# **Equation:**

$$p_{\rm player} = (110 \text{ kg})(8.00 \text{ m/s}) = 880 \text{ kg} \cdot \text{m/s}$$

# Solution for (b)

To determine the momentum of the ball, substitute the known values for the ball's mass and speed into the equation.

# **Equation:**

$$p_{\mathrm{ball}} = (0.410 \ \mathrm{kg})(25.0 \ \mathrm{m/s}) = 10.3 \ \mathrm{kg \cdot m/s}$$

The ratio of the player's momentum to that of the ball is

# **Equation:**

$$rac{p_{
m player}}{p_{
m ball}} = rac{880}{10.3} = 85.9.$$

### Discussion

Although the ball has greater velocity, the player has a much greater mass. Thus the momentum of the player is much greater than the momentum of the football, as you might guess. As a result, the player's motion is only slightly affected if he catches the ball. We shall quantify what happens in such collisions in terms of momentum in later sections.

### Momentum and Newton's Second Law

The importance of momentum, unlike the importance of energy, was recognized early in the development of classical physics. Momentum was deemed so important that it was called the "quantity of motion." Newton actually stated his **second law of motion** in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes. Using symbols, this law is

### **Equation:**

$$\mathbf{F}_{ ext{net}} = rac{\Delta \mathbf{p}}{\Delta t},$$

where  $\mathbf{F}_{\mathrm{net}}$  is the net external force,  $\Delta \mathbf{p}$  is the change in momentum, and  $\Delta t$  is the change in time.

### Note:

### Newton's Second Law of Motion in Terms of Momentum

The net external force equals the change in momentum of a system divided by the time over which it changes.

# **Equation:**

$$\mathbf{F}_{ ext{net}} = rac{\Delta \mathbf{p}}{\Delta t}$$

#### Note:

# Making Connections: Force and Momentum

Force and momentum are intimately related. Force acting over time can change momentum, and Newton's second law of motion, can be stated in its most broadly applicable form in terms of momentum. Momentum continues to be a key concept in the study of atomic and subatomic particles in quantum mechanics.

This statement of Newton's second law of motion includes the more familiar  $\mathbf{F}_{\rm net} = m\mathbf{a}$  as a special case. We can derive this form as follows. First, note that the change in momentum  $\Delta \mathbf{p}$  is given by

### **Equation:**

$$\Delta \mathbf{p} = \Delta(m\mathbf{v}).$$

If the mass of the system is constant, then

### **Equation:**

$$\Delta(m\mathbf{v})=m\Delta\mathbf{v}.$$

So that for constant mass, Newton's second law of motion becomes **Equation:** 

$$\mathbf{F}_{
m net} = rac{\Delta \mathbf{p}}{\Delta t} = rac{m \Delta \mathbf{v}}{\Delta t}.$$

Because  $\frac{\Delta \mathbf{v}}{\Delta t} = \mathbf{a}$ , we get the familiar equation **Equation**:

$$\mathbf{F}_{\mathrm{net}} = m\mathbf{a}$$

when the mass of the system is constant.

Newton's second law of motion stated in terms of momentum is more generally applicable because it can be applied to systems where the mass is changing, such as rockets, as well as to systems of constant mass. We will consider systems with varying mass in some detail; however, the relationship between momentum and force remains useful when mass is constant, such as in the following example.

# **Example:**

# Calculating Force: Venus Williams' Racquet

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet, assuming that the ball's speed just after impact is 58 m/s, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms (milliseconds)?

### Strategy

This problem involves only one dimension because the ball starts from having no horizontal velocity component before impact. Newton's second law stated in terms of momentum is then written as

# **Equation:**

$$\mathbf{F}_{ ext{net}} = rac{\Delta \mathbf{p}}{\Delta t}.$$

As noted above, when mass is constant, the change in momentum is given by

# **Equation:**

$$\Delta p = m \Delta v = m (v_{
m f} - v_{
m i}).$$

In this example, the velocity just after impact and the change in time are given; thus, once  $\Delta p$  is calculated,  $F_{\rm net}=\frac{\Delta p}{\Delta t}$  can be used to find the force.

#### Solution

To determine the change in momentum, substitute the values for the initial and final velocities into the equation above.

### **Equation:**

$$egin{array}{lll} \Delta p &=& m(v_{
m f}\!-\!v_{
m i}) \ &=& (0.057~{
m kg})(58~{
m m/s}\!-\!0~{
m m/s}) \ &=& 3.306~{
m kg}\cdot{
m m/s} pprox 3.3~{
m kg}\cdot{
m m/s} \end{array}$$

Now the magnitude of the net external force can determined by using  $F_{\rm net} = \frac{\Delta p}{\Delta t}$ :

# **Equation:**

$$egin{array}{lcl} F_{
m net} & = & rac{\Delta p}{\Delta t} = rac{3.306 \ {
m kg \cdot m/s}}{5.0 imes 10^{-3} \ {
m s}} \ & = & 661 \ {
m N} pprox 660 \ {
m N}, \end{array}$$

where we have retained only two significant figures in the final step.

#### Discussion

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact (note that the ball also experienced the 0.56-N force of gravity, but that force was not due to the racquet). This problem could also be solved by first finding the acceleration and then using  $F_{\rm net} = {\rm ma}$ , but one additional step would be required compared with the strategy used in this example.

# **Section Summary**

- Linear momentum (*momentum* for brevity) is defined as the product of a system's mass multiplied by its velocity.
- In symbols, linear momentum p is defined to be Equation:

$$\mathbf{p} = m\mathbf{v}$$
,

where m is the mass of the system and  $\mathbf{v}$  is its velocity.

• The SI unit for momentum is  $kg \cdot m/s$ .

- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes.
- In symbols, Newton's second law of motion is defined to be Equation:

$$\mathbf{F}_{ ext{net}} = rac{\Delta \mathbf{p}}{\Delta t},$$

 ${\bf F}_{\rm net}$  is the net external force,  $\Delta {\bf p}$  is the change in momentum, and  $\Delta t$  is the change time.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

### **Exercise:**

#### **Problem:**

An object that has a small mass and an object that has a large mass have the same kinetic energy. Which mass has the largest momentum?

#### Exercise:

# **Problem: Professional Application**

Football coaches advise players to block, hit, and tackle with their feet on the ground rather than by leaping through the air. Using the concepts of momentum, work, and energy, explain how a football player can be more effective with his feet on the ground.

#### **Exercise:**

### **Problem:**

How can a small force impart the same momentum to an object as a large force?

### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

(a) Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of  $7.50~\mathrm{m/s}$ . (b) Compare the elephant's momentum with the momentum of a 0.0400-kg tranquilizer dart fired at a speed of  $600~\mathrm{m/s}$ . (c) What is the momentum of the 90.0-kg hunter running at  $7.40~\mathrm{m/s}$  after missing the elephant?

### **Solution:**

- (a)  $1.50 \times 10^4 \ \mathrm{kg \cdot m/s}$
- (b) 625 to 1
- (c)  $6.66 \times 10^2 \text{ kg} \cdot \text{m/s}$

#### **Exercise:**

### **Problem:**

(a) What is the mass of a large ship that has a momentum of  $1.60 \times 10^9~{\rm kg\cdot m/s}$ , when the ship is moving at a speed of  $48.0~{\rm km/h?}$  (b) Compare the ship's momentum to the momentum of a 1100-kg artillery shell fired at a speed of  $1200~{\rm m/s}$ .

### Exercise:

### **Problem:**

(a) At what speed would a  $2.00 \times 10^4$ -kg airplane have to fly to have a momentum of  $1.60 \times 10^9$  kg  $\cdot$  m/s (the same as the ship's momentum in the problem above)? (b) What is the plane's momentum when it is taking off at a speed of 60.0 m/s? (c) If the ship is an aircraft carrier that launches these airplanes with a catapult, discuss the implications of your answer to (b) as it relates to recoil effects of the catapult on the ship.

#### **Solution:**

- (a)  $8.00 \times 10^4 \text{ m/s}$
- (b)  $1.20 \times 10^6 \text{ kg} \cdot \text{m/s}$
- (c) Because the momentum of the airplane is 3 orders of magnitude smaller than of the ship, the ship will not recoil very much. The recoil would be -0.0100 m/s, which is probably not noticeable.

#### Exercise:

#### **Problem:**

(a) What is the momentum of a garbage truck that is  $1.20\times10^4~{\rm kg}$  and is moving at  $10.0~{\rm m/s?}$  (b) At what speed would an 8.00-kg trash can have the same momentum as the truck?

### **Exercise:**

#### Problem:

A runaway train car that has a mass of 15,000 kg travels at a speed of 5.4 m/s down a track. Compute the time required for a force of 1500 N to bring the car to rest.

### **Solution:**

### **Exercise:**

### **Problem:**

The mass of Earth is  $5.972\times10^{24}~{\rm kg}$  and its orbital radius is an average of  $1.496\times10^{11}~{\rm m}$ . Calculate its linear momentum.

# Glossary

linear momentum the product of mass and velocity

# second law of motion

physical law that states that the net external force equals the change in momentum of a system divided by the time over which it changes

# **Impulse**

- Define impulse.
- Describe effects of impulses in everyday life.
- Determine the average effective force using graphical representation.
- Calculate average force and impulse given mass, velocity, and time.

The effect of a force on an object depends on how long it acts, as well as how great the force is. In [link], a very large force acting for a short time had a great effect on the momentum of the tennis ball. A small force could cause the same **change in momentum**, but it would have to act for a much longer time. For example, if the ball were thrown upward, the gravitational force (which is much smaller than the tennis racquet's force) would eventually reverse the momentum of the ball. Quantitatively, the effect we are talking about is the change in momentum  $\Delta \mathbf{p}$ .

By rearranging the equation  $\mathbf{F}_{\mathrm{net}} = rac{\Delta \mathbf{p}}{\Delta t}$  to be

# **Equation:**

$$\Delta \mathbf{p} = \mathbf{F}_{\rm net} \Delta t$$

we can see how the change in momentum equals the average net external force multiplied by the time this force acts. The quantity  $\mathbf{F}_{\rm net}\Delta t$  is given the name **impulse**. Impulse is the same as the change in momentum.

#### Note:

Impulse: Change in Momentum

Change in momentum equals the average net external force multiplied by the time this force acts.

# **Equation:**

$$\Delta \mathbf{p} = \mathbf{F}_{\mathrm{net}} \Delta t$$

The quantity  $\mathbf{F}_{\mathrm{net}}\Delta t$  is given the name impulse.

There are many ways in which an understanding of impulse can save lives, or at least limbs. The dashboard padding in a car, and certainly the airbags, allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant, whether an air bag is deployed or not, but the force (to bring the occupant to a stop) will be much less if it acts over a larger time. Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

Bones in a body will fracture if the force on them is too large. If you jump onto the floor from a table, the force on your legs can be immense if you land stiff-legged on a hard surface. Rolling on the ground after jumping from the table, or landing with a parachute, extends the time over which the force (on you from the ground) acts.

# **Example:**

# Calculating Magnitudes of Impulses: Two Billiard Balls Striking a Rigid Wall

Two identical billiard balls strike a rigid wall with the same speed, and are reflected without any change of speed. The first ball strikes perpendicular to the wall. The second ball strikes the wall at an angle of 30° from the perpendicular, and bounces off at an angle of 30° from perpendicular to the wall.

- (a) Determine the direction of the force on the wall due to each ball.
- (b) Calculate the ratio of the magnitudes of impulses on the two balls by the wall.

# Strategy for (a)

In order to determine the force on the wall, consider the force on the ball due to the wall using Newton's second law and then apply Newton's third law to determine the direction. Assume the x-axis to be normal to the wall and to be positive in the initial direction of motion. Choose the y-axis to be

along the wall in the plane of the second ball's motion. The momentum direction and the velocity direction are the same.

# Solution for (a)

The first ball bounces directly into the wall and exerts a force on it in the +x direction. Therefore the wall exerts a force on the ball in the -x direction. The second ball continues with the same momentum component in the y direction, but reverses its x-component of momentum, as seen by sketching a diagram of the angles involved and keeping in mind the proportionality between velocity and momentum.

These changes mean the change in momentum for both balls is in the -x direction, so the force of the wall on each ball is along the -x direction.

# Strategy for (b)

Calculate the change in momentum for each ball, which is equal to the impulse imparted to the ball.

# Solution for (b)

Let u be the speed of each ball before and after collision with the wall, and m the mass of each ball. Choose the x-axis and y-axis as previously described, and consider the change in momentum of the first ball which strikes perpendicular to the wall.

# **Equation:**

$$p_{
m xi}={
m mu}; p_{
m yi}=0$$

# **Equation:**

$$p_{\mathrm{xf}} = -\mathrm{mu}; p_{\mathrm{vf}} = 0$$

Impulse is the change in momentum vector. Therefore the x-component of impulse is equal to -2mu and the y-component of impulse is equal to zero.

Now consider the change in momentum of the second ball.

# **Equation:**

$$p_{
m xi} = mu \cos 30^{
m o}; p_{
m yi} = -mu \sin 30^{
m o}$$

# **Equation:**

$$p_{
m xf} = -mu \cos 30^{
m o}; p_{
m yf} = -mu \sin 30^{
m o}$$

It should be noted here that while  $p_{\rm x}$  changes sign after the collision,  $p_{\rm y}$  does not. Therefore the x-component of impulse is equal to  $-2mu\cos 30^{\rm o}$  and the y-component of impulse is equal to zero.

The ratio of the magnitudes of the impulse imparted to the balls is

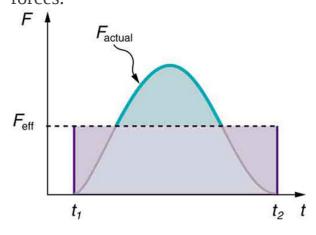
# **Equation:**

$$\frac{2\text{mu}}{2mu\cos 30^{\circ}} = \frac{2}{\sqrt{3}} = 1.155.$$

### Discussion

The direction of impulse and force is the same as in the case of (a); it is normal to the wall and along the negative x-direction. Making use of Newton's third law, the force on the wall due to each ball is normal to the wall along the positive x-direction.

Our definition of impulse includes an assumption that the force is constant over the time interval  $\Delta t$ . Forces are usually not constant. Forces vary considerably even during the brief time intervals considered. It is, however, possible to find an average effective force  $F_{\rm eff}$  that produces the same result as the corresponding time-varying force. [link] shows a graph of what an actual force looks like as a function of time for a ball bouncing off the floor. The area under the curve has units of momentum and is equal to the impulse or change in momentum between times  $t_1$  and  $t_2$ . That area is equal to the area inside the rectangle bounded by  $F_{\rm eff}$ ,  $t_1$ , and  $t_2$ . Thus the impulses and their effects are the same for both the actual and effective forces.



A graph of force versus time with time along the x-axis and force along the y-axis for an actual force and an equivalent effective force. The areas under the two curves are equal.

### **Note:**

Making Connections: Take-Home Investigation—Hand Movement and Impulse

Try catching a ball while "giving" with the ball, pulling your hands toward your body. Then, try catching a ball while keeping your hands still. Hit water in a tub with your full palm. After the water has settled, hit the water again by diving your hand with your fingers first into the water. (Your full palm represents a swimmer doing a belly flop and your diving hand represents a swimmer doing a dive.) Explain what happens in each case and why. Which orientations would you advise people to avoid and why?

#### Note:

Making Connections: Constant Force and Constant Acceleration
The assumption of a constant force in the definition of impulse is
analogous to the assumption of a constant acceleration in kinematics. In
both cases, nature is adequately described without the use of calculus.

# **Section Summary**

• Impulse, or change in momentum, equals the average net external force multiplied by the time this force acts:

# **Equation:**

$$\Delta \mathbf{p} = \mathbf{F}_{\rm net} \Delta t.$$

• Forces are usually not constant over a period of time.

# **Conceptual Questions**

#### **Exercise:**

# **Problem: Professional Application**

Explain in terms of impulse how padding reduces forces in a collision. State this in terms of a real example, such as the advantages of a carpeted vs. tile floor for a day care center.

#### **Exercise:**

### **Problem:**

While jumping on a trampoline, sometimes you land on your back and other times on your feet. In which case can you reach a greater height and why?

#### **Exercise:**

# **Problem:** Professional Application

Tennis racquets have "sweet spots." If the ball hits a sweet spot then the player's arm is not jarred as much as it would be otherwise. Explain why this is the case.

### **Problems & Exercises**

### **Exercise:**

### **Problem:**

A bullet is accelerated down the barrel of a gun by hot gases produced in the combustion of gun powder. What is the average force exerted on a 0.0300-kg bullet to accelerate it to a speed of 600 m/s in a time of 2.00 ms (milliseconds)?

#### **Solution:**

 $9.00 \times 10^{3} \text{ N}$ 

#### **Exercise:**

# **Problem: Professional Application**

A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seat belt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.

### **Exercise:**

#### **Problem:**

A person slaps her leg with her hand, bringing her hand to rest in 2.50 milliseconds from an initial speed of 4.00 m/s. (a) What is the average force exerted on the leg, taking the effective mass of the hand and forearm to be 1.50 kg? (b) Would the force be any different if the woman clapped her hands together at the same speed and brought them to rest in the same time? Explain why or why not.

#### **Solution:**

- a)  $2.40 \times 10^3~N$  toward the leg
- b) The force on each hand would have the same magnitude as that found in part (a) (but in opposite directions by Newton's third law) because the change in momentum and the time interval are the same.

#### **Exercise:**

# **Problem: Professional Application**

A professional boxer hits his opponent with a 1000-N horizontal blow that lasts for 0.150 s. (a) Calculate the impulse imparted by this blow. (b) What is the opponent's final velocity, if his mass is 105 kg and he is motionless in midair when struck near his center of mass? (c) Calculate the recoil velocity of the opponent's 10.0-kg head if hit in this manner, assuming the head does not initially transfer significant momentum to the boxer's body. (d) Discuss the implications of your answers for parts (b) and (c).

### **Exercise:**

# **Problem: Professional Application**

Suppose a child drives a bumper car head on into the side rail, which exerts a force of 4000 N on the car for 0.200 s. (a) What impulse is imparted by this force? (b) Find the final velocity of the bumper car if its initial velocity was 2.80 m/s and the car plus driver have a mass of 200 kg. You may neglect friction between the car and floor.

#### **Solution:**

- a)  $800 \text{ kg} \cdot \text{m/s}$  away from the wall
- b) 1.20 m/s away from the wall

#### **Exercise:**

# **Problem: Professional Application**

One hazard of space travel is debris left by previous missions. There are several thousand objects orbiting Earth that are large enough to be detected by radar, but there are far greater numbers of very small objects, such as flakes of paint. Calculate the force exerted by a 0.100-mg chip of paint that strikes a spacecraft window at a relative speed of  $4.00 \times 10^3$  m/s, given the collision lasts  $6.00 \times 10^{-8}$  s.

#### **Exercise:**

# **Problem: Professional Application**

A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment. (a) Calculate the average force on the person if he is stopped by a padded dashboard that compresses an average of 1.00 cm. (b) Calculate the average force on the person if he is stopped by an air bag that compresses an average of 15.0 cm.

### **Solution:**

- (a)  $1.50 \times 10^6~N$  away from the dashboard
- (b)  $1.00 \times 10^5~\mathrm{N}$  away from the dashboard

### **Exercise:**

# **Problem: Professional Application**

Military rifles have a mechanism for reducing the recoil forces of the gun on the person firing it. An internal part recoils over a relatively large distance and is stopped by damping mechanisms in the gun. The larger distance reduces the average force needed to stop the internal part. (a) Calculate the recoil velocity of a 1.00-kg plunger that directly interacts with a 0.0200-kg bullet fired at 600 m/s from the gun. (b) If this part is stopped over a distance of 20.0 cm, what average force is exerted upon it by the gun? (c) Compare this to the force exerted on the gun if the bullet is accelerated to its velocity in 10.0 ms (milliseconds).

#### **Exercise:**

### **Problem:**

A cruise ship with a mass of  $1.00 \times 10^7$  kg strikes a pier at a speed of 0.750 m/s. It comes to rest 6.00 m later, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (Hint: First calculate the time it took to bring the ship to rest.)

#### **Solution:**

 $4.69 \times 10^5 \; \mathrm{N}$  in the boat's original direction of motion

#### **Exercise:**

#### **Problem:**

Calculate the final speed of a 110-kg rugby player who is initially running at 8.00 m/s but collides head-on with a padded goalpost and experiences a backward force of  $1.76 \times 10^4$  N for  $5.50 \times 10^{-2}$  s.

#### **Exercise:**

### **Problem:**

Water from a fire hose is directed horizontally against a wall at a rate of 50.0 kg/s and a speed of 42.0 m/s. Calculate the magnitude of the force exerted on the wall, assuming the water's horizontal momentum is reduced to zero.

#### **Solution:**

 $2.10 \times 10^3 \ N$  away from the wall

#### **Exercise:**

#### **Problem:**

A 0.450-kg hammer is moving horizontally at 7.00 m/s when it strikes a nail and comes to rest after driving the nail 1.00 cm into a board. (a) Calculate the duration of the impact. (b) What was the average force exerted on the nail?

#### **Exercise:**

#### **Problem:**

Starting with the definitions of momentum and kinetic energy, derive an equation for the kinetic energy of a particle expressed as a function of its momentum.

# **Solution:**

# **Equation:**

$$egin{aligned} \mathbf{p} &= m\mathbf{v} \Rightarrow p^2 = m^2v^2 \Rightarrow rac{p^2}{m} = mv^2 \ &\Rightarrow rac{p^2}{2m} = rac{1}{2}mv^2 = ext{KE} \ KE &= rac{p^2}{2m} \end{aligned}$$

#### **Exercise:**

### **Problem:**

A ball with an initial velocity of 10 m/s moves at an angle  $60^{\circ}$  above the +x-direction. The ball hits a vertical wall and bounces off so that it is moving  $60^{\circ}$  above the -x-direction with the same speed. What is the impulse delivered by the wall?

#### **Exercise:**

#### **Problem:**

When serving a tennis ball, a player hits the ball when its velocity is zero (at the highest point of a vertical toss). The racquet exerts a force of 540 N on the ball for 5.00 ms, giving it a final velocity of 45.0 m/s. Using these data, find the mass of the ball.

#### **Solution:**

60.0 g

### **Exercise:**

### **Problem:**

A punter drops a ball from rest vertically 1 meter down onto his foot. The ball leaves the foot with a speed of 18 m/s at an angle 55° above the horizontal. What is the impulse delivered by the foot (magnitude and direction)?

# Glossary

### change in momentum

the difference between the final and initial momentum; the mass times the change in velocity

# impulse

the average net external force times the time it acts; equal to the change in momentum

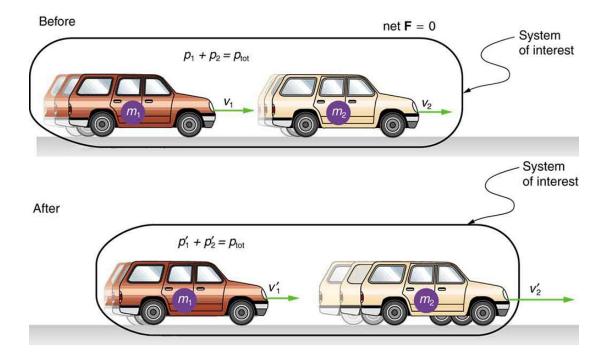
#### Conservation of Momentum

- Describe the principle of conservation of momentum.
- Derive an expression for the conservation of momentum.
- Explain conservation of momentum with examples.
- Explain the principle of conservation of momentum as it relates to atomic and subatomic particles.

Momentum is an important quantity because it is conserved. Yet it was not conserved in the examples in <u>Impulse</u> and <u>Linear Momentum and Force</u>, where large changes in momentum were produced by forces acting on the system of interest. Under what circumstances is momentum conserved?

The answer to this question entails considering a sufficiently large system. It is always possible to find a larger system in which total momentum is constant, even if momentum changes for components of the system. If a football player runs into the goalpost in the end zone, there will be a force on him that causes him to bounce backward. However, the Earth also recoils —conserving momentum—because of the force applied to it through the goalpost. Because Earth is many orders of magnitude more massive than the player, its recoil is immeasurably small and can be neglected in any practical sense, but it is real nevertheless.

Consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—for example, one car bumping into another, as shown in [link]. Both cars are coasting in the same direction when the lead car (labeled  $m_2$ ) is bumped by the trailing car (labeled  $m_1$ ). The only unbalanced force on each car is the force of the collision. (Assume that the effects due to friction are negligible.) Car 1 slows down as a result of the collision, losing some momentum, while car 2 speeds up and gains some momentum. We shall now show that the total momentum of the two-car system remains constant.



A car of mass  $m_1$  moving with a velocity of  $v_1$  bumps into another car of mass  $m_2$  and velocity  $v_2$  that it is following. As a result, the first car slows down to a velocity of  $v_1$  and the second speeds up to a velocity of  $v_2$ . The momentum of each car is changed, but the total momentum  $p_{\text{tot}}$  of the two cars is the same before and after the collision (if you assume friction is negligible).

Using the definition of impulse, the change in momentum of car 1 is given by

# **Equation:**

$$\Delta p_1 = F_1 \Delta t$$
,

where  $F_1$  is the force on car 1 due to car 2, and  $\Delta t$  is the time the force acts (the duration of the collision). Intuitively, it seems obvious that the collision time is the same for both cars, but it is only true for objects traveling at ordinary speeds. This assumption must be modified for objects travelling

near the speed of light, without affecting the result that momentum is conserved.

Similarly, the change in momentum of car 2 is

### **Equation:**

$$\Delta p_2 = F_2 \Delta t$$
,

where  $F_2$  is the force on car 2 due to car 1, and we assume the duration of the collision  $\Delta t$  is the same for both cars. We know from Newton's third law that  $F_2 = -F_1$ , and so

### **Equation:**

$$\Delta p_2 = -F_1 \Delta t = -\Delta p_1.$$

Thus, the changes in momentum are equal and opposite, and **Equation:** 

$$\Delta p_1 + \Delta p_2 = 0.$$

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

# **Equation:**

$$p_1 + p_2 = \text{constant},$$

# **Equation:**

$$p_1 + p_2 = p\prime_1 + p\prime_2,$$

where  $p_1$  and  $p_2$  are the momenta of cars 1 and 2 after the collision. (We often use primes to denote the final state.)

This result—that momentum is conserved—has validity far beyond the preceding one-dimensional case. It can be similarly shown that total momentum is conserved for any isolated system, with any number of

objects in it. In equation form, the **conservation of momentum principle** for an isolated system is written

## **Equation:**

$$\mathbf{p}_{\mathrm{tot}} = \mathrm{constant},$$

or

### **Equation:**

$$\mathbf{p}_{\mathrm{tot}} = \mathbf{p}\prime_{\mathrm{tot}},$$

where  $\mathbf{p}_{\mathrm{tot}}$  is the total momentum (the sum of the momenta of the individual objects in the system) and  $\mathbf{p}\prime_{\mathrm{tot}}$  is the total momentum some time later. (The total momentum can be shown to be the momentum of the center of mass of the system.) An **isolated system** is defined to be one for which the net external force is zero ( $\mathbf{F}_{\mathrm{net}}=0$ ).

### Note:

Conservation of Momentum Principle

# **Equation:**

$$\mathbf{p}_{\mathrm{tot}} = \mathrm{constant}$$
 $\mathbf{p}_{\mathrm{tot}} = \mathbf{p}\prime_{\mathrm{tot}} \; (\mathrm{isolated \; system})$ 

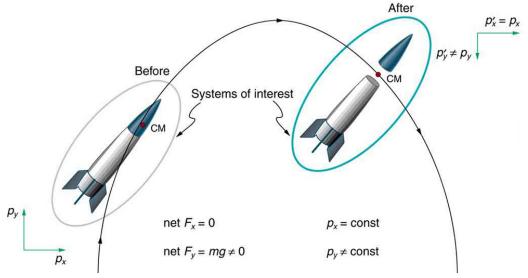
### Note:

# **Isolated System**

An isolated system is defined to be one for which the net external force is zero ( $\mathbf{F}_{net} = 0$ ).

Perhaps an easier way to see that momentum is conserved for an isolated system is to consider Newton's second law in terms of momentum,  $\mathbf{F}_{\rm net} = \frac{\Delta \mathbf{p}_{\rm tot}}{\Delta t}$ . For an isolated system, ( $\mathbf{F}_{\rm net} = 0$ ); thus,  $\Delta \mathbf{p}_{\rm tot} = 0$ , and  $\mathbf{p}_{\rm tot}$  is constant.

We have noted that the three length dimensions in nature—x, y, and z—are independent, and it is interesting to note that momentum can be conserved in different ways along each dimension. For example, during projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero and momentum is unchanged. But along the vertical direction, the net vertical force is not zero and the momentum of the projectile is not conserved. (See [link].) However, if the momentum of the projectile-Earth system is considered in the vertical direction, we find that the total momentum is conserved.



The horizontal component of a projectile's momentum is conserved if air resistance is negligible, even in this case where a space probe separates. The forces causing the separation are internal to the system, so that the net external horizontal force  $F_{x-\mathrm{net}}$  is still zero. The vertical component of the momentum is not conserved, because the net vertical force  $F_{y-\mathrm{net}}$  is not zero. In the vertical direction, the space probe-Earth system needs to be considered and we find that the total momentum is conserved. The center of mass of the

# space probe takes the same path it would if the separation did not occur.

The conservation of momentum principle can be applied to systems as different as a comet striking Earth and a gas containing huge numbers of atoms and molecules. Conservation of momentum is violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

#### Note:

Making Connections: Take-Home Investigation—Drop of Tennis Ball and a Basketball

Hold a tennis ball side by side and in contact with a basketball. Drop the balls together. (Be careful!) What happens? Explain your observations. Now hold the tennis ball above and in contact with the basketball. What happened? Explain your observations. What do you think will happen if the basketball ball is held above and in contact with the tennis ball?

#### Note:

Making Connections: Take-Home Investigation—Two Tennis Balls in a Ballistic Trajectory

Tie two tennis balls together with a string about a foot long. Hold one ball and let the other hang down and throw it in a ballistic trajectory. Explain your observations. Now mark the center of the string with bright ink or attach a brightly colored sticker to it and throw again. What happened? Explain your observations.

Some aquatic animals such as jellyfish move around based on the principles of conservation of momentum. A jellyfish fills its umbrella section with water and then pushes the water out resulting in motion in the opposite direction to that of the jet of water. Squids propel themselves in a

similar manner but, in contrast with jellyfish, are able to control the direction in which they move by aiming their nozzle forward or backward. Typical squids can move at speeds of 8 to 12 km/h.

The ballistocardiograph (BCG) was a diagnostic tool used in the second half of the 20th century to study the strength of the heart. About once a second, your heart beats, forcing blood into the aorta. A force in the opposite direction is exerted on the rest of your body (recall Newton's third law). A ballistocardiograph is a device that can measure this reaction force. This measurement is done by using a sensor (resting on the person) or by using a moving table suspended from the ceiling. This technique can gather information on the strength of the heart beat and the volume of blood passing from the heart. However, the electrocardiogram (ECG or EKG) and the echocardiogram (cardiac ECHO or ECHO; a technique that uses ultrasound to see an image of the heart) are more widely used in the practice of cardiology.

#### Note:

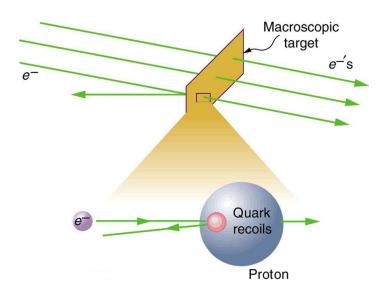
Making Connections: Conservation of Momentum and Collision Conservation of momentum is quite useful in describing collisions. Momentum is crucial to our understanding of atomic and subatomic particles because much of what we know about these particles comes from collision experiments.

# **Subatomic Collisions and Momentum**

The conservation of momentum principle not only applies to the macroscopic objects, it is also essential to our explorations of atomic and subatomic particles. Giant machines hurl subatomic particles at one another, and researchers evaluate the results by assuming conservation of momentum (among other things).

On the small scale, we find that particles and their properties are invisible to the naked eye but can be measured with our instruments, and models of these subatomic particles can be constructed to describe the results. Momentum is found to be a property of all subatomic particles including massless particles such as photons that compose light. Momentum being a property of particles hints that momentum may have an identity beyond the description of an object's mass multiplied by the object's velocity. Indeed, momentum relates to wave properties and plays a fundamental role in what measurements are taken and how we take these measurements.

Furthermore, we find that the conservation of momentum principle is valid when considering systems of particles. We use this principle to analyze the masses and other properties of previously undetected particles, such as the nucleus of an atom and the existence of quarks that make up particles of nuclei. [link] below illustrates how a particle scattering backward from another implies that its target is massive and dense. Experiments seeking evidence that quarks make up protons (one type of particle that makes up nuclei) scattered high-energy electrons off of protons (nuclei of hydrogen atoms). Electrons occasionally scattered straight backward in a manner that implied a very small and very dense particle makes up the proton—this observation is considered nearly direct evidence of quarks. The analysis was based partly on the same conservation of momentum principle that works so well on the large scale.



A subatomic particle scatters straight backward from a target particle. In experiments seeking evidence for

quarks, electrons were observed to occasionally scatter straight backward from a proton.

# **Section Summary**

• The conservation of momentum principle is written **Equation:** 

$$\mathbf{p}_{\mathrm{tot}} = \mathrm{constant}$$

or

**Equation:** 

$$\mathbf{p}_{\text{tot}} = \mathbf{p}\prime_{\text{tot}}$$
 (isolated system),

 $\mathbf{p}_{\mathrm{tot}}$  is the initial total momentum and  $\mathbf{p}\prime_{\mathrm{tot}}$  is the total momentum some time later.

- An isolated system is defined to be one for which the net external force is zero ( $\mathbf{F}_{net}=0$ ).
- During projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero.
- Conservation of momentum applies only when the net external force is zero.
- The conservation of momentum principle is valid when considering systems of particles.

# **Conceptual Questions**

#### **Exercise:**

**Problem: Professional Application** 

If you dive into water, you reach greater depths than if you do a belly flop. Explain this difference in depth using the concept of conservation of energy. Explain this difference in depth using what you have learned in this chapter.

#### **Exercise:**

**Problem:** Under what circumstances is momentum conserved?

### **Exercise:**

### **Problem:**

Can momentum be conserved for a system if there are external forces acting on the system? If so, under what conditions? If not, why not?

### **Exercise:**

### **Problem:**

Momentum for a system can be conserved in one direction while not being conserved in another. What is the angle between the directions? Give an example.

#### **Exercise:**

# **Problem: Professional Application**

Explain in terms of momentum and Newton's laws how a car's air resistance is due in part to the fact that it pushes air in its direction of motion.

#### **Exercise:**

#### **Problem:**

Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.

#### Exercise:

### **Problem:**

Must the total energy of a system be conserved whenever its momentum is conserved? Explain why or why not.

### **Problems & Exercises**

#### **Exercise:**

### **Problem: Professional Application**

Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 150,000 kg and a velocity of 0.300 m/s, and the second having a mass of 110,000 kg and a velocity of -0.120 m/s. (The minus indicates direction of motion.) What is their final velocity?

### **Solution:**

0.122 m/s

#### **Exercise:**

#### **Problem:**

Suppose a clay model of a koala bear has a mass of 0.200 kg and slides on ice at a speed of 0.750 m/s. It runs into another clay model, which is initially motionless and has a mass of 0.350 kg. Both being soft clay, they naturally stick together. What is their final velocity?

#### **Exercise:**

# **Problem:** Professional Application

Consider the following question: A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seatbelt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg. Would the answer to this question be different if the car with

the 70-kg passenger had collided with a car that has a mass equal to and is traveling in the opposite direction and at the same speed? Explain your answer.

### **Solution:**

In a collision with an identical car, momentum is conserved. Afterwards  $v_{\rm f}=0$  for both cars. The change in momentum will be the same as in the crash with the tree. However, the force on the body is not determined since the time is not known. A padded stop will reduce injurious force on body.

### **Exercise:**

### **Problem:**

What is the velocity of a 900-kg car initially moving at 30.0 m/s, just after it hits a 150-kg deer initially running at 12.0 m/s in the same direction? Assume the deer remains on the car.

### **Exercise:**

### **Problem:**

A 1.80-kg falcon catches a 0.650-kg dove from behind in midair. What is their velocity after impact if the falcon's velocity is initially 28.0 m/s and the dove's velocity is 7.00 m/s in the same direction?

#### **Solution:**

22.4 m/s in the same direction as the original motion

# Glossary

conservation of momentum principle
when the net external force is zero, the total momentum of the system
is conserved or constant

isolated system

a system in which the net external force is zero

quark

fundamental constituent of matter and an elementary particle

### Elastic Collisions in One Dimension

- Describe an elastic collision of two objects in one dimension.
- Define internal kinetic energy.
- Derive an expression for conservation of internal kinetic energy in a one dimensional collision.
- Determine the final velocities in an elastic collision given masses and initial velocities.

Let us consider various types of two-object collisions. These collisions are the easiest to analyze, and they illustrate many of the physical principles involved in collisions. The conservation of momentum principle is very useful here, and it can be used whenever the net external force on a system is zero.

We start with the elastic collision of two objects moving along the same line—a one-dimensional problem. An **elastic collision** is one that also conserves internal kinetic energy. **Internal kinetic energy** is the sum of the kinetic energies of the objects in the system. [link] illustrates an elastic collision in which internal kinetic energy and momentum are conserved.

Truly elastic collisions can only be achieved with subatomic particles, such as electrons striking nuclei. Macroscopic collisions can be very nearly, but not quite, elastic—some kinetic energy is always converted into other forms of energy such as heat transfer due to friction and sound. One macroscopic collision that is nearly elastic is that of two steel blocks on ice. Another nearly elastic collision is that between two carts with spring bumpers on an air track. Icy surfaces and air tracks are nearly frictionless, more readily allowing nearly elastic collisions on them.

#### Note:

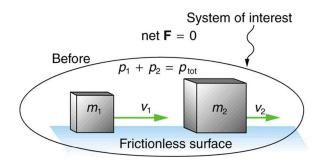
**Elastic Collision** 

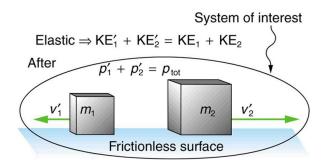
An **elastic collision** is one that conserves internal kinetic energy.

#### Note:

Internal Kinetic Energy

**Internal kinetic energy** is the sum of the kinetic energies of the objects in the system.





An elastic one-dimensional two-object collision.

Momentum and internal kinetic energy are conserved.

Now, to solve problems involving one-dimensional elastic collisions between two objects we can use the equations for conservation of momentum and conservation of internal kinetic energy. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

### **Equation:**

$$p_1 + p_2 = p\prime_1 + p\prime_2 \ \ (F_{
m net} = 0)$$

### **Equation:**

$$m_1 v_1 + m_2 v_2 = m_1 v \prime_1 + m_2 v \prime_2 ~~(F_{
m net} = 0),$$

where the primes (') indicate values after the collision. By definition, an elastic collision conserves internal kinetic energy, and so the sum of kinetic energies before the collision equals the sum after the collision. Thus,

### **Equation:**

$$rac{1}{2}{m_1{v_1}^2} + rac{1}{2}{m_2{v_2}^2} = rac{1}{2}{m_1}{v{\prime_1}^2} + rac{1}{2}{m_2}{v{\prime_2}^2} \; ext{(two-object elastic collision)}$$

expresses the equation for conservation of internal kinetic energy in a onedimensional collision.

### **Example:**

### **Calculating Velocities Following an Elastic Collision**

Calculate the velocities of two objects following an elastic collision, given that

# **Equation:**

$$m_1 = 0.500 \text{ kg}, \ m_2 = 3.50 \text{ kg}, \ v_1 = 4.00 \text{ m/s}, \text{ and } v_2 = 0.$$

### **Strategy and Concept**

First, visualize what the initial conditions mean—a small object strikes a larger object that is initially at rest. This situation is slightly simpler than the situation shown in [link] where both objects are initially moving. We are asked to find two unknowns (the final velocities  $vl_1$  and  $vl_2$ ). To find two unknowns, we must use two independent equations. Because this collision is elastic, we can use the above two equations. Both can be simplified by the fact that object 2 is initially at rest, and thus  $v_2 = 0$ . Once we simplify these equations, we combine them algebraically to solve for the unknowns.

### **Solution**

For this problem, note that  $v_2 = 0$  and use conservation of momentum. Thus, **Equation:** 

$$p_1 = p\prime_1 + p\prime_2$$

or

### **Equation:**

$$m_1v_1=m_1v_1+m_2v_2.$$

Using conservation of internal kinetic energy and that  $v_2 = 0$ ,

### **Equation:**

$$rac{1}{2}m_{1}{v_{1}}^{2}=rac{1}{2}m_{1}{v{\prime}_{1}}^{2}+rac{1}{2}m_{2}{v{\prime}_{2}}^{2}.$$

Solving the first equation (momentum equation) for  $v_2$ , we obtain

### **Equation:**

$$v\prime_2 = rac{m_1}{m_2}(v_1 - v\prime_1).$$

Substituting this expression into the second equation (internal kinetic energy equation) eliminates the variable  $v_2$ , leaving only  $v_1$  as an unknown (the algebra is left as an exercise for the reader). There are two solutions to any quadratic equation; in this example, they are

# **Equation:**

$$v_1 = 4.00 \text{ m/s}$$

and

# **Equation:**

$$v_{1} = -3.00 \text{ m/s}.$$

As noted when quadratic equations were encountered in earlier chapters, both solutions may or may not be meaningful. In this case, the first solution is the same as the initial condition. The first solution thus represents the situation before the collision and is discarded. The second solution  $(vl_1 = -3.00 \text{ m/s})$  is negative, meaning that the first object bounces backward. When this negative value of  $vl_1$  is used to find the velocity of the second object after the collision, we get

# **Equation:**

$$v\prime_2 = rac{m_1}{m_2}(v_1 - v\prime_1) = rac{0.500 ext{ kg}}{3.50 ext{ kg}} [4.00 - (-3.00)] ext{ m/s}$$

or

### **Equation:**

$$v_2 = 1.00 \text{ m/s}.$$

#### Discussion

The result of this example is intuitively reasonable. A small object strikes a larger one at rest and bounces backward. The larger one is knocked forward, but with a low speed. (This is like a compact car bouncing backward off a full-size SUV that is initially at rest.) As a check, try calculating the internal kinetic energy before and after the collision. You will see that the internal kinetic energy is unchanged at 4.00 J. Also check the total momentum before and after the collision; you will find it, too, is unchanged.

The equations for conservation of momentum and internal kinetic energy as

The equations for conservation of momentum and internal kinetic energy as written above can be used to describe any one-dimensional elastic collision of two objects. These equations can be extended to more objects if needed.

#### Note:

# Making Connections: Take-Home Investigation—Ice Cubes and Elastic Collision

Find a few ice cubes which are about the same size and a smooth kitchen tabletop or a table with a glass top. Place the ice cubes on the surface several centimeters away from each other. Flick one ice cube toward a stationary ice cube and observe the path and velocities of the ice cubes after the collision. Try to avoid edge-on collisions and collisions with rotating ice cubes. Have you created approximately elastic collisions? Explain the speeds and directions of the ice cubes using momentum.

#### Note:

# PhET Explorations: Collision Lab

Investigate collisions on an air hockey table. Set up your own experiments: vary the number of discs, masses and initial conditions. Is momentum

conserved? Is kinetic energy conserved? Vary the elasticity and see what happens.

https://phet.colorado.edu/sims/collision-lab/collision-lab en.html

# **Section Summary**

- An elastic collision is one that conserves internal kinetic energy.
- Conservation of kinetic energy and momentum together allow the final velocities to be calculated in terms of initial velocities and masses in one dimensional two-body collisions.

# **Conceptual Questions**

#### **Exercise:**

**Problem:** What is an elastic collision?

### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

Two identical objects (such as billiard balls) have a one-dimensional collision in which one is initially motionless. After the collision, the moving object is stationary and the other moves with the same speed as the other originally had. Show that both momentum and kinetic energy are conserved.

#### **Exercise:**

# **Problem: Professional Application**

Two manned satellites approach one another at a relative speed of 0.250 m/s, intending to dock. The first has a mass of  $4.00 \times 10^3$  kg, and the

second a mass of  $7.50\times10^3$  kg. If the two satellites collide elastically rather than dock, what is their final relative velocity?

### **Solution:**

0.250 m/s

### **Exercise:**

### **Problem:**

A 70.0-kg ice hockey goalie, originally at rest, catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. Suppose the goalie and the ice puck have an elastic collision and the puck is reflected back in the direction from which it came. What would their final velocities be in this case?

# **Glossary**

elastic collision

a collision that also conserves internal kinetic energy

internal kinetic energy

the sum of the kinetic energies of the objects in a system

#### Inelastic Collisions in One Dimension

- Define inelastic collision.
- Explain perfectly inelastic collision.
- Apply an understanding of collisions to sports.
- Determine recoil velocity and loss in kinetic energy given mass and initial velocity.

We have seen that in an elastic collision, internal kinetic energy is conserved. An **inelastic collision** is one in which the internal kinetic energy changes (it is not conserved). This lack of conservation means that the forces between colliding objects may remove or add internal kinetic energy. Work done by internal forces may change the forms of energy within a system. For inelastic collisions, such as when colliding objects stick together, this internal work may transform some internal kinetic energy into heat transfer. Or it may convert stored energy into internal kinetic energy, such as when exploding bolts separate a satellite from its launch vehicle.

### Note:

### Inelastic Collision

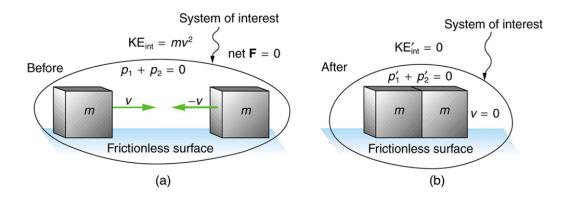
An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).

[link] shows an example of an inelastic collision. Two objects that have equal masses head toward one another at equal speeds and then stick together. Their total internal kinetic energy is initially  $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$ . The two objects come to rest after sticking together, conserving momentum. But the internal kinetic energy is zero after the collision. A collision in which the objects stick together is sometimes called a **perfectly inelastic collision** because it reduces internal kinetic energy more than does any other type of inelastic collision. In fact, such a collision reduces internal kinetic energy to the minimum it can have while still conserving momentum.

### Note:

# Perfectly Inelastic Collision

A collision in which the objects stick together is sometimes called "perfectly inelastic."



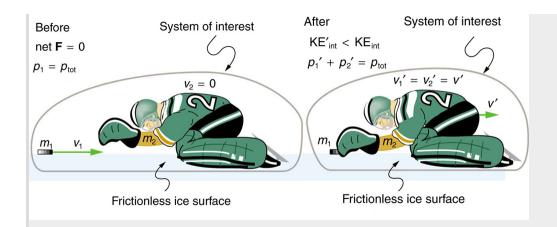
An inelastic one-dimensional two-object collision.

Momentum is conserved, but internal kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward one another at the same speed. (b) The objects stick together (a perfectly inelastic collision), and so their final velocity is zero. The internal kinetic energy of the system changes in any inelastic collision and is reduced to zero in this example.

# **Example:**

# Calculating Velocity and Change in Kinetic Energy: Inelastic Collision of a Puck and a Goalie

(a) Find the recoil velocity of a 70.0-kg ice hockey goalie, originally at rest, who catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. (b) How much kinetic energy is lost during the collision? Assume friction between the ice and the puck-goalie system is negligible. (See [link])



An ice hockey goalie catches a hockey puck and recoils backward. The initial kinetic energy of the puck is almost entirely converted to thermal energy and sound in this inelastic collision.

### Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. We can thus use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same. Once the final velocity is found, the kinetic energies can be calculated before and after the collision and compared as requested.

# Solution for (a)

Momentum is conserved because the net external force on the puck-goalie system is zero.

Conservation of momentum is

# **Equation:**

$$p_1 + p_2 = p\prime_1 + p\prime_2$$

or

# **Equation:**

$$m_1v_1 + m_2v_2 = m_1v_1 + m_2v_2.$$

Because the goalie is initially at rest, we know  $v_2 = 0$ . Because the goalie catches the puck, the final velocities are equal, or  $v'_1 = v'_2 = v'$ . Thus, the

conservation of momentum equation simplifies to

## **Equation:**

$$m_1v_1 = (m_1 + m_2)v'$$
.

Solving for *v*/ yields

### **Equation:**

$$v\prime = rac{m_1}{m_1+m_2}v_1.$$

Entering known values in this equation, we get

### **Equation:**

$$v\prime = \left(rac{0.150 ext{ kg}}{0.150 ext{ kg} + 70.0 ext{ kg}}
ight) (35.0 ext{ m/s}) = 7.48 imes 10^{-2} ext{ m/s}.$$

### Discussion for (a)

This recoil velocity is small and in the same direction as the puck's original velocity, as we might expect.

### Solution for (b)

Before the collision, the internal kinetic energy  $KE_{\rm int}$  of the system is that of the hockey puck, because the goalie is initially at rest. Therefore,  $KE_{\rm int}$  is initially

# **Equation:**

KE<sub>int</sub> = 
$$\frac{1}{2}mv^2 = \frac{1}{2}(0.150 \text{ kg})(35.0 \text{ m/s})^2$$
  
= 91.9 J.

After the collision, the internal kinetic energy is

# **Equation:**

KE'<sub>int</sub> = 
$$\frac{1}{2}(m+M)v^2 = \frac{1}{2}(70.15 \text{ kg})(7.48 \times 10^{-2} \text{ m/s})^2$$
  
= 0.196 J.

The change in internal kinetic energy is thus

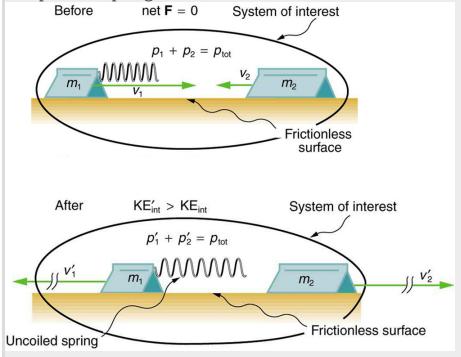
# **Equation:**

$$KE_{int} - KE_{int} = 0.196 J - 91.9 J$$
  
= -91.7 J

where the minus sign indicates that the energy was lost.

### Discussion for (b)

Nearly all of the initial internal kinetic energy is lost in this perfectly inelastic collision.  $KE_{\rm int}$  is mostly converted to thermal energy and sound. During some collisions, the objects do not stick together and less of the internal kinetic energy is removed—such as happens in most automobile accidents. Alternatively, stored energy may be converted into internal kinetic energy during a collision. [link] shows a one-dimensional example in which two carts on an air track collide, releasing potential energy from a compressed spring. [link] deals with data from such a collision.



An air track is nearly frictionless, so that momentum is conserved. Motion is one-dimensional. In this collision, examined in [link], the potential energy of a compressed spring is released during the collision and is converted to internal kinetic energy.

Collisions are particularly important in sports and the sporting and leisure industry utilizes elastic and inelastic collisions. Let us look briefly at tennis. Recall that in a collision, it is momentum and not force that is important. So, a heavier tennis racquet will have the advantage over a lighter one. This conclusion also holds true for other sports—a lightweight bat (such as a softball bat) cannot hit a hardball very far.

The location of the impact of the tennis ball on the racquet is also important, as is the part of the stroke during which the impact occurs. A smooth motion results in the maximizing of the velocity of the ball after impact and reduces sports injuries such as tennis elbow. A tennis player tries to hit the ball on the "sweet spot" on the racquet, where the vibration and impact are minimized and the ball is able to be given more velocity. Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

### Note:

### Take-Home Experiment—Bouncing of Tennis Ball

- 1. Find a racquet (a tennis, badminton, or other racquet will do). Place the racquet on the floor and stand on the handle. Drop a tennis ball on the strings from a measured height. Measure how high the ball bounces. Now ask a friend to hold the racquet firmly by the handle and drop a tennis ball from the same measured height above the racquet. Measure how high the ball bounces and observe what happens to your friend's hand during the collision. Explain your observations and measurements.
- 2. The coefficient of restitution (c) is a measure of the elasticity of a collision between a ball and an object, and is defined as the ratio of the speeds after and before the collision. A perfectly elastic collision has a c of 1. For a ball bouncing off the floor (or a racquet on the floor), c can be shown to be  $c = (h/H)^{1/2}$  where h is the height to which the ball bounces and H is the height from which the ball is dropped. Determine c for the cases in Part 1 and for the case of a tennis ball bouncing off a concrete or wooden floor (c = 0.85 for new tennis balls used on a tennis court).

# **Example:**

Calculating Final Velocity and Energy Release: Two Carts Collide

In the collision pictured in [link], two carts collide inelastically. Cart 1 (denoted  $m_1$  carries a spring which is initially compressed. During the collision, the spring releases its potential energy and converts it to internal kinetic energy. The mass of cart 1 and the spring is 0.350 kg, and the cart and the spring together have an initial velocity of 2.00 m/s. Cart 2 (denoted  $m_2$  in [link]) has a mass of 0.500 kg and an initial velocity of -0.500 m/s. After the collision, cart 1 is observed to recoil with a velocity of -4.00 m/s. (a) What is the final velocity of cart 2? (b) How much energy was released by the spring (assuming all of it was converted into internal kinetic energy)?

### Strategy

We can use conservation of momentum to find the final velocity of cart 2, because  $F_{\rm net}=0$  (the track is frictionless and the force of the spring is internal). Once this velocity is determined, we can compare the internal kinetic energy before and after the collision to see how much energy was released by the spring.

# Solution for (a)

As before, the equation for conservation of momentum in a two-object system is

### **Equation:**

$$m_1v_1 + m_2v_2 = m_1v_1 + m_2v_2.$$

The only unknown in this equation is  $v_2$ . Solving for  $v_2$  and substituting known values into the previous equation yields

# **Equation:**

$$v_{12} = \frac{m_1v_1 + m_2v_2 - m_1v_1}{m_2}$$

$$= \frac{(0.350 \text{ kg})(2.00 \text{ m/s}) + (0.500 \text{ kg})(-0.500 \text{ m/s})}{0.500 \text{ kg}} - \frac{(0.350 \text{ kg})(-4.00 \text{ m/s})}{0.500 \text{ kg}}$$

$$= 3.70 \text{ m/s}.$$

### Solution for (b)

The internal kinetic energy before the collision is

### **Equation:**

KE<sub>int</sub> = 
$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$
  
=  $\frac{1}{2}(0.350 \text{ kg})(2.00 \text{ m/s})^2 + \frac{1}{2}(0.500 \text{ kg})(-0.500 \text{ m/s})^2$   
= 0.763 J.

After the collision, the internal kinetic energy is

### **Equation:**

KE
$$\prime_{\text{int}} = \frac{1}{2} m_1 v \prime_1^2 + \frac{1}{2} m_2 v \prime_2^2$$
  
=  $\frac{1}{2} (0.350 \text{ kg}) (-4.00 \text{ m/s})^2 + \frac{1}{2} (0.500 \text{ kg}) (3.70 \text{ m/s})^2$   
= 6.22 J.

The change in internal kinetic energy is thus

### **Equation:**

$${
m KE}\prime_{
m int} - {
m KE}_{
m int} = 6.22~{
m J} - 0.763~{
m J} \ = 5.46~{
m J}.$$

### Discussion

The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision. The internal kinetic energy in this collision increases by 5.46 J. That energy was released by the spring.

# **Section Summary**

- An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).
- A collision in which the objects stick together is sometimes called perfectly inelastic because it reduces internal kinetic energy more than does any other type of inelastic collision.

• Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

# **Conceptual Questions**

#### **Exercise:**

### **Problem:**

What is an inelastic collision? What is a perfectly inelastic collision?

#### **Exercise:**

### Problem:

Mixed-pair ice skaters performing in a show are standing motionless at arms length just before starting a routine. They reach out, clasp hands, and pull themselves together by only using their arms. Assuming there is no friction between the blades of their skates and the ice, what is their velocity after their bodies meet?

#### **Exercise:**

#### **Problem:**

A small pickup truck that has a camper shell slowly coasts toward a red light with negligible friction. Two dogs in the back of the truck are moving and making various inelastic collisions with each other and the walls. What is the effect of the dogs on the motion of the center of mass of the system (truck plus entire load)? What is their effect on the motion of the truck?

### **Problems & Exercises**

#### **Exercise:**

### **Problem:**

A 0.240-kg billiard ball that is moving at 3.00 m/s strikes the bumper of a pool table and bounces straight back at 2.40 m/s (80% of its original speed). The collision lasts 0.0150 s. (a) Calculate the average force exerted on the ball by the bumper. (b) How much kinetic energy in joules is lost during the collision? (c) What percent of the original energy is left?

#### **Solution:**

- (a) 86.4 N perpendicularly away from the bumper
- (b) 0.389 J
- (c) 64.0%

#### **Exercise:**

### **Problem:**

During an ice show, a 60.0-kg skater leaps into the air and is caught by an initially stationary 75.0-kg skater. (a) What is their final velocity assuming negligible friction and that the 60.0-kg skater's original horizontal velocity is 4.00 m/s? (b) How much kinetic energy is lost?

#### **Exercise:**

# **Problem: Professional Application**

Using mass and speed data from [link] and assuming that the football player catches the ball with his feet off the ground with both of them moving horizontally, calculate: (a) the final velocity if the ball and player are going in the same direction and (b) the loss of kinetic energy in this case. (c) Repeat parts (a) and (b) for the situation in which the ball and the player are going in opposite directions. Might the loss of kinetic energy be related to how much it hurts to catch the pass?

#### **Solution:**

- (a) 8.06 m/s
- (b) -56.0 J
- (c)(i) 7.88 m/s; (ii) -223 J

### **Exercise:**

### **Problem:**

A battleship that is  $6.00 \times 10^7$  kg and is originally at rest fires a 1100-kg artillery shell horizontally with a velocity of 575 m/s. (a) If the shell is fired straight aft (toward the rear of the ship), there will be negligible friction opposing the ship's recoil. Calculate its recoil velocity. (b) Calculate the increase in internal kinetic energy (that is, for the ship and the shell). This energy is less than the energy released by the gun powder—significant heat transfer occurs.

### **Exercise:**

### **Problem: Professional Application**

Two manned satellites approaching one another, at a relative speed of 0.250 m/s, intending to dock. The first has a mass of  $4.00 \times 10^3$  kg, and the second a mass of  $7.50 \times 10^3$  kg. (a) Calculate the final velocity (after docking) by using the frame of reference in which the first satellite was originally at rest. (b) What is the loss of kinetic energy in this inelastic collision? (c) Repeat both parts by using the frame of reference in which the second satellite was originally at rest. Explain why the change in velocity is different in the two frames, whereas the change in kinetic energy is the same in both.

### **Solution:**

- (a) 0.163 m/s in the direction of motion of the more massive satellite
- (b) 81.6 J

(c)  $8.70 \times 10^{-2} \,\mathrm{m/s}$  in the direction of motion of the less massive satellite,  $81.5 \,\mathrm{J}$ . Because there are no external forces, the velocity of the center of mass of the two-satellite system is unchanged by the collision. The two velocities calculated above are the velocity of the center of mass in each of the two different individual reference frames. The loss in KE is the same in both reference frames because the KE lost to internal forces (heat, friction, etc.) is the same regardless of the coordinate system chosen.

### **Exercise:**

# **Problem: Professional Application**

A 30,000-kg freight car is coasting at 0.850 m/s with negligible friction under a hopper that dumps 110,000 kg of scrap metal into it. (a) What is the final velocity of the loaded freight car? (b) How much kinetic energy is lost?

#### **Exercise:**

# **Problem: Professional Application**

Space probes may be separated from their launchers by exploding bolts. (They bolt away from one another.) Suppose a 4800-kg satellite uses this method to separate from the 1500-kg remains of its launcher, and that 5000 J of kinetic energy is supplied to the two parts. What are their subsequent velocities using the frame of reference in which they were at rest before separation?

### **Solution:**

0.704 m/s

-2.25 m/s

#### **Exercise:**

### **Problem:**

A 0.0250-kg bullet is accelerated from rest to a speed of 550 m/s in a 3.00-kg rifle. The pain of the rifle's kick is much worse if you hold the gun loosely a few centimeters from your shoulder rather than holding it tightly against your shoulder. (a) Calculate the recoil velocity of the rifle if it is held loosely away from the shoulder. (b) How much kinetic energy does the rifle gain? (c) What is the recoil velocity if the rifle is held tightly against the shoulder, making the effective mass 28.0 kg? (d) How much kinetic energy is transferred to the rifle-shoulder combination? The pain is related to the amount of kinetic energy, which is significantly less in this latter situation. (e) Calculate the momentum of a 110-kg football player running at 8.00 m/s. Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s. Discuss its relationship to this problem.

### **Solution:**

- (a) 4.58 m/s away from the bullet
- (b) 31.5 J
- (c) -0.491 m/s
- (d) 3.38 J

#### **Exercise:**

### **Problem: Professional Application**

One of the waste products of a nuclear reactor is plutonium-239 ( $^{239}\mathrm{Pu}$ ). This nucleus is radioactive and decays by splitting into a helium-4 nucleus and a uranium-235 nucleus ( $^{4}\mathrm{He}+^{235}\mathrm{U}$ ), the latter of which is also radioactive and will itself decay some time later. The energy emitted in the plutonium decay is  $8.40\times10^{-13}~\mathrm{J}$  and is entirely converted to kinetic energy of the helium and uranium nuclei.

The mass of the helium nucleus is  $6.68 \times 10^{-27}$  kg, while that of the uranium is  $3.92 \times 10^{-25}$  kg (note that the ratio of the masses is 4 to 235). (a) Calculate the velocities of the two nuclei, assuming the plutonium nucleus is originally at rest. (b) How much kinetic energy does each nucleus carry away? Note that the data given here are accurate to three digits only.

#### **Exercise:**

### **Problem: Professional Application**

The Moon's craters are remnants of meteorite collisions. Suppose a fairly large asteroid that has a mass of  $5.00 \times 10^{12}$  kg (about a kilometer across) strikes the Moon at a speed of 15.0 km/s. (a) At what speed does the Moon recoil after the perfectly inelastic collision (the mass of the Moon is  $7.36 \times 10^{22}$  kg)? (b) How much kinetic energy is lost in the collision? Such an event may have been observed by medieval English monks who reported observing a red glow and subsequent haze about the Moon. (c) In October 2009, NASA crashed a rocket into the Moon, and analyzed the plume produced by the impact. (Significant amounts of water were detected.) Answer part (a) and (b) for this real-life experiment. The mass of the rocket was 2000 kg and its speed upon impact was 9000 km/h. How does the plume produced alter these results?

#### **Solution:**

(a) 
$$1.02 \times 10^{-6} \text{ m/s}$$

(b) 
$$5.63 \times 10^{20}~\mathrm{J}$$
 (almost all KE lost)

(c) Recoil speed is  $6.79 \times 10^{-17}$  m/s, energy lost is  $6.25 \times 10^9$  J. The plume will not affect the momentum result because the plume is still part of the Moon system. The plume may affect the kinetic energy result because a significant part of the initial kinetic energy may be transferred to the kinetic energy of the plume particles.

#### **Exercise:**

### **Problem: Professional Application**

Two football players collide head-on in midair while trying to catch a thrown football. The first player is 95.0 kg and has an initial velocity of 6.00 m/s, while the second player is 115 kg and has an initial velocity of –3.50 m/s. What is their velocity just after impact if they cling together?

### **Exercise:**

#### **Problem:**

What is the speed of a garbage truck that is  $1.20\times10^4~{\rm kg}$  and is initially moving at 25.0 m/s just after it hits and adheres to a trash can that is 80.0 kg and is initially at rest?

#### **Solution:**

24.8 m/s

### **Exercise:**

#### **Problem:**

During a circus act, an elderly performer thrills the crowd by catching a cannon ball shot at him. The cannon ball has a mass of 10.0 kg and the horizontal component of its velocity is 8.00 m/s when the 65.0-kg performer catches it. If the performer is on nearly frictionless roller skates, what is his recoil velocity?

#### **Exercise:**

#### **Problem:**

(a) During an ice skating performance, an initially motionless 80.0-kg clown throws a fake barbell away. The clown's ice skates allow her to recoil frictionlessly. If the clown recoils with a velocity of 0.500 m/s and the barbell is thrown with a velocity of 10.0 m/s, what is the mass of the barbell? (b) How much kinetic energy is gained by this maneuver? (c) Where does the kinetic energy come from?

### **Solution:**

- (a) 4.00 kg
- (b) 210 J
- (c) The clown does work to throw the barbell, so the kinetic energy comes from the muscles of the clown. The muscles convert the chemical potential energy of ATP into kinetic energy.

# Glossary

inelastic collision

a collision in which internal kinetic energy is not conserved

perfectly inelastic collision

a collision in which the colliding objects stick together

# Introduction to Rocket Propulsion

- State Newton's third law of motion.
- Explain the principle involved in propulsion of rockets and jet engines.
- Derive an expression for the acceleration of the rocket and discuss the factors that affect the acceleration.
- Describe the function of a space shuttle.

Rockets range in size from fireworks so small that ordinary people use them to immense Saturn Vs that once propelled massive payloads toward the Moon. The propulsion of all rockets, jet engines, deflating balloons, and even squids and octopuses is explained by the same physical principle—Newton's third law of motion. Matter is forcefully ejected from a system, producing an equal and opposite reaction on what remains. Another common example is the recoil of a gun. The gun exerts a force on a bullet to accelerate it and consequently experiences an equal and opposite force, causing the gun's recoil or kick.

#### Note:

Making Connections: Take-Home Experiment—Propulsion of a Balloon Hold a balloon and fill it with air. Then, let the balloon go. In which direction does the air come out of the balloon and in which direction does the balloon get propelled? If you fill the balloon with water and then let the balloon go, does the balloon's direction change? Explain your answer.

[link] shows a rocket accelerating straight up. In part (a), the rocket has a mass m and a velocity v relative to Earth, and hence a momentum mv. In part (b), a time  $\Delta t$  has elapsed in which the rocket has ejected a mass  $\Delta m$  of hot gas at a velocity  $v_e$  relative to the rocket. The remainder of the mass  $(m-\Delta m)$  now has a greater velocity  $(v+\Delta v)$ . The momentum of the entire system (rocket plus expelled gas) has actually decreased because the force of gravity has acted for a time  $\Delta t$ , producing a negative impulse  $\Delta p = -mg\Delta t$ . (Remember that impulse is the net external force on a system multiplied by the time it acts, and it equals the change in momentum

of the system.) So, the center of mass of the system is in free fall but, by rapidly expelling mass, part of the system can accelerate upward. It is a commonly held misconception that the rocket exhaust pushes on the ground. If we consider thrust; that is, the force exerted on the rocket by the exhaust gases, then a rocket's thrust is greater in outer space than in the atmosphere or on the launch pad. In fact, gases are easier to expel into a vacuum.

By calculating the change in momentum for the entire system over  $\Delta t$ , and equating this change to the impulse, the following expression can be shown to be a good approximation for the acceleration of the rocket.

### **Equation:**

$$a=rac{v_{
m e}}{m}\;rac{\Delta m}{\Delta t}-g$$

"The rocket" is that part of the system remaining after the gas is ejected, and g is the acceleration due to gravity.

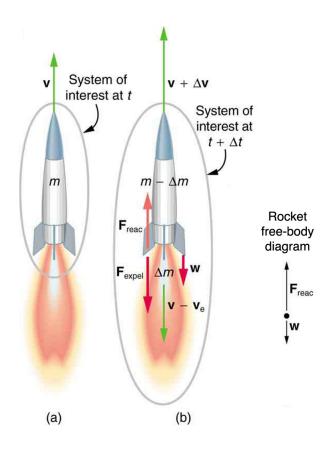
### Note:

Acceleration of a Rocket Acceleration of a rocket is

## **Equation:**

$$a=rac{v_{
m e}}{m}\;rac{\Delta m}{\Delta t}-g,$$

where a is the acceleration of the rocket,  $v_{\rm e}$  is the exhaust velocity, m is the mass of the rocket,  $\Delta m$  is the mass of the ejected gas, and  $\Delta t$  is the time in which the gas is ejected.



(a) This rocket has a mass m and an upward velocity v. The net external force on the system is -mg, if air resistance is neglected. (b) A time  $\Delta t$  later the system has two main parts, the ejected gas and the remainder of the rocket. The reaction force on the rocket is what overcomes the gravitational force and accelerates it upward.

A rocket's acceleration depends on three major factors, consistent with the equation for acceleration of a rocket . First, the greater the exhaust velocity of the gases relative to the rocket,  $v_{\rm e}$ , the greater the acceleration is. The

practical limit for  $v_{\rm e}$  is about  $2.5 \times 10^3$  m/s for conventional (non-nuclear) hot-gas propulsion systems. The second factor is the rate at which mass is ejected from the rocket. This is the factor  $\Delta m/\Delta t$  in the equation. The quantity  $(\Delta m/\Delta t)v_{\rm e}$ , with units of newtons, is called "thrust." The faster the rocket burns its fuel, the greater its thrust, and the greater its acceleration. The third factor is the mass m of the rocket. The smaller the mass is (all other factors being the same), the greater the acceleration. The rocket mass m decreases dramatically during flight because most of the rocket is fuel to begin with, so that acceleration increases continuously, reaching a maximum just before the fuel is exhausted.

### Note:

# Factors Affecting a Rocket's Acceleration

- The greater the exhaust velocity  $v_{\rm e}$  of the gases relative to the rocket, the greater the acceleration.
- The faster the rocket burns its fuel, the greater its acceleration.
- The smaller the rocket's mass (all other factors being the same), the greater the acceleration.

### **Example:**

# Calculating Acceleration: Initial Acceleration of a Moon Launch

A Saturn V's mass at liftoff was  $2.80 \times 10^6$  kg, its fuel-burn rate was  $1.40 \times 10^4$  kg/s, and the exhaust velocity was  $2.40 \times 10^3$  m/s. Calculate its initial acceleration.

### Strategy

This problem is a straightforward application of the expression for acceleration because a is the unknown and all of the terms on the right side of the equation are given.

### Solution

Substituting the given values into the equation for acceleration yields

### **Equation:**

$$egin{array}{lll} a & = & rac{v_{
m e}}{m} rac{\Delta m}{\Delta t} - g \ & = & rac{2.40 imes 10^3 \, {
m m/s}}{2.80 imes 10^6 \, {
m kg}} ig( 1.40 imes 10^4 \, {
m kg/s} ig) - 9.80 \, {
m m/s}^2 \ & = & 2.20 \, {
m m/s}^2. \end{array}$$

#### Discussion

This value is fairly small, even for an initial acceleration. The acceleration does increase steadily as the rocket burns fuel, because m decreases while  $v_{\rm e}$  and  $\frac{\Delta m}{\Delta t}$  remain constant. Knowing this acceleration and the mass of the rocket, you can show that the thrust of the engines was  $3.36 \times 10^7~{\rm N}.$ 

To achieve the high speeds needed to hop continents, obtain orbit, or escape Earth's gravity altogether, the mass of the rocket other than fuel must be as small as possible. It can be shown that, in the absence of air resistance and neglecting gravity, the final velocity of a one-stage rocket initially at rest is

### **Equation:**

$$v=v_{
m e} \ln rac{m_0}{m_{
m r}},$$

where  $\ln(m_0/m_{\rm r})$  is the natural logarithm of the ratio of the initial mass of the rocket  $(m_0)$  to what is left  $(m_{\rm r})$  after all of the fuel is exhausted. (Note that v is actually the change in velocity, so the equation can be used for any segment of the flight. If we start from rest, the change in velocity equals the final velocity.) For example, let us calculate the mass ratio needed to escape Earth's gravity starting from rest, given that the escape velocity from Earth is about  $11.2 \times 10^3$  m/s, and assuming an exhaust velocity  $v_{\rm e} = 2.5 \times 10^3$  m/s.

# **Equation:**

$$\ln rac{m_0}{m_{
m r}} = rac{v}{v_{
m e}} = rac{11.2 imes 10^3 {
m m/s}}{2.5 imes 10^3 {
m m/s}} = 4.48$$

Solving for  $m_0/m_{
m r}$  gives

### **Equation:**

$$\frac{m_0}{m_{
m r}} = e^{4.48} = 88.$$

Thus, the mass of the rocket is

### **Equation:**

$$m_{
m r}=rac{m_0}{88}.$$

This result means that only 1/88 of the mass is left when the fuel is burnt, and 87/88 of the initial mass was fuel. Expressed as percentages, 98.9% of the rocket is fuel, while payload, engines, fuel tanks, and other components make up only 1.10%. Taking air resistance and gravitational force into account, the mass  $m_{\rm r}$  remaining can only be about  $m_0/180$ . It is difficult to build a rocket in which the fuel has a mass 180 times everything else. The solution is multistage rockets. Each stage only needs to achieve part of the final velocity and is discarded after it burns its fuel. The result is that each successive stage can have smaller engines and more payload relative to its fuel. Once out of the atmosphere, the ratio of payload to fuel becomes more favorable, too.

The space shuttle was an attempt at an economical vehicle with some reusable parts, such as the solid fuel boosters and the craft itself. (See [link]) The shuttle's need to be operated by humans, however, made it at least as costly for launching satellites as expendable, unmanned rockets. Ideally, the shuttle would only have been used when human activities were required for the success of a mission, such as the repair of the Hubble space telescope. Rockets with satellites can also be launched from airplanes. Using airplanes has the double advantage that the initial velocity is significantly above zero and a rocket can avoid most of the atmosphere's resistance.



The space shuttle had a number of reusable parts. Solid fuel boosters on either side were recovered and refueled after each flight, and the entire orbiter returned to Earth for use in subsequent flights. The large liquid fuel tank was expended. The space shuttle was a complex assemblage of technologies, employing both solid and liquid fuel and pioneering ceramic tiles as reentry heat shields. As a result, it permitted multiple launches as opposed to

single-use rockets. (credit: NASA)

#### Note:

### PhET Explorations: Lunar Lander

Can you avoid the boulder field and land safely, just before your fuel runs out, as Neil Armstrong did in 1969? Our version of this classic video game accurately simulates the real motion of the lunar lander with the correct mass, thrust, fuel consumption rate, and lunar gravity. The real lunar lander is very hard to control.

https://phet.colorado.edu/sims/lunar-lander/lunar-lander en.html

# **Section Summary**

- Newton's third law of motion states that to every action, there is an equal and opposite reaction.
- Acceleration of a rocket is  $a = \frac{v_{\rm e}}{m} \, \frac{\Delta m}{\Delta t} g$ .
- A rocket's acceleration depends on three main factors. They are
  - 1. The greater the exhaust velocity of the gases, the greater the acceleration.
  - 2. The faster the rocket burns its fuel, the greater its acceleration.
  - 3. The smaller the rocket's mass, the greater the acceleration.

# **Conceptual Questions**

#### **Exercise:**

# **Problem: Professional Application**

Suppose a fireworks shell explodes, breaking into three large pieces for which air resistance is negligible. How is the motion of the center

of mass affected by the explosion? How would it be affected if the pieces experienced significantly more air resistance than the intact shell?

### **Exercise:**

### **Problem: Professional Application**

During a visit to the International Space Station, an astronaut was positioned motionless in the center of the station, out of reach of any solid object on which he could exert a force. Suggest a method by which he could move himself away from this position, and explain the physics involved.

#### **Exercise:**

# **Problem: Professional Application**

It is possible for the velocity of a rocket to be greater than the exhaust velocity of the gases it ejects. When that is the case, the gas velocity and gas momentum are in the same direction as that of the rocket. How is the rocket still able to obtain thrust by ejecting the gases?

### **Problems & Exercises**

### **Exercise:**

# **Problem:** Professional Application

Antiballistic missiles (ABMs) are designed to have very large accelerations so that they may intercept fast-moving incoming missiles in the short time available. What is the takeoff acceleration of a 10,000-kg ABM that expels 196 kg of gas per second at an exhaust velocity of  $2.50 \times 10^3 \ m/s$ ?

### **Solution:**

 $39.2 \text{ m/s}^2$ 

### **Exercise:**

**Problem: Professional Application** 

What is the acceleration of a 5000-kg rocket taking off from the Moon, where the acceleration due to gravity is only 1.6  $\rm m/s^2$ , if the rocket expels 8.00 kg of gas per second at an exhaust velocity of  $\rm 2.20 \times 10^3 \ m/s?$ 

### **Exercise:**

**Problem: Professional Application** 

Calculate the increase in velocity of a 4000-kg space probe that expels 3500 kg of its mass at an exhaust velocity of  $2.00 \times 10^3$  m/s. You may assume the gravitational force is negligible at the probe's location.

### **Solution:**

$$4.16 \times 10^{3} \text{ m/s}$$

### **Exercise:**

**Problem: Professional Application** 

Ion-propulsion rockets have been proposed for use in space. They employ atomic ionization techniques and nuclear energy sources to produce extremely high exhaust velocities, perhaps as great as  $8.00 \times 10^6 \, \mathrm{m/s}$ . These techniques allow a much more favorable payload-to-fuel ratio. To illustrate this fact: (a) Calculate the increase in velocity of a 20,000-kg space probe that expels only 40.0-kg of its mass at the given exhaust velocity. (b) These engines are usually designed to produce a very small thrust for a very long time—the type of engine that might be useful on a trip to the outer planets, for example. Calculate the acceleration of such an engine if it expels

 $4.50 imes 10^{-6}~{
m kg/s}$  at the given velocity, assuming the acceleration due to gravity is negligible.

#### **Exercise:**

**Problem:** Derive the equation for the vertical acceleration of a rocket.

#### **Solution:**

The force needed to give a small mass  $\Delta m$  an acceleration  $a_{\Delta m}$  is  $F = \Delta m a_{\Delta m}$ . To accelerate this mass in the small time interval  $\Delta t$  at a speed  $v_{
m e}$  requires  $v_{
m e}=a_{\Delta m}\Delta t$ , so  $F=v_{
m e}{\Delta t\over \Delta t}$  . By Newton's third law, this force is equal in magnitude to the thrust force acting on the rocket, so  $F_{
m thrust} = v_{
m e} {\Delta m \over \Delta t}$  , where all quantities are positive. Applying Newton's second law to the rocket gives  $F_{\mathrm{thrust}} - \mathrm{mg} = \mathrm{ma} \Rightarrow a = \frac{v_{\mathrm{e}}}{m} \frac{\Delta m}{\Delta t} - g$ , where m is the mass of the

rocket and unburnt fuel.

#### **Exercise:**

# **Problem: Professional Application**

(a) Calculate the maximum rate at which a rocket can expel gases if its acceleration cannot exceed seven times that of gravity. The mass of the rocket just as it runs out of fuel is 75,000-kg, and its exhaust velocity is  $2.40 \times 10^3$  m/s. Assume that the acceleration of gravity is the same as on Earth's surface  $(9.80 \text{ m/s}^2)$ . (b) Why might it be necessary to limit the acceleration of a rocket?

### **Exercise:**

### **Problem:**

Given the following data for a fire extinguisher-toy wagon rocket experiment, calculate the average exhaust velocity of the gases expelled from the extinguisher. Starting from rest, the final velocity is 10.0 m/s. The total mass is initially 75.0 kg and is 70.0 kg after the extinguisher is fired.

#### **Exercise:**

### **Problem:**

How much of a single-stage rocket that is 100,000 kg can be anything but fuel if the rocket is to have a final speed of 8.00 km/s, given that it expels gases at an exhaust velocity of  $2.20 \times 10^3 \text{ m/s}$ ?

### **Solution:**

$$2.63 \times 10^3 \text{ kg}$$

### **Exercise:**

# **Problem: Professional Application**

(a) A 5.00-kg squid initially at rest ejects 0.250-kg of fluid with a velocity of 10.0 m/s. What is the recoil velocity of the squid if the ejection is done in 0.100 s and there is a 5.00-N frictional force opposing the squid's movement. (b) How much energy is lost to work done against friction?

### **Solution:**

- (a) 0.421 m/s away from the ejected fluid.
- (b) 0.237 J.

#### **Exercise:**

**Problem:** Unreasonable Results

Squids have been reported to jump from the ocean and travel 30.0 m (measured horizontally) before re-entering the water. (a) Calculate the initial speed of the squid if it leaves the water at an angle of  $20.0^{\circ}$ , assuming negligible lift from the air and negligible air resistance. (b) The squid propels itself by squirting water. What fraction of its mass would it have to eject in order to achieve the speed found in the previous part? The water is ejected at 12.0 m/s; gravitational force and friction are neglected. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?

### **Exercise:**

### **Problem:** Construct Your Own Problem

Consider an astronaut in deep space cut free from her space ship and needing to get back to it. The astronaut has a few packages that she can throw away to move herself toward the ship. Construct a problem in which you calculate the time it takes her to get back by throwing all the packages at one time compared to throwing them one at a time. Among the things to be considered are the masses involved, the force she can exert on the packages through some distance, and the distance to the ship.

### **Exercise:**

#### Problem: Construct Your Own Problem

Consider an artillery projectile striking armor plating. Construct a problem in which you find the force exerted by the projectile on the plate. Among the things to be considered are the mass and speed of the projectile and the distance over which its speed is reduced. Your instructor may also wish for you to consider the relative merits of depleted uranium versus lead projectiles based on the greater density of uranium.

## Introduction to Circular Motion class="introduction"

```
The mention of
   a tornado
  conjures up
images of raw
  destructive
    power.
  Tornadoes
 blow houses
away as if they
were made of
paper and have
been known to
  pierce tree
  trunks with
pieces of straw.
They descend
from clouds in
  funnel-like
shapes that spin
   violently,
particularly at
  the bottom
where they are
 most narrow,
  producing
 winds as high
 as 500 km/h.
(credit: Daphne
  Zaras, U.S.
   National
 Oceanic and
 Atmospheric
Administration
```



Why do tornadoes spin at all? And why do tornados spin so rapidly? The answer is that air masses that produce tornadoes are themselves rotating, and when the radii of the air masses decrease, their rate of rotation increases. An ice skater increases her spin in an exactly analogous manner as seen in [link]. The skater starts her rotation with outstretched limbs and increases her spin by pulling them in toward her body. The same physics describes the exhilarating spin of a skater and the wrenching force of a tornado.

Clearly, force, energy, and power are associated with rotational motion. These and other aspects of rotational motion are covered in this chapter. We shall see that all important aspects of rotational motion either have already been defined for linear motion or have exact analogs in linear motion. First, we look at angular acceleration—the rotational analog of linear acceleration.



This figure skater increases her rate of spin by pulling her arms and her extended leg closer to her axis of rotation. (credit: Luu, Wikimedia Commons)

### Rotation Angle and Angular Velocity

- Define arc length, rotation angle, radius of curvature and angular velocity.
- Calculate the angular velocity of a car wheel spin.

In <u>Kinematics</u>, we studied motion along a straight line and introduced such concepts as displacement, velocity, and acceleration. <u>Two-Dimensional Kinematics</u> dealt with motion in two dimensions. Projectile motion is a special case of two-dimensional kinematics in which the object is projected into the air, while being subject to the gravitational force, and lands a distance away. In this chapter, we consider situations where the object does not land but moves in a curve. We begin the study of uniform circular motion by defining two angular quantities needed to describe rotational motion.

### **Rotation Angle**

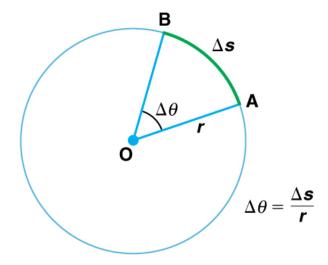
When objects rotate about some axis—for example, when the CD (compact disc) in [link] rotates about its center—each point in the object follows a circular arc. Consider a line from the center of the CD to its edge. Each **pit** used to record sound along this line moves through the same angle in the same amount of time. The rotation angle is the amount of rotation and is analogous to linear distance. We define the **rotation angle**  $\Delta\theta$  to be the ratio of the arc length to the radius of curvature:

### **Equation:**

$$\Delta heta = rac{\Delta s}{r}.$$



All points on a CD travel in circular arcs. The pits along a line from the center to the edge all move through the same angle  $\Delta \theta$  in a time  $\Delta t$ .



The radius of a circle is rotated through an angle  $\Delta\theta$ . The arc

length  $\Delta s$  is described on the circumference.

The **arc length**  $\Delta s$  is the distance traveled along a circular path as shown in [link] Note that r is the **radius of curvature** of the circular path.

We know that for one complete revolution, the arc length is the circumference of a circle of radius r. The circumference of a circle is  $2\pi r$ . Thus for one complete revolution the rotation angle is

### **Equation:**

$$\Delta heta = rac{2\pi r}{r} = 2\pi.$$

This result is the basis for defining the units used to measure rotation angles,  $\Delta\theta$  to be **radians** (rad), defined so that

### **Equation:**

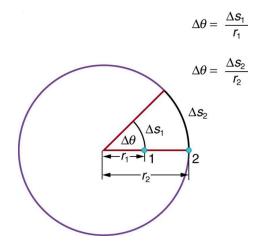
$$2\pi \text{ rad} = 1 \text{ revolution}.$$

A comparison of some useful angles expressed in both degrees and radians is shown in [link].

Degree Measures	Radian Measure
30°	$\frac{\pi}{6}$

Degree Measures	Radian Measure
60°	$rac{\pi}{3}$
90°	$rac{\pi}{2}$
120°	$\frac{2\pi}{3}$
135°	$\frac{3\pi}{4}$
180°	$\pi$

Comparison of Angular Units



Points 1 and 2 rotate through the same angle  $(\Delta\theta)$ , but point 2 moves through a greater arc length  $(\Delta s)$  because it is at a greater distance from the center of rotation (r).

If  $\Delta\theta=2\pi$  rad, then the CD has made one complete revolution, and every point on the CD is back at its original position. Because there are  $360^{\circ}$  in a circle or one revolution, the relationship between radians and degrees is thus

### **Equation:**

$$2\pi~{\rm rad}=360^{\rm o}$$

so that

### **Equation:**

$$1~{
m rad}=rac{360^{
m o}}{2\pi}pprox 57.3^{
m o}.$$

### **Angular Velocity**

How fast is an object rotating? We define **angular velocity**  $\omega$  as the rate of change of an angle. In symbols, this is

### **Equation:**

$$\omega = rac{\Delta heta}{\Delta t},$$

where an angular rotation  $\Delta\theta$  takes place in a time  $\Delta t$ . The greater the rotation angle in a given amount of time, the greater the angular velocity. The units for angular velocity are radians per second (rad/s).

Angular velocity  $\omega$  is analogous to linear velocity v. To get the precise relationship between angular and linear velocity, we again consider a pit on the rotating CD. This pit moves an arc length  $\Delta s$  in a time  $\Delta t$ , and so it has a linear velocity

### **Equation:**

$$v = \frac{\Delta s}{\Delta t}$$
.

From  $\Delta\theta=\frac{\Delta s}{r}$  we see that  $\Delta s=r\Delta\theta$ . Substituting this into the expression for v gives

### **Equation:**

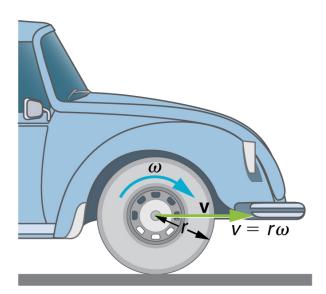
$$v=rac{r\Delta heta}{\Delta t}=r\omega.$$

We write this relationship in two different ways and gain two different insights:

### **Equation:**

$$v = r\omega \text{ or } \omega = \frac{v}{r}.$$

The first relationship in  $v=r\omega$  or  $\omega=\frac{v}{r}$  states that the linear velocity v is proportional to the distance from the center of rotation, thus, it is largest for a point on the rim (largest r), as you might expect. We can also call this linear speed v of a point on the rim the *tangential speed*. The second relationship in  $v=r\omega$  or  $\omega=\frac{v}{r}$  can be illustrated by considering the tire of a moving car. Note that the speed of a point on the rim of the tire is the same as the speed v of the car. See [link]. So the faster the car moves, the faster the tire spins—large v means a large  $\omega$ , because  $v=r\omega$ . Similarly, a larger-radius tire rotating at the same angular velocity  $(\omega)$  will produce a greater linear speed (v) for the car.



A car moving at a velocity v to the right has a tire rotating with an angular velocity  $\omega$ . The speed of the tread of the tire relative to the axle is v, the same as if the car were jacked up. Thus the car moves forward at linear velocity  $v=r\omega$ , where r is the tire radius. A larger angular velocity for the tire means a greater velocity for the car.

### **Example:**

### **How Fast Does a Car Tire Spin?**

Calculate the angular velocity of a 0.300 m radius car tire when the car travels at 15.0 m/s (about 54 km/h). See [link].

#### Strategy

Because the linear speed of the tire rim is the same as the speed of the car, we have  $v=15.0~\mathrm{m/s}$ . The radius of the tire is given to be  $r=0.300~\mathrm{m}$ . Knowing v and r, we can use the second relationship in  $v=r\omega$ ,  $\omega=\frac{v}{r}$  to calculate the angular velocity.

#### Solution

To calculate the angular velocity, we will use the following relationship:

### **Equation:**

$$\omega=rac{v}{r}.$$

Substituting the knowns,

### **Equation:**

$$\omega = rac{15.0 ext{ m/s}}{0.300 ext{ m}} = 50.0 ext{ rad/s}.$$

#### Discussion

When we cancel units in the above calculation, we get 50.0/s. But the angular velocity must have units of rad/s. Because radians are actually unitless (radians are defined as a ratio of distance), we can simply insert them into the answer for the angular velocity. Also note that if an earth mover with much larger tires, say 1.20 m in radius, were moving at the same speed of 15.0 m/s, its tires would rotate more slowly. They would have an angular velocity

### **Equation:**

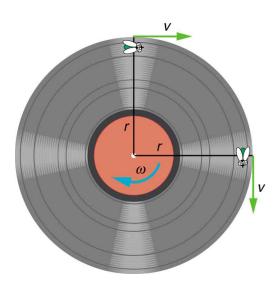
$$\omega = (15.0 \text{ m/s})/(1.20 \text{ m}) = 12.5 \text{ rad/s}.$$

Both  $\omega$  and v have directions (hence they are angular and linear *velocities*, respectively). Angular velocity has only two directions with respect to the axis of rotation—it is either clockwise or counterclockwise. Linear velocity is tangent to the path, as illustrated in [link].

#### Note:

### Take-Home Experiment

Tie an object to the end of a string and swing it around in a horizontal circle above your head (swing at your wrist). Maintain uniform speed as the object swings and measure the angular velocity of the motion. What is the approximate speed of the object? Identify a point close to your hand and take appropriate measurements to calculate the linear speed at this point. Identify other circular motions and measure their angular velocities.



As an object moves in a circle, here a fly on the edge of an old-fashioned vinyl record, its instantaneous velocity is

always tangent to the circle. The direction of the angular velocity is clockwise in this case.

#### Note:

PhET Explorations: Ladybug Revolution

<u>Ladybug</u> <u>Revolutio</u> n

Ш

Join the ladybug in an exploration of rotational motion. Rotate the merry-go-round to change its angle, or choose a constant angular velocity or angular acceleration. Explore how circular motion relates to the bug's x,y position, velocity, and acceleration using vectors or graphs.

### **Section Summary**

• Uniform circular motion is motion in a circle at constant speed. The rotation angle  $\Delta \theta$  is defined as the ratio of the arc length to the radius of curvature:

### **Equation:**

$$\Delta heta = rac{\Delta s}{r},$$

where arc length  $\Delta s$  is distance traveled along a circular path and r is the radius of curvature of the circular path. The quantity  $\Delta \theta$  is

measured in units of radians (rad), for which **Equation:** 

$$2\pi \text{ rad} = 360^{\circ} = 1 \text{ revolution}.$$

- The conversion between radians and degrees is  $1 \text{ rad} = 57.3^{\circ}$ .
- Angular velocity  $\omega$  is the rate of change of an angle, **Equation:**

$$\omega = rac{\Delta heta}{\Delta t},$$

where a rotation  $\Delta\theta$  takes place in a time  $\Delta t$ . The units of angular velocity are radians per second (rad/s). Linear velocity v and angular velocity  $\omega$  are related by

**Equation:** 

$$v = r\omega \text{ or } \omega = \frac{v}{r}.$$

### **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

There is an analogy between rotational and linear physical quantities. What rotational quantities are analogous to distance and velocity?

#### **Problem Exercises**

#### **Exercise:**

#### **Problem:**

Semi-trailer trucks have an odometer on one hub of a trailer wheel. The hub is weighted so that it does not rotate, but it contains gears to count the number of wheel revolutions—it then calculates the distance traveled. If the wheel has a 1.15 m diameter and goes through 200,000 rotations, how many kilometers should the odometer read?

#### **Solution:**

723 km

#### **Exercise:**

#### **Problem:**

Microwave ovens rotate at a rate of about 6 rev/min. What is this in revolutions per second? What is the angular velocity in radians per second?

#### **Exercise:**

#### **Problem:**

An automobile with 0.260 m radius tires travels 80,000 km before wearing them out. How many revolutions do the tires make, neglecting any backing up and any change in radius due to wear?

#### **Solution:**

 $5 \times 10^7$  rotations

#### **Exercise:**

#### **Problem:**

(a) What is the period of rotation of Earth in seconds? (b) What is the angular velocity of Earth? (c) Given that Earth has a radius of  $6.4 \times 10^6$  m at its equator, what is the linear velocity at Earth's surface?

#### **Exercise:**

#### **Problem:**

A baseball pitcher brings his arm forward during a pitch, rotating the forearm about the elbow. If the velocity of the ball in the pitcher's hand is 35.0 m/s and the ball is 0.300 m from the elbow joint, what is the angular velocity of the forearm?

#### **Solution:**

117 rad/s

#### **Exercise:**

#### **Problem:**

In lacrosse, a ball is thrown from a net on the end of a stick by rotating the stick and forearm about the elbow. If the angular velocity of the ball about the elbow joint is 30.0 rad/s and the ball is 1.30 m from the elbow joint, what is the velocity of the ball?

#### **Exercise:**

#### Problem:

A truck with 0.420-m-radius tires travels at 32.0 m/s. What is the angular velocity of the rotating tires in radians per second? What is this in rev/min?

#### **Solution:**

76.2 rad/s

728 rpm

#### **Exercise:**

#### **Problem:**

**Integrated Concepts** When kicking a football, the kicker rotates his leg about the hip joint.

- (a) If the velocity of the tip of the kicker's shoe is 35.0 m/s and the hip joint is 1.05 m from the tip of the shoe, what is the shoe tip's angular velocity?
- (b) The shoe is in contact with the initially stationary 0.500 kg football for 20.0 ms. What average force is exerted on the football to give it a velocity of 20.0 m/s?
- (c) Find the maximum range of the football, neglecting air resistance.

#### **Solution:**

- (a) 33.3 rad/s
- (b) 500 N
- (c) 40.8 m

#### **Exercise:**

#### **Problem:**Construct Your Own Problem

Consider an amusement park ride in which participants are rotated about a vertical axis in a cylinder with vertical walls. Once the angular velocity reaches its full value, the floor drops away and friction between the walls and the riders prevents them from sliding down. Construct a problem in which you calculate the necessary angular velocity that assures the riders will not slide down the wall. Include a free body diagram of a single rider. Among the variables to consider are the radius of the cylinder and the coefficients of friction between the riders' clothing and the wall.

### **Glossary**

arc length

 $\Delta s$ , the distance traveled by an object along a circular path

pit

a tiny indentation on the spiral track moulded into the top of the polycarbonate layer of CD

rotation angle

the ratio of the arc length to the radius of curvature on a circular path:

$$\Delta \theta = \frac{\Delta s}{r}$$

radius of curvature radius of a circular path

radians

a unit of angle measurement

angular velocity

 $\omega$ , the rate of change of the angle with which an object moves on a circular path

### **Angular Acceleration**

- Describe uniform circular motion.
- Explain non-uniform circular motion.
- Calculate angular acceleration of an object.
- Observe the link between linear and angular acceleration.

Uniform Circular Motion and Gravitation discussed only uniform circular motion, which is motion in a circle at constant speed and, hence, constant angular velocity. Recall that angular velocity  $\omega$  was defined as the time rate of change of angle  $\theta$ :

### **Equation:**

$$\omega = rac{\Delta heta}{\Delta t},$$

where  $\theta$  is the angle of rotation as seen in [link]. The relationship between angular velocity  $\omega$  and linear velocity v was also defined in Rotation Angle and Angular Velocity as

### **Equation:**

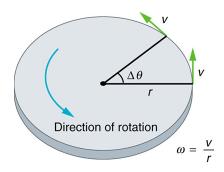
$$v=r\omega$$

or

### **Equation:**

$$\omega=rac{v}{r},$$

where r is the radius of curvature, also seen in [link]. According to the sign convention, the counter clockwise direction is considered as positive direction and clockwise direction as negative



This figure shows uniform circular motion and some of its defined quantities.

Angular velocity is not constant when a skater pulls in her arms, when a child starts up a merry-go-round from rest, or when a computer's hard disk slows to a halt when switched off. In all these cases, there is an **angular acceleration**, in which  $\omega$  changes. The faster the change occurs, the greater the angular acceleration. Angular acceleration  $\alpha$  is defined as the rate of change of angular velocity. In equation form, angular acceleration is expressed as follows:

### **Equation:**

$$lpha = rac{\Delta \omega}{\Delta t},$$

where  $\Delta\omega$  is the **change in angular velocity** and  $\Delta t$  is the change in time. The units of angular acceleration are (rad/s)/s, or  $rad/s^2$ . If  $\omega$  increases, then  $\alpha$  is positive. If  $\omega$  decreases, then  $\alpha$  is negative.

### **Example:**

Calculating the Angular Acceleration and Deceleration of a Bike Wheel

Suppose a teenager puts her bicycle on its back and starts the rear wheel spinning from rest to a final angular velocity of 250 rpm in 5.00 s. (a) Calculate the angular acceleration in  $rad/s^2$ . (b) If she now slams on the brakes, causing an angular acceleration of  $-87.3 \text{ rad/s}^2$ , how long does it take the wheel to stop?

### Strategy for (a)

The angular acceleration can be found directly from its definition in  $\alpha = \frac{\Delta \omega}{\Delta t}$  because the final angular velocity and time are given. We see that  $\Delta \omega$  is 250 rpm and  $\Delta t$  is 5.00 s.

### Solution for (a)

Entering known information into the definition of angular acceleration, we get

#### **Equation:**

$$lpha = rac{\Delta \omega}{\Delta t} \ = rac{250 ext{ rpm}}{5.00 ext{ s}}.$$

Because  $\Delta\omega$  is in revolutions per minute (rpm) and we want the standard units of  $\mathrm{rad/s}^2$  for angular acceleration, we need to convert  $\Delta\omega$  from rpm to rad/s:

### **Equation:**

$$egin{array}{lcl} \Delta\omega &=& 250rac{\mathrm{rev}}{\mathrm{min}}\cdotrac{2\pi\,\mathrm{rad}}{\mathrm{rev}}\cdotrac{1\,\mathrm{min}}{60\,\mathrm{sec}} \ &=& 26.2rac{\mathrm{rad}}{\mathrm{s}}. \end{array}$$

Entering this quantity into the expression for  $\alpha$ , we get

### **Equation:**

$$egin{array}{lll} lpha &=& rac{\Delta \omega}{\Delta t} \ &=& rac{26.2 \ \mathrm{rad/s}}{5.00 \ \mathrm{s}} \ &=& 5.24 \ \mathrm{rad/s}^2. \end{array}$$

### Strategy for (b)

In this part, we know the angular acceleration and the initial angular velocity. We can find the stoppage time by using the definition of angular acceleration and solving for  $\Delta t$ , yielding

### **Equation:**

$$\Delta t = rac{\Delta \omega}{lpha}.$$

### Solution for (b)

Here the angular velocity decreases from 26.2 rad/s (250 rpm) to zero, so that  $\Delta\omega$  is -26.2 rad/s, and  $\alpha$  is given to be  $-87.3 \text{ rad/s}^2$ . Thus,

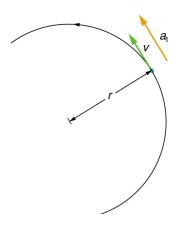
### **Equation:**

$$egin{array}{lll} \Delta t & = & rac{-26.2 \ {
m rad/s}}{-87.3 \ {
m rad/s}^2} \ & = & 0.300 \ {
m s}. \end{array}$$

#### Discussion

Note that the angular acceleration as the girl spins the wheel is small and positive; it takes 5 s to produce an appreciable angular velocity. When she hits the brake, the angular acceleration is large and negative. The angular velocity quickly goes to zero. In both cases, the relationships are analogous to what happens with linear motion. For example, there is a large deceleration when you crash into a brick wall—the velocity change is large in a short time interval.

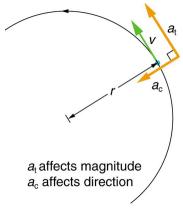
If the bicycle in the preceding example had been on its wheels instead of upside-down, it would first have accelerated along the ground and then come to a stop. This connection between circular motion and linear motion needs to be explored. For example, it would be useful to know how linear and angular acceleration are related. In circular motion, linear acceleration is *tangent* to the circle at the point of interest, as seen in [link]. Thus, linear acceleration is called **tangential acceleration**  $a_t$ .



In circular motion, linear acceleration a, occurs as the magnitude of the velocity changes: a is tangent to the motion. In the context of circular motion, linear acceleration is also called tangential acceleration  $a_{\mathsf{t}}.$ 

Linear or tangential acceleration refers to changes in the magnitude of velocity but not its direction. We know from <u>Uniform Circular Motion and Gravitation</u> that in circular motion centripetal acceleration,  $a_{\rm c}$ , refers to changes in the direction of the velocity but not its magnitude. An object undergoing circular motion experiences centripetal acceleration, as seen in [<u>link</u>]. Thus,  $a_{\rm t}$  and  $a_{\rm c}$  are perpendicular and independent of one another.

Tangential acceleration  $a_t$  is directly related to the angular acceleration  $\alpha$  and is linked to an increase or decrease in the velocity, but not its direction.



Centripetal acceleration  $a_c$  occurs as the direction of velocity changes; it is perpendicular to the circular motion.

Centripetal and tangential acceleration are thus perpendicular to each other.

Now we can find the exact relationship between linear acceleration  $a_{\rm t}$  and angular acceleration  $\alpha$ . Because linear acceleration is proportional to a change in the magnitude of the velocity, it is defined (as it was in One-Dimensional Kinematics) to be

### **Equation:**

$$a_{
m t} = rac{\Delta v}{\Delta t}.$$

For circular motion, note that  $v = r\omega$ , so that

### **Equation:**

$$a_{
m t} = rac{\Delta(r\omega)}{\Delta t}.$$

The radius r is constant for circular motion, and so  $\Delta(r\omega)=r(\Delta\omega)$ . Thus, **Equation:** 

$$a_{
m t} = r rac{\Delta \omega}{\Delta t}.$$

By definition,  $\alpha = \frac{\Delta \omega}{\Delta t}$ . Thus,

### **Equation:**

$$a_{
m t}=rlpha,$$

or

### **Equation:**

$$\alpha = \frac{a_{\mathrm{t}}}{r}$$
.

These equations mean that linear acceleration and angular acceleration are directly proportional. The greater the angular acceleration is, the larger the linear (tangential) acceleration is, and vice versa. For example, the greater the angular acceleration of a car's drive wheels, the greater the acceleration of the car. The radius also matters. For example, the smaller a wheel, the smaller its linear acceleration for a given angular acceleration  $\alpha$ .

### **Example:**

### Calculating the Angular Acceleration of a Motorcycle Wheel

A powerful motorcycle can accelerate from 0 to 30.0 m/s (about 108 km/h) in 4.20 s. What is the angular acceleration of its 0.320-m-radius wheels? (See [link].)



The linear acceleration of a motorcycle is accompanied by an angular acceleration of its wheels.

### Strategy

We are given information about the linear velocities of the motorcycle. Thus, we can find its linear acceleration  $a_t$ . Then, the expression  $\alpha = \frac{a_t}{r}$  can be used to find the angular acceleration.

#### Solution

The linear acceleration is

### **Equation:**

$$egin{array}{lcl} a_{
m t} & = & rac{\Delta v}{\Delta t} \ & = & rac{30.0 \ {
m m/s}}{4.20 \ {
m s}} \ & = & 7.14 \ {
m m/s}^2. \end{array}$$

We also know the radius of the wheels. Entering the values for  $a_{\rm t}$  and r into  $lpha=rac{a_{
m t}}{r}$  , we get

### **Equation:**

$$egin{array}{lll} lpha & = & rac{a_{
m t}}{r} \ & = & rac{7.14 \ {
m m/s}^2}{0.320 \ {
m m}} \ & = & 22.3 \ {
m rad/s}^2. \end{array}$$

### Discussion

Units of radians are dimensionless and appear in any relationship between angular and linear quantities.

So far, we have defined three rotational quantities— $\theta$ ,  $\omega$ , and  $\alpha$ . These quantities are analogous to the translational quantities x, v, and a. [link] displays rotational quantities, the analogous translational quantities, and the relationships between them.

Rotational	Translational	Relationship
heta	x	$ heta=rac{x}{r}$
$\omega$	v	$\omega=rac{v}{r}$
$\alpha$	a	$lpha=rac{a_t}{r}$

Rotational and Translational Quantities

#### Note:

### Making Connections: Take-Home Experiment

Sit down with your feet on the ground on a chair that rotates. Lift one of your legs such that it is unbent (straightened out). Using the other leg, begin to rotate yourself by pushing on the ground. Stop using your leg to push the ground but allow the chair to rotate. From the origin where you began, sketch the angle, angular velocity, and angular acceleration of your leg as a function of time in the form of three separate graphs. Estimate the magnitudes of these quantities.

#### **Exercise:**

### **Check Your Understanding**

#### Problem:

Angular acceleration is a vector, having both magnitude and direction. How do we denote its magnitude and direction? Illustrate with an example.

#### **Solution:**

The magnitude of angular acceleration is  $\alpha$  and its most common units are  $\mathrm{rad/s}^2$ . The direction of angular acceleration along a fixed axis is denoted by a + or a – sign, just as the direction of linear acceleration in one dimension is denoted by a + or a – sign. For example, consider a gymnast doing a forward flip. Her angular momentum would be parallel to the mat and to her left. The magnitude of her angular acceleration would be proportional to her angular velocity (spin rate) and her moment of inertia about her spin axis.

#### Note:

### PhET Explorations: Ladybug Revolution

Join the ladybug in an exploration of rotational motion. Rotate the merry-go-round to change its angle, or choose a constant angular velocity or

angular acceleration. Explore how circular motion relates to the bug's x,y position, velocity, and acceleration using vectors or graphs.

Ladybug Revolutio n

### **Section Summary**

- Uniform circular motion is the motion with a constant angular velocity  $\omega=rac{\Delta heta}{\Delta t}.$
- In non-uniform circular motion, the velocity changes with time and the rate of change of angular velocity (i.e. angular acceleration) is  $\alpha = \frac{\Delta \omega}{\Delta t}$ .
- Linear or tangential acceleration refers to changes in the magnitude of velocity but not its direction, given as  $a_{\rm t} = \frac{\Delta v}{\Delta t}$ .
- For circular motion, note that  $v=r\omega$ , so that **Equation:**

$$a_{
m t} = rac{\Delta(r\omega)}{\Delta t}.$$

• The radius r is constant for circular motion, and so  $\Delta(r\omega)=r\Delta\omega$ . Thus,

### **Equation:**

$$a_{
m t} = r rac{\Delta \omega}{\Delta t}.$$

• By definition,  $\Delta \omega/\Delta t = \alpha$ . Thus, **Equation:** 

$$a_{
m t}=rlpha$$

or

### **Equation:**

$$\alpha = \frac{a_{\mathrm{t}}}{r}$$
.

### **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

Analogies exist between rotational and translational physical quantities. Identify the rotational term analogous to each of the following: acceleration, force, mass, work, translational kinetic energy, linear momentum, impulse.

#### **Exercise:**

#### **Problem:**

Explain why centripetal acceleration changes the direction of velocity in circular motion but not its magnitude.

#### **Exercise:**

#### **Problem:**

In circular motion, a tangential acceleration can change the magnitude of the velocity but not its direction. Explain your answer.

#### **Exercise:**

#### **Problem:**

Suppose a piece of food is on the edge of a rotating microwave oven plate. Does it experience nonzero tangential acceleration, centripetal acceleration, or both when: (a) The plate starts to spin? (b) The plate rotates at constant angular velocity? (c) The plate slows to a halt?

#### **Problems & Exercises**

#### **Exercise:**

#### Problem:

At its peak, a tornado is 60.0 m in diameter and carries 500 km/h winds. What is its angular velocity in revolutions per second?

#### **Solution:**

 $\omega = 0.737 \ \mathrm{rev/s}$ 

#### **Exercise:**

### **Problem:** Integrated Concepts

An ultracentrifuge accelerates from rest to 100,000 rpm in 2.00 min. (a) What is its angular acceleration in  $rad/s^2$ ? (b) What is the tangential acceleration of a point 9.50 cm from the axis of rotation? (c) What is the radial acceleration in  $m/s^2$  and multiples of g of this point at full rpm?

#### **Exercise:**

### **Problem:** Integrated Concepts

You have a grindstone (a disk) that is 90.0 kg, has a 0.340-m radius, and is turning at 90.0 rpm, and you press a steel axe against it with a radial force of 20.0 N. (a) Assuming the kinetic coefficient of friction

between steel and stone is 0.20, calculate the angular acceleration of the grindstone. (b) How many turns will the stone make before coming to rest?

#### **Solution:**

- (a)  $-0.26 \text{ rad/s}^2$
- (b) 27 rev

#### **Exercise:**

#### **Problem:** Unreasonable Results

You are told that a basketball player spins the ball with an angular acceleration of  $100 \, \mathrm{rad/s^2}$ . (a) What is the ball's final angular velocity if the ball starts from rest and the acceleration lasts 2.00 s? (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?

### Glossary

angular acceleration

the rate of change of angular velocity with time

change in angular velocity

the difference between final and initial values of angular velocity

tangential acceleration

the acceleration in a direction tangent to the circle at the point of interest in circular motion

# Introduction to Uniform Circular Motion class="introduction"

This Australian **Grand Prix** Formula 1 race car moves in a circular path as it makes the turn. Its wheels also spin rapidly —the latter completing many revolutions, the former only part of one (a circular arc). The same physical principles are involved in each. (credit: Richard Munckton)



Many motions, such as the arc of a bird's flight or Earth's path around the Sun, are curved. Recall that Newton's first law tells us that motion is along a straight line at constant speed unless there is a net external force. We will therefore study not only motion along curves, but also the forces that cause it, including gravitational forces. In some ways, this chapter is a continuation of <u>Dynamics: Newton's Laws of Motion</u> as we study more applications of Newton's laws of motion.

This chapter deals with the simplest form of curved motion, **uniform circular motion**, motion in a circular path at constant speed. Studying this topic illustrates most concepts associated with rotational motion and leads to the study of many new topics we group under the name *rotation*. Pure *rotational motion* occurs when points in an object move in circular paths centered on one point. Pure *translational motion* is motion with no rotation. Some motion combines both types, such as a rotating hockey puck moving along ice.

## Glossary

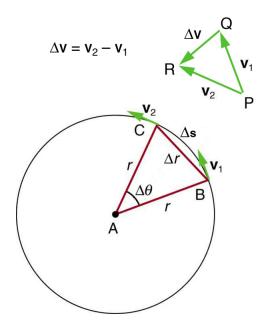
uniform circular motion the motion of an object in a circular path at constant speed

### Centripetal Acceleration

- Establish the expression for centripetal acceleration.
- Explain the centrifuge.

We know from kinematics that acceleration is a change in velocity, either in its magnitude or in its direction, or both. In uniform circular motion, the direction of the velocity changes constantly, so there is always an associated acceleration, even though the magnitude of the velocity might be constant. You experience this acceleration yourself when you turn a corner in your car. (If you hold the wheel steady during a turn and move at constant speed, you are in uniform circular motion.) What you notice is a sideways acceleration because you and the car are changing direction. The sharper the curve and the greater your speed, the more noticeable this acceleration will become. In this section we examine the direction and magnitude of that acceleration.

[link] shows an object moving in a circular path at constant speed. The direction of the instantaneous velocity is shown at two points along the path. Acceleration is in the direction of the change in velocity, which points directly toward the center of rotation (the center of the circular path). This pointing is shown with the vector diagram in the figure. We call the acceleration of an object moving in uniform circular motion (resulting from a net external force) the **centripetal acceleration**( $a_c$ ); centripetal means "toward the center" or "center seeking."



The directions of the velocity of an object at two different points are shown, and the change in velocity  $\Delta \mathbf{v}$  is seen to point directly toward the center of curvature. (See small inset.) Because  $\mathbf{a}_{\mathrm{c}} = \Delta \mathbf{v}/\Delta t$ , the acceleration is also toward the center;  $\mathbf{a}_c$  is called centripetal acceleration. (Because  $\Delta\theta$  is very small, the arc length  $\Delta s$  is equal to the chord length  $\Delta r$  for small time differences.)

The direction of centripetal acceleration is toward the center of curvature, but what is its magnitude? Note that the triangle formed by the velocity vectors and the one formed by the radii r and  $\Delta s$  are similar. Both the

triangles ABC and PQR are isosceles triangles (two equal sides). The two equal sides of the velocity vector triangle are the speeds  $v_1 = v_2 = v$ . Using the properties of two similar triangles, we obtain

### **Equation:**

$$rac{\Delta v}{v} = rac{\Delta s}{r}.$$

Acceleration is  $\Delta v/\Delta t$ , and so we first solve this expression for  $\Delta v$ :

# **Equation:**

$$\Delta v = rac{v}{r} \Delta s.$$

Then we divide this by  $\Delta t$ , yielding

### **Equation:**

$$rac{\Delta v}{\Delta t} = rac{v}{r} imes rac{\Delta s}{\Delta t}.$$

Finally, noting that  $\Delta v/\Delta t=a_{\rm c}$  and that  $\Delta s/\Delta t=v$ , the linear or tangential speed, we see that the magnitude of the centripetal acceleration is **Equation:** 

$$a_{
m c}=rac{v^2}{r},$$

which is the acceleration of an object in a circle of radius r at a speed v. So, centripetal acceleration is greater at high speeds and in sharp curves (smaller radius), as you have noticed when driving a car. But it is a bit surprising that  $a_{\rm c}$  is proportional to speed squared, implying, for example, that it is four times as hard to take a curve at 100 km/h than at 50 km/h. A sharp corner has a small radius, so that  $a_{\rm c}$  is greater for tighter turns, as you have probably noticed.

It is also useful to express  $a_{\rm c}$  in terms of angular velocity. Substituting  $v=r\omega$  into the above expression, we find  $a_{\rm c}=(r\omega)^2/r=r\omega^2$ . We can express the magnitude of centripetal acceleration using either of two equations:

# **Equation:**

$$a_{
m c}=rac{v^2}{r};\,\,a_{
m c}=r\omega^2.$$

Recall that the direction of  $a_c$  is toward the center. You may use whichever expression is more convenient, as illustrated in examples below.

A **centrifuge** (see [link]b) is a rotating device used to separate specimens of different densities. High centripetal acceleration significantly decreases the time it takes for separation to occur, and makes separation possible with small samples. Centrifuges are used in a variety of applications in science and medicine, including the separation of single cell suspensions such as bacteria, viruses, and blood cells from a liquid medium and the separation of macromolecules, such as DNA and protein, from a solution. Centrifuges are often rated in terms of their centripetal acceleration relative to acceleration due to gravity (g); maximum centripetal acceleration of several hundred thousand g is possible in a vacuum. Human centrifuges, extremely large centrifuges, have been used to test the tolerance of astronauts to the effects of accelerations larger than that of Earth's gravity.

# **Example:**

# How Does the Centripetal Acceleration of a Car Around a Curve Compare with That Due to Gravity?

What is the magnitude of the centripetal acceleration of a car following a curve of radius 500 m at a speed of 25.0 m/s (about 90 km/h)? Compare the acceleration with that due to gravity for this fairly gentle curve taken at highway speed. See [link](a).

# Strategy

Because v and r are given, the first expression in  $a_c = \frac{v^2}{r}$ ;  $a_c = r\omega^2$  is the most convenient to use.

### Solution

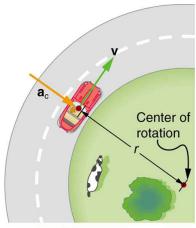
Entering the given values of  $v=25.0~\mathrm{m/s}$  and  $r=500~\mathrm{m}$  into the first expression for  $a_{\mathrm{c}}$  gives

# **Equation:**

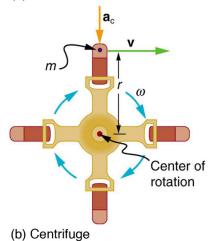
$$a_{
m c} = rac{v^2}{r} = rac{(25.0 \ {
m m/s})^2}{500 \ {
m m}} = 1.25 \ {
m m/s}^2.$$

### **Discussion**

To compare this with the acceleration due to gravity  $(g=9.80~{\rm m/s}^2)$ , we take the ratio of  $a_{\rm c}/g=\left(1.25~{\rm m/s}^2\right)/\left(9.80~{\rm m/s}^2\right)=0.128$ . Thus,  $a_{\rm c}=0.128~{\rm g}$  and is noticeable especially if you were not wearing a seat belt.



(a) Car around corner



(a) The car following a circular path at constant speed is accelerated perpendicular to its velocity, as shown. The magnitude of this centripetal acceleration is found in [link]. (b) A particle of mass in a centrifuge is rotating at constant angular velocity. It

must be accelerated perpendicular to its velocity or it would continue in a straight line. The magnitude of the necessary acceleration is found in [link].

# **Example:**

# How Big Is the Centripetal Acceleration in an Ultracentrifuge?

Calculate the centripetal acceleration of a point 7.50 cm from the axis of an **ultracentrifuge** spinning at  $7.5 \times 10^4 \, \mathrm{rev/min}$ . Determine the ratio of this acceleration to that due to gravity. See [link](b).

# Strategy

The term rev/min stands for revolutions per minute. By converting this to radians per second, we obtain the angular velocity  $\omega$ . Because r is given, we can use the second expression in the equation  $a_{\rm c}=\frac{v^2}{r}$ ;  $a_{\rm c}=r\omega^2$  to calculate the centripetal acceleration.

### Solution

To convert  $7.50 \times 10^4 \, \mathrm{rev/min}$  to radians per second, we use the facts that one revolution is  $2\pi \mathrm{rad}$  and one minute is 60.0 s. Thus,

# **Equation:**

$$\omega = 7.50 imes 10^4 \, rac{ ext{rev}}{ ext{min}} imes rac{2\pi ext{ rad}}{1 ext{ rev}} imes rac{1 ext{ min}}{60.0 ext{ s}} = 7854 ext{ rad/s}.$$

Now the centripetal acceleration is given by the second expression in  $a_{\rm c}=rac{v^2}{r}$ ;  $a_{\rm c}=r\omega^2$  as

# **Equation:**

$$a_{
m c}=r\omega^2$$
.

Converting 7.50 cm to meters and substituting known values gives **Equation:** 

$$a_{\rm c} = (0.0750~{
m m})(7854~{
m rad/s})^2 = 4.63 imes 10^6~{
m m/s}^2.$$

Note that the unitless radians are discarded in order to get the correct units for centripetal acceleration. Taking the ratio of  $a_c$  to g yields

### **Equation:**

$$rac{a_{
m c}}{g} = rac{4.63 imes 10^6}{9.80} = 4.72 imes 10^5.$$

#### Discussion

This last result means that the centripetal acceleration is 472,000 times as strong as g. It is no wonder that such high  $\omega$  centrifuges are called ultracentrifuges. The extremely large accelerations involved greatly decrease the time needed to cause the sedimentation of blood cells or other materials.

Of course, a net external force is needed to cause any acceleration, just as Newton proposed in his second law of motion. So a net external force is needed to cause a centripetal acceleration. In <u>Centripetal Force</u>, we will consider the forces involved in circular motion.

#### Note:

# PhET Explorations: Ladybug Motion 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

https://archive.cnx.org/specials/317a2b1e-2fbd-11e5-99b5-e38ffb545fe6/ladybug-motion/#sim-ladybug-motion

# **Section Summary**

• Centripetal acceleration  $a_{\rm c}$  is the acceleration experienced while in uniform circular motion. It always points toward the center of rotation. It is perpendicular to the linear velocity v and has the magnitude **Equation:** 

$$a_{
m c}=rac{v^2}{r}; a_{
m c}=r\omega^2.$$

• The unit of centripetal acceleration is  $m/s^2$ .

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

Can centripetal acceleration change the speed of circular motion? Explain.

# **Problem Exercises**

#### **Exercise:**

#### **Problem:**

A fairground ride spins its occupants inside a flying saucer-shaped container. If the horizontal circular path the riders follow has an 8.00 m radius, at how many revolutions per minute will the riders be subjected to a centripetal acceleration whose magnitude is 1.50 times that due to gravity?

#### **Solution:**

12.9 rev/min

A runner taking part in the 200 m dash must run around the end of a track that has a circular arc with a radius of curvature of 30 m. If he completes the 200 m dash in 23.2 s and runs at constant speed throughout the race, what is the magnitude of his centripetal acceleration as he runs the curved portion of the track?

#### **Exercise:**

### **Problem:**

Taking the age of Earth to be about  $4 \times 10^9$  years and assuming its orbital radius of  $1.5 \times 10^{11}$  m has not changed and is circular, calculate the approximate total distance Earth has traveled since its birth (in a frame of reference stationary with respect to the Sun).

#### **Solution:**

$$4 \times 10^{21} \,\mathrm{m}$$

### **Exercise:**

#### **Problem:**

The propeller of a World War II fighter plane is 2.30 m in diameter.

- (a) What is its angular velocity in radians per second if it spins at 1200 rev/min?
- (b) What is the linear speed of its tip at this angular velocity if the plane is stationary on the tarmac?
- (c) What is the centripetal acceleration of the propeller tip under these conditions? Calculate it in meters per second squared and convert to multiples of g.

An ordinary workshop grindstone has a radius of 7.50 cm and rotates at 6500 rev/min.

- (a) Calculate the magnitude of the centripetal acceleration at its edge in meters per second squared and convert it to multiples of g.
- (b) What is the linear speed of a point on its edge?

### **Solution:**

a) 
$$3.47 \times 10^4 \,\mathrm{m/s^2}$$
,  $3.55 \times 10^3 \,\mathrm{g}$ 

b) 
$$51.1 \text{ m/s}$$

#### **Exercise:**

#### **Problem:**

Helicopter blades withstand tremendous stresses. In addition to supporting the weight of a helicopter, they are spun at rapid rates and experience large centripetal accelerations, especially at the tip.

- (a) Calculate the magnitude of the centripetal acceleration at the tip of a 4.00 m long helicopter blade that rotates at 300 rev/min.
- (b) Compare the linear speed of the tip with the speed of sound (taken to be 340 m/s).

#### **Exercise:**

**Problem:** Olympic ice skaters are able to spin at about 5 rev/s.

- (a) What is their angular velocity in radians per second?
- (b) What is the centripetal acceleration of the skater's nose if it is 0.120 m from the axis of rotation?

- (c) An exceptional skater named Dick Button was able to spin much faster in the 1950s than anyone since—at about 9 rev/s. What was the centripetal acceleration of the tip of his nose, assuming it is at 0.120 m radius?
- (d) Comment on the magnitudes of the accelerations found. It is reputed that Button ruptured small blood vessels during his spins.

#### **Solution:**

- a) 31.4 rad/s
- b) 118 m/s
- c) 384 m/s
- d)The centripetal acceleration felt by Olympic skaters is 12 times larger than the acceleration due to gravity. That's quite a lot of acceleration in itself. The centripetal acceleration felt by Button's nose was 39.2 times larger than the acceleration due to gravity. It is no wonder that he ruptured small blood vessels in his spins.

### **Exercise:**

#### **Problem:**

What percentage of the acceleration at Earth's surface is the acceleration due to gravity at the position of a satellite located 300 km above Earth?

#### **Exercise:**

#### **Problem:**

Verify that the linear speed of an ultracentrifuge is about 0.50 km/s, and Earth in its orbit is about 30 km/s by calculating:

(a) The linear speed of a point on an ultracentrifuge 0.100 m from its center, rotating at 50,000 rev/min.

(b) The linear speed of Earth in its orbit about the Sun (use data from the text on the radius of Earth's orbit and approximate it as being circular).

### **Solution:**

- a) 0.524 km/s
- b) 29.7 km/s

#### **Exercise:**

#### **Problem:**

A rotating space station is said to create "artificial gravity"—a loosely-defined term used for an acceleration that would be crudely similar to gravity. The outer wall of the rotating space station would become a floor for the astronauts, and centripetal acceleration supplied by the floor would allow astronauts to exercise and maintain muscle and bone strength more naturally than in non-rotating space environments. If the space station is 200 m in diameter, what angular velocity would produce an "artificial gravity" of  $9.80 \, \mathrm{m/s^2}$  at the rim?

#### **Exercise:**

#### Problem:

At takeoff, a commercial jet has a 60.0 m/s speed. Its tires have a diameter of 0.850 m.

- (a) At how many rev/min are the tires rotating?
- (b) What is the centripetal acceleration at the edge of the tire?
- (c) With what force must a determined  $1.00 \times 10^{-15}$  kg bacterium cling to the rim?
- (d) Take the ratio of this force to the bacterium's weight.

- (a)  $1.35 \times 10^3 \text{ rpm}$
- (b)  $8.47 \times 10^3 \text{ m/s}^2$
- (c)  $8.47 \times 10^{-12} \,\mathrm{N}$
- (d) 865

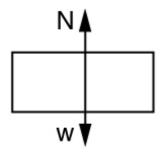
### **Exercise:**

# **Problem:**Integrated Concepts

Riders in an amusement park ride shaped like a Viking ship hung from a large pivot are rotated back and forth like a rigid pendulum. Sometime near the middle of the ride, the ship is momentarily motionless at the top of its circular arc. The ship then swings down under the influence of gravity.

- (a) Assuming negligible friction, find the speed of the riders at the bottom of its arc, given the system's center of mass travels in an arc having a radius of 14.0 m and the riders are near the center of mass.
- (b) What is the centripetal acceleration at the bottom of the arc?
- (c) Draw a free body diagram of the forces acting on a rider at the bottom of the arc.
- (d) Find the force exerted by the ride on a 60.0 kg rider and compare it to her weight.
- (e) Discuss whether the answer seems reasonable.

- (a) 16.6 m/s
- (b)  $19.6 \text{ m/s}^2$
- (c)



- (d)  $1.76 \times 10^3 \, \mathrm{N} \ \mathrm{or} \ 3.00 \, w$  , that is, the normal force (upward) is three times her weight.
- (e) This answer seems reasonable, since she feels like she's being forced into the chair MUCH stronger than just by gravity.

### **Exercise:**

### **Problem:** Unreasonable Results

A mother pushes her child on a swing so that his speed is 9.00 m/s at the lowest point of his path. The swing is suspended 2.00 m above the child's center of mass.

- (a) What is the magnitude of the centripetal acceleration of the child at the low point?
- (b) What is the magnitude of the force the child exerts on the seat if his mass is 18.0 kg?
- (c) What is unreasonable about these results?
- (d) Which premises are unreasonable or inconsistent?

- a)  $40.5 \text{ m/s}^2$
- b) 905 N

- c) The force in part (b) is very large. The acceleration in part (a) is too much, about 4 g.
- d) The speed of the swing is too large. At the given velocity at the bottom of the swing, there is enough kinetic energy to send the child all the way over the top, ignoring friction.

# **Glossary**

## centripetal acceleration

the acceleration of an object moving in a circle, directed toward the center

# ultracentrifuge

a centrifuge optimized for spinning a rotor at very high speeds

# Centripetal Force

- Calculate coefficient of friction on a car tire.
- Calculate ideal speed and angle of a car on a turn.

Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, friction between roller skates and a rink floor, a banked roadway's force on a car, and forces on the tube of a spinning centrifuge.

Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton's second law of motion, net force is mass times acceleration: net F = ma. For uniform circular motion, the acceleration is the centripetal acceleration  $-a = a_c$ . Thus, the magnitude of centripetal force  $F_c$  is

### **Equation:**

$$F_c = ma_c$$
.

By using the expressions for centripetal acceleration  $a_c$  from  $a_c=\frac{v^2}{r}$ ;  $a_c=r\omega^2$ , we get two expressions for the centripetal force  $F_c$  in terms of mass, velocity, angular velocity, and radius of curvature:

# **Equation:**

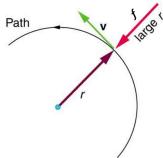
$$F_c=mrac{v^2}{r}; F_c={
m mr}\omega^2.$$

You may use whichever expression for centripetal force is more convenient. Centripetal force  $F_c$  is always perpendicular to the path and pointing to the center of curvature, because  $\mathbf{a}_c$  is perpendicular to the velocity and pointing to the center of curvature.

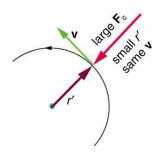
Note that if you solve the first expression for r, you get **Equation:** 

$$r=rac{mv^2}{F_c}.$$

This implies that for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve.



 $f = \mathbf{F}_{c}$  is parallel to  $\mathbf{a}_{c}$  since  $\mathbf{F}_{c} = m\mathbf{a}_{c}$ 



The frictional force supplies the centripetal force and is numerically equal to it.

Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the F<sub>c</sub>, the smaller the radius of curvature r and the sharper the curve.

The second curve

has the same v, but a larger  $F_c$  produces a smaller r'.

### **Example:**

#### What Coefficient of Friction Do Car Tires Need on a Flat Curve?

- (a) Calculate the centripetal force exerted on a 900 kg car that negotiates a 500 m radius curve at 25.0 m/s.
- (b) Assuming an unbanked curve, find the minimum static coefficient of friction, between the tires and the road, static friction being the reason that keeps the car from slipping (see [link]).

# Strategy and Solution for (a)

We know that  $F_{
m c}=rac{mv^2}{r}.$  Thus,

### **Equation:**

$$F_{
m c} = rac{mv^2}{r} = rac{(900~{
m kg})(25.0~{
m m/s})^2}{(500~{
m m})} = 1125~{
m N}.$$

# Strategy for (b)

[link] shows the forces acting on the car on an unbanked (level ground) curve. Friction is to the left, keeping the car from slipping, and because it is the only horizontal force acting on the car, the friction is the centripetal force in this case. We know that the maximum static friction (at which the tires roll but do not slip) is  $\mu_s N$ , where  $\mu_s$  is the static coefficient of friction and N is the normal force. The normal force equals the car's weight on level ground, so that N=mg. Thus the centripetal force in this situation is

# **Equation:**

$$F_{
m c}=f=\mu_{
m s}N=\mu_{
m s}{
m mg}.$$

Now we have a relationship between centripetal force and the coefficient of friction. Using the first expression for  $F_c$  from the equation

# **Equation:**

$$\left.egin{aligned} F_{
m c} = mrac{v^2}{r} \ F_{
m c} = mr\omega^2 \end{aligned}
ight\},$$

# **Equation:**

$$mrac{v^2}{r}=\mu_{
m s}{
m mg}.$$

We solve this for  $\mu_{\rm s}$ , noting that mass cancels, and obtain

# **Equation:**

$$\mu_{
m s} = rac{v^2}{{
m rg}}.$$

### Solution for (b)

Substituting the knowns,

# **Equation:**

$$\mu_{
m s} = rac{(25.0~{
m m/s})^2}{(500~{
m m})(9.80~{
m m/s}^2)} = 0.13.$$

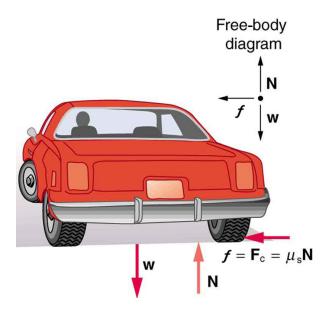
(Because coefficients of friction are approximate, the answer is given to only two digits.)

### Discussion

We could also solve part (a) using the first expression in  $rac{F_{
m c}=mrac{v^2}{r}}{F_{
m c}=mr\omega^2} 
ight\}$  ,

because m,v, and r are given. The coefficient of friction found in part (b) is much smaller than is typically found between tires and roads. The car will still negotiate the curve if the coefficient is greater than 0.13, because static friction is a responsive force, being able to assume a value less than but no more than  $\mu_s N$ . A higher coefficient would also allow the car to negotiate the curve at a higher speed, but if the coefficient of friction is

less, the safe speed would be less than 25 m/s. Note that mass cancels, implying that in this example, it does not matter how heavily loaded the car is to negotiate the turn. Mass cancels because friction is assumed proportional to the normal force, which in turn is proportional to mass. If the surface of the road were banked, the normal force would be less as will be discussed below.



This car on level ground is moving away and turning to the left. The centripetal force causing the car to turn in a circular path is due to friction between the tires and the road. A minimum coefficient of friction is needed, or the car will move in a larger-radius curve and leave the roadway.

Let us now consider **banked curves**, where the slope of the road helps you negotiate the curve. See [link]. The greater the angle  $\theta$ , the faster you can

take the curve. Race tracks for bikes as well as cars, for example, often have steeply banked curves. In an "ideally banked curve," the angle  $\theta$  is such that you can negotiate the curve at a certain speed without the aid of friction between the tires and the road. We will derive an expression for  $\theta$  for an ideally banked curve and consider an example related to it.

For **ideal banking**, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force N in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively. In cases in which forces are not parallel, it is most convenient to consider components along perpendicular axes—in this case, the vertical and horizontal directions.

[link] shows a free body diagram for a car on a frictionless banked curve. If the angle  $\theta$  is ideal for the speed and radius, then the net external force will equal the necessary centripetal force. The only two external forces acting on the car are its weight  $\mathbf{w}$  and the normal force of the road  $\mathbf{N}$ . (A frictionless surface can only exert a force perpendicular to the surface—that is, a normal force.) These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude  $\mathrm{mv}^2/\mathrm{r}$ . Because this is the crucial force and it is horizontal, we use a coordinate system with vertical and horizontal axes. Only the normal force has a horizontal component, and so this must equal the centripetal force—that is, **Equation:** 

-

$$N\sin heta=rac{mv^2}{r}.$$

Because the car does not leave the surface of the road, the net vertical force must be zero, meaning that the vertical components of the two external forces must be equal in magnitude and opposite in direction. From the figure, we see that the vertical component of the normal force is  $N\cos\theta$ , and the only other vertical force is the car's weight. These must be equal in magnitude; thus,

# **Equation:**

$$N\cos\theta=\mathrm{mg}.$$

Now we can combine the last two equations to eliminate N and get an expression for  $\theta$ , as desired. Solving the second equation for  $N = mg/(\cos \theta)$ , and substituting this into the first yields

# **Equation:**

$$\operatorname{mg} \frac{\sin \theta}{\cos \theta} = \frac{\operatorname{mv}^2}{r}$$

# **Equation:**

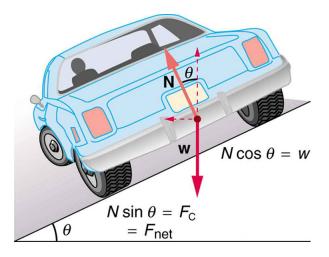
$$mg an( heta) = rac{mv^2}{r} \ an heta = rac{v^2}{
m rg.}$$

Taking the inverse tangent gives

# **Equation:**

$$\theta = \tan^{-1} \left( \frac{v^2}{\text{rg}} \right)$$
 (ideally banked curve, no friction).

This expression can be understood by considering how  $\theta$  depends on v and r. A large  $\theta$  will be obtained for a large v and a small r. That is, roads must be steeply banked for high speeds and sharp curves. Friction helps, because it allows you to take the curve at greater or lower speed than if the curve is frictionless. Note that  $\theta$  does not depend on the mass of the vehicle.



The car on this banked curve is moving away and turning to the left.

# **Example:**

# What Is the Ideal Speed to Take a Steeply Banked Tight Curve?

Curves on some test tracks and race courses, such as the Daytona International Speedway in Florida, are very steeply banked. This banking, with the aid of tire friction and very stable car configurations, allows the curves to be taken at very high speed. To illustrate, calculate the speed at which a 100 m radius curve banked at 65.0° should be driven if the road is frictionless.

# Strategy

We first note that all terms in the expression for the ideal angle of a banked curve except for speed are known; thus, we need only rearrange it so that speed appears on the left-hand side and then substitute known quantities.

# Solution

Starting with

# **Equation:**

$$an heta=rac{v^2}{ ext{rg}}$$

we get

### **Equation:**

$$v = (\operatorname{rg} \tan \theta)^{1/2}$$
.

Noting that  $\tan 65.0^{\circ} = 2.14$ , we obtain

# **Equation:**

$$v = \left[ (100 \text{ m})(9.80 \text{ m/s}^2)(2.14) \right]^{1/2}$$
  
= 45.8 m/s.

#### Discussion

This is just about 165 km/h, consistent with a very steeply banked and rather sharp curve. Tire friction enables a vehicle to take the curve at significantly higher speeds.

Calculations similar to those in the preceding examples can be performed for a host of interesting situations in which centripetal force is involved—a number of these are presented in this chapter's Problems and Exercises.

#### Note:

# Take-Home Experiment

Ask a friend or relative to swing a golf club or a tennis racquet. Take appropriate measurements to estimate the centripetal acceleration of the end of the club or racquet. You may choose to do this in slow motion.

### Note:

# PhET Explorations: Gravity and Orbits

Move the sun, earth, moon and space station to see how it affects their gravitational forces and orbital paths. Visualize the sizes and distances between different heavenly bodies, and turn off gravity to see what would happen without it!

https://phet.colorado.edu/sims/html/gravity-and-orbits/latest/gravity-and-orbits en.html

# **Section Summary**

• Centripetal force  $F_c$  is any force causing uniform circular motion. It is a "center-seeking" force that always points toward the center of rotation. It is perpendicular to linear velocity v and has magnitude **Equation:** 

$$F_{\rm c} = ma_{\rm c}$$

which can also be expressed as **Equation:** 

$$\left.egin{aligned} F_{
m c} = mrac{v^2}{r} \ {
m or} \ F_{
m c} = mr\omega^2 \end{aligned}
ight\}$$

# **Conceptual Questions**

#### **Exercise:**

### **Problem:**

If you wish to reduce the stress (which is related to centripetal force) on high-speed tires, would you use large- or small-diameter tires? Explain.

Define centripetal force. Can any type of force (for example, tension, gravitational force, friction, and so on) be a centripetal force? Can any combination of forces be a centripetal force?

### **Exercise:**

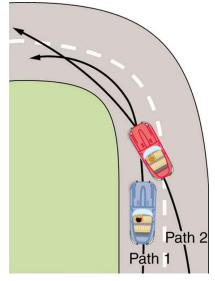
#### **Problem:**

If centripetal force is directed toward the center, why do you feel that you are 'thrown' away from the center as a car goes around a curve? Explain.

### **Exercise:**

### **Problem:**

Race car drivers routinely cut corners as shown in [link]. Explain how this allows the curve to be taken at the greatest speed.



Two paths around a race track curve are shown. Race car drivers will take the inside path (called cutting the corner)

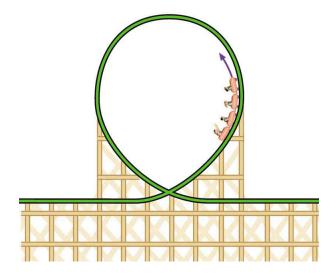
whenever possible because it allows them to take the curve at the highest speed.

### **Exercise:**

### **Problem:**

A number of amusement parks have rides that make vertical loops like the one shown in [link]. For safety, the cars are attached to the rails in such a way that they cannot fall off. If the car goes over the top at just the right speed, gravity alone will supply the centripetal force. What other force acts and what is its direction if:

- (a) The car goes over the top at faster than this speed?
- (b)The car goes over the top at slower than this speed?



Amusement rides with a vertical loop are an example of a form of curved motion.

### **Exercise:**

#### **Problem:**

What is the direction of the force exerted by the car on the passenger as the car goes over the top of the amusement ride pictured in [link] under the following circumstances:

- (a) The car goes over the top at such a speed that the gravitational force is the only force acting?
- (b) The car goes over the top faster than this speed?
- (c) The car goes over the top slower than this speed?

#### **Exercise:**

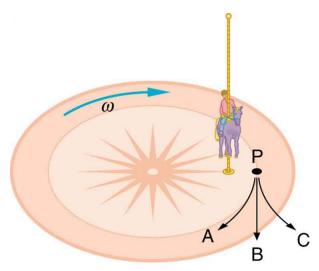
### **Problem:**

As a skater forms a circle, what force is responsible for making her turn? Use a free body diagram in your answer.

#### **Exercise:**

#### **Problem:**

Suppose a child is riding on a merry-go-round at a distance about halfway between its center and edge. She has a lunch box resting on wax paper, so that there is very little friction between it and the merry-go-round. Which path shown in [link] will the lunch box take when she lets go? The lunch box leaves a trail in the dust on the merry-go-round. Is that trail straight, curved to the left, or curved to the right? Explain your answer.



Merry-go-round's rotating frame of reference

A child riding on a merry-goround releases her lunch box at point P. This is a view from above the clockwise rotation.

Assuming it slides with negligible friction, will it follow path A, B, or C, as viewed from Earth's frame of reference?

What will be the shape of the path it leaves in the dust on the merry-go-round?

### **Exercise:**

### **Problem:**

Do you feel yourself thrown to either side when you negotiate a curve that is ideally banked for your car's speed? What is the direction of the force exerted on you by the car seat?

Suppose a mass is moving in a circular path on a frictionless table as shown in figure. In the Earth's frame of reference, there is no centrifugal force pulling the mass away from the centre of rotation, yet there is a very real force stretching the string attaching the mass to the nail. Using concepts related to centripetal force and Newton's third law, explain what force stretches the string, identifying its physical origin.

String

A mass attached to a nail on a frictionless table moves in a circular path. The force stretching the string is real and not fictional. What is the physical origin of the force on the string?

# **Problems Exercise**

- (a) A 22.0 kg child is riding a playground merry-go-round that is rotating at 40.0 rev/min. What centripetal force must she exert to stay on if she is 1.25 m from its center?
- (b) What centripetal force does she need to stay on an amusement park merry-go-round that rotates at 3.00 rev/min if she is 8.00 m from its center?
- (c) Compare each force with her weight.

### **Solution:**

- a) 483 N
- b) 17.4 N
- c) 2.24 times her weight, 0.0807 times her weight

#### **Exercise:**

#### **Problem:**

Calculate the centripetal force on the end of a 100 m (radius) wind turbine blade that is rotating at 0.5 rev/s. Assume the mass is 4 kg.

#### **Exercise:**

#### Problem:

What is the ideal banking angle for a gentle turn of 1.20 km radius on a highway with a 105 km/h speed limit (about 65 mi/h), assuming everyone travels at the limit?

#### **Solution:**

 $4.14^{\circ}$ 

What is the ideal speed to take a 100 m radius curve banked at a 20.0° angle?

### **Exercise:**

#### **Problem:**

- (a) What is the radius of a bobsled turn banked at 75.0° and taken at 30.0 m/s, assuming it is ideally banked?
- (b) Calculate the centripetal acceleration.
- (c) Does this acceleration seem large to you?

### **Solution:**

- a) 24.6 m
- b)  $36.6 \text{ m/s}^2$
- c)  $a_{\rm c}=3.73~g$ . This does not seem too large, but it is clear that bobsledders feel a lot of force on them going through sharply banked turns.

#### **Exercise:**

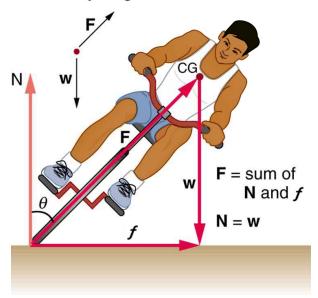
#### **Problem:**

Part of riding a bicycle involves leaning at the correct angle when making a turn, as seen in [link]. To be stable, the force exerted by the ground must be on a line going through the center of gravity. The force on the bicycle wheel can be resolved into two perpendicular components—friction parallel to the road (this must supply the centripetal force), and the vertical normal force (which must equal the system's weight).

(a) Show that  $\theta$  (as defined in the figure) is related to the speed v and radius of curvature r of the turn in the same way as for an ideally

banked roadway—that is,  $heta= an^{-1}v^2/rg$ 

(b) Calculate  $\theta$  for a 12.0 m/s turn of radius 30.0 m (as in a race). Free-body diagram



A bicyclist negotiating a turn on level ground must lean at the correct angle—the ability to do this becomes instinctive. The force of the ground on the wheel needs to be on a line through the center of gravity. The net external force on the system is the centripetal force. The vertical component of the force on the wheel cancels the weight of the system while its horizontal component must supply the centripetal force. This process produces a relationship among the angle  $\theta$ , the speed v, and the radius of curvature *r* of the turn similar to

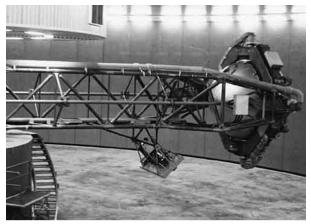
# that for the ideal banking of roadways.

#### **Exercise:**

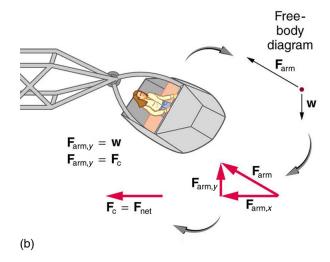
#### **Problem:**

A large centrifuge, like the one shown in [link](a), is used to expose aspiring astronauts to accelerations similar to those experienced in rocket launches and atmospheric reentries.

- (a) At what angular velocity is the centripetal acceleration 10 g if the rider is 15.0 m from the center of rotation?
- (b) The rider's cage hangs on a pivot at the end of the arm, allowing it to swing outward during rotation as shown in [link](b). At what angle  $\theta$  below the horizontal will the cage hang when the centripetal acceleration is 10~g? (Hint: The arm supplies centripetal force and supports the weight of the cage. Draw a free body diagram of the forces to see what the angle  $\theta$  should be.)



(a) NASA centrifuge and ride



(a) NASA centrifuge used to subject trainees to accelerations similar to those experienced in rocket launches and reentries. (credit: NASA) (b) Rider in cage showing how the cage pivots outward during rotation. This allows the total force exerted on the rider by the cage to be along its axis at all times.

- a) 2.56 rad/s
- b) 5.71°

### **Exercise:**

# **Problem:** Integrated Concepts

If a car takes a banked curve at less than the ideal speed, friction is needed to keep it from sliding toward the inside of the curve (a real problem on icy mountain roads). (a) Calculate the ideal speed to take a 100 m radius curve banked at 15.0°. (b) What is the minimum coefficient of friction needed for a frightened driver to take the same curve at 20.0 km/h?

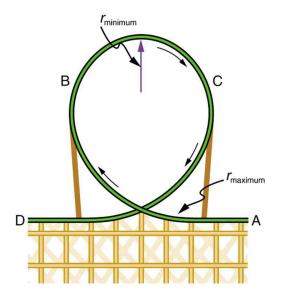
### **Solution:**

- a) 16.2 m/s
- b) 0.234

### **Exercise:**

### **Problem:**

Modern roller coasters have vertical loops like the one shown in [link]. The radius of curvature is smaller at the top than on the sides so that the downward centripetal acceleration at the top will be greater than the acceleration due to gravity, keeping the passengers pressed firmly into their seats. What is the speed of the roller coaster at the top of the loop if the radius of curvature there is 15.0 m and the downward acceleration of the car is 1.50 g?



Teardrop-shaped loops are used in the latest roller coasters so that the radius of curvature gradually decreases to a minimum at the top. This means that the centripetal acceleration builds from zero to a maximum at the top and gradually decreases again. A circular loop would cause a jolting change in acceleration at entry, a disadvantage discovered long ago in railroad curve design. With a small radius of curvature at the top, the centripetal acceleration can more easily be kept greater than *q* so that the passengers do not lose contact with their seats nor do they

# need seat belts to keep them in place.

#### **Exercise:**

#### **Problem:** Unreasonable Results

- (a) Calculate the minimum coefficient of friction needed for a car to negotiate an unbanked 50.0 m radius curve at 30.0 m/s.
- (b) What is unreasonable about the result?
- (c) Which premises are unreasonable or inconsistent?

#### **Solution:**

- a) 1.84
- b) A coefficient of friction this much greater than 1 is unreasonable .
- c) The assumed speed is too great for the tight curve.

# Glossary

# centripetal force

any net force causing uniform circular motion

# ideal banking

the sloping of a curve in a road, where the angle of the slope allows the vehicle to negotiate the curve at a certain speed without the aid of friction between the tires and the road; the net external force on the vehicle equals the horizontal centripetal force in the absence of friction

# ideal speed

the maximum safe speed at which a vehicle can turn on a curve without the aid of friction between the tire and the road

## ideal angle

the angle at which a car can turn safely on a steep curve, which is in proportion to the ideal speed

### banked curve

the curve in a road that is sloping in a manner that helps a vehicle negotiate the curve

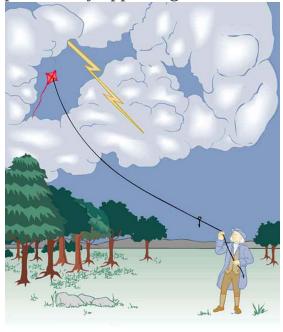
# Introduction to Electric Charge and Electric Field class="introduction"

Static electricity from this plastic slide causes the child's hair to stand on end. The sliding motion stripped electrons away from the child's body, leaving an excess of positive charges, which repel each other along each strand of hair. (credit: Ken Bosma/Wikimedi a Commons)



The image of American politician and scientist Benjamin Franklin (1706–1790) flying a kite in a thunderstorm is familiar to every schoolchild. (See [link].) In this experiment, Franklin demonstrated a connection between lightning and **static electricity**. Sparks were drawn from a key hung on a kite string during an electrical storm. These sparks were like those produced by static electricity, such as the spark that jumps from your finger to a metal doorknob after you walk across a wool carpet. What Franklin demonstrated in his dangerous experiment was a connection between phenomena on two different scales: one the grand power of an electrical storm, the other an effect of more human proportions. Connections like this one reveal the underlying unity of the laws of nature, an aspect we humans find

particularly appealing.



When Benjamin Franklin demonstrated that lightning was related to static electricity, he made a connection that is now part of the evidence that all directly experienced forces except the gravitational force are manifestations of the electromagnetic force.

Much has been written about Franklin. His experiments were only part of the life of a man who was a scientist, inventor, revolutionary, statesman, and writer. Franklin's experiments were not performed in isolation, nor were they the only ones to reveal connections.

For example, the Italian scientist Luigi Galvani (1737–1798) performed a series of experiments in which static electricity was used to stimulate contractions of leg muscles of dead frogs, an effect already known in humans subjected to static discharges. But Galvani also found that if he joined two metal wires (say copper and zinc) end to end and touched the other ends to muscles, he produced the same effect in frogs as static discharge. Alessandro Volta (1745–1827), partly inspired by Galvani's work, experimented with various combinations of metals and developed the battery.

During the same era, other scientists made progress in discovering fundamental connections. The periodic table was developed as the systematic properties of the elements were discovered. This influenced the development and refinement of the concept of atoms as the basis of matter. Such submicroscopic descriptions of matter also help explain a great deal more.

Atomic and molecular interactions, such as the forces of friction, cohesion, and adhesion, are now known to be manifestations of the **electromagnetic force**. Static electricity is just one aspect of the electromagnetic force, which also includes moving electricity and magnetism.

All the macroscopic forces that we experience directly, such as the sensations of touch and the tension in a rope, are due to the electromagnetic force, one of the four fundamental forces in nature. The gravitational force, another fundamental force, is actually sensed through the electromagnetic interaction of molecules, such as between those in our feet and those on the top of a bathroom scale. (The other two fundamental forces, the strong nuclear force and the weak nuclear force, cannot be sensed on the human scale.)

This chapter begins the study of electromagnetic phenomena at a fundamental level. The next several chapters will cover static electricity, moving electricity, and magnetism—collectively known as electromagnetism. In this chapter, we begin with the study of electric phenomena due to charges that are at least temporarily stationary, called electrostatics, or static electricity.

# Glossary

static electricity

a buildup of electric charge on the surface of an object

electromagnetic force

one of the four fundamental forces of nature; the electromagnetic force consists of static electricity, moving electricity and magnetism

# Static Electricity and Charge: Conservation of Charge

- Define electric charge, and describe how the two types of charge interact.
- Describe three common situations that generate static electricity.
- State the law of conservation of charge.



Borneo amber was mined in Sabah, Malaysia, from shale-sandstone-mudstone veins. When a piece of amber is rubbed with a piece of silk, the amber gains more electrons, giving it a net negative charge. At the same time, the silk, having lost electrons, becomes positively charged. (credit: Sebakoamber, Wikimedia Commons)

What makes plastic wrap cling? Static electricity. Not only are applications of static electricity common these days, its existence has been known since ancient times. The first record of its effects dates to ancient Greeks who noted more than 500 years B.C. that polishing amber temporarily enabled it

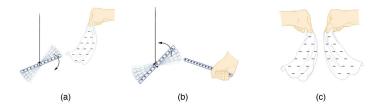
to attract bits of straw (see [link]). The very word *electric* derives from the Greek word for amber (*electron*).

Many of the characteristics of static electricity can be explored by rubbing things together. Rubbing creates the spark you get from walking across a wool carpet, for example. Static cling generated in a clothes dryer and the attraction of straw to recently polished amber also result from rubbing. Similarly, lightning results from air movements under certain weather conditions. You can also rub a balloon on your hair, and the static electricity created can then make the balloon cling to a wall. We also have to be cautious of static electricity, especially in dry climates. When we pump gasoline, we are warned to discharge ourselves (after sliding across the seat) on a metal surface before grabbing the gas nozzle. Attendants in hospital operating rooms must wear booties with aluminum foil on the bottoms to avoid creating sparks which may ignite the oxygen being used.

Some of the most basic characteristics of static electricity include:

- The effects of static electricity are explained by a physical quantity not previously introduced, called electric charge.
- There are only two types of charge, one called positive and the other called negative.
- Like charges repel, whereas unlike charges attract.
- The force between charges decreases with distance.

How do we know there are two types of **electric charge**? When various materials are rubbed together in controlled ways, certain combinations of materials always produce one type of charge on one material and the opposite type on the other. By convention, we call one type of charge "positive", and the other type "negative." For example, when glass is rubbed with silk, the glass becomes positively charged and the silk negatively charged. Since the glass and silk have opposite charges, they attract one another like clothes that have rubbed together in a dryer. Two glass rods rubbed with silk in this manner will repel one another, since each rod has positive charge on it. Similarly, two silk cloths so rubbed will repel, since both cloths have negative charge. [link] shows how these simple materials can be used to explore the nature of the force between charges.



A glass rod becomes positively charged when rubbed with silk, while the silk becomes negatively charged.

(a) The glass rod is attracted to the silk because their charges are opposite. (b) Two similarly charged glass rods repel. (c) Two similarly charged silk cloths repel.

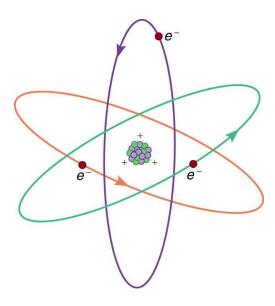
More sophisticated questions arise. Where do these charges come from? Can you create or destroy charge? Is there a smallest unit of charge? Exactly how does the force depend on the amount of charge and the distance between charges? Such questions obviously occurred to Benjamin Franklin and other early researchers, and they interest us even today.

# **Charge Carried by Electrons and Protons**

Franklin wrote in his letters and books that he could see the effects of electric charge but did not understand what caused the phenomenon. Today we have the advantage of knowing that normal matter is made of atoms, and that atoms contain positive and negative charges, usually in equal amounts.

[link] shows a simple model of an atom with negative **electrons** orbiting its positive nucleus. The nucleus is positive due to the presence of positively charged **protons**. Nearly all charge in nature is due to electrons and protons, which are two of the three building blocks of most matter. (The third is the neutron, which is neutral, carrying no charge.) Other charge-carrying particles are observed in cosmic rays and nuclear decay, and are created in

particle accelerators. All but the electron and proton survive only a short time and are quite rare by comparison.



This simplified (and not to scale) view of an atom is called the planetary model of the atom. Negative electrons orbit a much heavier positive nucleus, as the planets orbit the much heavier sun. There the similarity ends, because forces in the atom are electromagnetic, whereas those in the planetary system are gravitational. Normal macroscopic amounts of matter contain immense numbers of atoms and molecules and, hence, even greater numbers of individual

negative and positive charges.

The charges of electrons and protons are identical in magnitude but opposite in sign. Furthermore, all charged objects in nature are integral multiples of this basic quantity of charge, meaning that all charges are made of combinations of a basic unit of charge. Usually, charges are formed by combinations of electrons and protons. The magnitude of this basic charge is

# **Equation:**

$$\mid q_e \mid = 1.60 imes 10^{-19} \ {
m C}.$$

The symbol q is commonly used for charge and the subscript e indicates the charge of a single electron (or proton).

The SI unit of charge is the coulomb (C). The number of protons needed to make a charge of 1.00 C is

## **Equation:**

$$1.00~{
m C} imes rac{1~{
m proton}}{1.60 imes 10^{-19}~{
m C}} = 6.25 imes 10^{18}~{
m protons}.$$

Similarly,  $6.25 \times 10^{18}$  electrons have a combined charge of -1.00 coulomb. Just as there is a smallest bit of an element (an atom), there is a smallest bit of charge. There is no directly observed charge smaller than  $|q_e|$  (see Things Great and Small: The Submicroscopic Origin of Charge), and all observed charges are integral multiples of  $|q_e|$ .

#### Note:

Things Great and Small: The Submicroscopic Origin of Charge

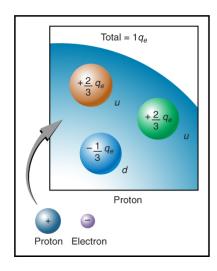
With the exception of exotic, short-lived particles, all charge in nature is carried by electrons and protons. Electrons carry the charge we have named negative. Protons carry an equal-magnitude charge that we call positive. (See [link].) Electron and proton charges are considered fundamental building blocks, since all other charges are integral multiples of those carried by electrons and protons. Electrons and protons are also two of the three fundamental building blocks of ordinary matter. The neutron is the third and has zero total charge.

[link] shows a person touching a Van de Graaff generator and receiving excess positive charge. The expanded view of a hair shows the existence of both types of charges but an excess of positive. The repulsion of these positive like charges causes the strands of hair to repel other strands of hair and to stand up. The further blowup shows an artist's conception of an electron and a proton perhaps found in an atom in a strand of hair.



When this person touches a Van de Graaff generator, she receives an excess of positive charge, causing her hair to stand on end. The charges in one hair are shown. An artist's conception of an electron and a proton illustrate the particles carrying the negative and positive charges. We cannot really see these particles with visible light because they are so small (the electron seems to be an infinitesimal point), but we know a great deal about their measurable properties, such as the charges they carry.

The electron seems to have no substructure; in contrast, when the substructure of protons is explored by scattering extremely energetic electrons from them, it appears that there are point-like particles inside the proton. These sub-particles, named quarks, have never been directly observed, but they are believed to carry fractional charges as seen in [link]. Charges on electrons and protons and all other directly observable particles are unitary, but these quark substructures carry charges of either  $-\frac{1}{3}$  or  $+\frac{2}{3}$ . There are continuing attempts to observe fractional charge directly and to learn of the properties of quarks, which are perhaps the ultimate substructure of matter.

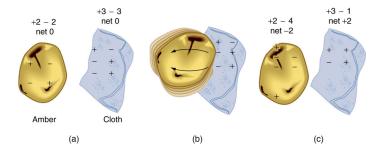


Artist's conception of fractional quark charges inside a proton. A group of three quark charges add up to the single positive charge on the proton:

 $-\frac{1}{3}q_e + \frac{2}{3}q_e + \frac{2}{3}q_e = +1q_e$ 

# **Separation of Charge in Atoms**

Charges in atoms and molecules can be separated—for example, by rubbing materials together. Some atoms and molecules have a greater affinity for electrons than others and will become negatively charged by close contact in rubbing, leaving the other material positively charged. (See [link].) Positive charge can similarly be induced by rubbing. Methods other than rubbing can also separate charges. Batteries, for example, use combinations of substances that interact in such a way as to separate charges. Chemical interactions may transfer negative charge from one substance to the other, making one battery terminal negative and leaving the first one positive.



When materials are rubbed together, charges can be separated, particularly if one material has a greater affinity for electrons than another. (a) Both the amber and cloth are originally neutral, with equal positive and negative charges. Only a tiny fraction of the charges are involved, and only a few of them are shown here. (b) When rubbed together, some negative charge is transferred to the amber, leaving the cloth with a net positive charge. (c) When separated, the amber and cloth now have net charges, but the absolute value of the net positive and negative charges will be equal.

No charge is actually created or destroyed when charges are separated as we have been discussing. Rather, existing charges are moved about. In fact, in all situations the total amount of charge is always constant. This universally obeyed law of nature is called the **law of conservation of charge**.

#### **Note:**

Law of Conservation of Charge

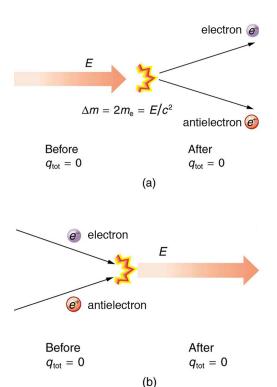
Total charge is constant in any process.

In more exotic situations, such as in particle accelerators, mass,  $\Delta m$ , can be created from energy in the amount  $\Delta m = \frac{E}{c^2}$ . Sometimes, the created mass is charged, such as when an electron is created. Whenever a charged particle is created, another having an opposite charge is always created along with it, so that the total charge created is zero. Usually, the two particles are "matter-antimatter" counterparts. For example, an antielectron would usually be created at the same time as an electron. The antielectron has a positive charge (it is called a positron), and so the total charge created is zero. (See [link].) All particles have antimatter counterparts with opposite signs. When matter and antimatter counterparts are brought together, they completely annihilate one another. By annihilate, we mean that the mass of the two particles is converted to energy E, again obeying the relationship  $\Delta m = \frac{E}{c^2}$ . Since the two particles have equal and opposite charge, the total charge is zero before and after the annihilation; thus, total charge is conserved.

#### Note:

## Making Connections: Conservation Laws

Only a limited number of physical quantities are universally conserved. Charge is one—energy, momentum, and angular momentum are others. Because they are conserved, these physical quantities are used to explain more phenomena and form more connections than other, less basic quantities. We find that conserved quantities give us great insight into the rules followed by nature and hints to the organization of nature. Discoveries of conservation laws have led to further discoveries, such as the weak nuclear force and the quark substructure of protons and other particles.



(a) When enough energy is present, it can be converted into matter. Here the matter created is an electron—antielectron pair. ( $m_e$  is the electron's mass.) The total charge before and after this event is zero. (b) When matter and antimatter collide, they annihilate each other; the total charge is conserved at zero before and after the annihilation.

The law of conservation of charge is absolute—it has never been observed to be violated. Charge, then, is a special physical quantity, joining a very

short list of other quantities in nature that are always conserved. Other conserved quantities include energy, momentum, and angular momentum.

#### Note:

## PhET Explorations: Balloons and Static Electricity

Why does a balloon stick to your sweater? Rub a balloon on a sweater, then let go of the balloon and it flies over and sticks to the sweater. View the charges in the sweater, balloons, and the wall.

https://phet.colorado.edu/sims/html/balloons-and-static-electricity/latest/balloons-and-static-electricity\_en.html

# **Section Summary**

- There are only two types of charge, which we call positive and negative.
- Like charges repel, unlike charges attract, and the force between charges decreases with the square of the distance.
- The vast majority of positive charge in nature is carried by protons, while the vast majority of negative charge is carried by electrons.
- The electric charge of one electron is equal in magnitude and opposite in sign to the charge of one proton.
- An ion is an atom or molecule that has nonzero total charge due to having unequal numbers of electrons and protons.
- The SI unit for charge is the coulomb (C), with protons and electrons having charges of opposite sign but equal magnitude; the magnitude of this basic charge  $\mid q_e \mid$  is

# **Equation:**

$$|q_e| = 1.60 \times 10^{-19} \text{ C}.$$

- Whenever charge is created or destroyed, equal amounts of positive and negative are involved.
- Most often, existing charges are separated from neutral objects to obtain some net charge.

- Both positive and negative charges exist in neutral objects and can be separated by rubbing one object with another. For macroscopic objects, negatively charged means an excess of electrons and positively charged means a depletion of electrons.
- The law of conservation of charge ensures that whenever a charge is created, an equal charge of the opposite sign is created at the same time.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?

#### **Exercise:**

#### **Problem:**

Why do most objects tend to contain nearly equal numbers of positive and negative charges?

#### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

Common static electricity involves charges ranging from nanocoulombs to microcoulombs. (a) How many electrons are needed to form a charge of  $-2.00~\rm nC$  (b) How many electrons must be removed from a neutral object to leave a net charge of  $0.500~\mu\rm C$ ?

#### **Solution:**

(a) 
$$1.25 \times 10^{10}$$

(b)  $3.13 \times 10^{12}$ 

#### **Exercise:**

#### **Problem:**

If  $1.80 \times 10^{20}$  electrons move through a pocket calculator during a full day's operation, how many coulombs of charge moved through it?

#### **Exercise:**

#### **Problem:**

To start a car engine, the car battery moves  $3.75\times10^{21}$  electrons through the starter motor. How many coulombs of charge were moved?

#### **Solution:**

-600 C

#### **Exercise:**

#### **Problem:**

A certain lightning bolt moves 40.0 C of charge. How many fundamental units of charge  $\mid q_e \mid$  is this?

# Glossary

## electric charge

a physical property of an object that causes it to be attracted toward or repelled from another charged object; each charged object generates and is influenced by a force called an electromagnetic force

# law of conservation of charge

states that whenever a charge is created, an equal amount of charge with the opposite sign is created simultaneously

#### electron

a particle orbiting the nucleus of an atom and carrying the smallest unit of negative charge

## proton

a particle in the nucleus of an atom and carrying a positive charge equal in magnitude and opposite in sign to the amount of negative charge carried by an electron

#### Conductors and Insulators

- Define conductor and insulator, explain the difference, and give examples of each.
- Describe three methods for charging an object.
- Explain what happens to an electric force as you move farther from the source.
- Define polarization.

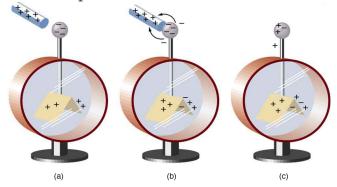


This power adapter uses metal wires and connectors to conduct electricity from the wall socket to a laptop computer. The conducting wires allow electrons to move freely through the cables, which are shielded by rubber and plastic. These materials act as insulators that don't allow electric charge to escape outward. (credit: Evan-Amos, Wikimedia Commons)

Some substances, such as metals and salty water, allow charges to move through them with relative ease. Some of the electrons in metals and similar conductors are not bound to individual atoms or sites in the material. These **free electrons** can move through the material much as air moves through loose sand. Any substance that has free electrons and allows charge to move

relatively freely through it is called a **conductor**. The moving electrons may collide with fixed atoms and molecules, losing some energy, but they can move in a conductor. Superconductors allow the movement of charge without any loss of energy. Salty water and other similar conducting materials contain free ions that can move through them. An ion is an atom or molecule having a positive or negative (nonzero) total charge. In other words, the total number of electrons is not equal to the total number of protons.

Other substances, such as glass, do not allow charges to move through them. These are called **insulators**. Electrons and ions in insulators are bound in the structure and cannot move easily—as much as  $10^{23}$  times more slowly than in conductors. Pure water and dry table salt are insulators, for example, whereas molten salt and salty water are conductors.



An electroscope is a favorite instrument in physics demonstrations and student laboratories. It is typically made with gold foil leaves hung from a (conducting) metal stem and is insulated from the room air in a glass-walled container. (a) A positively charged glass rod is brought near the tip of the electroscope, attracting electrons to the top and leaving a net positive charge on the leaves. Like charges in the light flexible gold leaves

repel, separating them. (b) When the rod is touched against the ball, electrons are attracted and transferred, reducing the net charge on the glass rod but leaving the electroscope positively charged. (c) The excess charges are evenly distributed in the stem and leaves of the electroscope once the glass rod is removed.

# **Charging by Contact**

[link] shows an electroscope being charged by touching it with a positively charged glass rod. Because the glass rod is an insulator, it must actually touch the electroscope to transfer charge to or from it. (Note that the extra positive charges reside on the surface of the glass rod as a result of rubbing it with silk before starting the experiment.) Since only electrons move in metals, we see that they are attracted to the top of the electroscope. There, some are transferred to the positive rod by touch, leaving the electroscope with a net positive charge.

**Electrostatic repulsion** in the leaves of the charged electroscope separates them. The electrostatic force has a horizontal component that results in the leaves moving apart as well as a vertical component that is balanced by the gravitational force. Similarly, the electroscope can be negatively charged by contact with a negatively charged object.

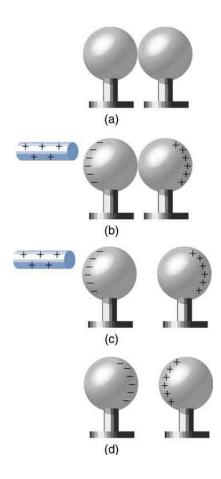
# **Charging by Induction**

It is not necessary to transfer excess charge directly to an object in order to charge it. [link] shows a method of **induction** wherein a charge is created in a nearby object, without direct contact. Here we see two neutral metal spheres in contact with one another but insulated from the rest of the world.

A positively charged rod is brought near one of them, attracting negative charge to that side, leaving the other sphere positively charged.

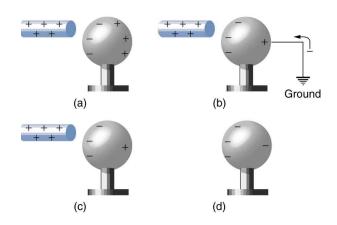
This is an example of induced **polarization** of neutral objects. Polarization is the separation of charges in an object that remains neutral. If the spheres are now separated (before the rod is pulled away), each sphere will have a net charge. Note that the object closest to the charged rod receives an opposite charge when charged by induction. Note also that no charge is removed from the charged rod, so that this process can be repeated without depleting the supply of excess charge.

Another method of charging by induction is shown in [link]. The neutral metal sphere is polarized when a charged rod is brought near it. The sphere is then grounded, meaning that a conducting wire is run from the sphere to the ground. Since the earth is large and most ground is a good conductor, it can supply or accept excess charge easily. In this case, electrons are attracted to the sphere through a wire called the ground wire, because it supplies a conducting path to the ground. The ground connection is broken before the charged rod is removed, leaving the sphere with an excess charge opposite to that of the rod. Again, an opposite charge is achieved when charging by induction and the charged rod loses none of its excess charge.



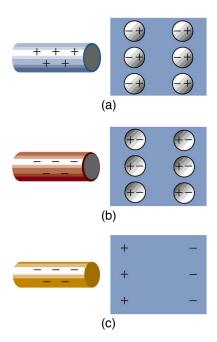
Charging by induction. (a) Two uncharged or neutral metal spheres are in contact with each other but insulated from the rest of the world. (b) A positively charged glass rod is brought near the sphere on the left, attracting negative charge and leaving the other sphere positively charged. (c) The

spheres are separated before the rod is removed, thus separating negative and positive charge. (d) The spheres retain net charges after the inducing rod is removed—without ever having been touched by a charged object.



Charging by induction, using a ground connection. (a) A positively charged rod is brought near a neutral metal sphere, polarizing it. (b) The sphere is grounded, allowing electrons to be attracted from the earth's ample supply. (c) The ground connection is broken. (d) The positive rod is

removed, leaving the sphere with an induced negative charge.



Both positive and negative objects attract a neutral object by polarizing its molecules. (a) A positive object brought near a neutral insulator polarizes its molecules. There is a slight shift in the distribution of the electrons orbiting the molecule, with

unlike charges being brought nearer and like charges moved away. Since the electrostatic force decreases with distance, there is a net attraction. (b) A negative object produces the opposite polarization, but again attracts the neutral object. (c) The same effect occurs for a conductor; since the unlike charges are closer, there is a net attraction.

Neutral objects can be attracted to any charged object. The pieces of straw attracted to polished amber are neutral, for example. If you run a plastic comb through your hair, the charged comb can pick up neutral pieces of paper. [link] shows how the polarization of atoms and molecules in neutral objects results in their attraction to a charged object.

When a charged rod is brought near a neutral substance, an insulator in this case, the distribution of charge in atoms and molecules is shifted slightly. Opposite charge is attracted nearer the external charged rod, while like charge is repelled. Since the electrostatic force decreases with distance, the repulsion of like charges is weaker than the attraction of unlike charges, and so there is a net attraction. Thus a positively charged glass rod attracts neutral pieces of paper, as will a negatively charged rubber rod. Some

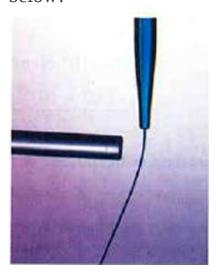
molecules, like water, are polar molecules. Polar molecules have a natural or inherent separation of charge, although they are neutral overall. Polar molecules are particularly affected by other charged objects and show greater polarization effects than molecules with naturally uniform charge distributions.

#### **Exercise:**

## **Check Your Understanding**

#### **Problem:**

Can you explain the attraction of water to the charged rod in the figure below?



### Solution: Answer

Water molecules are polarized, giving them slightly positive and slightly negative sides. This makes water even more susceptible to a charged rod's attraction. As the water flows downward, due to the force of gravity, the charged conductor exerts a net attraction to the opposite charges in the stream of water, pulling it closer.

#### Note:

PhET Explorations: John Travoltage

Make sparks fly with John Travoltage. Wiggle Johnnie's foot and he picks up charges from the carpet. Bring his hand close to the door knob and get rid of the excess charge.

https://phet.colorado.edu/sims/html/john-travoltage/latest/john-travoltage\_en.html

# **Section Summary**

- Polarization is the separation of positive and negative charges in a neutral object.
- A conductor is a substance that allows charge to flow freely through its atomic structure.
- An insulator holds charge within its atomic structure.
- Objects with like charges repel each other, while those with unlike charges attract each other.
- A conducting object is said to be grounded if it is connected to the Earth through a conductor. Grounding allows transfer of charge to and from the earth's large reservoir.
- Objects can be charged by contact with another charged object and obtain the same sign charge.
- If an object is temporarily grounded, it can be charged by induction, and obtains the opposite sign charge.
- Polarized objects have their positive and negative charges concentrated in different areas, giving them a non-symmetrical charge.
- Polar molecules have an inherent separation of charge.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

An eccentric inventor attempts to levitate by first placing a large negative charge on himself and then putting a large positive charge on the ceiling of his workshop. Instead, while attempting to place a large negative charge on himself, his clothes fly off. Explain.

#### **Exercise:**

#### **Problem:**

If you have charged an electroscope by contact with a positively charged object, describe how you could use it to determine the charge of other objects. Specifically, what would the leaves of the electroscope do if other charged objects were brought near its knob?

#### **Exercise:**

#### **Problem:**

When a glass rod is rubbed with silk, it becomes positive and the silk becomes negative—yet both attract dust. Does the dust have a third type of charge that is attracted to both positive and negative? Explain.

#### **Exercise:**

#### **Problem:**

Why does a car always attract dust right after it is polished? (Note that car wax and car tires are insulators.)

#### **Exercise:**

#### **Problem:**

Describe how a positively charged object can be used to give another object a negative charge. What is the name of this process?

#### **Exercise:**

#### **Problem:**

What is grounding? What effect does it have on a charged conductor? On a charged insulator?

#### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

Suppose a speck of dust in an electrostatic precipitator has  $1.0000 \times 10^{12}$  protons in it and has a net charge of -5.00 nC (a very large charge for a small speck). How many electrons does it have?

#### **Solution:**

 $1.03 \times 10^{12}$ 

#### **Exercise:**

#### **Problem:**

An amoeba has  $1.00 \times 10^{16}$  protons and a net charge of 0.300 pC. (a) How many fewer electrons are there than protons? (b) If you paired them up, what fraction of the protons would have no electrons?

#### **Exercise:**

#### **Problem:**

A 50.0 g ball of copper has a net charge of  $2.00 \,\mu\text{C}$ . What fraction of the copper's electrons has been removed? (Each copper atom has 29 protons, and copper has an atomic mass of 63.5.)

#### **Solution:**

$$9.09 \times 10^{-13}$$

#### **Exercise:**

#### **Problem:**

What net charge would you place on a 100 g piece of sulfur if you put an extra electron on 1 in  $10^{12}$  of its atoms? (Sulfur has an atomic mass of 32.1.)

#### **Exercise:**

#### **Problem:**

How many coulombs of positive charge are there in 4.00 kg of plutonium, given its atomic mass is 244 and that each plutonium atom has 94 protons?

#### **Solution:**

 $1.48 \times 10^{8} \, {\rm C}$ 

# Glossary

#### free electron

an electron that is free to move away from its atomic orbit

#### conductor

a material that allows electrons to move separately from their atomic orbits

#### insulator

a material that holds electrons securely within their atomic orbits

# grounded

when a conductor is connected to the Earth, allowing charge to freely flow to and from Earth's unlimited reservoir

#### induction

the process by which an electrically charged object brought near a neutral object creates a charge in that object

# polarization

slight shifting of positive and negative charges to opposite sides of an atom or molecule

## electrostatic repulsion

the phenomenon of two objects with like charges repelling each other

#### Coulomb's Law

- State Coulomb's law in terms of how the electrostatic force changes with the distance between two objects.
- Calculate the electrostatic force between two charged point forces, such as electrons or protons.
- Compare the electrostatic force to the gravitational attraction for a proton and an electron; for a human and the Earth.



This NASA image of Arp 87 shows the result of a strong gravitational attraction between two galaxies. In contrast, at the subatomic level, the electrostatic attraction between two objects, such as an electron and a proton, is far greater than their mutual attraction due to gravity. (credit: NASA/HST)

Through the work of scientists in the late 18th century, the main features of the **electrostatic force**—the existence of two types of charge, the observation that like charges repel, unlike charges attract, and the decrease of force with distance—were eventually refined, and expressed as a mathematical formula. The mathematical formula for the electrostatic force is called **Coulomb's law** after the French physicist Charles Coulomb (1736–1806), who performed experiments and first proposed a formula to calculate it.

Note:

Coulomb's Law

**Equation:** 

$$F=krac{|q_1q_2|}{r^2}.$$

Coulomb's law calculates the magnitude of the force F between two point charges,  $q_1$  and  $q_2$ , separated by a distance r. In SI units, the constant k is equal to

#### **Equation:**

$$k = 8.988 imes 10^9 rac{ ext{N} \cdot ext{m}^2}{ ext{C}^2} pprox 8.99 imes 10^9 rac{ ext{N} \cdot ext{m}^2}{ ext{C}^2}.$$

The electrostatic force is a vector quantity and is expressed in units of newtons. The force is understood to be along the line joining the two charges. (See [link].)

Although the formula for Coulomb's law is simple, it was no mean task to prove it. The experiments Coulomb did, with the primitive equipment then available, were difficult. Modern experiments have verified Coulomb's law to great precision. For example, it has been shown that the force is inversely proportional to distance between two objects squared  $(F \propto 1/r^2)$  to an accuracy of 1 part in  $10^{16}$ . No exceptions have ever been found, even at the small distances within the atom.

$$F_{21} \longrightarrow F_{12} \longrightarrow F_{12} \longrightarrow F_{21} \longrightarrow F_{21} \longrightarrow F_{22} \longrightarrow F_{22} \longrightarrow F_{21} \longrightarrow F_{22} \longrightarrow F_{22} \longrightarrow F_{21} \longrightarrow F_{22} \longrightarrow F$$

The magnitude of the electrostatic force F between point charges  $q_1$  and  $q_2$  separated by a distance r is given by Coulomb's law. Note that Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force on  $q_1$  is equal in magnitude and opposite in direction to the force it exerts on  $q_2$ . (a) Like charges. (b) Unlike charges.

#### **Example:**

## How Strong is the Coulomb Force Relative to the Gravitational Force?

Compare the electrostatic force between an electron and proton separated by  $0.530 \times 10^{-10}$  m with the gravitational force between them. This distance is their average separation in a hydrogen atom.

#### Strategy

To compare the two forces, we first compute the electrostatic force using Coulomb's law,  $F=krac{|q_1q_2|}{r^2}$ . We then calculate the gravitational force using Newton's universal law of

gravitation. Finally, we take a ratio to see how the forces compare in magnitude.

#### Solution

Entering the given and known information about the charges and separation of the electron and proton into the expression of Coulomb's law yields

#### **Equation:**

$$F=krac{|q_1q_2|}{r^2}$$

### **Equation:**

$$= \left(8.99 \times 10^9 \ \text{N} \cdot \text{m}^2/\text{C}^2\right) \times \tfrac{(1.60 \times 10^{-19} \ \text{C})(1.60 \times 10^{-19} \ \text{C})}{(0.530 \times 10^{-10} \ \text{m})^2}$$

Thus the Coulomb force is

### **Equation:**

$$F = 8.19 \times 10^{-8} \text{ N}.$$

The charges are opposite in sign, so this is an attractive force. This is a very large force for an electron—it would cause an acceleration of  $8.99 \times 10^{22} \, \mathrm{m/s^2}$  (verification is left as an end-of-section problem). The gravitational force is given by Newton's law of gravitation as:

### **Equation:**

$$F_G=Grac{mM}{r^2},$$

where  $G=6.67\times 10^{-11}~{
m N\cdot m^2/kg^2}$ . Here m and M represent the electron and proton masses, which can be found in the appendices. Entering values for the knowns yields

#### **Equation:**

$$F_G = (6.67 imes 10^{-11} \ ext{N} \cdot ext{m}^2/ ext{kg}^2) imes rac{(9.11 imes 10^{-31} \ ext{kg})(1.67 imes 10^{-27} \ ext{kg})}{(0.530 imes 10^{-10} \ ext{m})^2} = 3.61 imes 10^{-47} \ ext{N}$$

This is also an attractive force, although it is traditionally shown as positive since gravitational force is always attractive. The ratio of the magnitude of the electrostatic force to gravitational force in this case is, thus,

### **Equation:**

$$rac{F}{F_G} = 2.27 imes 10^{39}.$$

#### Discussion

This is a remarkably large ratio! Note that this will be the ratio of electrostatic force to gravitational force for an electron and a proton at any distance (taking the ratio before entering numerical values shows that the distance cancels). This ratio gives some indication

of just how much larger the Coulomb force is than the gravitational force between two of the most common particles in nature.

As the example implies, gravitational force is completely negligible on a small scale, where the interactions of individual charged particles are important. On a large scale, such as between the Earth and a person, the reverse is true. Most objects are nearly electrically neutral, and so attractive and repulsive **Coulomb forces** nearly cancel. Gravitational force on a large scale dominates interactions between large objects because it is always attractive, while Coulomb forces tend to cancel.

## **Section Summary**

- Frenchman Charles Coulomb was the first to publish the mathematical equation that describes the electrostatic force between two objects.
- Coulomb's law gives the magnitude of the force between point charges. It is **Equation:**

$$F=krac{|q_1q_2|}{r^2},$$

where  $q_1$  and  $q_2$  are two point charges separated by a distance r, and  $k \approx 8.99 \times 10^9~{
m N}\cdot{
m m}^2/{
m C}^2$ 

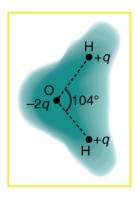
- This Coulomb force is extremely basic, since most charges are due to point-like particles. It is responsible for all electrostatic effects and underlies most macroscopic forces.
- The Coulomb force is extraordinarily strong compared with the gravitational force, another basic force—but unlike gravitational force it can cancel, since it can be either attractive or repulsive.
- The electrostatic force between two subatomic particles is far greater than the gravitational force between the same two particles.

# **Conceptual Questions**

#### **Exercise:**

#### Problem:

[link] shows the charge distribution in a water molecule, which is called a polar molecule because it has an inherent separation of charge. Given water's polar character, explain what effect humidity has on removing excess charge from objects.



Schematic representation of the outer electron cloud of a neutral water molecule. The electrons spend more time near the oxygen than the hydrogens, giving a permanent charge separation as shown. Water is thus a *polar* molecule. It is more easily affected by electrostatic forces than molecules with uniform charge distributions.

## **Exercise:**

#### **Problem:**

Using [link], explain, in terms of Coulomb's law, why a polar molecule (such as in [link]) is attracted by both positive and negative charges.

### **Problem:**

Given the polar character of water molecules, explain how ions in the air form nucleation centers for rain droplets.

#### **Problems & Exercises**

#### **Exercise:**

### **Problem:**

What is the repulsive force between two pith balls that are 8.00 cm apart and have equal charges of -30.0 nC?

#### **Exercise:**

#### **Problem:**

(a) How strong is the attractive force between a glass rod with a  $0.700~\mu\mathrm{C}$  charge and a silk cloth with a  $-0.600~\mu\mathrm{C}$  charge, which are 12.0 cm apart, using the approximation that they act like point charges? (b) Discuss how the answer to this problem might be affected if the charges are distributed over some area and do not act like point charges.

#### **Solution:**

- (a) 0.263 N
- (b) If the charges are distributed over some area, there will be a concentration of charge along the side closest to the oppositely charged object. This effect will increase the net force.

#### **Exercise:**

#### **Problem:**

Two point charges exert a 5.00 N force on each other. What will the force become if the distance between them is increased by a factor of three?

#### Exercise:

### **Problem:**

Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?

#### **Solution:**

The separation decreased by a factor of 5.

### **Problem:**

How far apart must two point charges of 75.0 nC (typical of static electricity) be to have a force of 1.00 N between them?

#### **Exercise:**

#### **Problem:**

If two equal charges each of 1 C each are separated in air by a distance of 1 km, what is the magnitude of the force acting between them? You will see that even at a distance as large as 1 km, the repulsive force is substantial because 1 C is a very significant amount of charge.

#### **Exercise:**

#### **Problem:**

A test charge of  $+2~\mu\mathrm{C}$  is placed halfway between a charge of  $+6~\mu\mathrm{C}$  and another of  $+4~\mu\mathrm{C}$  separated by 10 cm. (a) What is the magnitude of the force on the test charge? (b) What is the direction of this force (away from or toward the  $+6~\mu\mathrm{C}$  charge)?

#### **Exercise:**

#### **Problem:**

Bare free charges do not remain stationary when close together. To illustrate this, calculate the acceleration of two isolated protons separated by 2.00 nm (a typical distance between gas atoms). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

#### **Solution:**

$$egin{array}{lll} F &=& krac{|q_1q_2|}{r^2} = ma \Rightarrow a = rac{kq^2}{mr^2} \ &=& rac{\left(9.00 imes10^9\,\mathrm{N\cdot m^2/C^2}
ight)\left(1.60 imes10^{-19}\,\mathrm{m}
ight)^2}{\left(1.67 imes10^{-27}\,\mathrm{kg}
ight)\left(2.00 imes10^{-9}\,\mathrm{m}
ight)^2} \ &=& 3.45 imes10^{16}\,\mathrm{m/s^2} \end{array}$$

#### **Exercise:**

#### **Problem:**

(a) By what factor must you change the distance between two point charges to change the force between them by a factor of 10? (b) Explain how the distance can either increase or decrease by this factor and still cause a factor of 10 change in the force.

#### **Solution:**

- (a) 3.2
- (b) If the distance increases by 3.2, then the force will decrease by a factor of 10; if the distance decreases by 3.2, then the force will increase by a factor of 10. Either way, the force changes by a factor of 10.

#### **Problem:**

Suppose you have a total charge  $q_{\text{tot}}$  that you can split in any manner. Once split, the separation distance is fixed. How do you split the charge to achieve the greatest force?

#### **Exercise:**

#### **Problem:**

(a) Common transparent tape becomes charged when pulled from a dispenser. If one piece is placed above another, the repulsive force can be great enough to support the top piece's weight. Assuming equal point charges (only an approximation), calculate the magnitude of the charge if electrostatic force is great enough to support the weight of a 10.0 mg piece of tape held 1.00 cm above another. (b) Discuss whether the magnitude of this charge is consistent with what is typical of static electricity.

#### **Solution:**

- (a)  $1.04 \times 10^{-9}$  C
- (b) This charge is approximately 1 nC, which is consistent with the magnitude of charge typical for static electricity

#### **Exercise:**

#### Problem:

(a) Find the ratio of the electrostatic to gravitational force between two electrons. (b) What is this ratio for two protons? (c) Why is the ratio different for electrons and protons?

### **Exercise:**

#### **Problem:**

At what distance is the electrostatic force between two protons equal to the weight of one proton?

#### Exercise:

#### **Problem:**

A certain five cent coin contains 5.00 g of nickel. What fraction of the nickel atoms' electrons, removed and placed 1.00 m above it, would support the weight of this coin? The atomic mass of nickel is 58.7, and each nickel atom contains 28 electrons and 28 protons.

#### **Solution:**

 $1.02 \times 10^{-11}$ 

#### **Exercise:**

#### **Problem:**

(a) Two point charges totaling  $8.00~\mu\mathrm{C}$  exert a repulsive force of  $0.150~\mathrm{N}$  on one another when separated by  $0.500~\mathrm{m}$ . What is the charge on each? (b) What is the charge on each if the force is attractive?

#### **Exercise:**

#### **Problem:**

Point charges of  $5.00~\mu\mathrm{C}$  and  $-3.00~\mu\mathrm{C}$  are placed 0.250 m apart. (a) Where can a third charge be placed so that the net force on it is zero? (b) What if both charges are positive?

#### **Solution:**

- a. 0.859 m beyond negative charge on line connecting two charges
- b. 0.109 m from lesser charge on line connecting two charges

### **Exercise:**

#### Problem:

Two point charges  $q_1$  and  $q_2$  are 3.00 m apart, and their total charge is  $20 \,\mu\text{C}$ . (a) If the force of repulsion between them is 0.075N, what are magnitudes of the two charges? (b) If one charge attracts the other with a force of 0.525N, what are the magnitudes of the two charges? Note that you may need to solve a quadratic equation to reach your answer.

## Glossary

#### Coulomb's law

the mathematical equation calculating the electrostatic force vector between two charged particles

#### Coulomb force

another term for the electrostatic force

## electrostatic force

the amount and direction of attraction or repulsion between two charged bodies

## Electric Field: Concept of a Field Revisited

- Describe a force field and calculate the strength of an electric field due to a point charge.
- Calculate the force exerted on a test charge by an electric field.
- Explain the relationship between electrical force (F) on a test charge and electrical field strength (E).

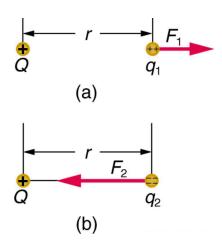
Contact forces, such as between a baseball and a bat, are explained on the small scale by the interaction of the charges in atoms and molecules in close proximity. They interact through forces that include the **Coulomb force**. Action at a distance is a force between objects that are not close enough for their atoms to "touch." That is, they are separated by more than a few atomic diameters.

For example, a charged rubber comb attracts neutral bits of paper from a distance via the Coulomb force. It is very useful to think of an object being surrounded in space by a **force field**. The force field carries the force to another object (called a test object) some distance away.

# Concept of a Field

A field is a way of conceptualizing and mapping the force that surrounds any object and acts on another object at a distance without apparent physical connection. For example, the gravitational field surrounding the earth (and all other masses) represents the gravitational force that would be experienced if another mass were placed at a given point within the field.

In the same way, the Coulomb force field surrounding any charge extends throughout space. Using Coulomb's law,  $F = k|q_1q_2|/r^2$ , its magnitude is given by the equation  $F = k|qQ|/r^2$ , for a **point charge** (a particle having a charge Q) acting on a **test charge** q at a distance r (see [link]). Both the magnitude and direction of the Coulomb force field depend on Q and the test charge q.



The Coulomb force field due to a positive charge Qis shown acting on two different charges. Both charges are the same distance from Q. (a) Since  $q_1$  is positive, the force  $F_1$  acting on it is repulsive. (b) The charge  $q_2$  is negative and greater in magnitude than  $q_1$ , and so the force  $F_2$ acting on it is attractive and stronger than  $F_1$ . The Coulomb force field is thus not unique at any point in space, because it depends on the test charges  $q_1$  and  $q_2$ 

as well as the charge Q.

To simplify things, we would prefer to have a field that depends only on Q and not on the test charge q. The electric field is defined in such a manner that it represents only the charge creating it and is unique at every point in space. Specifically, the electric field E is defined to be the ratio of the Coulomb force to the test charge:

## **Equation:**

$$\mathbf{E}=rac{\mathbf{F}}{q},$$

where  $\mathbf{F}$  is the electrostatic force (or Coulomb force) exerted on a positive test charge q. It is understood that  $\mathbf{E}$  is in the same direction as  $\mathbf{F}$ . It is also assumed that q is so small that it does not alter the charge distribution creating the electric field. The units of electric field are newtons per coulomb (N/C). If the electric field is known, then the electrostatic force on any charge q is simply obtained by multiplying charge times electric field, or  $\mathbf{F} = q\mathbf{E}$ . Consider the electric field due to a point charge q. According to Coulomb's law, the force it exerts on a test charge q is  $F = k|qQ|/r^2$ . Thus the magnitude of the electric field, E, for a point charge is

# **Equation:**

$$E=\left|rac{F}{q}
ight|=k\left|rac{\mathrm{q}\mathrm{Q}}{qr^2}
ight|=krac{|Q|}{r^2}.$$

Since the test charge cancels, we see that

# **Equation:**

$$E = k rac{|Q|}{r^2}.$$

The electric field is thus seen to depend only on the charge Q and the distance r; it is completely independent of the test charge q.

## **Example:**

## Calculating the Electric Field of a Point Charge

Calculate the strength and direction of the electric field E due to a point charge of 2.00 nC (nano-Coulombs) at a distance of 5.00 mm from the charge.

## Strategy

We can find the electric field created by a point charge by using the equation  $E=\mathrm{kQ}/r^2$ .

## Solution

Here  $Q=2.00\times 10^{-9}$  C and  $r=5.00\times 10^{-3}$  m. Entering those values into the above equation gives

## **Equation:**

$$egin{array}{lcl} E &=& k rac{Q}{r^2} \ &=& (8.99 imes 10^9 \ {
m N} \cdot {
m m}^2/{
m C}^2) imes rac{(2.00 imes 10^{-9} \ {
m C})}{(5.00 imes 10^{-3} \ {
m m})^2} \ &=& 7.19 imes 10^5 \ {
m N/C}. \end{array}$$

### Discussion

This **electric field strength** is the same at any point 5.00 mm away from the charge Q that creates the field. It is positive, meaning that it has a direction pointing away from the charge Q.

## **Example:**

# Calculating the Force Exerted on a Point Charge by an Electric Field

What force does the electric field found in the previous example exert on a point charge of  $-0.250~\mu\mathrm{C}$ ?

## **Strategy**

Since we know the electric field strength and the charge in the field, the force on that charge can be calculated using the definition of electric field

 $\mathbf{E} = \mathbf{F}/q$  rearranged to  $\mathbf{F} = q\mathbf{E}$ .

## Solution

The magnitude of the force on a charge  $q=-0.250~\mu\mathrm{C}$  exerted by a field of strength  $E=7.20\times10^5~\mathrm{N/C}$  is thus,

## **Equation:**

$$egin{array}{lll} F &=& -qE \ &=& (0.250 imes 10^{-6} \; \mathrm{C}) (7.20 imes 10^{5} \; \mathrm{N/C}) \ &=& 0.180 \; \mathrm{N}. \end{array}$$

Because q is negative, the force is directed opposite to the direction of the field.

### Discussion

The force is attractive, as expected for unlike charges. (The field was created by a positive charge and here acts on a negative charge.) The charges in this example are typical of common static electricity, and the modest attractive force obtained is similar to forces experienced in static cling and similar situations.

### Note:

# PhET Explorations: Electric Field of Dreams

Play ball! Add charges to the Field of Dreams and see how they react to the electric field. Turn on a background electric field and adjust the direction and magnitude.

https://archive.cnx.org/specials/ca9a78b4-06a7-11e6-b638-3bb71d1f0b42/electric-field-of-dreams/#sim-electric-field-of-dreams

# **Section Summary**

- The electrostatic force field surrounding a charged object extends out into space in all directions.
- The electrostatic force exerted by a point charge on a test charge at a distance r depends on the charge of both charges, as well as the

distance between the two.

The electric field **E** is defined to be **Equation**:

$$\mathbf{E}=rac{\mathbf{F}}{q,}$$

where  $\mathbf{F}$  is the Coulomb or electrostatic force exerted on a small positive test charge q.  $\mathbf{E}$  has units of N/C.

• The magnitude of the electric field  ${\bf E}$  created by a point charge Q is **Equation:** 

$$\mathbf{E}=krac{|Q|}{r^2}.$$

where r is the distance from Q. The electric field  $\mathbf{E}$  is a vector and fields due to multiple charges add like vectors.

# **Conceptual Questions**

### **Exercise:**

### **Problem:**

Why must the test charge q in the definition of the electric field be vanishingly small?

### **Exercise:**

### **Problem:**

Are the direction and magnitude of the Coulomb force unique at a given point in space? What about the electric field?

## **Problem Exercises**

### **Exercise:**

## **Problem:**

What is the magnitude and direction of an electric field that exerts a  $2.00 \times 10^{-5}$  N upward force on a  $-1.75~\mu C$  charge?

## **Exercise:**

## **Problem:**

What is the magnitude and direction of the force exerted on a  $3.50~\mu\mathrm{C}$  charge by a 250 N/C electric field that points due east?

## **Solution:**

$$8.75 \times 10^{-4} \text{ N}$$

## **Exercise:**

## **Problem:**

Calculate the magnitude of the electric field 2.00 m from a point charge of 5.00 mC (such as found on the terminal of a Van de Graaff).

## **Exercise:**

## **Problem:**

(a) What magnitude point charge creates a 10,000 N/C electric field at a distance of 0.250 m? (b) How large is the field at 10.0 m?

## **Solution:**

(a) 
$$6.94 \times 10^{-8}$$
 C

(b) 
$$6.25 \text{ N/C}$$

### **Exercise:**

## **Problem:**

Calculate the initial (from rest) acceleration of a proton in a  $5.00 \times 10^6 \ \mathrm{N/C}$  electric field (such as created by a research Van de Graaff). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

## **Exercise:**

## **Problem:**

(a) Find the magnitude and direction of an electric field that exerts a  $4.80\times 10^{-17}~\mathrm{N}$  westward force on an electron. (b) What magnitude and direction force does this field exert on a proton?

## **Solution:**

- (a) 300 N/C (east)
- (b)  $4.80 \times 10^{-17} \text{ N (east)}$

# **Glossary**

field

a map of the amount and direction of a force acting on other objects, extending out into space

point charge

A charged particle, designated Q, generating an electric field

test charge

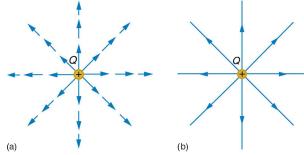
A particle (designated q) with either a positive or negative charge set down within an electric field generated by a point charge

## Electric Field Lines: Multiple Charges

- Calculate the total force (magnitude and direction) exerted on a test charge from more than one charge
- Describe an electric field diagram of a positive point charge; of a negative point charge with twice the magnitude of positive charge
- Draw the electric field lines between two points of the same charge; between two points of opposite charge.

Drawings using lines to represent **electric fields** around charged objects are very useful in visualizing field strength and direction. Since the electric field has both magnitude and direction, it is a vector. Like all **vectors**, the electric field can be represented by an arrow that has length proportional to its magnitude and that points in the correct direction. (We have used arrows extensively to represent force vectors, for example.)

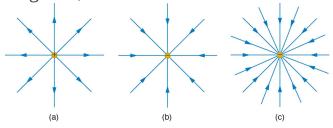
[link] shows two pictorial representations of the same electric field created by a positive point charge Q. [link] (b) shows the standard representation using continuous lines. [link] (a) shows numerous individual arrows with each arrow representing the force on a test charge q. Field lines are essentially a map of infinitesimal force vectors.



Two equivalent representations of the electric field due to a positive charge Q. (a) Arrows representing the electric field's magnitude and direction. (b) In the standard representation, the arrows are replaced by continuous field lines having the same direction at any point

as the electric field. The closeness of the lines is directly related to the strength of the electric field. A test charge placed anywhere will feel a force in the direction of the field line; this force will have a strength proportional to the density of the lines (being greater near the charge, for example).

Note that the electric field is defined for a positive test charge q, so that the field lines point away from a positive charge and toward a negative charge. (See [link].) The electric field strength is exactly proportional to the number of field lines per unit area, since the magnitude of the electric field for a point charge is  $E = k|Q|/r^2$  and area is proportional to  $r^2$ . This pictorial representation, in which field lines represent the direction and their closeness (that is, their areal density or the number of lines crossing a unit area) represents strength, is used for all fields: electrostatic, gravitational, magnetic, and others.



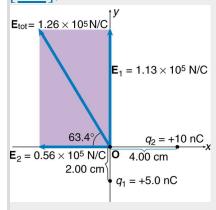
The electric field surrounding three different point charges. (a) A positive charge. (b) A negative charge of equal magnitude. (c) A larger negative charge.

In many situations, there are multiple charges. The total electric field created by multiple charges is the vector sum of the individual fields created by each charge. The following example shows how to add electric field vectors.

## **Example:**

## **Adding Electric Fields**

Find the magnitude and direction of the total electric field due to the two point charges,  $q_1$  and  $q_2$ , at the origin of the coordinate system as shown in [link].



The electric fields  $\mathbf{E}_1$  and  $\mathbf{E}_2$  at the origin O add to  $\mathbf{E}_{tot}$ .

## Strategy

Since the electric field is a vector (having magnitude and direction), we add electric fields with the same vector techniques used for other types of vectors. We first must find the electric field due to each charge at the point of interest, which is the origin of the coordinate system (O) in this instance. We pretend that there is a positive test charge, q, at point O, which allows us to determine the direction of the fields  $\mathbf{E}_1$  and  $\mathbf{E}_2$ . Once those fields are found, the total field can be determined using **vector addition**.

#### Solution

The electric field strength at the origin due to  $q_1$  is labeled  $E_1$  and is calculated:

## **Equation:**

$$E_1 = krac{q_1}{r_1^2} = \left(8.99 imes 10^9 \ ext{N} \cdot ext{m}^2/ ext{C}^2
ight) rac{(5.00 imes 10^{-9} \ ext{C})}{\left(2.00 imes 10^{-2} \ ext{m}
ight)^2} 
onumber \ E_1 = 1.124 imes 10^5 \ ext{N/C}.$$

Similarly,  $E_2$  is

## **Equation:**

$$egin{aligned} E_2 &= krac{q_2}{r_2^2} = \left(8.99 imes 10^9 \; ext{N} \cdot ext{m}^2/ ext{C}^2
ight) rac{\left(10.0 imes 10^{-9} \; ext{C}
ight)}{\left(4.00 imes 10^{-2} \; ext{m}
ight)^2} \ E_2 &= 0.5619 imes 10^5 \; ext{N/C}. \end{aligned}$$

Four digits have been retained in this solution to illustrate that  $E_1$  is exactly twice the magnitude of  $E_2$ . Now arrows are drawn to represent the magnitudes and directions of  $\mathbf{E}_1$  and  $\mathbf{E}_2$ . (See [link].) The direction of the electric field is that of the force on a positive charge so both arrows point directly away from the positive charges that create them. The arrow for  $\mathbf{E}_1$  is exactly twice the length of that for  $\mathbf{E}_2$ . The arrows form a right triangle in this case and can be added using the Pythagorean theorem. The magnitude of the total field  $E_{\mathrm{tot}}$  is

# **Equation:**

$$egin{array}{lcl} E_{
m tot} &=& (E_1^2+E_2^2)^{1/2} \ &=& \left\{ (1.124 imes10^5~{
m N/C})^2 + (0.5619 imes10^5~{
m N/C})^2 
ight\}^{1/2} \ &=& 1.26 imes10^5~{
m N/C}. \end{array}$$

The direction is

# **Equation:**

$$egin{array}{lcl} heta &=& an^{-1}\Big(rac{E_1}{E_2}\Big) \ &=& an^{-1}\Big(rac{1.124 imes10^5~ ext{N/C}}{0.5619 imes10^5~ ext{N/C}}\Big) \ &=& ext{63.4}^{ ext{o}}, \end{array}$$

or  $63.4^{\circ}$  above the *x*-axis.

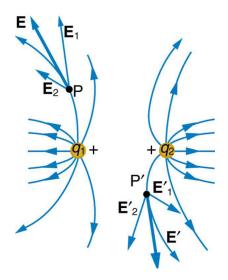
### Discussion

In cases where the electric field vectors to be added are not perpendicular, vector components or graphical techniques can be used. The total electric field found in this example is the total electric field at only one point in space. To find the total electric field due to these two charges over an entire region, the same technique must be repeated for each point in the region. This impossibly lengthy task (there are an infinite number of points in space) can be avoided by calculating the total field at representative points and using some of the unifying features noted next.

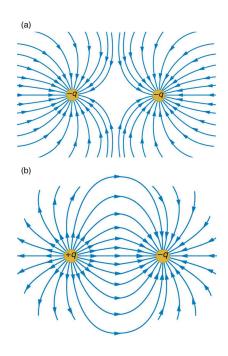
[link] shows how the electric field from two point charges can be drawn by finding the total field at representative points and drawing electric field lines consistent with those points. While the electric fields from multiple charges are more complex than those of single charges, some simple features are easily noticed.

For example, the field is weaker between like charges, as shown by the lines being farther apart in that region. (This is because the fields from each charge exert opposing forces on any charge placed between them.) (See [link] and [link](a).) Furthermore, at a great distance from two like charges, the field becomes identical to the field from a single, larger charge.

[link](b) shows the electric field of two unlike charges. The field is stronger between the charges. In that region, the fields from each charge are in the same direction, and so their strengths add. The field of two unlike charges is weak at large distances, because the fields of the individual charges are in opposite directions and so their strengths subtract. At very large distances, the field of two unlike charges looks like that of a smaller single charge.



Two positive point charges  $q_1$  and  $q_2$  produce the resultant electric field shown. The field is calculated at representative points and then smooth field lines drawn following the rules outlined in the text.



(a) Two negative charges produce the fields shown. It is very similar to the field produced by two positive charges, except that the directions are reversed. The field is clearly weaker between the charges. The individual forces on a test charge in that region are in opposite directions. (b) Two opposite charges produce the field shown, which is stronger in the region between the charges.

We use electric field lines to visualize and analyze electric fields (the lines are a pictorial tool, not a physical entity in themselves). The properties of electric field lines for any charge distribution can be summarized as follows:

- 1. Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
- 2. The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
- 3. The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
- 4. The direction of the electric field is tangent to the field line at any point in space.
- 5. Field lines can never cross.

The last property means that the field is unique at any point. The field line represents the direction of the field; so if they crossed, the field would have two directions at that location (an impossibility if the field is unique).

### Note:

# PhET Explorations: Charges and Fields

Move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more. It's colorful, it's dynamic, it's free.

Click here for the simulation

•

# **Section Summary**

- Drawings of electric field lines are useful visual tools. The properties of electric field lines for any charge distribution are that:
- Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
- The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
- The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
- The direction of the electric field is tangent to the field line at any point in space.
- Field lines can never cross.

# **Conceptual Questions**

## **Exercise:**

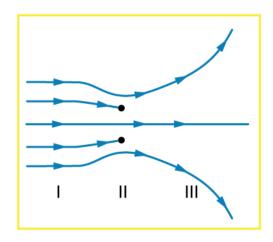
### **Problem:**

Compare and contrast the Coulomb force field and the electric field. To do this, make a list of five properties for the Coulomb force field analogous to the five properties listed for electric field lines. Compare each item in your list of Coulomb force field properties with those of the electric field—are they the same or different? (For example, electric field lines cannot cross. Is the same true for Coulomb field lines?)

### **Exercise:**

### **Problem:**

[link] shows an electric field extending over three regions, labeled I, II, and III. Answer the following questions. (a) Are there any isolated charges? If so, in what region and what are their signs? (b) Where is the field strongest? (c) Where is it weakest? (d) Where is the field the most uniform?



## **Problem Exercises**

## **Exercise:**

## **Problem:**

(a) Sketch the electric field lines near a point charge +q. (b) Do the same for a point charge -3.00q.

## **Exercise:**

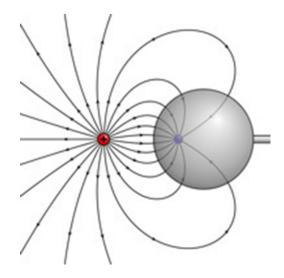
## **Problem:**

Sketch the electric field lines a long distance from the charge distributions shown in [link] (a) and (b)

## **Exercise:**

## **Problem:**

[link] shows the electric field lines near two charges  $q_1$  and  $q_2$ . What is the ratio of their magnitudes? (b) Sketch the electric field lines a long distance from the charges shown in the figure.



The electric field near two charges.

## **Problem:**

Sketch the electric field lines in the vicinity of two opposite charges, where the negative charge is three times greater in magnitude than the positive. (See [link] for a similar situation).

# **Glossary**

## electric field

a three-dimensional map of the electric force extended out into space from a point charge

## electric field lines

a series of lines drawn from a point charge representing the magnitude and direction of force exerted by that charge

### vector

a quantity with both magnitude and direction

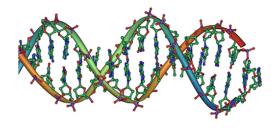
# vector addition

mathematical combination of two or more vectors, including their magnitudes, directions, and positions

# Electric Forces in Biology

- Describe how a water molecule is polar.
- Explain electrostatic screening by a water molecule within a living cell.

Classical electrostatics has an important role to play in modern molecular biology. Large molecules such as proteins, nucleic acids, and so on—so important to life—are usually electrically charged. DNA itself is highly charged; it is the electrostatic force that not only holds the molecule together but gives the molecule structure and strength. [link] is a schematic of the DNA double helix.



DNA is a highly charged molecule. The DNA double helix shows the two coiled strands each containing a row of nitrogenous bases, which "code" the genetic information needed by a living organism. The strands are connected by bonds between pairs of bases. While pairing combinations between certain bases are fixed (C-G and A-T), the sequence of nucleotides in the strand varies. (credit: Jerome Walker)

The four nucleotide bases are given the symbols A (adenine), C (cytosine), G (guanine), and T (thymine). The order of the four bases varies in each strand, but the pairing between bases is always the same. C and G are always paired and A and T are always paired, which helps to preserve the order of bases in cell division (mitosis) so as to pass on the correct genetic information. Since the Coulomb force drops with distance ( $F \propto 1/r^2$ ), the distances between the base pairs must be small enough that the electrostatic force is sufficient to hold them together.

DNA is a highly charged molecule, with about  $2q_{\rm e}$  (fundamental charge) per  $0.3 \times 10^{-9}$  m. The distance separating the two strands that make up the DNA structure is about 1 nm, while the distance separating the individual atoms within each base is about 0.3 nm.

One might wonder why electrostatic forces do not play a larger role in biology than they do if we have so many charged molecules. The reason is that the electrostatic force is "diluted" due to **screening** between molecules. This is due to the presence of other charges in the cell.

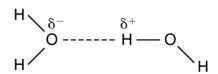
# **Polarity of Water Molecules**

The best example of this charge screening is the water molecule, represented as  $H_2O$ . Water is a strongly **polar molecule**. Its 10 electrons (8 from the oxygen atom and 2 from the two hydrogen atoms) tend to remain closer to the oxygen nucleus than the hydrogen nuclei. This creates two centers of equal and opposite charges—what is called a **dipole**, as illustrated in [link]. The magnitude of the dipole is called the dipole moment.

These two centers of charge will terminate some of the electric field lines coming from a free charge, as on a DNA molecule. This results in a reduction in the strength of the **Coulomb interaction**. One might say that screening makes the Coulomb force a short range force rather than long range.

Other ions of importance in biology that can reduce or screen Coulomb interactions are  $Na^+$ , and  $K^+$ , and  $Cl^-$ . These ions are located both inside and outside of living cells. The movement of these ions through cell membranes is crucial to the motion of nerve impulses through nerve axons.

Recent studies of electrostatics in biology seem to show that electric fields in cells can be extended over larger distances, in spite of screening, by "microtubules" within the cell. These microtubules are hollow tubes composed of proteins that guide the movement of chromosomes when cells divide, the motion of other organisms within the cell, and provide mechanisms for motion of some cells (as motors).



This schematic shows water  $(H_2O)$ as a polar molecule. Unequal sharing of electrons between the oxygen (O) and hydrogen (H) atoms leads to a net separation of positive and negative charge forming a dipole. The symbols  $\delta^$ and  $\delta^+$  indicate that the oxygen side of the H<sub>2</sub>O molecule tends to be more negative, while the hydrogen ends tend

to be more positive.

This leads to an
attraction of
opposite charges
between molecules.

# **Section Summary**

- Many molecules in living organisms, such as DNA, carry a charge.
- An uneven distribution of the positive and negative charges within a polar molecule produces a dipole.
- The effect of a Coulomb field generated by a charged object may be reduced or blocked by other nearby charged objects.
- Biological systems contain water, and because water molecules are polar, they have a strong effect on other molecules in living systems.

# **Conceptual Question**

### **Exercise:**

### **Problem:**

A cell membrane is a thin layer enveloping a cell. The thickness of the membrane is much less than the size of the cell. In a static situation the membrane has a charge distribution of  $-2.5 \times 10^{-6}$  C/m  $^2$  on its inner surface and  $+2.5 \times 10^{-6}$  C/m $^2$  on its outer surface. Draw a diagram of the cell and the surrounding cell membrane. Include on this diagram the charge distribution and the corresponding electric field. Is there any electric field inside the cell? Is there any electric field outside the cell?

# **Glossary**

dipole

a molecule's lack of symmetrical charge distribution, causing one side to be more positive and another to be more negative

# polar molecule

a molecule with an asymmetrical distribution of positive and negative charge

## screening

the dilution or blocking of an electrostatic force on a charged object by the presence of other charges nearby

## Coulomb interaction

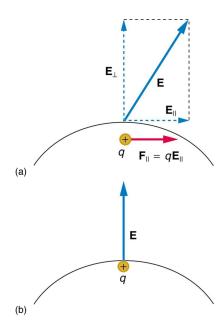
the interaction between two charged particles generated by the Coulomb forces they exert on one another

# Conductors and Electric Fields in Static Equilibrium

- List the three properties of a conductor in electrostatic equilibrium.
- Explain the effect of an electric field on free charges in a conductor.
- Explain why no electric field may exist inside a conductor.
- Describe the electric field surrounding Earth.
- Explain what happens to an electric field applied to an irregular conductor.
- Describe how a lightning rod works.
- Explain how a metal car may protect passengers inside from the dangerous electric fields caused by a downed line touching the car.

**Conductors** contain **free charges** that move easily. When excess charge is placed on a conductor or the conductor is put into a static electric field, charges in the conductor quickly respond to reach a steady state called **electrostatic equilibrium**.

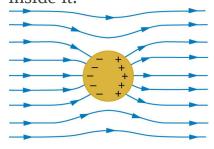
[link] shows the effect of an electric field on free charges in a conductor. The free charges move until the field is perpendicular to the conductor's surface. There can be no component of the field parallel to the surface in electrostatic equilibrium, since, if there were, it would produce further movement of charge. A positive free charge is shown, but free charges can be either positive or negative and are, in fact, negative in metals. The motion of a positive charge is equivalent to the motion of a negative charge in the opposite direction.



When an electric field  ${f E}$  is applied to a conductor, free charges inside the conductor move until the field is perpendicular to the surface. (a) The electric field is a vector quantity, with both parallel and perpendicular components. The parallel component  $(\mathbf{E}_{\parallel})$  exerts a force  $(\mathbf{F}_{\parallel})$  on the free charge q, which moves the charge until  $\mathbf{F}_{\parallel}=0$ . (b) The resulting field is perpendicular to the surface. The free charge has

been brought to the conductor's surface, leaving electrostatic forces in equilibrium.

A conductor placed in an **electric field** will be **polarized**. [link] shows the result of placing a neutral conductor in an originally uniform electric field. The field becomes stronger near the conductor but entirely disappears inside it.



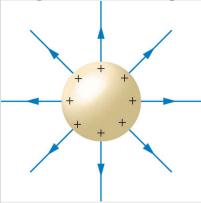
This illustration shows a spherical conductor in static equilibrium with an originally uniform electric field. Free charges move within the conductor, polarizing it, until the electric field lines are perpendicular to the surface. The field lines end on excess negative charge on one section of the surface and begin

again on excess positive charge on the opposite side. No electric field exists inside the conductor, since free charges in the conductor would continue moving in response to any field until it was neutralized.

### Note:

# Misconception Alert: Electric Field inside a Conductor

Excess charges placed on a spherical conductor repel and move until they are evenly distributed, as shown in [link]. Excess charge is forced to the surface until the field inside the conductor is zero. Outside the conductor, the field is exactly the same as if the conductor were replaced by a point charge at its center equal to the excess charge.



The mutual repulsion of excess positive charges on

a spherical conductor distributes them uniformly on its surface. The resulting electric field is perpendicular to the surface and zero inside. Outside the conductor, the field is identical to that of a point charge at the center equal to the excess charge.

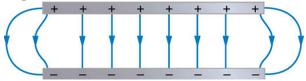
## Note:

# Properties of a Conductor in Electrostatic Equilibrium

- 1. The electric field is zero inside a conductor.
- 2. Just outside a conductor, the electric field lines are perpendicular to its surface, ending or beginning on charges on the surface.
- 3. Any excess charge resides entirely on the surface or surfaces of a conductor.

The properties of a conductor are consistent with the situations already discussed and can be used to analyze any conductor in electrostatic equilibrium. This can lead to some interesting new insights, such as described below.

How can a very uniform electric field be created? Consider a system of two metal plates with opposite charges on them, as shown in [link]. The properties of conductors in electrostatic equilibrium indicate that the electric field between the plates will be uniform in strength and direction. Except near the edges, the excess charges distribute themselves uniformly, producing field lines that are uniformly spaced (hence uniform in strength) and perpendicular to the surfaces (hence uniform in direction, since the plates are flat). The edge effects are less important when the plates are close together.



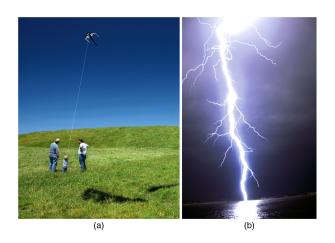
Two metal plates with equal, but opposite, excess charges.
The field between them is uniform in strength and direction except near the edges.
One use of such a field is to produce uniform acceleration of charges between the plates, such as in the electron gun of a TV tube.

# Earth's Electric Field

A near uniform electric field of approximately 150 N/C, directed downward, surrounds Earth, with the magnitude increasing slightly as we get closer to the surface. What causes the electric field? At around 100 km above the surface of Earth we have a layer of charged particles, called the **ionosphere**. The ionosphere is responsible for a range of phenomena including the electric field surrounding Earth. In fair weather the ionosphere is positive and the Earth largely negative, maintaining the electric field ([link](a)).

In storm conditions clouds form and localized electric fields can be larger and reversed in direction ([link](b)). The exact charge distributions depend on the local conditions, and variations of [link](b) are possible.

If the electric field is sufficiently large, the insulating properties of the surrounding material break down and it becomes conducting. For air this occurs at around  $3\times 10^6$  N/C. Air ionizes ions and electrons recombine, and we get discharge in the form of lightning sparks and corona discharge.



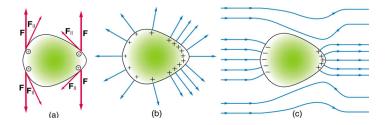
Earth's electric field. (a) Fair weather field. Earth and the ionosphere (a layer of charged particles) are both conductors. They produce a uniform electric field of about 150 N/C. (credit: D. H. Parks) (b) Storm fields. In the presence of storm clouds, the local electric fields can be larger. At very high fields, the insulating properties of the air break down and lightning can occur. (credit: Jan-Joost Verhoef)

## **Electric Fields on Uneven Surfaces**

So far we have considered excess charges on a smooth, symmetrical conductor surface. What happens if a conductor has sharp corners or is pointed? Excess charges on a nonuniform conductor become concentrated at the sharpest points. Additionally, excess charge may move on or off the conductor at the sharpest points.

To see how and why this happens, consider the charged conductor in [link]. The electrostatic repulsion of like charges is most effective in moving them apart on the flattest surface, and so they become least concentrated there. This is because the forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surfaces are different. The component parallel to the surface is greatest on the flattest surface and, hence, more effective in moving the charge.

The same effect is produced on a conductor by an externally applied electric field, as seen in [link] (c). Since the field lines must be perpendicular to the surface, more of them are concentrated on the most curved parts.



Excess charge on a nonuniform conductor becomes most concentrated at the location of greatest curvature.

(a) The forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surface are different. It is  $\mathbf{F}_{\parallel}$  that moves the charges apart once they

have reached the surface. (b)  $\mathbf{F}_{\parallel}$  is smallest at the more pointed end, the charges are left closer together, producing the electric field shown. (c) An uncharged conductor in an originally uniform electric field is polarized, with the most concentrated charge at its most pointed end.

# **Applications of Conductors**

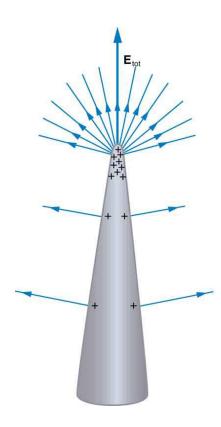
On a very sharply curved surface, such as shown in [link], the charges are so concentrated at the point that the resulting electric field can be great enough to remove them from the surface. This can be useful.

Lightning rods work best when they are most pointed. The large charges created in storm clouds induce an opposite charge on a building that can result in a lightning bolt hitting the building. The induced charge is bled away continually by a lightning rod, preventing the more dramatic lightning strike.

Of course, we sometimes wish to prevent the transfer of charge rather than to facilitate it. In that case, the conductor should be very smooth and have as large a radius of curvature as possible. (See [link].) Smooth surfaces are used on high-voltage transmission lines, for example, to avoid leakage of charge into the air.

Another device that makes use of some of these principles is a **Faraday cage**. This is a metal shield that encloses a volume. All electrical charges will reside on the outside surface of this shield, and there will be no electrical field inside. A Faraday cage is used to prohibit stray electrical fields in the environment from interfering with sensitive measurements, such as the electrical signals inside a nerve cell.

During electrical storms if you are driving a car, it is best to stay inside the car as its metal body acts as a Faraday cage with zero electrical field inside. If in the vicinity of a lightning strike, its effect is felt on the outside of the car and the inside is unaffected, provided you remain totally inside. This is also true if an active ("hot") electrical wire was broken (in a storm or an accident) and fell on your car.



A very pointed conductor has a large charge concentration at the point. The electric field is very strong at the point and can exert a force large enough to transfer charge on or off the conductor.

Lightning rods are used to prevent the buildup of large excess charges on structures and, thus, are pointed.



(a) A lightning rod is pointed to facilitate the transfer of charge.
 (credit: Romaine, Wikimedia
 Commons) (b) This Van de Graaff generator has a smooth surface with a large radius of curvature to prevent the transfer of charge and allow a large voltage to be generated. The mutual repulsion of like charges is evident in the person's hair while touching the metal sphere. (credit: Jon 'ShakataGaNai' Davis/Wikimedia Commons).

# **Section Summary**

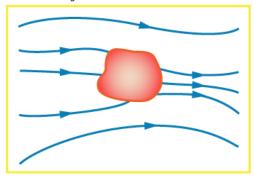
- A conductor allows free charges to move about within it.
- The electrical forces around a conductor will cause free charges to move around inside the conductor until static equilibrium is reached.
- Any excess charge will collect along the surface of a conductor.
- Conductors with sharp corners or points will collect more charge at those points.
- A lightning rod is a conductor with sharply pointed ends that collect excess charge on the building caused by an electrical storm and allow it to dissipate back into the air.
- Electrical storms result when the electrical field of Earth's surface in certain locations becomes more strongly charged, due to changes in the insulating effect of the air.
- A Faraday cage acts like a shield around an object, preventing electric charge from penetrating inside.

# **Conceptual Questions**

### **Exercise:**

# **Problem:**

Is the object in [link] a conductor or an insulator? Justify your answer.



## **Exercise:**

#### **Problem:**

If the electric field lines in the figure above were perpendicular to the object, would it necessarily be a conductor? Explain.

The discussion of the electric field between two parallel conducting plates, in this module states that edge effects are less important if the plates are close together. What does close mean? That is, is the actual plate separation crucial, or is the ratio of plate separation to plate area crucial?

## **Exercise:**

### **Problem:**

Would the self-created electric field at the end of a pointed conductor, such as a lightning rod, remove positive or negative charge from the conductor? Would the same sign charge be removed from a neutral pointed conductor by the application of a similar externally created electric field? (The answers to both questions have implications for charge transfer utilizing points.)

### **Exercise:**

### **Problem:**

Why is a golfer with a metal club over her shoulder vulnerable to lightning in an open fairway? Would she be any safer under a tree?

#### **Exercise:**

### **Problem:**

Can the belt of a Van de Graaff accelerator be a conductor? Explain.

### **Exercise:**

### **Problem:**

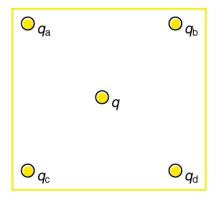
Are you relatively safe from lightning inside an automobile? Give two reasons.

Discuss pros and cons of a lightning rod being grounded versus simply being attached to a building.

## **Exercise:**

## **Problem:**

Using the symmetry of the arrangement, show that the net Coulomb force on the charge q at the center of the square below ( $[\underline{link}]$ ) is zero if the charges on the four corners are exactly equal.



Four point charges  $q_a$ ,  $q_b$ ,  $q_c$ , and  $q_d$  lie on the corners of a square and q is located at its center.

(a) Using the symmetry of the arrangement, show that the electric field at the center of the square in [link] is zero if the charges on the four corners are exactly equal. (b) Show that this is also true for any combination of charges in which  $q_a = q_d$  and  $q_b = q_c$ 

### **Exercise:**

## **Problem:**

(a) What is the direction of the total Coulomb force on q in [link] if q is negative,  $q_a = q_c$  and both are negative, and  $q_b = q_c$  and both are positive? (b) What is the direction of the electric field at the center of the square in this situation?

### **Exercise:**

#### **Problem:**

Considering [link], suppose that  $q_a = q_d$  and  $q_b = q_c$ . First show that q is in static equilibrium. (You may neglect the gravitational force.) Then discuss whether the equilibrium is stable or unstable, noting that this may depend on the signs of the charges and the direction of displacement of q from the center of the square.

#### **Exercise:**

### **Problem:**

If  $q_a = 0$  in [link], under what conditions will there be no net Coulomb force on q?

## **Exercise:**

#### **Problem:**

In regions of low humidity, one develops a special "grip" when opening car doors, or touching metal door knobs. This involves placing as much of the hand on the device as possible, not just the ends of one's fingers. Discuss the induced charge and explain why this is done.

### **Exercise:**

### **Problem:**

Tollbooth stations on roadways and bridges usually have a piece of wire stuck in the pavement before them that will touch a car as it approaches. Why is this done?

### **Exercise:**

### **Problem:**

Suppose a woman carries an excess charge. To maintain her charged status can she be standing on ground wearing just any pair of shoes? How would you discharge her? What are the consequences if she simply walks away?

## **Problems & Exercises**

### **Exercise:**

### **Problem:**

Sketch the electric field lines in the vicinity of the conductor in [link] given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?



### **Exercise:**

### **Problem:**

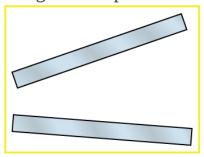
Sketch the electric field lines in the vicinity of the conductor in [link] given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?



# **Exercise:**

# **Problem:**

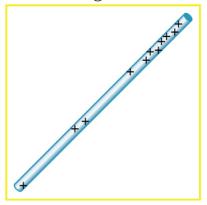
Sketch the electric field between the two conducting plates shown in [link], given the top plate is positive and an equal amount of negative charge is on the bottom plate. Be certain to indicate the distribution of charge on the plates.



# **Exercise:**

# **Problem:**

Sketch the electric field lines in the vicinity of the charged insulator in [link] noting its nonuniform charge distribution.



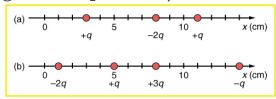
A charged insulating rod such as might be used in

a classroom demonstration.

## **Exercise:**

## **Problem:**

What is the force on the charge located at x = 8.00 cm in [link](a) given that  $q = 1.00 \mu C$ ?



(a) Point charges located at 3.00, 8.00, and 11.0 cm along the *x*-axis. (b) Point charges located at 1.00, 5.00, 8.00, and 14.0 cm along the *x*-axis.

### **Exercise:**

## **Problem:**

(a) Find the total electric field at x = 1.00 cm in [link](b) given that q = 5.00 nC. (b) Find the total electric field at x = 11.00 cm in [link] (b). (c) If the charges are allowed to move and eventually be brought to rest by friction, what will the final charge configuration be? (That is, will there be a single charge, double charge, etc., and what will its value(s) be?)

### **Solution:**

(a) 
$$E_{x=1.00~{
m cm}} = -\infty$$

- (b)  $2.12 \times 10^5 \text{ N/C}$
- (c) one charge of +q

## **Exercise:**

## **Problem:**

(a) Find the electric field at x = 5.00 cm in [link](a), given that  $q = 1.00 \,\mu\text{C}$ . (b) At what position between 3.00 and 8.00 cm is the total electric field the same as that for -2q alone? (c) Can the electric field be zero anywhere between 0.00 and 8.00 cm? (d) At very large positive or negative values of x, the electric field approaches zero in both (a) and (b). In which does it most rapidly approach zero and why? (e) At what position to the right of 11.0 cm is the total electric field zero, other than at infinity? (Hint: A graphing calculator can yield considerable insight in this problem.)

#### **Exercise:**

## **Problem:**

(a) Find the total Coulomb force on a charge of 2.00 nC located at x = 4.00 cm in [link] (b), given that q = 1.00  $\mu$ C. (b) Find the *x*-position at which the electric field is zero in [link] (b).

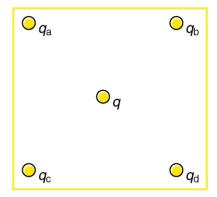
### **Solution:**

- (a) 0.252 N to the left
- (b) x = 6.07 cm

### **Exercise:**

### **Problem:**

Using the symmetry of the arrangement, determine the direction of the force on q in the figure below, given that  $q_a = q_b = +7.50 \ \mu\text{C}$  and  $q_c = q_d = -7.50 \ \mu\text{C}$ . (b) Calculate the magnitude of the force on the charge q, given that the square is 10.0 cm on a side and  $q = 2.00 \ \mu\text{C}$ .



## **Exercise:**

### **Problem:**

(a) Using the symmetry of the arrangement, determine the direction of the electric field at the center of the square in [link], given that  $q_a = q_b = -1.00 \ \mu\text{C}$  and  $q_c = q_d = +1.00 \ \mu\text{C}$ . (b) Calculate the magnitude of the electric field at the location of q, given that the square is 5.00 cm on a side.

### **Solution:**

(a)The electric field at the center of the square will be straight up, since  $q_a$  and  $q_b$  are positive and  $q_c$  and  $q_d$  are negative and all have the same magnitude.

(b) 
$$2.04 \times 10^7 \text{ N/C (upward)}$$

## **Exercise:**

### **Problem:**

Find the electric field at the location of  $q_a$  in [link] given that  $q_b = q_c = q_d = +2.00 \text{ nC}$ , q = -1.00 nC, and the square is 20.0 cm on a side.

Find the total Coulomb force on the charge q in [link], given that  $q=1.00~\mu\text{C},\,q_a=2.00~\mu\text{C},\,q_b=-3.00~\mu\text{C},\,q_c=-4.00~\mu\text{C}$ , and  $q_d{=}{+}1.00~\mu\text{C}$ . The square is 50.0 cm on a side.

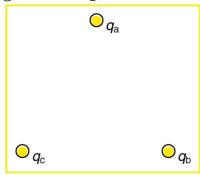
## **Solution:**

 $0.102 \mathrm{\ N}$ , in the -y direction

# **Exercise:**

### **Problem:**

(a) Find the electric field at the location of  $q_a$  in [link], given that  $q_b = +10.00~\mu\text{C}$  and  $q_c = -5.00~\mu\text{C}$ . (b) What is the force on  $q_a$ , given that  $q_a = +1.50~\text{nC}$ ?



Point charges located at the corners of an equilateral triangle 25.0 cm on a side.

(a) Find the electric field at the center of the triangular configuration of charges in [link], given that  $q_a$ =+2.50 nC,  $q_b$  = -8.00 nC, and  $q_c$ =+1.50 nC. (b) Is there any combination of charges, other than  $q_a = q_b = q_c$ , that will produce a zero strength electric field at the center of the triangular configuration?

### **Solution:**

- (a)  $ec{E}=4.36 imes10^3~\mathrm{N/C},\,35.0^\circ$ , below the horizontal.
- (b) No

# Glossary

#### conductor

an object with properties that allow charges to move about freely within it

# free charge

an electrical charge (either positive or negative) which can move about separately from its base molecule

# electrostatic equilibrium

an electrostatically balanced state in which all free electrical charges have stopped moving about

# polarized

a state in which the positive and negative charges within an object have collected in separate locations

# ionosphere

a layer of charged particles located around 100 km above the surface of Earth, which is responsible for a range of phenomena including the electric field surrounding Earth

# Faraday cage

a metal shield which prevents electric charge from penetrating its surface

# Applications of Electrostatics

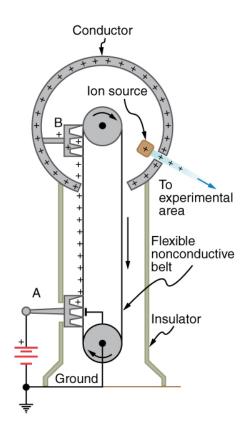
• Name several real-world applications of the study of electrostatics.

The study of **electrostatics** has proven useful in many areas. This module covers just a few of the many applications of electrostatics.

## The Van de Graaff Generator

Van de Graaff generators (or Van de Graaffs) are not only spectacular devices used to demonstrate high voltage due to static electricity—they are also used for serious research. The first was built by Robert Van de Graaff in 1931 (based on original suggestions by Lord Kelvin) for use in nuclear physics research. [link] shows a schematic of a large research version. Van de Graaffs utilize both smooth and pointed surfaces, and conductors and insulators to generate large static charges and, hence, large voltages.

A very large excess charge can be deposited on the sphere, because it moves quickly to the outer surface. Practical limits arise because the large electric fields polarize and eventually ionize surrounding materials, creating free charges that neutralize excess charge or allow it to escape. Nevertheless, voltages of 15 million volts are well within practical limits.



Schematic of Van de Graaff generator. A battery (A) supplies excess positive charge to a pointed conductor, the points of which spray the charge onto a moving insulating belt near the bottom. The pointed conductor (B) on top in the large sphere picks up the charge. (The induced electric field at the points is so large that it removes the charge from the belt.) This can be done because the charge does not

remain inside the conducting sphere but moves to its outside surface. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere to high velocities.

#### Note:

# Take-Home Experiment: Electrostatics and Humidity

Rub a comb through your hair and use it to lift pieces of paper. It may help to tear the pieces of paper rather than cut them neatly. Repeat the exercise in your bathroom after you have had a long shower and the air in the bathroom is moist. Is it easier to get electrostatic effects in dry or moist air? Why would torn paper be more attractive to the comb than cut paper? Explain your observations.

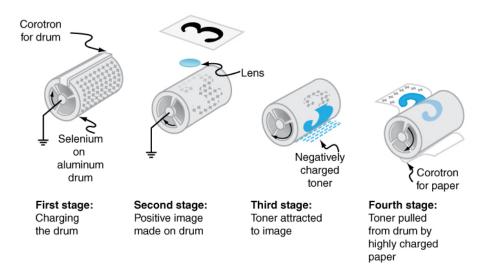
# Xerography

Most copy machines use an electrostatic process called **xerography**—a word coined from the Greek words *xeros* for dry and *graphos* for writing. The heart of the process is shown in simplified form in [link].

A selenium-coated aluminum drum is sprayed with positive charge from points on a device called a corotron. Selenium is a substance with an interesting property—it is a **photoconductor**. That is, selenium is an insulator when in the dark and a conductor when exposed to light.

In the first stage of the xerography process, the conducting aluminum drum is **grounded** so that a negative charge is induced under the thin layer of uniformly positively charged selenium. In the second stage, the surface of the drum is exposed to the image of whatever is to be copied. Where the image is light, the selenium becomes conducting, and the positive charge is neutralized. In dark areas, the positive charge remains, and so the image has been transferred to the drum.

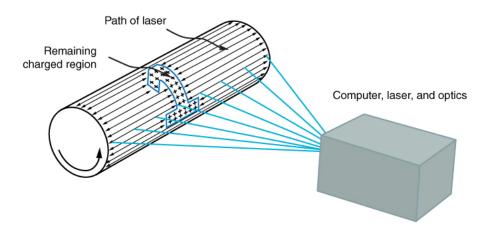
The third stage takes a dry black powder, called toner, and sprays it with a negative charge so that it will be attracted to the positive regions of the drum. Next, a blank piece of paper is given a greater positive charge than on the drum so that it will pull the toner from the drum. Finally, the paper and electrostatically held toner are passed through heated pressure rollers, which melt and permanently adhere the toner within the fibers of the paper.



Xerography is a dry copying process based on electrostatics. The major steps in the process are the charging of the photoconducting drum, transfer of an image creating a positive charge duplicate, attraction of toner to the charged parts of the drum, and transfer of toner to the paper. Not shown are heat treatment of the paper and cleansing of the drum for the next copy.

### **Laser Printers**

Laser printers use the xerographic process to make high-quality images on paper, employing a laser to produce an image on the photoconducting drum as shown in [link]. In its most common application, the laser printer receives output from a computer, and it can achieve high-quality output because of the precision with which laser light can be controlled. Many laser printers do significant information processing, such as making sophisticated letters or fonts, and may contain a computer more powerful than the one giving them the raw data to be printed.

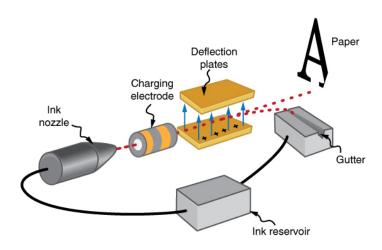


In a laser printer, a laser beam is scanned across a photoconducting drum, leaving a positive charge image. The other steps for charging the drum and transferring the image to paper are the same as in xerography. Laser light can be very precisely controlled, enabling laser printers to produce high-quality images.

# **Ink Jet Printers and Electrostatic Painting**

The **ink jet printer**, commonly used to print computer-generated text and graphics, also employs electrostatics. A nozzle makes a fine spray of tiny ink droplets, which are then given an electrostatic charge. (See [link].)

Once charged, the droplets can be directed, using pairs of charged plates, with great precision to form letters and images on paper. Ink jet printers can produce color images by using a black jet and three other jets with primary colors, usually cyan, magenta, and yellow, much as a color television produces color. (This is more difficult with xerography, requiring multiple drums and toners.)



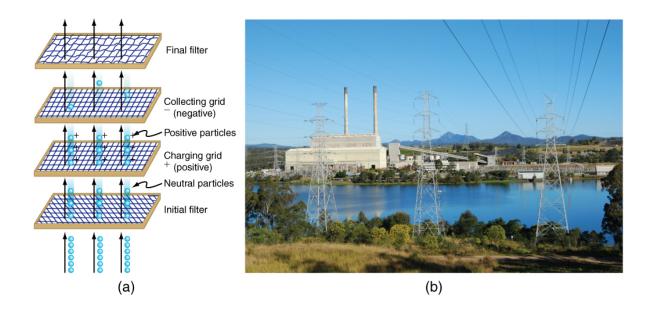
The nozzle of an ink-jet printer produces small ink droplets, which are sprayed with electrostatic charge. Various computer-driven devices are then used to direct the droplets to the correct positions on a page.

Electrostatic painting employs electrostatic charge to spray paint onto oddshaped surfaces. Mutual repulsion of like charges causes the paint to fly away from its source. Surface tension forms drops, which are then attracted by unlike charges to the surface to be painted. Electrostatic painting can reach those hard-to-get at places, applying an even coat in a controlled manner. If the object is a conductor, the electric field is perpendicular to the surface, tending to bring the drops in perpendicularly. Corners and points on conductors will receive extra paint. Felt can similarly be applied.

# **Smoke Precipitators and Electrostatic Air Cleaning**

Another important application of electrostatics is found in air cleaners, both large and small. The electrostatic part of the process places excess (usually positive) charge on smoke, dust, pollen, and other particles in the air and then passes the air through an oppositely charged grid that attracts and retains the charged particles. (See [link].)

Large **electrostatic precipitators** are used industrially to remove over 99% of the particles from stack gas emissions associated with the burning of coal and oil. Home precipitators, often in conjunction with the home heating and air conditioning system, are very effective in removing polluting particles, irritants, and allergens.



(a) Schematic of an electrostatic precipitator. Air is passed through grids of opposite charge. The first grid charges airborne particles, while the second attracts and collects them. (b) The dramatic effect of

electrostatic precipitators is seen by the absence of smoke from this power plant. (credit: Cmdalgleish, Wikimedia Commons)

## Note:

# Problem-Solving Strategies for Electrostatics

- 1. Examine the situation to determine if static electricity is involved. This may concern separated stationary charges, the forces among them, and the electric fields they create.
- 2. Identify the system of interest. This includes noting the number, locations, and types of charges involved.
- 3. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful. Determine whether the Coulomb force is to be considered directly—if so, it may be useful to draw a free-body diagram, using electric field lines.
- 4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns). It is important to distinguish the Coulomb force F from the electric field E, for example.
- 5. Solve the appropriate equation for the quantity to be determined (the unknown) or draw the field lines as requested.
- 6. Examine the answer to see if it is reasonable: Does it make sense? Are units correct and the numbers involved reasonable?

# **Integrated Concepts**

The Integrated Concepts exercises for this module involve concepts such as electric charges, electric fields, and several other topics. Physics is most interesting when applied to general situations involving more than a narrow set of physical principles. The electric field exerts force on charges, for example, and hence the relevance of <a href="Dynamics: Force and Newton's Laws">Dynamics: Force and Newton's Laws</a> of <a href="Motion">Motion</a>. The following topics are involved in some or all of the problems labeled "Integrated Concepts":

- Kinematics
- Two-Dimensional Kinematics
- Dynamics: Force and Newton's Laws of Motion
- Uniform Circular Motion and Gravitation
- Statics and Torque
- Fluid Statics

The following worked example illustrates how this strategy is applied to an Integrated Concept problem:

# **Example:**

# Acceleration of a Charged Drop of Gasoline

If steps are not taken to ground a gasoline pump, static electricity can be placed on gasoline when filling your car's tank. Suppose a tiny drop of gasoline has a mass of  $4.00 \times 10^{-15}$  kg and is given a positive charge of  $3.20 \times 10^{-19}$  C. (a) Find the weight of the drop. (b) Calculate the electric force on the drop if there is an upward electric field of strength  $3.00 \times 10^5$  N/C due to other static electricity in the vicinity. (c) Calculate the drop's acceleration.

# Strategy

To solve an integrated concept problem, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example asks for weight. This is a topic of dynamics and is defined in <a href="Dynamics: Force and Newton's Laws of Motion">Dynamics: Force and Newton's Laws of Motion</a>. Part (b) deals with electric force on a charge, a topic of <a href="Electric Charge and Electric Field">Electric Charge and Electric Field</a>. Part (c) asks for acceleration, knowing forces and mass. These are part of Newton's laws, also found in <a href="Dynamics: Force and Newton's Laws of Motion">Dynamics: Force and Newton's Laws of Motion</a>.

The following solutions to each part of the example illustrate how the specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so on.

# Solution for (a)

Weight is mass times the acceleration due to gravity, as first expressed in

# **Equation:**

$$w = mg$$
.

Entering the given mass and the average acceleration due to gravity yields **Equation:** 

$$w = (4.00 \times 10^{-15} \ \mathrm{kg})(9.80 \ \mathrm{m/s}^2) = 3.92 \times 10^{-14} \ \mathrm{N}.$$

## Discussion for (a)

This is a small weight, consistent with the small mass of the drop.

# Solution for (b)

The force an electric field exerts on a charge is given by rearranging the following equation:

# **Equation:**

$$F = qE$$
.

Here we are given the charge  $(3.20 \times 10^{-19} \ \mathrm{C})$  is twice the fundamental unit of charge) and the electric field strength, and so the electric force is found to be

# **Equation:**

$$F = (3.20 \times 10^{-19} \ \mathrm{C})(3.00 \times 10^5 \ \mathrm{N/C}) = 9.60 \times 10^{-14} \ \mathrm{N}.$$

# Discussion for (b)

While this is a small force, it is greater than the weight of the drop.

# Solution for (c)

The acceleration can be found using Newton's second law, provided we can identify all of the external forces acting on the drop. We assume only the drop's weight and the electric force are significant. Since the drop has a positive charge and the electric field is given to be upward, the electric force is upward. We thus have a one-dimensional (vertical direction) problem, and we can state Newton's second law as

# **Equation:**

$$a=rac{F_{
m net}}{m}.$$

where  $F_{\text{net}} = F - w$ . Entering this and the known values into the expression for Newton's second law yields

## **Equation:**

$$a = \frac{F-w}{m}$$

$$= \frac{9.60 \times 10^{-14} \text{ N} - 3.92 \times 10^{-14} \text{ N}}{4.00 \times 10^{-15} \text{ kg}}$$

$$= 14.2 \text{ m/s}^{2}.$$

# Discussion for (c)

This is an upward acceleration great enough to carry the drop to places where you might not wish to have gasoline.

This worked example illustrates how to apply problem-solving strategies to situations that include topics in different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These are found throughout the text, and many worked examples show how to use them for single topics. In this integrated concepts example, you can see how to apply them across several topics. You will find these techniques useful in applications of physics outside a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

# Note:

## Unreasonable Results

The Unreasonable Results exercises for this module have results that are unreasonable because some premise is unreasonable or because certain of the premises are inconsistent with one another. Physical principles applied correctly then produce unreasonable results. The purpose of these problems is to give practice in assessing whether nature is being accurately described, and if it is not to trace the source of difficulty.

### Note:

# **Problem-Solving Strategy**

To determine if an answer is reasonable, and to determine the cause if it is not, do the following.

- 1. Solve the problem using strategies as outlined above. Use the format followed in the worked examples in the text to solve the problem as usual.
- 2. Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, and so on?
- 3. If the answer is unreasonable, look for what specifically could cause the identified difficulty. Usually, the manner in which the answer is unreasonable is an indication of the difficulty. For example, an extremely large Coulomb force could be due to the assumption of an excessively large separated charge.

# **Section Summary**

- Electrostatics is the study of electric fields in static equilibrium.
- In addition to research using equipment such as a Van de Graaff generator, many practical applications of electrostatics exist, including photocopiers, laser printers, ink-jet printers and electrostatic air filters.

## **Problems & Exercises**

(a) What is the electric field 5.00 m from the center of the terminal of a Van de Graaff with a 3.00 mC charge, noting that the field is equivalent to that of a point charge at the center of the terminal? (b) At this distance, what force does the field exert on a  $2.00~\mu\mathrm{C}$  charge on the Van de Graaff's belt?

#### **Exercise:**

### **Problem:**

(a) What is the direction and magnitude of an electric field that supports the weight of a free electron near the surface of Earth? (b) Discuss what the small value for this field implies regarding the relative strength of the gravitational and electrostatic forces.

## **Solution:**

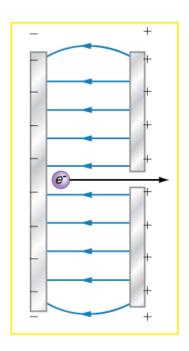
(a) 
$$5.58 \times 10^{-11} \text{ N/C}$$

(b)the coulomb force is extraordinarily stronger than gravity

#### **Exercise:**

### **Problem:**

A simple and common technique for accelerating electrons is shown in [link], where there is a uniform electric field between two plates. Electrons are released, usually from a hot filament, near the negative plate, and there is a small hole in the positive plate that allows the electrons to continue moving. (a) Calculate the acceleration of the electron if the field strength is  $2.50 \times 10^4 \ N/C$ . (b) Explain why the electron will not be pulled back to the positive plate once it moves through the hole.



Parallel conducting plates with opposite charges on them create a relatively uniform electric field used to accelerate electrons to the right. Those that go through the hole can be used to make a TV or computer screen glow or to produce X-rays.

Earth has a net charge that produces an electric field of approximately 150 N/C downward at its surface. (a) What is the magnitude and sign of the excess charge, noting the electric field of a conducting sphere is equivalent to a point charge at its center? (b) What acceleration will the field produce on a free electron near Earth's surface? (c) What mass object with a single extra electron will have its weight supported by this field?

## **Solution:**

(a) 
$$-6.76 \times 10^5 \text{ C}$$

(b) 
$$2.63 \times 10^{13} \text{ m/s}^2 \text{ (upward)}$$

(c) 
$$2.45 \times 10^{-18} \text{ kg}$$

## **Exercise:**

## **Problem:**

Point charges of  $25.0~\mu\mathrm{C}$  and  $45.0~\mu\mathrm{C}$  are placed 0.500 m apart. (a) At what point along the line between them is the electric field zero? (b) What is the electric field halfway between them?

# **Exercise:**

## **Problem:**

What can you say about two charges  $q_1$  and  $q_2$ , if the electric field one-fourth of the way from  $q_1$  to  $q_2$  is zero?

## **Solution:**

The charge  $q_2$  is 9 times greater than  $q_1$ .

### **Exercise:**

**Problem:** Integrated Concepts

Calculate the angular velocity  $\omega$  of an electron orbiting a proton in the hydrogen atom, given the radius of the orbit is  $0.530 \times 10^{-10}$  m. You may assume that the proton is stationary and the centripetal force is supplied by Coulomb attraction.

#### **Exercise:**

# **Problem:** Integrated Concepts

An electron has an initial velocity of  $5.00 \times 10^6$  m/s in a uniform  $2.00 \times 10^5$  N/C strength electric field. The field accelerates the electron in the direction opposite to its initial velocity. (a) What is the direction of the electric field? (b) How far does the electron travel before coming to rest? (c) How long does it take the electron to come to rest? (d) What is the electron's velocity when it returns to its starting point?

#### **Exercise:**

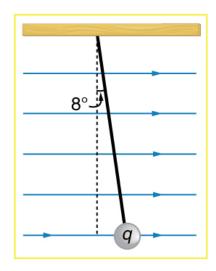
### **Problem:** Integrated Concepts

The practical limit to an electric field in air is about  $3.00 \times 10^6 \ N/C$ . Above this strength, sparking takes place because air begins to ionize and charges flow, reducing the field. (a) Calculate the distance a free proton must travel in this field to reach 3.00% of the speed of light, starting from rest. (b) Is this practical in air, or must it occur in a vacuum?

#### **Exercise:**

# **Problem:** Integrated Concepts

A 5.00 g charged insulating ball hangs on a 30.0 cm long string in a uniform horizontal electric field as shown in [link]. Given the charge on the ball is 1.00  $\mu$ C, find the strength of the field.

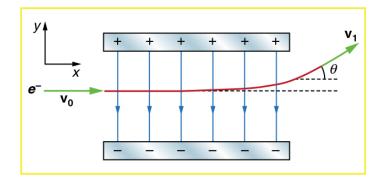


A horizontal electric field causes the charged ball to hang at an angle of 8.00°.

#### **Exercise:**

# **Problem:** Integrated Concepts

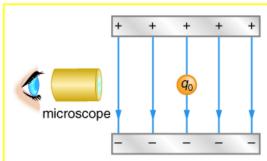
[link] shows an electron passing between two charged metal plates that create an 100 N/C vertical electric field perpendicular to the electron's original horizontal velocity. (These can be used to change the electron's direction, such as in an oscilloscope.) The initial speed of the electron is  $3.00 \times 10^6$  m/s, and the horizontal distance it travels in the uniform field is 4.00 cm. (a) What is its vertical deflection? (b) What is the vertical component of its final velocity? (c) At what angle does it exit? Neglect any edge effects.



#### **Exercise:**

### **Problem:** Integrated Concepts

The classic Millikan oil drop experiment was the first to obtain an accurate measurement of the charge on an electron. In it, oil drops were suspended against the gravitational force by a vertical electric field. (See [link].) Given the oil drop to be  $1.00~\mu m$  in radius and have a density of  $920~kg/m^3$ : (a) Find the weight of the drop. (b) If the drop has a single excess electron, find the electric field strength needed to balance its weight.



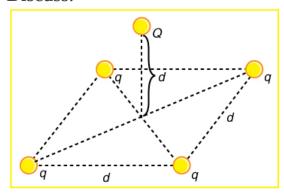
In the Millikan oil drop experiment, small drops can be suspended in an electric field by the force exerted on a single excess electron. Classically, this experiment was used to determine the electron charge  $q_{\rm e}$  by

measuring the electric field and mass of the drop.

#### **Exercise:**

### **Problem:** Integrated Concepts

(a) In [link], four equal charges q lie on the corners of a square. A fifth charge Q is on a mass m directly above the center of the square, at a height equal to the length d of one side of the square. Determine the magnitude of q in terms of Q, m, and d, if the Coulomb force is to equal the weight of m. (b) Is this equilibrium stable or unstable? Discuss.



Four equal charges on the corners of a horizontal square support the weight of a fifth charge located directly above the center of the square.

#### **Exercise:**

#### **Problem:** Unreasonable Results

(a) Calculate the electric field strength near a 10.0 cm diameter conducting sphere that has 1.00 C of excess charge on it. (b) What is

unreasonable about this result? (c) Which assumptions are responsible?

#### **Exercise:**

#### **Problem:** Unreasonable Results

(a) Two 0.500 g raindrops in a thunderhead are 1.00 cm apart when they each acquire 1.00 mC charges. Find their acceleration. (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

#### **Exercise:**

#### **Problem:** Unreasonable Results

A wrecking yard inventor wants to pick up cars by charging a 0.400 m diameter ball and inducing an equal and opposite charge on the car. If a car has a 1000 kg mass and the ball is to be able to lift it from a distance of 1.00 m: (a) What minimum charge must be used? (b) What is the electric field near the surface of the ball? (c) Why are these results unreasonable? (d) Which premise or assumption is responsible?

#### **Exercise:**

#### **Problem:** Construct Your Own Problem

Consider two insulating balls with evenly distributed equal and opposite charges on their surfaces, held with a certain distance between the centers of the balls. Construct a problem in which you calculate the electric field (magnitude and direction) due to the balls at various points along a line running through the centers of the balls and extending to infinity on either side. Choose interesting points and comment on the meaning of the field at those points. For example, at what points might the field be just that due to one ball and where does the field become negligibly small? Among the things to be considered are the magnitudes of the charges and the distance between the centers of the balls. Your instructor may wish for you to consider the electric

field off axis or for a more complex array of charges, such as those in a water molecule.

#### **Exercise:**

#### **Problem:** Construct Your Own Problem

Consider identical spherical conducting space ships in deep space where gravitational fields from other bodies are negligible compared to the gravitational attraction between the ships. Construct a problem in which you place identical excess charges on the space ships to exactly counter their gravitational attraction. Calculate the amount of excess charge needed. Examine whether that charge depends on the distance between the centers of the ships, the masses of the ships, or any other factors. Discuss whether this would be an easy, difficult, or even impossible thing to do in practice.

# **Glossary**

### Van de Graaff generator

a machine that produces a large amount of excess charge, used for experiments with high voltage

#### electrostatics

the study of electric forces that are static or slow-moving

### photoconductor

a substance that is an insulator until it is exposed to light, when it becomes a conductor

# xerography

a dry copying process based on electrostatics

### grounded

connected to the ground with a conductor, so that charge flows freely to and from the Earth to the grounded object

# laser printer

uses a laser to create a photoconductive image on a drum, which attracts dry ink particles that are then rolled onto a sheet of paper to print a high-quality copy of the image

### ink-jet printer

small ink droplets sprayed with an electric charge are controlled by electrostatic plates to create images on paper

# electrostatic precipitators

filters that apply charges to particles in the air, then attract those charges to a filter, removing them from the airstream

# Introduction to Electric Potential and Electric Energy class="introduction"

Automated
external
defibrillato
r unit
(AED)
(credit:
U.S.
Defense
Department
photo/Tech.
Sgt.
Suzanne
M. Day)



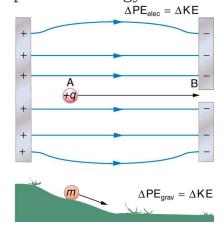
In <u>Electric Charge and Electric Field</u>, we just scratched the surface (or at least rubbed it) of electrical phenomena. Two of the most familiar aspects of

electricity are its energy and *voltage*. We know, for example, that great amounts of electrical energy can be stored in batteries, are transmitted cross-country through power lines, and may jump from clouds to explode the sap of trees. In a similar manner, at molecular levels, *ions* cross cell membranes and transfer information. We also know about voltages associated with electricity. Batteries are typically a few volts, the outlets in your home produce 120 volts, and power lines can be as high as hundreds of thousands of volts. But energy and voltage are not the same thing. A motorcycle battery, for example, is small and would not be very successful in replacing the much larger car battery, yet each has the same voltage. In this chapter, we shall examine the relationship between voltage and electrical energy and begin to explore some of the many applications of electricity.

### Electric Potential Energy: Potential Difference

- Define electric potential and electric potential energy.
- Describe the relationship between potential difference and electrical potential energy.
- Explain electron volt and its usage in submicroscopic process.
- Determine electric potential energy given potential difference and amount of charge.

When a free positive charge q is accelerated by an electric field, such as shown in  $[\underline{link}]$ , it is given kinetic energy. The process is analogous to an object being accelerated by a gravitational field. It is as if the charge is going down an electrical hill where its electric potential energy is converted to kinetic energy. Let us explore the work done on a charge q by the electric field in this process, so that we may develop a definition of electric potential energy.



A charge accelerated by an electric field is analogous to a mass going down a hill. In both cases potential energy is converted to another form. Work is done by a force, but since this force

is conservative, we can write  $W = -\Delta PE$ .

The electrostatic or Coulomb force is conservative, which means that the work done on q is independent of the path taken. This is exactly analogous to the gravitational force in the absence of dissipative forces such as friction. When a force is conservative, it is possible to define a potential energy associated with the force, and it is usually easier to deal with the potential energy (because it depends only on position) than to calculate the work directly.

We use the letters PE to denote electric potential energy, which has units of joules (J). The change in potential energy,  $\Delta PE$ , is crucial, since the work done by a conservative force is the negative of the change in potential energy; that is,  $W = -\Delta PE$ . For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative  $\Delta PE$ . There must be a minus sign in front of  $\Delta PE$  to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

#### **Note:**

### Potential Energy

 $W = -\Delta PE$ . For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative  $\Delta PE$ . There must be a minus sign in front of  $\Delta PE$  to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Gravitational potential energy and electric potential energy are quite analogous. Potential energy accounts for work done by a conservative force and gives added insight regarding energy and energy transformation without the necessity of dealing with the force directly. It is much more common, for example, to use the concept of voltage (related to electric potential energy) than to deal with the Coulomb force directly.

Calculating the work directly is generally difficult, since  $W=\mathrm{Fd}\cos\theta$  and the direction and magnitude of F can be complex for multiple charges, for odd-shaped objects, and along arbitrary paths. But we do know that, since  $F=\mathrm{qE}$ , the work, and hence  $\Delta\mathrm{PE}$ , is proportional to the test charge q. To have a physical quantity that is independent of test charge, we define **electric potential** V (or simply potential, since electric is understood) to be the potential energy per unit charge:

### **Equation:**

$$V = rac{ ext{PE}}{q}.$$

#### Note:

Electric Potential

This is the electric potential energy per unit charge.

# **Equation:**

$$V = rac{ ext{PE}}{q}$$

Since PE is proportional to q, the dependence on q cancels. Thus V does not depend on q. The change in potential energy  $\Delta PE$  is crucial, and so we are concerned with the difference in potential or potential difference  $\Delta V$  between two points, where

$$\Delta V = V_{
m B} - V_{
m A} = rac{\Delta {
m PE}}{q}.$$

The **potential difference** between points A and B,  $V_{\rm B}-V_{\rm A}$ , is thus defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

### **Equation:**

$$1~ ext{V} = 1~rac{ ext{J}}{ ext{C}}$$

#### Note:

### Potential Difference

The potential difference between points A and B,  $V_{\rm B}-V_{\rm A}$ , is defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

### **Equation:**

$$1 V = 1 \frac{J}{C}$$

The familiar term **voltage** is the common name for potential difference. Keep in mind that whenever a voltage is quoted, it is understood to be the potential difference between two points. For example, every battery has two terminals, and its voltage is the potential difference between them. More fundamentally, the point you choose to be zero volts is arbitrary. This is analogous to the fact that gravitational potential energy has an arbitrary zero, such as sea level or perhaps a lecture hall floor.

In summary, the relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = rac{\Delta ext{PE}}{q} ext{ and } \Delta ext{PE} = q \Delta V.$$

#### Note:

### Potential Difference and Electrical Potential Energy

The relationship between potential difference (or voltage) and electrical potential energy is given by

### **Equation:**

$$\Delta V = rac{\Delta ext{PE}}{q} ext{ and } \Delta ext{PE} = q \Delta V.$$

The second equation is equivalent to the first.

Voltage is not the same as energy. Voltage is the energy per unit charge. Thus a motorcycle battery and a car battery can both have the same voltage (more precisely, the same potential difference between battery terminals), yet one stores much more energy than the other since  $\Delta PE = q\Delta V$ . The car battery can move more charge than the motorcycle battery, although both are 12 V batteries.

# **Example:**

# **Calculating Energy**

Suppose you have a 12.0 V motorcycle battery that can move 5000 C of charge, and a 12.0 V car battery that can move 60,000 C of charge. How much energy does each deliver? (Assume that the numerical value of each charge is accurate to three significant figures.)

### Strategy

To say we have a 12.0 V battery means that its terminals have a 12.0 V potential difference. When such a battery moves charge, it puts the charge

through a potential difference of 12.0 V, and the charge is given a change in potential energy equal to  $\Delta PE = q\Delta V$ .

So to find the energy output, we multiply the charge moved by the potential difference.

#### Solution

For the motorcycle battery, q = 5000 C and  $\Delta V = 12.0$  V. The total energy delivered by the motorcycle battery is

### **Equation:**

$$\begin{array}{lll} \Delta PE_{cycle} & = & (5000~C)(12.0~V) \\ & = & (5000~C)(12.0~J/C) \\ & = & 6.00 \times 10^4~J. \end{array}$$

Similarly, for the car battery, q = 60,000 C and

### **Equation:**

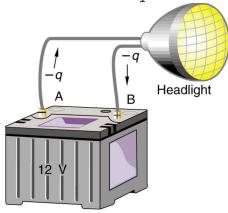
$$\Delta PE_{car} = (60,000 \text{ C})(12.0 \text{ V})$$
  
=  $7.20 \times 10^5 \text{ J}.$ 

#### Discussion

While voltage and energy are related, they are not the same thing. The voltages of the batteries are identical, but the energy supplied by each is quite different. Note also that as a battery is discharged, some of its energy is used internally and its terminal voltage drops, such as when headlights dim because of a low car battery. The energy supplied by the battery is still calculated as in this example, but not all of the energy is available for external use.

Note that the energies calculated in the previous example are absolute values. The change in potential energy for the battery is negative, since it loses energy. These batteries, like many electrical systems, actually move negative charge—electrons in particular. The batteries repel electrons from their negative terminals (A) through whatever circuitry is involved and attract them to their positive terminals (B) as shown in [link]. The change in potential is  $\Delta V = V_{\rm B} - V_{\rm A} = +12$  V and the charge q is negative, so that

 $\Delta PE = q\Delta V$  is negative, meaning the potential energy of the battery has decreased when q has moved from A to B.



A battery moves negative charge from its negative terminal through a headlight to its positive terminal.

Appropriate combinations of chemicals in the battery separate charges so that the negative terminal has an excess of negative charge, which is repelled by it and attracted to the excess positive charge on the other terminal. In terms of potential, the positive terminal is at a higher voltage than the negative. Inside the battery, both positive and negative charges move.

# **Example:**

# How Many Electrons Move through a Headlight Each Second?

When a 12.0 V car battery runs a single 30.0 W headlight, how many electrons pass through it each second?

### Strategy

To find the number of electrons, we must first find the charge that moved in 1.00 s. The charge moved is related to voltage and energy through the equation  $\Delta PE = q\Delta V$ . A 30.0 W lamp uses 30.0 joules per second. Since the battery loses energy, we have  $\Delta PE = -30.0$  J and, since the electrons are going from the negative terminal to the positive, we see that  $\Delta V = +12.0$  V.

#### Solution

To find the charge q moved, we solve the equation  $\Delta PE = q\Delta V$ :

### **Equation:**

$$q = rac{\Delta ext{PE}}{\Delta V}.$$

Entering the values for  $\Delta PE$  and  $\Delta V$ , we get

# **Equation:**

$$q = rac{-30.0 ext{ J}}{+12.0 ext{ V}} = rac{-30.0 ext{ J}}{+12.0 ext{ J/C}} = -2.50 ext{ C}.$$

The number of electrons  $\mathbf{n}_{\rm e}$  is the total charge divided by the charge per electron. That is,

# **Equation:**

$$n_{e} = rac{-2.50~C}{-1.60 imes 10^{-19}~C/e^{-}} = 1.56 imes 10^{19}~electrons.$$

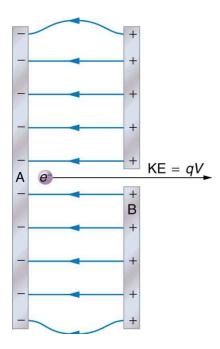
#### **Discussion**

This is a very large number. It is no wonder that we do not ordinarily observe individual electrons with so many being present in ordinary

systems. In fact, electricity had been in use for many decades before it was determined that the moving charges in many circumstances were negative. Positive charge moving in the opposite direction of negative charge often produces identical effects; this makes it difficult to determine which is moving or whether both are moving.

#### The Electron Volt

The energy per electron is very small in macroscopic situations like that in the previous example—a tiny fraction of a joule. But on a submicroscopic scale, such energy per particle (electron, proton, or ion) can be of great importance. For example, even a tiny fraction of a joule can be great enough for these particles to destroy organic molecules and harm living tissue. The particle may do its damage by direct collision, or it may create harmful x rays, which can also inflict damage. It is useful to have an energy unit related to submicroscopic effects. [link] shows a situation related to the definition of such an energy unit. An electron is accelerated between two charged metal plates as it might be in an old-model television tube or oscilloscope. The electron is given kinetic energy that is later converted to another form—light in the television tube, for example. (Note that downhill for the electron is uphill for a positive charge.) Since energy is related to voltage by  $\Delta PE = q\Delta V$ , we can think of the joule as a coulomb-volt.



A typical electron gun accelerates electrons using a potential difference between two metal plates. The energy of the electron in electron volts is numerically the same as the voltage between the plates. For example, a 5000 V potential difference produces 5000 eV electrons.

On the submicroscopic scale, it is more convenient to define an energy unit called the **electron volt** (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form, **Equation:** 

$$1 \text{ eV} = (1.60 \times 10^{-19} \text{ C})(1 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C})$$
  
=  $1.60 \times 10^{-19} \text{ J}$ .

#### Note:

#### Electron Volt

On the submicroscopic scale, it is more convenient to define an energy unit called the electron volt (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

### **Equation:**

$$1 \text{ eV} = (1.60 \times 10^{-19} \text{ C})(1 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C})$$
  
=  $1.60 \times 10^{-19} \text{ J}$ .

An electron accelerated through a potential difference of 1 V is given an energy of 1 eV. It follows that an electron accelerated through 50 V is given 50 eV. A potential difference of 100,000 V (100 kV) will give an electron an energy of 100,000 eV (100 keV), and so on. Similarly, an ion with a double positive charge accelerated through 100 V will be given 200 eV of energy. These simple relationships between accelerating voltage and particle charges make the electron volt a simple and convenient energy unit in such circumstances.

#### **Note:**

### Connections: Energy Units

The electron volt (eV) is the most common energy unit for submicroscopic processes. This will be particularly noticeable in the chapters on modern physics. Energy is so important to so many subjects that there is a tendency to define a special energy unit for each major topic. There are, for example,

calories for food energy, kilowatt-hours for electrical energy, and therms for natural gas energy.

The electron volt is commonly employed in submicroscopic processes—chemical valence energies and molecular and nuclear binding energies are among the quantities often expressed in electron volts. For example, about 5 eV of energy is required to break up certain organic molecules. If a proton is accelerated from rest through a potential difference of 30 kV, it is given an energy of 30 keV (30,000 eV) and it can break up as many as 6000 of these molecules (30,000 eV  $\div$  5 eV per molecule = 6000 molecules). Nuclear decay energies are on the order of 1 MeV (1,000,000 eV) per event and can, thus, produce significant biological damage.

# **Conservation of Energy**

The total energy of a system is conserved if there is no net addition (or subtraction) of work or heat transfer. For conservative forces, such as the electrostatic force, conservation of energy states that mechanical energy is a constant.

**Mechanical energy** is the sum of the kinetic energy and potential energy of a system; that is, KE + PE = constant. A loss of PE of a charged particle becomes an increase in its KE. Here PE is the electric potential energy. Conservation of energy is stated in equation form as

# **Equation:**

$$KE + PE = constant$$

or

$$KE_i + PE_i = KE_f + PE_f$$

where i and f stand for initial and final conditions. As we have found many times before, considering energy can give us insights and facilitate problem solving.

### **Example:**

### **Electrical Potential Energy Converted to Kinetic Energy**

Calculate the final speed of a free electron accelerated from rest through a potential difference of 100 V. (Assume that this numerical value is accurate to three significant figures.)

### Strategy

We have a system with only conservative forces. Assuming the electron is accelerated in a vacuum, and neglecting the gravitational force (we will check on this assumption later), all of the electrical potential energy is converted into kinetic energy. We can identify the initial and final forms of energy to be  $\mathrm{KE_i} = 0$ ,  $\mathrm{KE_f} = \frac{1}{2} \, mv^2$ ,  $\mathrm{PE_i} = qV$ , and  $\mathrm{PE_f} = 0$ .

#### Solution

Conservation of energy states that

# **Equation:**

$$KE_i + PE_i = KE_f + PE_f$$
.

Entering the forms identified above, we obtain

# **Equation:**

$$qV = rac{mv^2}{2}.$$

We solve this for v:

# **Equation:**

$$v = \sqrt{rac{2 \mathrm{qV}}{m}}.$$

Entering values for q, V, and m gives

$$egin{array}{lcl} v & = & \sqrt{rac{2 \left(-1.60 imes 10^{-19} \; \mathrm{C}
ight) \left(-100 \; \mathrm{J/C}
ight)}{9.11 imes 10^{-31} \; \mathrm{kg}}} \ & = & 5.93 imes 10^6 \; \mathrm{m/s}. \end{array}$$

#### Discussion

Note that both the charge and the initial voltage are negative, as in [link]. From the discussions in Electric Charge and Electric Field, we know that electrostatic forces on small particles are generally very large compared with the gravitational force. The large final speed confirms that the gravitational force is indeed negligible here. The large speed also indicates how easy it is to accelerate electrons with small voltages because of their very small mass. Voltages much higher than the 100 V in this problem are typically used in electron guns. Those higher voltages produce electron speeds so great that relativistic effects must be taken into account. That is why a low voltage is considered (accurately) in this example.

# **Section Summary**

- Electric potential is potential energy per unit charge.
- The potential difference between points A and B,  $V_{\rm B}-V_{\rm A}$ , defined to be the change in potential energy of a charge q moved from A to B, is equal to the change in potential energy divided by the charge, Potential difference is commonly called voltage, represented by the symbol  $\Delta V$ . **Equation:**

$$\Delta V = rac{\Delta \mathrm{PE}}{q} ext{ and } \Delta \mathrm{PE} = q \Delta V.$$

• An electron volt is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form, **Equation:** 

$$\begin{array}{lll} 1~{\rm eV} &=& \left(1.60\times 10^{-19}~{\rm C}\right)(1~{\rm V}) = \left(1.60\times 10^{-19}~{\rm C}\right)(1~{\rm J/C}) \\ &=& 1.60\times 10^{-19}~{\rm J}. \end{array}$$

• Mechanical energy is the sum of the kinetic energy and potential energy of a system, that is, KE + PE. This sum is a constant.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

Voltage is the common word for potential difference. Which term is more descriptive, voltage or potential difference?

#### **Exercise:**

#### **Problem:**

If the voltage between two points is zero, can a test charge be moved between them with zero net work being done? Can this necessarily be done without exerting a force? Explain.

#### Exercise:

#### **Problem:**

What is the relationship between voltage and energy? More precisely, what is the relationship between potential difference and electric potential energy?

#### **Exercise:**

**Problem:** Voltages are always measured between two points. Why?

#### **Exercise:**

#### **Problem:**

How are units of volts and electron volts related? How do they differ?

### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

Find the ratio of speeds of an electron and a negative hydrogen ion (one having an extra electron) accelerated through the same voltage, assuming non-relativistic final speeds. Take the mass of the hydrogen ion to be  $1.67 \times 10^{-27}$  kg.

#### **Solution:**

42.8

#### **Exercise:**

#### **Problem:**

An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce x rays. Non-relativistically, what would be the maximum speed of these electrons?

#### **Exercise:**

#### **Problem:**

A bare helium nucleus has two positive charges and a mass of  $6.64 \times 10^{-27}$  kg. (a) Calculate its kinetic energy in joules at 2.00% of the speed of light. (b) What is this in electron volts? (c) What voltage would be needed to obtain this energy?

#### **Exercise:**

# **Problem:** Integrated Concepts

Singly charged gas ions are accelerated from rest through a voltage of 13.0 V. At what temperature will the average kinetic energy of gas molecules be the same as that given these ions?

#### **Solution:**

$$1.00 \times 10^{5} \text{ K}$$

#### **Exercise:**

# **Problem:** Integrated Concepts

The temperature near the center of the Sun is thought to be 15 million degrees Celsius  $(1.5\times10^7 \, {}^{\circ}\text{C})$ . Through what voltage must a singly charged ion be accelerated to have the same energy as the average kinetic energy of ions at this temperature?

#### **Exercise:**

### **Problem:** Integrated Concepts

(a) What is the average power output of a heart defibrillator that dissipates 400 J of energy in 10.0 ms? (b) Considering the high-power output, why doesn't the defibrillator produce serious burns?

#### **Solution:**

(a) 
$$4 \times 10^4 \text{ W}$$

(b) A defibrillator does not cause serious burns because the skin conducts electricity well at high voltages, like those used in defibrillators. The gel used aids in the transfer of energy to the body, and the skin doesn't absorb the energy, but rather lets it pass through to the heart.

#### **Exercise:**

### **Problem:** Integrated Concepts

A lightning bolt strikes a tree, moving 20.0 C of charge through a potential difference of  $1.00 \times 10^2~\mathrm{MV}$ . (a) What energy was dissipated? (b) What mass of water could be raised from  $15^{\circ}\mathrm{C}$  to the boiling point and then boiled by this energy? (c) Discuss the damage that could be caused to the tree by the expansion of the boiling steam.

#### **Exercise:**

### **Problem:** Integrated Concepts

A 12.0 V battery-operated bottle warmer heats 50.0 g of glass,  $2.50 \times 10^2$  g of baby formula, and  $2.00 \times 10^2$  g of aluminum from  $20.0^{\circ}\text{C}$  to  $90.0^{\circ}\text{C}$ . (a) How much charge is moved by the battery? (b) How many electrons per second flow if it takes 5.00 min to warm the formula? (Hint: Assume that the specific heat of baby formula is about the same as the specific heat of water.)

#### **Solution:**

- (a)  $7.40 \times 10^3$  C
- (b)  $1.54 \times 10^{20}$  electrons per second

#### **Exercise:**

# **Problem:** Integrated Concepts

A battery-operated car utilizes a 12.0 V system. Find the charge the batteries must be able to move in order to accelerate the 750 kg car from rest to 25.0 m/s, make it climb a  $2.00 \times 10^2$  m high hill, and then cause it to travel at a constant 25.0 m/s by exerting a  $5.00 \times 10^2$  N force for an hour.

#### **Solution:**

$$3.89 \times 10^{6} \text{ C}$$

#### **Exercise:**

# **Problem:** Integrated Concepts

Fusion probability is greatly enhanced when appropriate nuclei are brought close together, but mutual Coulomb repulsion must be overcome. This can be done using the kinetic energy of high-temperature gas ions or by accelerating the nuclei toward one another.

(a) Calculate the potential energy of two singly charged nuclei separated by  $1.00\times10^{-12}$  m by finding the voltage of one at that distance and multiplying by the charge of the other. (b) At what temperature will atoms of a gas have an average kinetic energy equal to this needed electrical potential energy?

#### **Exercise:**

#### **Problem:** Unreasonable Results

(a) Find the voltage near a 10.0 cm diameter metal sphere that has 8.00 C of excess positive charge on it. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

#### **Solution:**

(a) 
$$1.44 \times 10^{12} \text{ V}$$

- (b) This voltage is very high. A 10.0 cm diameter sphere could never maintain this voltage; it would discharge.
- (c) An 8.00 C charge is more charge than can reasonably be accumulated on a sphere of that size.

#### **Exercise:**

#### **Problem:** Construct Your Own Problem

Consider a battery used to supply energy to a cellular phone. Construct a problem in which you determine the energy that must be supplied by the battery, and then calculate the amount of charge it must be able to move in order to supply this energy. Among the things to be considered are the energy needs and battery voltage. You may need to look ahead to interpret manufacturer's battery ratings in ampere-hours as energy in joules.

# Glossary

# electric potential

potential energy per unit charge

### potential difference (or voltage)

change in potential energy of a charge moved from one point to another, divided by the charge; units of potential difference are joules per coulomb, known as volt

### electron volt

the energy given to a fundamental charge accelerated through a potential difference of one volt

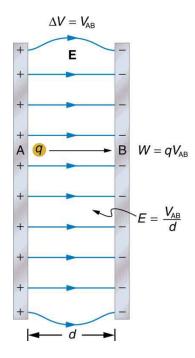
### mechanical energy

sum of the kinetic energy and potential energy of a system; this sum is a constant

#### Electric Potential in a Uniform Electric Field

- Describe the relationship between voltage and electric field.
- Derive an expression for the electric potential and electric field.
- Calculate electric field strength given distance and voltage.

In the previous section, we explored the relationship between voltage and energy. In this section, we will explore the relationship between voltage and electric field. For example, a uniform electric field  $\bf E$  is produced by placing a potential difference (or voltage)  $\Delta V$  across two parallel metal plates, labeled A and B. (See [link].) Examining this will tell us what voltage is needed to produce a certain electric field strength; it will also reveal a more fundamental relationship between electric potential and electric field. From a physicist's point of view, either  $\Delta V$  or **E** can be used to describe any charge distribution.  $\Delta V$  is most closely tied to energy, whereas **E** is most closely related to force.  $\Delta V$  is a scalar quantity and has no direction, while  $\mathbf{E}$  is a **vector** quantity, having both magnitude and direction. (Note that the magnitude of the electric field strength, a scalar quantity, is represented by E below.) The relationship between  $\Delta V$  and **E** is revealed by calculating the work done by the force in moving a charge from point A to point B. But, as noted in **Electric Potential Energy**: Potential Difference, this is complex for arbitrary charge distributions, requiring calculus. We therefore look at a uniform electric field as an interesting special case.



The relationship between V and E for parallel conducting plates is E=V/d. (Note that  $\Delta V=V_{\rm AB}$  in magnitude. For a charge that is moved from plate A at higher potential to plate B at lower potential, a minus sign needs to be included as follows:  $-\Delta V=V_{\rm A}-V_{\rm B}=V_{\rm AB}$ . See the text for details.)

The work done by the electric field in  $[\underline{link}]$  to move a positive charge q from A, the positive plate, higher potential, to B, the negative plate, lower potential, is

$$W = -\Delta {
m PE} = -q\Delta V.$$

The potential difference between points A and B is **Equation:** 

$$-\Delta \ V = -(V_{
m B} - V_{
m A}) = V_{
m A} - V_{
m B} = V_{
m AB}.$$

Entering this into the expression for work yields **Equation:** 

$$W=qV_{
m AB}.$$

Work is  $W = Fd \cos \theta$ ; here  $\cos \theta = 1$ , since the path is parallel to the field, and so W = Fd. Since F = qE, we see that W = qEd. Substituting this expression for work into the previous equation gives

### **Equation:**

$$qEd = qV_{\mathrm{AB}}.$$

The charge cancels, and so the voltage between points A and B is seen to be **Equation:** 

$$\left.egin{aligned} V_{ ext{AB}} = Ed \ E = rac{V_{ ext{AB}}}{d} \end{aligned}
ight\} ext{(uniform $E$ - field only)},$$

where d is the distance from A to B, or the distance between the plates in  $[\underline{link}]$ . Note that the above equation implies the units for electric field are volts per meter. We already know the units for electric field are newtons per coulomb; thus the following relation among units is valid:

$$1 \text{ N/C} = 1 \text{ V/m}.$$

#### Note:

Voltage between Points A and B

### **Equation:**

$$egin{aligned} V_{ ext{AB}} &= Ed \ E &= rac{V_{ ext{AB}}}{d} \end{aligned} iggl\} ext{(uniform $E$ - field only)},$$

where d is the distance from A to B, or the distance between the plates.

### **Example:**

# What Is the Highest Voltage Possible between Two Plates?

Dry air will support a maximum electric field strength of about  $3.0\times10^6~{\rm V/m}$ . Above that value, the field creates enough ionization in the air to make the air a conductor. This allows a discharge or spark that reduces the field. What, then, is the maximum voltage between two parallel conducting plates separated by 2.5 cm of dry air?

#### Strategy

We are given the maximum electric field E between the plates and the distance d between them. The equation  $V_{AB} = Ed$  can thus be used to calculate the maximum voltage.

#### Solution

The potential difference or voltage between the plates is

# **Equation:**

$$V_{AB} = Ed.$$

Entering the given values for E and d gives

### **Equation:**

$$V_{
m AB} = (3.0{ imes}10^6~{
m V/m})(0.025~{
m m}) = 7.5{ imes}10^4~{
m V}$$

or

$$V_{AB} = 75 \text{ kV}.$$

(The answer is quoted to only two digits, since the maximum field strength is approximate.)

#### Discussion

One of the implications of this result is that it takes about 75 kV to make a spark jump across a 2.5 cm (1 in.) gap, or 150 kV for a 5 cm spark. This limits the voltages that can exist between conductors, perhaps on a power transmission line. A smaller voltage will cause a spark if there are points on the surface, since points create greater fields than smooth surfaces. Humid air breaks down at a lower field strength, meaning that a smaller voltage will make a spark jump through humid air. The largest voltages can be built up, say with static electricity, on dry days.



A spark chamber is used to trace the paths of high-energy particles. Ionization created by the particles as they pass through the gas between the plates allows a spark to jump. The sparks are

perpendicular to the plates, following electric field lines between them. The potential difference between adjacent plates is not high enough to cause sparks without the ionization produced by particles from accelerator experiments (or cosmic rays). (credit: Daderot, Wikimedia Commons)

# **Example:**

### Field and Force inside an Electron Gun

(a) An electron gun has parallel plates separated by 4.00 cm and gives electrons 25.0 keV of energy. What is the electric field strength between the plates? (b) What force would this field exert on a piece of plastic with a  $0.500~\mu C$  charge that gets between the plates?

### **Strategy**

Since the voltage and plate separation are given, the electric field strength can be calculated directly from the expression  $E=\frac{V_{\rm AB}}{d}$ . Once the electric field strength is known, the force on a charge is found using  ${\bf F}=q~{\bf E}$ . Since the electric field is in only one direction, we can write this equation in terms of the magnitudes, F=q~E.

### Solution for (a)

The expression for the magnitude of the electric field between two uniform metal plates is

### **Equation:**

$$E = rac{V_{
m AB}}{d}.$$

Since the electron is a single charge and is given 25.0 keV of energy, the potential difference must be 25.0 kV. Entering this value for  $V_{\rm AB}$  and the plate separation of 0.0400 m, we obtain

### **Equation:**

$$E = rac{25.0 ext{ kV}}{0.0400 ext{ m}} = 6.25 imes 10^5 ext{ V/m}.$$

### Solution for (b)

The magnitude of the force on a charge in an electric field is obtained from the equation

### **Equation:**

$$F = qE$$
.

Substituting known values gives

# **Equation:**

$$F = (0.500{ imes}10^{-6}~{
m C})(6.25{ imes}10^5~{
m V/m}) = 0.313~{
m N}.$$

#### Discussion

Note that the units are newtons, since 1 V/m = 1 N/C. The force on the charge is the same no matter where the charge is located between the plates. This is because the electric field is uniform between the plates.

In more general situations, regardless of whether the electric field is uniform, it points in the direction of decreasing potential, because the force on a positive charge is in the direction of  $\mathbf{E}$  and also in the direction of lower potential V. Furthermore, the magnitude of  $\mathbf{E}$  equals the rate of decrease of V with distance. The faster V decreases over distance, the

greater the electric field. In equation form, the general relationship between voltage and electric field is

### **Equation:**

$$E = -rac{\Delta V}{\Delta s},$$

where  $\Delta s$  is the distance over which the change in potential,  $\Delta V$ , takes place. The minus sign tells us that  $\mathbf{E}$  points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

### Note:

Relationship between Voltage and Electric Field

In equation form, the general relationship between voltage and electric field is

### **Equation:**

$$E=-rac{\Delta V}{\Delta s},$$

where  $\Delta s$  is the distance over which the change in potential,  $\Delta V$ , takes place. The minus sign tells us that  $\mathbf{E}$  points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

For continually changing potentials,  $\Delta V$  and  $\Delta s$  become infinitesimals and differential calculus must be employed to determine the electric field.

# **Section Summary**

The voltage between points A and B is Equation:

$$egin{aligned} V_{ ext{AB}} &= Ed \ E &= rac{V_{ ext{AB}}}{d} \end{aligned} iggl\} ext{(uniform $E$ - field only)},$$

where d is the distance from A to B, or the distance between the plates.

• In equation form, the general relationship between voltage and electric field is

### **Equation:**

$$E = -\frac{\Delta V}{\Delta s},$$

where  $\Delta s$  is the distance over which the change in potential,  $\Delta V$ , takes place. The minus sign tells us that  $\mathbf{E}$  points in the direction of decreasing potential.) The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

Discuss how potential difference and electric field strength are related. Give an example.

#### **Exercise:**

#### **Problem:**

What is the strength of the electric field in a region where the electric potential is constant?

#### **Exercise:**

#### **Problem:**

Will a negative charge, initially at rest, move toward higher or lower potential? Explain why.

### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

Show that units of V/m and N/C for electric field strength are indeed equivalent.

#### **Exercise:**

#### **Problem:**

What is the strength of the electric field between two parallel conducting plates separated by 1.00 cm and having a potential difference (voltage) between them of  $1.50 \times 10^4$  V?

#### **Exercise:**

#### **Problem:**

The electric field strength between two parallel conducting plates separated by 4.00 cm is  $7.50 \times 10^4$  V/m. (a) What is the potential difference between the plates? (b) The plate with the lowest potential is taken to be at zero volts. What is the potential 1.00 cm from that plate (and 3.00 cm from the other)?

#### **Solution:**

- (a) 3.00 kV
- (b) 750 V

#### **Exercise:**

### **Problem:**

How far apart are two conducting plates that have an electric field strength of  $4.50 \times 10^3 \ V/m$  between them, if their potential difference is  $15.0 \ kV$ ?

#### **Exercise:**

#### **Problem:**

(a) Will the electric field strength between two parallel conducting plates exceed the breakdown strength for air  $(3.0 \times 10^6~V/m)$  if the plates are separated by 2.00 mm and a potential difference of  $5.0 \times 10^3~V$  is applied? (b) How close together can the plates be with this applied voltage?

#### **Solution:**

- (a) No. The electric field strength between the plates is  $2.5\times10^6~V/m$ , which is lower than the breakdown strength for air (  $3.0\times10^6~V/m$ ).
- (b) 1.7 mm

#### **Exercise:**

#### **Problem:**

The voltage across a membrane forming a cell wall is 80.0 mV and the membrane is 9.00 nm thick. What is the electric field strength? (The value is surprisingly large, but correct. Membranes are discussed in <a href="Capacitors and Dielectrics">Capacitors and Dielectrics</a> and <a href="Nerve Conduction">Nerve Conduction</a>—<a href="Electrocardiograms">Electrocardiograms</a>.) You may assume a uniform electric field.

#### **Exercise:**

#### **Problem:**

Membrane walls of living cells have surprisingly large electric fields across them due to separation of ions. (Membranes are discussed in some detail in <a href="Nerve Conduction—Electrocardiograms">Nerve Conduction—Electrocardiograms</a>.) What is the voltage across an 8.00 nm—thick membrane if the electric field strength across it is 5.50 MV/m? You may assume a uniform electric field.

#### **Solution:**

44.0 mV

#### **Exercise:**

#### **Problem:**

Two parallel conducting plates are separated by 10.0 cm, and one of them is taken to be at zero volts. (a) What is the electric field strength between them, if the potential 8.00 cm from the zero volt plate (and 2.00 cm from the other) is 450 V? (b) What is the voltage between the plates?

#### **Exercise:**

#### **Problem:**

Find the maximum potential difference between two parallel conducting plates separated by 0.500 cm of air, given the maximum sustainable electric field strength in air to be  $3.0 \times 10^6 \ V/m$ .

### **Solution:**

 $15 \mathrm{\; kV}$ 

#### **Exercise:**

#### **Problem:**

A doubly charged ion is accelerated to an energy of 32.0 keV by the electric field between two parallel conducting plates separated by 2.00 cm. What is the electric field strength between the plates?

#### **Exercise:**

#### **Problem:**

An electron is to be accelerated in a uniform electric field having a strength of  $2.00 \times 10^6~V/m$ . (a) What energy in keV is given to the electron if it is accelerated through 0.400 m? (b) Over what distance would it have to be accelerated to increase its energy by 50.0 GeV?

#### **Solution:**

(a) 800 KeV

# (b) 25.0 km

# Glossary

scalar

physical quantity with magnitude but no direction

vector

physical quantity with both magnitude and direction

### Electrical Potential Due to a Point Charge

- Explain point charges and express the equation for electric potential of a point charge.
- Distinguish between electric potential and electric field.
- Determine the electric potential of a point charge given charge and distance.

Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (like on a metal sphere) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider. Using calculus to find the work needed to move a test charge q from a large distance away to a distance of r from a point charge Q, and noting the connection between work and potential  $(W=-q\Delta V)$ , it can be shown that the *electric potential* V *of a point charge* is

### **Equation:**

$$V = \frac{kQ}{r}$$
 (Point Charge),

where *k* is a constant equal to  $9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$ .

#### Note:

Electric Potential V of a Point Charge

The electric potential V of a point charge is given by

# **Equation:**

$$V = \frac{kQ}{r}$$
 (Point Charge).

The potential at infinity is chosen to be zero. Thus V for a point charge decreases with distance, whereas  $\mathbf{E}$  for a point charge decreases with distance squared:

### **Equation:**

$$E=rac{F}{q}=rac{kQ}{r^2}.$$

Recall that the electric potential V is a scalar and has no direction, whereas the electric field  ${\bf E}$  is a vector. To find the voltage due to a combination of point charges, you add the individual voltages as numbers. To find the total electric field, you must add the individual fields as vectors, taking magnitude and direction into account. This is consistent with the fact that V is closely associated with energy, a scalar, whereas  ${\bf E}$  is closely associated with force, a vector.

### **Example:**

# What Voltage Is Produced by a Small Charge on a Metal Sphere?

Charges in static electricity are typically in the nanocoulomb (nC) to microcoulomb  $(\mu C)$  range. What is the voltage 5.00 cm away from the center of a 1-cm diameter metal sphere that has a -3.00 nC static charge? **Strategy** 

As we have discussed in Electric Charge and Electric Field, charge on a metal sphere spreads out uniformly and produces a field like that of a point charge located at its center. Thus we can find the voltage using the equation V = kQ/r.

### **Solution**

Entering known values into the expression for the potential of a point charge, we obtain

# **Equation:**

$$egin{array}{lcl} V & = & krac{Q}{r} \ & = & \left( 8.99 imes 10^9 \ {
m N} \cdot {
m m}^2/{
m C}^2 
ight) \left( rac{-3.00 imes 10^{-9} \ {
m C}}{5.00 imes 10^{-2} \ {
m m}} 
ight) \ & = & -539 \ {
m V}. \end{array}$$

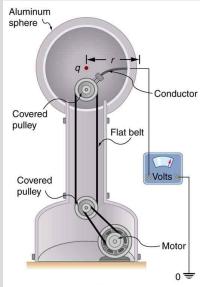
#### Discussion

The negative value for voltage means a positive charge would be attracted from a larger distance, since the potential is lower (more negative) than at larger distances. Conversely, a negative charge would be repelled, as expected.

### **Example:**

## What Is the Excess Charge on a Van de Graaff Generator

A demonstration Van de Graaff generator has a 25.0 cm diameter metal sphere that produces a voltage of 100 kV near its surface. (See [link].) What excess charge resides on the sphere? (Assume that each numerical value here is shown with three significant figures.)



The voltage of this demonstration Van de Graaff generator is measured between the charged sphere and ground. Earth's potential is taken to be zero as a reference. The potential of the charged conducting sphere is the same as that of an equal point charge at its center.

# **Strategy**

The potential on the surface will be the same as that of a point charge at the center of the sphere, 12.5 cm away. (The radius of the sphere is 12.5 cm.) We can thus determine the excess charge using the equation

### **Equation:**

$$V = rac{\mathrm{kQ}}{r}.$$

#### Solution

Solving for Q and entering known values gives

### **Equation:**

$$egin{array}{lll} Q & = & rac{{
m rV}}{k} \ & = & rac{(0.125\ {
m m})\left(100 imes10^3\ {
m V}
ight)}{8.99 imes10^9\ {
m N\cdot m^2/C^2}} \ & = & 1.39 imes10^{-6}\ {
m C} = 1.39\ {
m \mu C}. \end{array}$$

#### Discussion

This is a relatively small charge, but it produces a rather large voltage. We have another indication here that it is difficult to store isolated charges.

The voltages in both of these examples could be measured with a meter that compares the measured potential with ground potential. Ground potential is often taken to be zero (instead of taking the potential at infinity to be zero). It is the potential difference between two points that is of importance, and very often there is a tacit assumption that some reference point, such as Earth or a very distant point, is at zero potential. As noted in Electric Potential Energy: Potential Difference, this is analogous to taking sea level as h = 0 when considering gravitational potential energy,  $PE_g = mgh$ .

# **Section Summary**

- Electric potential of a point charge is V = kQ/r.
- Electric potential is a scalar, and electric field is a vector. Addition of voltages as numbers gives the voltage due to a combination of point charges, whereas addition of individual fields as vectors gives the total electric field.

# **Conceptual Questions**

#### **Exercise:**

#### Problem:

In what region of space is the potential due to a uniformly charged sphere the same as that of a point charge? In what region does it differ from that of a point charge?

#### **Exercise:**

#### **Problem:**

Can the potential of a non-uniformly charged sphere be the same as that of a point charge? Explain.

#### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

A 0.500 cm diameter plastic sphere, used in a static electricity demonstration, has a uniformly distributed 40.0 pC charge on its surface. What is the potential near its surface?

#### **Solution:**

144 V

#### **Exercise:**

#### **Problem:**

What is the potential  $0.530 \times 10^{-10}$  m from a proton (the average distance between the proton and electron in a hydrogen atom)?

#### **Exercise:**

#### **Problem:**

(a) A sphere has a surface uniformly charged with 1.00 C. At what distance from its center is the potential 5.00 MV? (b) What does your answer imply about the practical aspect of isolating such a large charge?

#### **Solution:**

- (a) 1.80 km
- (b) A charge of 1 C is a very large amount of charge; a sphere of radius 1.80 km is not practical.

### **Exercise:**

#### **Problem:**

How far from a 1.00  $\mu C$  point charge will the potential be 100 V? At what distance will it be  $2.00 \times 10^2$  V?

#### **Exercise:**

#### **Problem:**

What are the sign and magnitude of a point charge that produces a potential of -2.00 V at a distance of 1.00 mm?

#### **Solution:**

$$-2.22 \times 10^{-13} \text{ C}$$

### **Exercise:**

#### **Problem:**

If the potential due to a point charge is  $5.00 \times 10^2$  V at a distance of 15.0 m, what are the sign and magnitude of the charge?

#### **Exercise:**

#### Problem:

In nuclear fission, a nucleus splits roughly in half. (a) What is the potential  $2.00 \times 10^{-14}$  m from a fragment that has 46 protons in it? (b) What is the potential energy in MeV of a similarly charged fragment at this distance?

#### **Solution:**

- (a)  $3.31 \times 10^6 \text{ V}$
- (b) 152 MeV

#### **Exercise:**

#### **Problem:**

A research Van de Graaff generator has a 2.00-m-diameter metal sphere with a charge of 5.00 mC on it. (a) What is the potential near its surface? (b) At what distance from its center is the potential 1.00 MV? (c) An oxygen atom with three missing electrons is released near the Van de Graaff generator. What is its energy in MeV at this distance?

#### **Exercise:**

#### **Problem:**

An electrostatic paint sprayer has a 0.200-m-diameter metal sphere at a potential of 25.0 kV that repels paint droplets onto a grounded object. (a) What charge is on the sphere? (b) What charge must a 0.100-mg drop of paint have to arrive at the object with a speed of 10.0 m/s?

#### **Solution:**

(a) 
$$2.78 \times 10^{-7} \text{ C}$$

(b) 
$$2.00 \times 10^{-10} \text{ C}$$

#### **Exercise:**

#### **Problem:**

In one of the classic nuclear physics experiments at the beginning of the 20th century, an alpha particle was accelerated toward a gold nucleus, and its path was substantially deflected by the Coulomb interaction. If the energy of the doubly charged alpha nucleus was 5.00 MeV, how close to the gold nucleus (79 protons) could it come before being deflected?

#### **Exercise:**

#### **Problem:**

(a) What is the potential between two points situated 10 cm and 20 cm from a  $3.0~\mu C$  point charge? (b) To what location should the point at 20 cm be moved to increase this potential difference by a factor of two?

#### **Exercise:**

#### **Problem:** Unreasonable Results

(a) What is the final speed of an electron accelerated from rest through a voltage of 25.0 MV by a negatively charged Van de Graaff terminal?

- (b) What is unreasonable about this result?
- (c) Which assumptions are responsible?

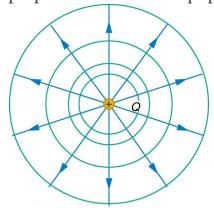
### **Solution:**

- (a)  $2.96 \times 10^9 \mathrm{\ m/s}$
- (b) This velocity is far too great. It is faster than the speed of light.
- (c) The assumption that the speed of the electron is far less than that of light and that the problem does not require a relativistic treatment produces an answer greater than the speed of light.

## **Equipotential Lines**

- Explain equipotential lines and equipotential surfaces.
- Describe the action of grounding an electrical appliance.
- Compare electric field and equipotential lines.

We can represent electric potentials (voltages) pictorially, just as we drew pictures to illustrate electric fields. Of course, the two are related. Consider [link], which shows an isolated positive point charge and its electric field lines. Electric field lines radiate out from a positive charge and terminate on negative charges. While we use blue arrows to represent the magnitude and direction of the electric field, we use green lines to represent places where the electric potential is constant. These are called **equipotential lines** in two dimensions, or *equipotential surfaces* in three dimensions. The term equipotential is also used as a noun, referring to an equipotential line or surface. The potential for a point charge is the same anywhere on an imaginary sphere of radius r surrounding the charge. This is true since the potential for a point charge is given by V = kQ/r and, thus, has the same value at any point that is a given distance r from the charge. An equipotential sphere is a circle in the two-dimensional view of [link]. Since the electric field lines point radially away from the charge, they are perpendicular to the equipotential lines.



An isolated point charge Q with its electric field lines in blue and equipotential lines

in green. The potential is the same along each equipotential line, meaning that no work is required to move a charge anywhere along one of those lines. Work is needed to move a charge from one equipotential line to another. Equipotential lines are perpendicular to electric field lines in every case.

It is important to note that equipotential lines are always perpendicular to electric field lines. No work is required to move a charge along an equipotential, since  $\Delta V=0$ . Thus the work is

### **Equation:**

$$W = -\Delta \ \mathrm{PE} = -q\Delta V = 0.$$

Work is zero if force is perpendicular to motion. Force is in the same direction as **E**, so that motion along an equipotential must be perpendicular to **E**. More precisely, work is related to the electric field by **Equation:** 

$$W = Fd \cos \theta = qEd \cos \theta = 0.$$

Note that in the above equation, E and F symbolize the magnitudes of the electric field strength and force, respectively. Neither q nor  $\mathbf{E}$  nor d is zero, and so  $\cos \theta$  must be 0, meaning  $\theta$  must be  $90^{\circ}$ . In other words, motion along an equipotential is perpendicular to  $\mathbf{E}$ .

One of the rules for static electric fields and conductors is that the electric field must be perpendicular to the surface of any conductor. This implies that a *conductor is an equipotential surface in static situations*. There can be no voltage difference across the surface of a conductor, or charges will flow. One of the uses of this fact is that a conductor can be fixed at zero volts by connecting it to the earth with a good conductor—a process called **grounding**. Grounding can be a useful safety tool. For example, grounding the metal case of an electrical appliance ensures that it is at zero volts relative to the earth.

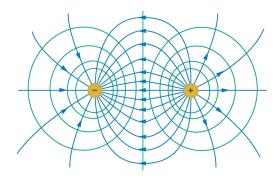
#### Note:

### Grounding

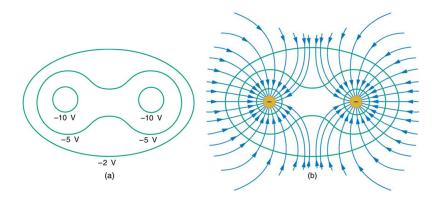
A conductor can be fixed at zero volts by connecting it to the earth with a good conductor—a process called grounding.

Because a conductor is an equipotential, it can replace any equipotential surface. For example, in [link] a charged spherical conductor can replace the point charge, and the electric field and potential surfaces outside of it will be unchanged, confirming the contention that a spherical charge distribution is equivalent to a point charge at its center.

[link] shows the electric field and equipotential lines for two equal and opposite charges. Given the electric field lines, the equipotential lines can be drawn simply by making them perpendicular to the electric field lines. Conversely, given the equipotential lines, as in [link](a), the electric field lines can be drawn by making them perpendicular to the equipotentials, as in [link](b).



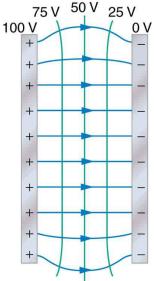
The electric field lines and equipotential lines for two equal but opposite charges. The equipotential lines can be drawn by making them perpendicular to the electric field lines, if those are known. Note that the potential is greatest (most positive) near the positive charge and least (most negative) near the negative charge.



(a) These equipotential lines might be measured with a voltmeter in a laboratory experiment. (b) The corresponding electric field lines are found by drawing them

perpendicular to the equipotentials. Note that these fields are consistent with two equal negative charges.

One of the most important cases is that of the familiar parallel conducting plates shown in [link]. Between the plates, the equipotentials are evenly spaced and parallel. The same field could be maintained by placing conducting plates at the equipotential lines at the potentials shown.



The electric field and equipotential lines between two metal plates.

An important application of electric fields and equipotential lines involves the heart. The heart relies on electrical signals to maintain its rhythm. The movement of electrical signals causes the chambers of the heart to contract and relax. When a person has a heart attack, the movement of these electrical signals may be disturbed. An artificial pacemaker and a defibrillator can be used to initiate the rhythm of electrical signals. The equipotential lines around the heart, the thoracic region, and the axis of the heart are useful ways of monitoring the structure and functions of the heart. An electrocardiogram (ECG) measures the small electric signals being generated during the activity of the heart. More about the relationship between electric fields and the heart is discussed in <a href="Energy Stored in Capacitors">Energy Stored in Capacitors</a>.

#### Note:

PhET Explorations: Charges and Fields

Move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more. It's colorful, it's dynamic, it's free.

https://phet.colorado.edu/sims/html/charges-and-fields/latest/charges-and-fields en.html

# **Section Summary**

- An equipotential line is a line along which the electric potential is constant.
- An equipotential surface is a three-dimensional version of equipotential lines.
- Equipotential lines are always perpendicular to electric field lines.
- The process by which a conductor can be fixed at zero volts by connecting it to the earth with a good conductor is called grounding.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

What is an equipotential line? What is an equipotential surface?

#### **Exercise:**

#### **Problem:**

Explain in your own words why equipotential lines and surfaces must be perpendicular to electric field lines.

#### **Exercise:**

**Problem:**Can different equipotential lines cross? Explain.

### **Problems & Exercises**

#### **Exercise:**

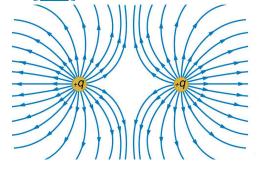
#### **Problem:**

(a) Sketch the equipotential lines near a point charge  $+\ q$ . Indicate the direction of increasing potential. (b) Do the same for a point charge  $-3\ q$ .

#### **Exercise:**

#### **Problem:**

Sketch the equipotential lines for the two equal positive charges shown in [link]. Indicate the direction of increasing potential.



The electric field near two equal positive charges is directed away from each of the charges.

#### **Exercise:**

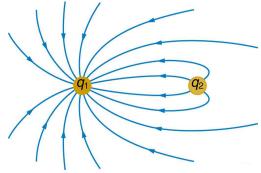
#### **Problem:**

[link] shows the electric field lines near two charges  $q_1$  and  $q_2$ , the first having a magnitude four times that of the second. Sketch the equipotential lines for these two charges, and indicate the direction of increasing potential.

#### **Exercise:**

#### **Problem:**

Sketch the equipotential lines a long distance from the charges shown in [link]. Indicate the direction of increasing potential.



The electric field near two charges.

#### **Exercise:**

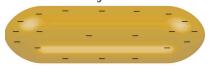
#### Problem:

Sketch the equipotential lines in the vicinity of two opposite charges, where the negative charge is three times as great in magnitude as the positive. See [link] for a similar situation. Indicate the direction of increasing potential.

#### **Exercise:**

#### **Problem:**

Sketch the equipotential lines in the vicinity of the negatively charged conductor in [link]. How will these equipotentials look a long distance from the object?

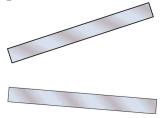


A negatively charged conductor.

#### **Exercise:**

### **Problem:**

Sketch the equipotential lines surrounding the two conducting plates shown in [link], given the top plate is positive and the bottom plate has an equal amount of negative charge. Be certain to indicate the distribution of charge on the plates. Is the field strongest where the plates are closest? Why should it be?



#### **Exercise:**

#### **Problem:**

(a) Sketch the electric field lines in the vicinity of the charged insulator in [link]. Note its non-uniform charge distribution. (b) Sketch equipotential lines surrounding the insulator. Indicate the direction of increasing potential.



A charged insulating rod such as might be used in a classroom demonstration.

#### **Exercise:**

#### **Problem:**

The naturally occurring charge on the ground on a fine day out in the open country is  $-1.00~{\rm nC/m^2}$ . (a) What is the electric field relative to ground at a height of 3.00 m? (b) Calculate the electric potential at this height. (c) Sketch electric field and equipotential lines for this scenario.

#### **Exercise:**

### **Problem:**

The lesser electric ray (*Narcine bancroftii*) maintains an incredible charge on its head and a charge equal in magnitude but opposite in sign on its tail ([link]). (a) Sketch the equipotential lines surrounding the ray. (b) Sketch the equipotentials when the ray is near a ship with a conducting surface. (c) How could this charge distribution be of use to the ray?



Lesser electric ray (*Narcine bancroftii*) (credit: National Oceanic and Atmospheric Administration, NOAA's Fisheries Collection).

# Glossary

equipotential line

a line along which the electric potential is constant

grounding

fixing a conductor at zero volts by connecting it to the earth or ground

# Capacitors and Dielectrics

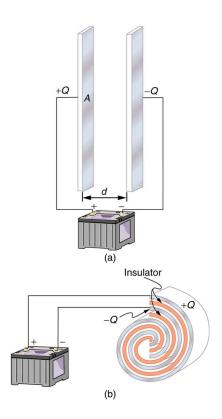
- Describe the action of a capacitor and define capacitance.
- Explain parallel plate capacitors and their capacitances.
- Discuss the process of increasing the capacitance of a dielectric.
- Determine capacitance given charge and voltage.

A **capacitor** is a device used to store electric charge. Capacitors have applications ranging from filtering static out of radio reception to energy storage in heart defibrillators. Typically, commercial capacitors have two conducting parts close to one another, but not touching, such as those in [link]. (Most of the time an insulator is used between the two plates to provide separation—see the discussion on dielectrics below.) When battery terminals are connected to an initially uncharged capacitor, equal amounts of positive and negative charge, +Q and -Q, are separated into its two plates. The capacitor remains neutral overall, but we refer to it as storing a charge Q in this circumstance.

#### Note:

Capacitor

A capacitor is a device used to store electric charge.



Both capacitors shown here were initially uncharged before being connected to a battery. They now have separated charges of +Q and -Q on their two halves. (a) A parallel plate capacitor. (b) A rolled capacitor with an insulating material between its two conducting sheets.

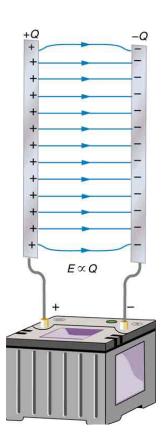
The amount of charge Q a *capacitor* can store depends on two major factors—the voltage applied and the capacitor's physical characteristics, such as its size.

#### Note:

### The Amount of Charge Q a Capacitor Can Store

The amount of charge Q a *capacitor* can store depends on two major factors—the voltage applied and the capacitor's physical characteristics, such as its size.

A system composed of two identical, parallel conducting plates separated by a distance, as in  $[\underline{link}]$ , is called a **parallel plate capacitor**. It is easy to see the relationship between the voltage and the stored charge for a parallel plate capacitor, as shown in  $[\underline{link}]$ . Each electric field line starts on an individual positive charge and ends on a negative one, so that there will be more field lines if there is more charge. (Drawing a single field line per charge is a convenience, only. We can draw many field lines for each charge, but the total number is proportional to the number of charges.) The electric field strength is, thus, directly proportional to Q.



Electric field lines in this parallel plate capacitor, as always, start on positive charges and end on negative charges. Since the electric field strength is proportional to the density of field lines, it is also proportional to the amount of charge on the capacitor.

The field is proportional to the charge:

### **Equation:**

$$E \propto Q$$
,

where the symbol  $\infty$  means "proportional to." From the discussion in Electric Potential in a Uniform Electric Field, we know that the voltage across parallel plates is  $V=\mathrm{Ed}$ . Thus,

### **Equation:**

$$V \propto E$$
.

It follows, then, that  $V \propto Q$ , and conversely,

### **Equation:**

$$Q \propto V$$
.

This is true in general: The greater the voltage applied to any capacitor, the greater the charge stored in it.

Different capacitors will store different amounts of charge for the same applied voltage, depending on their physical characteristics. We define their **capacitance** C to be such that the charge Q stored in a capacitor is proportional to C. The charge stored in a capacitor is given by

# **Equation:**

$$Q = CV$$
.

This equation expresses the two major factors affecting the amount of charge stored. Those factors are the physical characteristics of the capacitor,

C, and the voltage, V. Rearranging the equation, we see that *capacitance* C *is the amount of charge stored per volt*, or

### **Equation:**

$$C = rac{Q}{V}.$$

#### Note:

Capacitance

Capacitance C is the amount of charge stored per volt, or

### **Equation:**

$$C = rac{Q}{V}.$$

The unit of capacitance is the farad (F), named for Michael Faraday (1791–1867), an English scientist who contributed to the fields of electromagnetism and electrochemistry. Since capacitance is charge per unit voltage, we see that a farad is a coulomb per volt, or

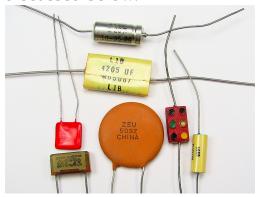
# **Equation:**

$$1 F = \frac{1 C}{1 V}.$$

A 1-farad capacitor would be able to store 1 coulomb (a very large amount of charge) with the application of only 1 volt. One farad is, thus, a very large capacitance. Typical capacitors range from fractions of a picofarad  $\left(1~\mathrm{pF}=10^{-12}~\mathrm{F}\right)$  to millifarads  $\left(1~\mathrm{mF}=10^{-3}~\mathrm{F}\right)$ .

[link] shows some common capacitors. Capacitors are primarily made of ceramic, glass, or plastic, depending upon purpose and size. Insulating

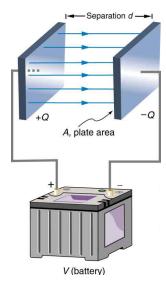
materials, called dielectrics, are commonly used in their construction, as discussed below.



Some typical capacitors.
Size and value of
capacitance are not
necessarily related.
(credit: Windell Oskay)

# **Parallel Plate Capacitor**

The parallel plate capacitor shown in [link] has two identical conducting plates, each having a surface area A, separated by a distance d (with no material between the plates). When a voltage V is applied to the capacitor, it stores a charge Q, as shown. We can see how its capacitance depends on A and d by considering the characteristics of the Coulomb force. We know that like charges repel, unlike charges attract, and the force between charges decreases with distance. So it seems quite reasonable that the bigger the plates are, the more charge they can store—because the charges can spread out more. Thus C should be greater for larger A. Similarly, the closer the plates are together, the greater the attraction of the opposite charges on them. So C should be greater for smaller d.



Parallel plate capacitor with plates separated by a distance d. Each plate has an area A

.

It can be shown that for a parallel plate capacitor there are only two factors (A and d) that affect its capacitance C. The capacitance of a parallel plate capacitor in equation form is given by

# **Equation:**

$$C = \varepsilon_0 rac{A}{d}$$
.

### Note:

Capacitance of a Parallel Plate Capacitor

## **Equation:**

$$C = \varepsilon_0 rac{A}{d}$$

A is the area of one plate in square meters, and d is the distance between the plates in meters. The constant  $\varepsilon_0$  is the permittivity of free space; its numerical value in SI units is  $\varepsilon_0 = 8.85 \times 10^{-12} \, \mathrm{F/m}$ . The units of F/m are equivalent to  $\mathrm{C^2/N \cdot m^2}$ . The small numerical value of  $\varepsilon_0$  is related to the large size of the farad. A parallel plate capacitor must have a large area to have a capacitance approaching a farad. (Note that the above equation is valid when the parallel plates are separated by air or free space. When another material is placed between the plates, the equation is modified, as discussed below.)

### **Example:**

# Capacitance and Charge Stored in a Parallel Plate Capacitor

(a) What is the capacitance of a parallel plate capacitor with metal plates, each of area  $1.00~{\rm m}^2$ , separated by  $1.00~{\rm mm}$ ? (b) What charge is stored in this capacitor if a voltage of  $3.00\times10^3~{\rm V}$  is applied to it?

# Strategy

Finding the capacitance C is a straightforward application of the equation  $C = \varepsilon_0 A/d$ . Once C is found, the charge stored can be found using the equation  $Q = \mathrm{CV}$ .

# Solution for (a)

Entering the given values into the equation for the capacitance of a parallel plate capacitor yields

# **Equation:**

$$egin{array}{lll} C &=& arepsilon_0 rac{A}{d} = \left( 8.85 imes 10^{-12} rac{\mathrm{F}}{\mathrm{m}} 
ight) rac{1.00 \ \mathrm{m}^2}{1.00 imes 10^{-3} \ \mathrm{m}} \ &=& 8.85 imes 10^{-9} \ \mathrm{F} = 8.85 \ \mathrm{nF}. \end{array}$$

### Discussion for (a)

This small value for the capacitance indicates how difficult it is to make a device with a large capacitance. Special techniques help, such as using very large area thin foils placed close together.

### Solution for (b)

The charge stored in any capacitor is given by the equation Q = CV. Entering the known values into this equation gives

## **Equation:**

$$\begin{array}{ll} Q & = & CV = \left(8.85 \times 10^{-9} \; F\right) \left(3.00 \times 10^{3} \; V\right) \\ & = & 26.6 \; \mu C. \end{array}$$

## Discussion for (b)

This charge is only slightly greater than those found in typical static electricity. Since air breaks down at about  $3.00 \times 10^6 \text{ V/m}$ , more charge cannot be stored on this capacitor by increasing the voltage.

Another interesting biological example dealing with electric potential is found in the cell's plasma membrane. The membrane sets a cell off from its surroundings and also allows ions to selectively pass in and out of the cell. There is a potential difference across the membrane of about  $-70~\rm mV$ . This is due to the mainly negatively charged ions in the cell and the predominance of positively charged sodium (Na $^+$ ) ions outside. Things change when a nerve cell is stimulated. Na $^+$  ions are allowed to pass through the membrane into the cell, producing a positive membrane potential—the nerve signal. The cell membrane is about 7 to 10 nm thick. An approximate value of the electric field across it is given by

# **Equation:**

$$E = rac{V}{d} = rac{-70 imes 10^{-3} \; ext{V}}{8 imes 10^{-9} \; ext{m}} = -9 imes 10^6 \; ext{V/m}.$$

This electric field is enough to cause a breakdown in air.

### **Dielectric**

The previous example highlights the difficulty of storing a large amount of charge in capacitors. If d is made smaller to produce a larger capacitance, then the maximum voltage must be reduced proportionally to avoid breakdown (since E=V/d). An important solution to this difficulty is to put an insulating material, called a **dielectric**, between the plates of a capacitor and allow d to be as small as possible. Not only does the smaller d make the capacitance greater, but many insulators can withstand greater electric fields than air before breaking down.

There is another benefit to using a dielectric in a capacitor. Depending on the material used, the capacitance is greater than that given by the equation  $C = \varepsilon_0 \frac{A}{d}$  by a factor  $\kappa$ , called the *dielectric constant*. A parallel plate capacitor with a dielectric between its plates has a capacitance given by **Equation:** 

$$C = \kappa \varepsilon_0 \frac{A}{d}$$
 (parallel plate capacitor with dielectric).

Values of the dielectric constant  $\kappa$  for various materials are given in [link]. Note that  $\kappa$  for vacuum is exactly 1, and so the above equation is valid in that case, too. If a dielectric is used, perhaps by placing Teflon between the plates of the capacitor in [link], then the capacitance is greater by the factor  $\kappa$ , which for Teflon is 2.1.

#### Note:

### Take-Home Experiment: Building a Capacitor

How large a capacitor can you make using a chewing gum wrapper? The plates will be the aluminum foil, and the separation (dielectric) in between will be the paper.

Material	Dielectric constant $\kappa$	Dielectric strength (V/m)
Vacuum	1.00000	_
Air	1.00059	$3 imes10^6$
Bakelite	4.9	$24 imes10^6$
Fused quartz	3.78	$8 imes10^6$
Neoprene rubber	6.7	$12 imes10^6$
Nylon	3.4	$14 imes10^6$
Paper	3.7	$16  imes 10^6$
Polystyrene	2.56	$24 imes10^6$
Pyrex glass	5.6	$14 imes10^6$
Silicon oil	2.5	$15 imes10^6$
Strontium titanate	233	$8 imes10^6$
Teflon	2.1	$60  imes 10^6$
Water	80	_

Dielectric Constants and Dielectric Strengths for Various Materials at 20°C

Note also that the dielectric constant for air is very close to 1, so that air-filled capacitors act much like those with vacuum between their plates *except* that the air can become conductive if the electric field strength

becomes too great. (Recall that E=V/d for a parallel plate capacitor.) Also shown in [link] are maximum electric field strengths in V/m, called **dielectric strengths**, for several materials. These are the fields above which the material begins to break down and conduct. The dielectric strength imposes a limit on the voltage that can be applied for a given plate separation. For instance, in [link], the separation is 1.00 mm, and so the voltage limit for air is

### **Equation:**

$$V = E \cdot d$$
  
=  $(3 \times 10^6 \text{ V/m})(1.00 \times 10^{-3} \text{ m})$   
=  $3000 \text{ V}.$ 

However, the limit for a 1.00 mm separation filled with Teflon is 60,000 V, since the dielectric strength of Teflon is  $60 \times 10^6$  V/m. So the same capacitor filled with Teflon has a greater capacitance and can be subjected to a much greater voltage. Using the capacitance we calculated in the above example for the air-filled parallel plate capacitor, we find that the Teflon-filled capacitor can store a maximum charge of

### **Equation:**

$$egin{array}{lll} Q &=& CV \ &=& \kappa C_{
m air} V \ &=& (2.1)(8.85~{
m nF})(6.0 imes 10^4~{
m V}) \ &=& 1.1~{
m mC}. \end{array}$$

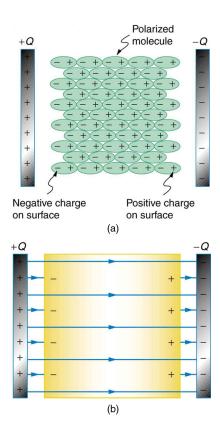
This is 42 times the charge of the same air-filled capacitor.

### Note:

### Dielectric Strength

The maximum electric field strength above which an insulating material begins to break down and conduct is called its dielectric strength.

Microscopically, how does a dielectric increase capacitance? Polarization of the insulator is responsible. The more easily it is polarized, the greater its dielectric constant  $\kappa$ . Water, for example, is a **polar molecule** because one end of the molecule has a slight positive charge and the other end has a slight negative charge. The polarity of water causes it to have a relatively large dielectric constant of 80. The effect of polarization can be best explained in terms of the characteristics of the Coulomb force. [link] shows the separation of charge schematically in the molecules of a dielectric material placed between the charged plates of a capacitor. The Coulomb force between the closest ends of the molecules and the charge on the plates is attractive and very strong, since they are very close together. This attracts more charge onto the plates than if the space were empty and the opposite charges were a distance d away.



(a) The molecules in the insulating material between

the plates of a capacitor are polarized by the charged plates. This produces a layer of opposite charge on the surface of the dielectric that attracts more charge onto the plate, increasing its capacitance. (b) The dielectric reduces the electric field strength inside the capacitor, resulting in a smaller voltage between the plates for the same charge. The capacitor stores the same charge for a smaller voltage, implying that it has a larger capacitance because of the dielectric.

Another way to understand how a dielectric increases capacitance is to consider its effect on the electric field inside the capacitor. [link](b) shows the electric field lines with a dielectric in place. Since the field lines end on charges in the dielectric, there are fewer of them going from one side of the capacitor to the other. So the electric field strength is less than if there were

a vacuum between the plates, even though the same charge is on the plates. The voltage between the plates is  $V=\operatorname{Ed}$ , so it too is reduced by the dielectric. Thus there is a smaller voltage V for the same charge Q; since C=Q/V, the capacitance C is greater.

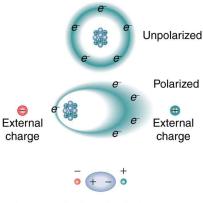
The dielectric constant is generally defined to be  $\kappa = E_0/E$ , or the ratio of the electric field in a vacuum to that in the dielectric material, and is intimately related to the polarizability of the material.

#### Note:

### Things Great and Small

### The Submicroscopic Origin of Polarization

Polarization is a separation of charge within an atom or molecule. As has been noted, the planetary model of the atom pictures it as having a positive nucleus orbited by negative electrons, analogous to the planets orbiting the Sun. Although this model is not completely accurate, it is very helpful in explaining a vast range of phenomena and will be refined elsewhere, such as in <u>Atomic Physics</u>. The submicroscopic origin of polarization can be modeled as shown in [link].



Large-scale view of polarized atom

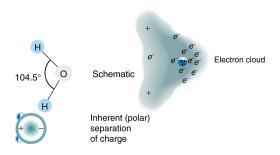
Artist's conception of a polarized atom.
The orbits of

electrons around the nucleus are shifted slightly by the external charges (shown exaggerated). The resulting separation of charge within the atom means that it is polarized. Note that the unlike charge is now closer to the external charges, causing the polarization.

We will find in <u>Atomic Physics</u> that the orbits of electrons are more properly viewed as electron clouds with the density of the cloud related to the probability of finding an electron in that location (as opposed to the definite locations and paths of planets in their orbits around the Sun). This cloud is shifted by the Coulomb force so that the atom on average has a separation of charge. Although the atom remains neutral, it can now be the source of a Coulomb force, since a charge brought near the atom will be closer to one type of charge than the other.

Some molecules, such as those of water, have an inherent separation of charge and are thus called polar molecules. [link] illustrates the separation of charge in a water molecule, which has two hydrogen atoms and one oxygen atom  $(H_2O)$ . The water molecule is not symmetric—the hydrogen atoms are repelled to one side, giving the molecule a boomerang shape. The electrons in a water molecule are more concentrated around the more highly charged oxygen nucleus than around the hydrogen nuclei. This makes the oxygen end of the molecule slightly negative and leaves the hydrogen ends slightly positive. The inherent separation of charge in polar molecules

makes it easier to align them with external fields and charges. Polar molecules therefore exhibit greater polarization effects and have greater dielectric constants. Those who study chemistry will find that the polar nature of water has many effects. For example, water molecules gather ions much more effectively because they have an electric field and a separation of charge to attract charges of both signs. Also, as brought out in the previous chapter, polar water provides a shield or screening of the electric fields in the highly charged molecules of interest in biological systems.



Artist's conception of a water molecule. There is an inherent separation of charge, and so water is a polar molecule. Electrons in the molecule are attracted to the oxygen nucleus and leave an excess of positive charge near the two hydrogen nuclei. (Note that the schematic on the right is a rough illustration of the distribution of electrons in the water molecule. It does not show the actual numbers of protons and electrons involved in the structure.)

### Note:

### PhET Explorations: Capacitor Lab

Explore how a capacitor works! Change the size of the plates and add a dielectric to see the effect on capacitance. Change the voltage and see charges built up on the plates. Observe the electric field in the capacitor. Measure the voltage and the electric field.

<u>Capacito</u> r <u>Lab</u>

# **Section Summary**

- A capacitor is a device used to store charge.
- The amount of charge *Q* a capacitor can store depends on two major factors—the voltage applied and the capacitor's physical characteristics, such as its size.
- The capacitance *C* is the amount of charge stored per volt, or **Equation:**

$$C = \frac{Q}{V}.$$

- The capacitance of a parallel plate capacitor is  $C=\varepsilon_0\,\frac{A}{d}$ , when the plates are separated by air or free space.  $\varepsilon_0$  is called the permittivity of free space.
- A parallel plate capacitor with a dielectric between its plates has a capacitance given by

# **Equation:**

$$C = \kappa \varepsilon_0 \frac{A}{d}$$

where  $\kappa$  is the dielectric constant of the material.

• The maximum electric field strength above which an insulating material begins to break down and conduct is called dielectric strength.

# **Conceptual Questions**

### **Exercise:**

### **Problem:**

Does the capacitance of a device depend on the applied voltage? What about the charge stored in it?

### **Exercise:**

### **Problem:**

Use the characteristics of the Coulomb force to explain why capacitance should be proportional to the plate area of a capacitor. Similarly, explain why capacitance should be inversely proportional to the separation between plates.

#### **Exercise:**

#### **Problem:**

Give the reason why a dielectric material increases capacitance compared with what it would be with air between the plates of a capacitor. What is the independent reason that a dielectric material also allows a greater voltage to be applied to a capacitor? (The dielectric thus increases C and permits a greater V.)

#### **Exercise:**

#### **Problem:**

How does the polar character of water molecules help to explain water's relatively large dielectric constant? ([link])

### **Exercise:**

#### **Problem:**

Sparks will occur between the plates of an air-filled capacitor at lower voltage when the air is humid than when dry. Explain why, considering the polar character of water molecules.

### **Exercise:**

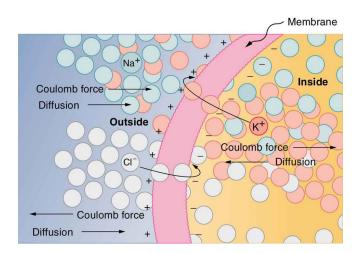
### **Problem:**

Water has a large dielectric constant, but it is rarely used in capacitors. Explain why.

### **Exercise:**

### **Problem:**

Membranes in living cells, including those in humans, are characterized by a separation of charge across the membrane. Effectively, the membranes are thus charged capacitors with important functions related to the potential difference across the membrane. Is energy required to separate these charges in living membranes and, if so, is its source the metabolization of food energy or some other source?



The semipermeable membrane of a cell has different concentrations of ions inside and out. Diffusion moves the K<sup>+</sup> (potassium) and Cl<sup>-</sup> (chloride) ions in the directions shown, until the Coulomb force halts further transfer. This results in a layer of positive charge on the outside, a layer of negative charge on the inside, and thus a voltage across the cell membrane. The membrane is normally impermeable to Na<sup>+</sup> (sodium ions).

### **Problems & Exercises**

### **Exercise:**

### **Problem:**

What charge is stored in a  $180~\mu F$  capacitor when 120~V is applied to it?

### **Solution:**

21.6 mC

#### **Exercise:**

#### **Problem:**

Find the charge stored when 5.50 V is applied to an 8.00 pF capacitor.

#### **Exercise:**

**Problem:** What charge is stored in the capacitor in [link]?

Solution:	
$80.0~\mathrm{mC}$	
Exercise:	
Problem:	
Calculate the voltage applied to a $2.00~\mu F$ capacitor when it h $3.10~\mu C$ of charge.	olds
Exercise:	
Problem:	
What voltage must be applied to an 8.00 nF capacitor to store mC of charge?	0.160
Solution:	
$20.0~\mathrm{kV}$	
Exercise:	
Problem:	
What capacitance is needed to store 3.00 $\mu C$ of charge at a vo 120 V?	ltage of
Exercise:	
Problem:	
What is the capacitance of a large Van de Graaff generator's tegiven that it stores 8.00 mC of charge at a voltage of 12.0 MV	
Solution:	
$667~\mathrm{pF}$	
Exercise:	

Find the capacitance of a parallel plate capacitor having plates of area  $5.00 \text{ m}^2$  that are separated by 0.100 mm of Teflon.

#### **Exercise:**

### **Problem:**

(a)What is the capacitance of a parallel plate capacitor having plates of area  $1.50~\rm m^2$  that are separated by 0.0200 mm of neoprene rubber? (b) What charge does it hold when 9.00 V is applied to it?

### **Solution:**

- (a)  $4.4 \mu F$
- (b)  $4.0 \times 10^{-5} \text{ C}$

### **Exercise:**

# **Problem:** Integrated Concepts

A prankster applies 450 V to an  $80.0~\mu F$  capacitor and then tosses it to an unsuspecting victim. The victim's finger is burned by the discharge of the capacitor through 0.200~g of flesh. What is the temperature increase of the flesh? Is it reasonable to assume no phase change?

### **Exercise:**

#### **Problem:** Unreasonable Results

(a) A certain parallel plate capacitor has plates of area  $4.00~\mathrm{m}^2$ , separated by  $0.0100~\mathrm{mm}$  of nylon, and stores  $0.170~\mathrm{C}$  of charge. What is the applied voltage? (b) What is unreasonable about this result? (c) Which assumptions are responsible or inconsistent?

#### **Solution:**

- (a) 14.2 kV
- (b) The voltage is unreasonably large, more than 100 times the breakdown voltage of nylon.
- (c) The assumed charge is unreasonably large and cannot be stored in a capacitor of these dimensions.

# **Glossary**

### capacitor

a device that stores electric charge

## capacitance

amount of charge stored per unit volt

### dielectric

an insulating material

# dielectric strength

the maximum electric field above which an insulating material begins to break down and conduct

# parallel plate capacitor

two identical conducting plates separated by a distance

# polar molecule

a molecule with inherent separation of charge

# Introduction to Electric Current, Resistance, and Ohm's Law class="introduction"

Electric energy in massive quantities is transmitted from this hydroelectri c facility, the Srisailam power station located along the Krishna River in India, by the movement of charge that is, by electric current. (credit: Chintohere, Wikimedia Commons)



The flicker of numbers on a handheld calculator, nerve impulses carrying signals of vision to the brain, an ultrasound device sending a signal to a computer screen, the brain sending a message for a baby to twitch its toes, an electric train pulling its load over a mountain pass, a hydroelectric plant sending energy to metropolitan and rural users—these and many other examples of electricity involve *electric current*, the movement of charge. Humankind has indeed harnessed electricity, the basis of technology, to improve our quality of life. Whereas the previous two chapters concentrated on static electricity and the fundamental force underlying its behavior, the next few chapters will be devoted to electric and magnetic phenomena involving current. In addition to exploring applications of electricity, we shall gain new insights into nature—in particular, the fact that all magnetism results from electric current.

#### Current

- Define electric current, ampere, and drift velocity
- Describe the direction of charge flow in conventional current.
- Use drift velocity to calculate current and vice versa.

### **Electric Current**

Electric current is defined to be the rate at which charge flows. A large current, such as that used to start a truck engine, moves a large amount of charge in a small time, whereas a small current, such as that used to operate a hand-held calculator, moves a small amount of charge over a long period of time. In equation form, **electric current** I is defined to be

### **Equation:**

$$I=rac{\Delta Q}{\Delta t},$$

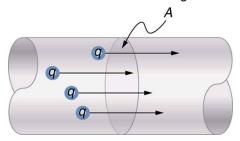
where  $\Delta Q$  is the amount of charge passing through a given area in time  $\Delta t$ . (As in previous chapters, initial time is often taken to be zero, in which case  $\Delta t = t$ .) (See [link].) The SI unit for current is the **ampere** (A), named for the French physicist André-Marie Ampère (1775–1836). Since  $I = \Delta Q/\Delta t$ , we see that an ampere is one coulomb per second:

# **Equation:**

$$1 A = 1 C/s$$

Not only are fuses and circuit breakers rated in amperes (or amps), so are many electrical appliances.

Current = flow of charge



The rate of flow of charge is current. An ampere is the flow of one coulomb through an area in one second.

### **Example:**

# Calculating Currents: Current in a Truck Battery and a Handheld Calculator

(a) What is the current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine? (b) How long does it take 1.00 C of charge to flow through a handheld calculator if a 0.300-mA current is flowing?

# Strategy

We can use the definition of current in the equation  $I = \Delta Q/\Delta t$  to find the current in part (a), since charge and time are given. In part (b), we rearrange the definition of current and use the given values of charge and current to find the time required.

### Solution for (a)

Entering the given values for charge and time into the definition of current gives

# **Equation:**

$$I = \frac{\Delta Q}{\Delta t} = \frac{720 \text{ C}}{4.00 \text{ s}} = 180 \text{ C/s}$$
  
= 180 A.

## Discussion for (a)

This large value for current illustrates the fact that a large charge is moved in a small amount of time. The currents in these "starter motors" are fairly large because large frictional forces need to be overcome when setting something in motion.

# Solution for (b)

Solving the relationship  $I = \Delta Q/\Delta t$  for time  $\Delta t$ , and entering the known values for charge and current gives

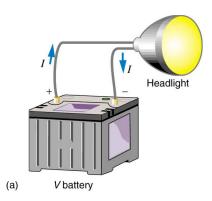
## **Equation:**

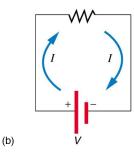
$$\Delta t = \frac{\Delta Q}{I} = \frac{1.00 \text{ C}}{0.300 \times 10^{-3} \text{ C/s}}$$
  
= 3.33×10<sup>3</sup> s.

### **Discussion for (b)**

This time is slightly less than an hour. The small current used by the handheld calculator takes a much longer time to move a smaller charge than the large current of the truck starter. So why can we operate our calculators only seconds after turning them on? It's because calculators require very little energy. Such small current and energy demands allow handheld calculators to operate from solar cells or to get many hours of use out of small batteries. Remember, calculators do not have moving parts in the same way that a truck engine has with cylinders and pistons, so the technology requires smaller currents.

[link] shows a simple circuit and the standard schematic representation of a battery, conducting path, and load (a resistor). Schematics are very useful in visualizing the main features of a circuit. A single schematic can represent a wide variety of situations. The schematic in [link] (b), for example, can represent anything from a truck battery connected to a headlight lighting the street in front of the truck to a small battery connected to a penlight lighting a keyhole in a door. Such schematics are useful because the analysis is the same for a wide variety of situations. We need to understand a few schematics to apply the concepts and analysis to many more situations.





(a) A simple electric circuit. A closed path for current to flow through is supplied by conducting wires connecting a load to the terminals of a battery. (b) In this schematic, the battery is represented by the two parallel red lines, conducting wires are shown as straight lines, and the zigzag represents the load. The schematic represents a wide

variety of similar circuits.

Note that the direction of current flow in [link] is from positive to negative. *The direction of conventional current is the direction that positive charge would flow.* Depending on the situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move. This is also true in nerve cells. A Van de Graaff generator used for nuclear research can produce a current of pure positive charges, such as protons. [link] illustrates the movement of charged particles that compose a current. The fact that conventional current is taken to be in the direction that positive charge would flow can be traced back to American politician and scientist Benjamin Franklin in the 1700s. He named the type of charge associated with electrons negative, long before they were known to carry current in so many situations. Franklin, in fact, was totally unaware of the small-scale structure of electricity.

It is important to realize that there is an electric field in conductors responsible for producing the current, as illustrated in [link]. Unlike static electricity, where a conductor in equilibrium cannot have an electric field in it, conductors carrying a current have an electric field and are not in static equilibrium. An electric field is needed to supply energy to move the charges.

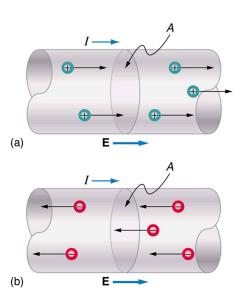
#### Note:

Making Connections: Take-Home Investigation—Electric Current Illustration

Find a straw and little peas that can move freely in the straw. Place the straw flat on a table and fill the straw with peas. When you pop one pea in at one end, a different pea should pop out the other end. This demonstration is an analogy for an electric current. Identify what compares

to the electrons and what compares to the supply of energy. What other analogies can you find for an electric current?

Note that the flow of peas is based on the peas physically bumping into each other; electrons flow due to mutually repulsive electrostatic forces.



Current *I* is the rate at which charge moves through an area A, such as the crosssection of a wire. Conventional current is defined to move in the direction of the electric field. (a) Positive charges move in the direction of the electric field and the same direction as conventional current. (b) Negative charges move in the direction opposite to the electric field. Conventional

current is in the direction opposite to the movement of negative charge. The flow of electrons is sometimes referred to as electronic flow.

### **Example:**

### Calculating the Number of Electrons that Move through a Calculator

If the 0.300-mA current through the calculator mentioned in the [link] example is carried by electrons, how many electrons per second pass through it?

### Strategy

The current calculated in the previous example was defined for the flow of positive charge. For electrons, the magnitude is the same, but the sign is opposite,  $I_{\rm electrons} = -0.300 \times 10^{-3} \, {\rm C/s}$  . Since each electron  $(e^-)$  has a charge of  $-1.60 \times 10^{-19}$  C, we can convert the current in coulombs per second to electrons per second.

#### Solution

Starting with the definition of current, we have

# **Equation:**

$$I_{
m electrons} = rac{\Delta Q_{
m electrons}}{\Delta t} = rac{-0.300 imes 10^{-3} {
m \ C}}{
m s}.$$

We divide this by the charge per electron, so that

# **Equation:**

$$\begin{array}{rcl} \frac{e^{-}}{s} & = & \frac{-0.300 \times 10^{-3} \text{ C}}{s} \times \frac{1 e^{-}}{-1.60 \times 10^{-19} \text{ C}} \\ & = & 1.88 \times 10^{15} \frac{e^{-}}{s}. \end{array}$$

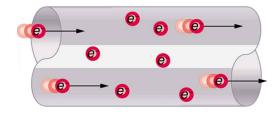
### Discussion

There are so many charged particles moving, even in small currents, that individual charges are not noticed, just as individual water molecules are not noticed in water flow. Even more amazing is that they do not always keep moving forward like soldiers in a parade. Rather they are like a crowd of people with movement in different directions but a general trend to move forward. There are lots of collisions with atoms in the metal wire and, of course, with other electrons.

# **Drift Velocity**

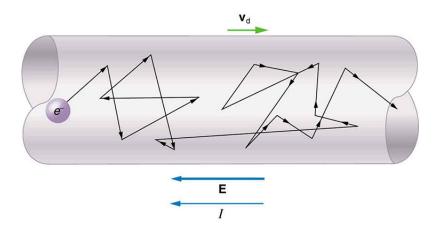
Electrical signals are known to move very rapidly. Telephone conversations carried by currents in wires cover large distances without noticeable delays. Lights come on as soon as a switch is flicked. Most electrical signals carried by currents travel at speeds on the order of  $10^8$  m/s, a significant fraction of the speed of light. Interestingly, the individual charges that make up the current move *much* more slowly on average, typically drifting at speeds on the order of  $10^{-4}$  m/s. How do we reconcile these two speeds, and what does it tell us about standard conductors?

The high speed of electrical signals results from the fact that the force between charges acts rapidly at a distance. Thus, when a free charge is forced into a wire, as in [link], the incoming charge pushes other charges ahead of it, which in turn push on charges farther down the line. The density of charge in a system cannot easily be increased, and so the signal is passed on rapidly. The resulting electrical shock wave moves through the system at nearly the speed of light. To be precise, this rapidly moving signal or shock wave is a rapidly propagating change in electric field.



When charged particles are forced into this volume of a conductor, an equal number are quickly forced to leave. The repulsion between like charges makes it difficult to increase the number of charges in a volume. Thus, as one charge enters, another leaves almost immediately, carrying the signal rapidly forward.

Good conductors have large numbers of free charges in them. In metals, the free charges are free electrons. [link] shows how free electrons move through an ordinary conductor. The distance that an individual electron can move between collisions with atoms or other electrons is quite small. The electron paths thus appear nearly random, like the motion of atoms in a gas. But there is an electric field in the conductor that causes the electrons to drift in the direction shown (opposite to the field, since they are negative). The **drift velocity**  $v_{\rm d}$  is the average velocity of the free charges. Drift velocity is quite small, since there are so many free charges. If we have an estimate of the density of free electrons in a conductor, we can calculate the drift velocity for a given current. The larger the density, the lower the velocity required for a given current.



Free electrons moving in a conductor make many collisions with other electrons and atoms. The path of one electron is shown. The average velocity of the free charges is called the drift velocity,  $v_{\rm d}$ , and it is in the direction opposite to the electric field for electrons. The collisions normally transfer energy to the conductor, requiring a constant supply of energy to maintain a steady current.

### Note:

### Conduction of Electricity and Heat

Good electrical conductors are often good heat conductors, too. This is because large numbers of free electrons can carry electrical current and can transport thermal energy.

The free-electron collisions transfer energy to the atoms of the conductor. The electric field does work in moving the electrons through a distance, but that work does not increase the kinetic energy (nor speed, therefore) of the electrons. The work is transferred to the conductor's atoms, possibly

increasing temperature. Thus a continuous power input is required to keep a current flowing. An exception, of course, is found in superconductors, for reasons we shall explore in a later chapter. Superconductors can have a steady current without a continual supply of energy—a great energy savings. In contrast, the supply of energy can be useful, such as in a lightbulb filament. The supply of energy is necessary to increase the temperature of the tungsten filament, so that the filament glows.

#### Note:

Making Connections: Take-Home Investigation—Filament Observations Find a lightbulb with a filament. Look carefully at the filament and describe its structure. To what points is the filament connected?

We can obtain an expression for the relationship between current and drift velocity by considering the number of free charges in a segment of wire, as illustrated in [link]. The number of free charges per unit volume is given the symbol n and depends on the material. The shaded segment has a volume Ax, so that the number of free charges in it is nAx. The charge  $\Delta Q$  in this segment is thus qnAx, where q is the amount of charge on each carrier. (Recall that for electrons, q is  $-1.60 \times 10^{-19}$  C.) Current is charge moved per unit time; thus, if all the original charges move out of this segment in time  $\Delta t$ , the current is

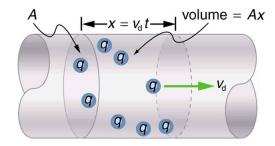
# **Equation:**

$$I = rac{\Delta Q}{\Delta t} = rac{ ext{qnAx}}{\Delta t}.$$

Note that  $x/\Delta t$  is the magnitude of the drift velocity,  $v_{\rm d}$ , since the charges move an average distance x in a time  $\Delta t$ . Rearranging terms gives **Equation:** 

$$I = \text{nqAv}_{d},$$

where I is the current through a wire of cross-sectional area A made of a material with a free charge density n. The carriers of the current each have charge q and move with a drift velocity of magnitude  $v_{\rm d}$ .



All the charges in the shaded volume of this wire move out in a time t, having a drift velocity of magnitude  $v_{\rm d}=x/t$ . See text for further discussion.

Note that simple drift velocity is not the entire story. The speed of an electron is much greater than its drift velocity. In addition, not all of the electrons in a conductor can move freely, and those that do might move somewhat faster or slower than the drift velocity. So what do we mean by free electrons? Atoms in a metallic conductor are packed in the form of a lattice structure. Some electrons are far enough away from the atomic nuclei that they do not experience the attraction of the nuclei as much as the inner electrons do. These are the free electrons. They are not bound to a single atom but can instead move freely among the atoms in a "sea" of electrons. These free electrons respond by accelerating when an electric field is applied. Of course as they move they collide with the atoms in the lattice and other electrons, generating thermal energy, and the conductor gets warmer. In an insulator, the organization of the atoms and the structure do not allow for such free electrons.

### **Example:**

# **Calculating Drift Velocity in a Common Wire**

Calculate the drift velocity of electrons in a 12-gauge copper wire (which has a diameter of 2.053 mm) carrying a 20.0-A current, given that there is one free electron per copper atom. (Household wiring often contains 12-gauge copper wire, and the maximum current allowed in such wire is usually 20 A.) The density of copper is  $8.80 \times 10^3 \text{ kg/m}^3$ .

### Strategy

We can calculate the drift velocity using the equation  $I = nqAv_{\rm d}$ . The current I = 20.0 A is given, and  $q = -1.60 \times 10^{-19} {\rm C}$  is the charge of an electron. We can calculate the area of a cross-section of the wire using the formula  $A = \pi r^2$ , where r is one-half the given diameter, 2.053 mm. We are given the density of copper,  $8.80 \times 10^3 {\rm ~kg/m^3}$ , and the periodic table shows that the atomic mass of copper is  $63.54 {\rm ~g/mol}$ . We can use these two quantities along with Avogadro's number,  $6.02 \times 10^{23} {\rm ~atoms/mol}$ , to determine n, the number of free electrons per cubic meter.

#### Solution

First, calculate the density of free electrons in copper. There is one free electron per copper atom. Therefore, is the same as the number of copper atoms per  $m^3$ . We can now find n as follows:

### **Equation:**

$$egin{array}{lll} n & = & rac{1 \ e^-}{
m atom} imes rac{6.02 imes 10^{23} \ 
m atoms}{
m mol} imes rac{1 \ 
m mol}{63.54 \ 
m g} imes rac{1000 \ 
m g}{
m kg} imes rac{8.80 imes 10^3 \ 
m kg}{1 \ 
m m^3} \ & = & 8.342 imes 10^{28} \ e^-/
m m^3. \end{array}$$

The cross-sectional area of the wire is

# **Equation:**

$$egin{array}{lcl} A & = & \pi r^2 \ & = & \pi \Big( rac{2.053 imes 10^{-3} \, \mathrm{m}}{2} \Big)^2 \ & = & 3.310 imes 10^{-6} \, \mathrm{m}^2. \end{array}$$

Rearranging  $I=nqAv_{
m d}$  to isolate drift velocity gives

### **Equation:**

$$egin{aligned} v_{
m d} &= rac{I}{nqA} \ &= rac{20.0 \ 
m A}{(8.342 imes 10^{28}/
m m^3)(-1.60 imes 10^{-19} \ 
m C)(3.310 imes 10^{-6} \ 
m m^2)} \ &= -4.53 imes 10^{-4} \ 
m m/s. \end{aligned}$$

### Discussion

The minus sign indicates that the negative charges are moving in the direction opposite to conventional current. The small value for drift velocity (on the order of  $10^{-4}$  m/s) confirms that the signal moves on the order of  $10^{12}$  times faster (about  $10^8$  m/s) than the charges that carry it.

# **Section Summary**

Electric current *I* is the rate at which charge flows, given by
 Equation:

$$I = \frac{\Delta Q}{\Delta t},$$

where  $\Delta Q$  is the amount of charge passing through an area in time  $\Delta t$ .

- The direction of conventional current is taken as the direction in which positive charge moves.
- The SI unit for current is the ampere (A), where 1 A = 1 C/s.
- Current is the flow of free charges, such as electrons and ions.
- Drift velocity  $v_{\rm d}$  is the average speed at which these charges move.
- Current I is proportional to drift velocity  $v_{\rm d}$ , as expressed in the relationship  $I={\rm nqAv_d}$ . Here, I is the current through a wire of cross-sectional area A. The wire's material has a free-charge density n, and each carrier has charge q and a drift velocity  $v_{\rm d}$ .
- Electrical signals travel at speeds about  $10^{12}$  times greater than the drift velocity of free electrons.

# **Conceptual Questions**

Can a wire carry a current and still be neutral—that is, have a total charge of zero? Explain.

#### **Exercise:**

#### **Problem:**

Car batteries are rated in ampere-hours  $(A \cdot h)$ . To what physical quantity do ampere-hours correspond (voltage, charge, . . .), and what relationship do ampere-hours have to energy content?

#### **Exercise:**

### **Problem:**

If two different wires having identical cross-sectional areas carry the same current, will the drift velocity be higher or lower in the better conductor? Explain in terms of the equation  $v_{\rm d}=\frac{I}{\rm nqA}$ , by considering how the density of charge carriers n relates to whether or not a material is a good conductor.

### **Exercise:**

#### **Problem:**

Why are two conducting paths from a voltage source to an electrical device needed to operate the device?

#### **Exercise:**

#### **Problem:**

In cars, one battery terminal is connected to the metal body. How does this allow a single wire to supply current to electrical devices rather than two wires?

Why isn't a bird sitting on a high-voltage power line electrocuted? Contrast this with the situation in which a large bird hits two wires simultaneously with its wings.

### **Problems & Exercises**

### **Exercise:**

### **Problem:**

What is the current in milliamperes produced by the solar cells of a pocket calculator through which 4.00 C of charge passes in 4.00 h?

### **Solution:**

 $0.278 \, \text{mA}$ 

#### **Exercise:**

### **Problem:**

A total of 600 C of charge passes through a flashlight in 0.500 h. What is the average current?

#### **Exercise:**

#### **Problem:**

What is the current when a typical static charge of  $0.250~\mu\mathrm{C}$  moves from your finger to a metal doorknob in  $1.00~\mu\mathrm{s}$ ?

### **Solution:**

0.250 A

Find the current when 2.00 nC jumps between your comb and hair over a 0.500 -  $\mu s$  time interval.

#### **Exercise:**

#### **Problem:**

A large lightning bolt had a 20,000-A current and moved 30.0 C of charge. What was its duration?

### **Solution:**

1.50ms

### **Exercise:**

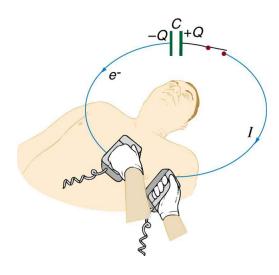
#### **Problem:**

The 200-A current through a spark plug moves 0.300 mC of charge. How long does the spark last?

#### **Exercise:**

#### **Problem:**

(a) A defibrillator sends a 6.00-A current through the chest of a patient by applying a 10,000-V potential as in the figure below. What is the resistance of the path? (b) The defibrillator paddles make contact with the patient through a conducting gel that greatly reduces the path resistance. Discuss the difficulties that would ensue if a larger voltage were used to produce the same current through the patient, but with the path having perhaps 50 times the resistance. (Hint: The current must be about the same, so a higher voltage would imply greater power. Use this equation for power:  $P = I^2 R$ .)



The capacitor in a defibrillation unit drives a current through the heart of a patient.

### **Solution:**

(a)  $1.67 \mathrm{k}\Omega$ 

(b) If a 50 times larger resistance existed, keeping the current about the same, the power would be increased by a factor of about 50 (based on the equation  $P=I^2R$ ), causing much more energy to be transferred to the skin, which could cause serious burns. The gel used reduces the resistance, and therefore reduces the power transferred to the skin.

### **Exercise:**

### **Problem:**

During open-heart surgery, a defibrillator can be used to bring a patient out of cardiac arrest. The resistance of the path is  $500~\Omega$  and a 10.0-mA current is needed. What voltage should be applied?

(a) A defibrillator passes 12.0 A of current through the torso of a person for 0.0100 s. How much charge moves? (b) How many electrons pass through the wires connected to the patient? (See figure two problems earlier.)

### **Solution:**

- (a) 0.120 C
- (b)  $7.50 \times 10^{17}$  electrons

### **Exercise:**

### **Problem:**

A clock battery wears out after moving 10,000 C of charge through the clock at a rate of 0.500 mA. (a) How long did the clock run? (b) How many electrons per second flowed?

### **Exercise:**

#### **Problem:**

The batteries of a submerged non-nuclear submarine supply 1000 A at full speed ahead. How long does it take to move Avogadro's number (  $6.02 \times 10^{23}$ ) of electrons at this rate?

### **Solution:**

96.3 s

Electron guns are used in X-ray tubes. The electrons are accelerated through a relatively large voltage and directed onto a metal target, producing X-rays. (a) How many electrons per second strike the target if the current is 0.500 mA? (b) What charge strikes the target in 0.750 s?

#### **Exercise:**

#### **Problem:**

A large cyclotron directs a beam of  $\mathrm{He^{++}}$  nuclei onto a target with a beam current of 0.250 mA. (a) How many  $\mathrm{He^{++}}$  nuclei per second is this? (b) How long does it take for 1.00 C to strike the target? (c) How long before 1.00 mol of  $\mathrm{He^{++}}$  nuclei strike the target?

#### **Solution:**

(a) 
$$7.81 \times 10^{14}~\mathrm{He^{++}}~\mathrm{nuclei/s}$$

(b) 
$$4.00 \times 10^3$$
 s

(c) 
$$7.71 \times 10^8 \text{ s}$$

#### **Exercise:**

#### **Problem:**

Repeat the above example on [link], but for a wire made of silver and given there is one free electron per silver atom.

#### **Exercise:**

#### **Problem:**

Using the results of the above example on [link], find the drift velocity in a copper wire of twice the diameter and carrying 20.0 A.

#### **Solution:**

$$-1.13 \times 10^{-4} \text{m/s}$$

#### **Exercise:**

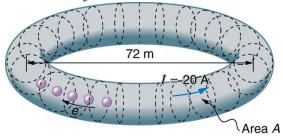
#### **Problem:**

A 14-gauge copper wire has a diameter of 1.628 mm. What magnitude current flows when the drift velocity is 1.00 mm/s? (See above example on [link] for useful information.)

### **Exercise:**

#### **Problem:**

SPEAR, a storage ring about 72.0 m in diameter at the Stanford Linear Accelerator (closed in 2009), has a 20.0-A circulating beam of electrons that are moving at nearly the speed of light. (See [link].) How many electrons are in the beam?



Electrons circulating in the storage ring called SPEAR constitute a 20.0-A current. Because they travel close to the speed of light, each electron completes many orbits in each second.

#### **Solution:**

 $9.42 \times 10^{13}$  electrons

# Glossary

electric current

the rate at which charge flows,  $I = \Delta Q/\Delta t$ 

ampere

(amp) the SI unit for current; 1 A = 1 C/s

drift velocity

the average velocity at which free charges flow in response to an electric field

### Ohm's Law: Resistance and Simple Circuits

- Explain the origin of Ohm's law.
- Calculate voltages, currents, or resistances with Ohm's law.
- Explain what an ohmic material is.
- Describe a simple circuit.

What drives current? We can think of various devices—such as batteries, generators, wall outlets, and so on—which are necessary to maintain a current. All such devices create a potential difference and are loosely referred to as voltage sources. When a voltage source is connected to a conductor, it applies a potential difference V that creates an electric field. The electric field in turn exerts force on charges, causing current.

#### Ohm's Law

The current that flows through most substances is directly proportional to the voltage V applied to it. The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is *directly proportional to the voltage applied*:

### **Equation:**

$$I \propto V$$
.

This important relationship is known as **Ohm's law**. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect. This is an empirical law like that for friction—an experimentally observed phenomenon. Such a linear relationship doesn't always occur.

# **Resistance and Simple Circuits**

If voltage drives current, what impedes it? The electric property that impedes current (crudely similar to friction and air resistance) is called **resistance** R. Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current. Resistance is defined as inversely proportional to current, or

### **Equation:**

$$I \propto \frac{1}{R}$$
.

Thus, for example, current is cut in half if resistance doubles. Combining the relationships of current to voltage and current to resistance gives **Equation:** 

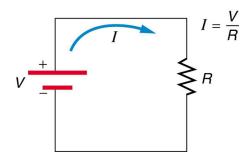
$$I = \frac{V}{R}$$
.

This relationship is also called Ohm's law. Ohm's law in this form really defines resistance for certain materials. Ohm's law (like Hooke's law) is not universally valid. The many substances for which Ohm's law holds are called **ohmic**. These include good conductors like copper and aluminum, and some poor conductors under certain circumstances. Ohmic materials have a resistance R that is independent of voltage V and current I. An object that has simple resistance is called a *resistor*, even if its resistance is small. The unit for resistance is an **ohm** and is given the symbol  $\Omega$  (upper case Greek omega). Rearranging I = V/R gives R = V/I, and so the units of resistance are 1 ohm = 1 volt per ampere:

# **Equation:**

$$1~\Omega=1rac{V}{A}.$$

[ $\underline{\text{link}}$ ] shows the schematic for a simple circuit. A **simple circuit** has a single voltage source and a single resistor. The wires connecting the voltage source to the resistor can be assumed to have negligible resistance, or their resistance can be included in R.



A simple electric circuit in which a closed path for current to flow is supplied by conductors (usually metal wires) connecting a load to the terminals of a battery, represented by the red parallel lines. The zigzag symbol represents the single resistor and includes any resistance in the connections to the voltage source.

# **Example:**

### Calculating Resistance: An Automobile Headlight

What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?

# Strategy

We can rearrange Ohm's law as stated by  $I=\mathrm{V/R}$  and use it to find the resistance.

### **Solution**

Rearranging I = V/R and substituting known values gives

### **Equation:**

$$R = rac{V}{I} = rac{12.0 \ ext{V}}{2.50 \ ext{A}} = 4.80 \ \Omega.$$

#### Discussion

This is a relatively small resistance, but it is larger than the cold resistance of the headlight. As we shall see in <u>Resistance and Resistivity</u>, resistance usually increases with temperature, and so the bulb has a lower resistance when it is first switched on and will draw considerably more current during its brief warm-up period.

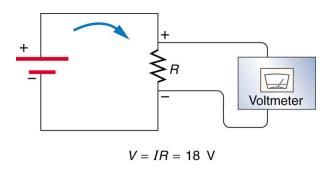
Resistances range over many orders of magnitude. Some ceramic insulators, such as those used to support power lines, have resistances of  $10^{12}~\Omega$  or more. A dry person may have a hand-to-foot resistance of  $10^{5}~\Omega$ , whereas the resistance of the human heart is about  $10^{3}~\Omega$ . A meter-long piece of large-diameter copper wire may have a resistance of  $10^{-5}~\Omega$ , and superconductors have no resistance at all (they are non-ohmic). Resistance is related to the shape of an object and the material of which it is composed, as will be seen in Resistance and Resistivity.

Additional insight is gained by solving I = V/R for V, yielding **Equation:** 

$$V = IR.$$

This expression for V can be interpreted as the *voltage drop across a* resistor produced by the flow of current I. The phrase IR drop is often used for this voltage. For instance, the headlight in [link] has an IR drop of 12.0 V. If voltage is measured at various points in a circuit, it will be seen to increase at the voltage source and decrease at the resistor. Voltage is similar to fluid pressure. The voltage source is like a pump, creating a pressure difference, causing current—the flow of charge. The resistor is like a pipe that reduces pressure and limits flow because of its resistance. Conservation of energy has important consequences here. The voltage source supplies

energy (causing an electric field and a current), and the resistor converts it to another form (such as thermal energy). In a simple circuit (one with a single simple resistor), the voltage supplied by the source equals the voltage drop across the resistor, since  $PE = q\Delta V$ , and the same q flows through each. Thus the energy supplied by the voltage source and the energy converted by the resistor are equal. (See [link].)



The voltage drop across a resistor in a simple circuit equals the voltage output of the battery.

### Note:

### Making Connections: Conservation of Energy

In a simple electrical circuit, the sole resistor converts energy supplied by the source into another form. Conservation of energy is evidenced here by the fact that all of the energy supplied by the source is converted to another form by the resistor alone. We will find that conservation of energy has other important applications in circuits and is a powerful tool in circuit analysis.

#### Note:

### PhET Explorations: Ohm's Law

See how the equation form of Ohm's law relates to a simple circuit. Adjust the voltage and resistance, and see the current change according to Ohm's law. The sizes of the symbols in the equation change to match the circuit diagram.

https://phet.colorado.edu/sims/html/ohms-law/latest/ohms-law\_en.html

# **Section Summary**

- A simple circuit *is* one in which there is a single voltage source and a single resistance.
- One statement of Ohm's law gives the relationship between current I, voltage V, and resistance R in a simple circuit to be  $I = \frac{V}{R}$ .
- Resistance has units of ohms ( $\Omega$ ), related to volts and amperes by  $1~\Omega=1~V/A$ .
- There is a voltage or IR drop across a resistor, caused by the current flowing through it, given by V = IR.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

The IR drop across a resistor means that there is a change in potential or voltage across the resistor. Is there any change in current as it passes through a resistor? Explain.

#### **Exercise:**

#### **Problem:**

How is the IR drop in a resistor similar to the pressure drop in a fluid flowing through a pipe?

#### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

What current flows through the bulb of a 3.00-V flashlight when its hot resistance is  $3.60 \Omega$ ?

#### **Solution:**

0.833 A

#### **Exercise:**

#### **Problem:**

Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.

#### **Exercise:**

### **Problem:**

What is the effective resistance of a car's starter motor when 150 A flows through it as the car battery applies 11.0 V to the motor?

#### **Solution:**

$$7.33 \times 10^{-2} \Omega$$

#### **Exercise:**

#### **Problem:**

How many volts are supplied to operate an indicator light on a DVD player that has a resistance of  $140~\Omega$ , given that 25.0 mA passes through it?

(a) Find the voltage drop in an extension cord having a 0.0600- $\Omega$  resistance and through which 5.00 A is flowing. (b) A cheaper cord utilizes thinner wire and has a resistance of 0.300  $\Omega$ . What is the voltage drop in it when 5.00 A flows? (c) Why is the voltage to whatever appliance is being used reduced by this amount? What is the effect on the appliance?

#### **Solution:**

- (a) 0.300 V
- (b) 1.50 V
- (c) The voltage supplied to whatever appliance is being used is reduced because the total voltage drop from the wall to the final output of the appliance is fixed. Thus, if the voltage drop across the extension cord is large, the voltage drop across the appliance is significantly decreased, so the power output by the appliance can be significantly decreased, reducing the ability of the appliance to work properly.

#### **Exercise:**

#### **Problem:**

A power transmission line is hung from metal towers with glass insulators having a resistance of  $1.00\times10^9~\Omega$ . What current flows through the insulator if the voltage is 200 kV? (Some high-voltage lines are DC.)

# Glossary

#### Ohm's law

an empirical relation stating that the current I is proportional to the potential difference V,  $\propto V$ ; it is often written as I = V/R, where R is the resistance

### resistance

the electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, R = V/I

### ohm

the unit of resistance, given by  $1\Omega = 1 \text{ V/A}$ 

# ohmic

a type of a material for which Ohm's law is valid

# simple circuit

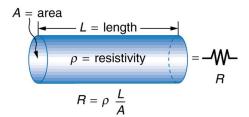
a circuit with a single voltage source and a single resistor

### Resistance and Resistivity

- Explain the concept of resistivity.
- Use resistivity to calculate the resistance of specified configurations of material.
- Use the thermal coefficient of resistivity to calculate the change of resistance with temperature.

# **Material and Shape Dependence of Resistance**

The resistance of an object depends on its shape and the material of which it is composed. The cylindrical resistor in [link] is easy to analyze, and, by so doing, we can gain insight into the resistance of more complicated shapes. As you might expect, the cylinder's electric resistance R is directly proportional to its length L, similar to the resistance of a pipe to fluid flow. The longer the cylinder, the more collisions charges will make with its atoms. The greater the diameter of the cylinder, the more current it can carry (again similar to the flow of fluid through a pipe). In fact, R is inversely proportional to the cylinder's cross-sectional area A.



A uniform cylinder of length L and crosssectional area A. Its resistance to the flow of current is similar to the resistance posed by a pipe to fluid flow. The longer the cylinder, the greater its

resistance. The larger its cross-sectional area A, the smaller its resistance.

For a given shape, the resistance depends on the material of which the object is composed. Different materials offer different resistance to the flow of charge. We define the **resistivity**  $\rho$  of a substance so that the **resistance** R of an object is directly proportional to  $\rho$ . Resistivity  $\rho$  is an *intrinsic* property of a material, independent of its shape or size. The resistance R of a uniform cylinder of length L, of cross-sectional area A, and made of a material with resistivity  $\rho$ , is

### **Equation:**

$$R = \frac{\rho L}{A}$$
.

[link] gives representative values of  $\rho$ . The materials listed in the table are separated into categories of conductors, semiconductors, and insulators, based on broad groupings of resistivities. Conductors have the smallest resistivities, and insulators have the largest; semiconductors have intermediate resistivities. Conductors have varying but large free charge densities, whereas most charges in insulators are bound to atoms and are not free to move. Semiconductors are intermediate, having far fewer free charges than conductors, but having properties that make the number of free charges depend strongly on the type and amount of impurities in the semiconductor. These unique properties of semiconductors are put to use in modern electronics, as will be explored in later chapters.

Material	Resistivity $ ho$ ( $\Omega \cdot \mathrm{m}$ )
Conductors	
Silver	$1.59\times10^{-8}$
Copper	$1.72\times10^{-8}$
Gold	$2.44\times10^{-8}$
Aluminum	$2.65\times10^{-8}$
Tungsten	$5.6\times10^{-8}$
Iron	$9.71\times10^{-8}$
Platinum	$10.6\times10^{-8}$
Steel	$20\times 10^{-8}$
Lead	$22\times 10^{-8}$

Material	Resistivity $ ho$ ( $\Omega \cdot \mathrm{m}$ )
Manganin (Cu, Mn, Ni alloy)	$44\times10^{-8}$
Constantan (Cu, Ni alloy)	$49\times10^{-8}$
Mercury	$96\times10^{-8}$
Nichrome (Ni, Fe, Cr alloy)	$100\times10^{-8}$
Semiconductors[footnote] Values depend strongly on amounts and types of impurities	
Carbon (pure)	$3.5\times 10^{-5}$
Carbon	$(3.5-60) imes 10^{-5}$
Germanium (pure)	$600\times 10^{-3}$
Germanium	$(1-600) imes 10^{-3}$

Material	Resistivity $ ho$ ( $\Omega \cdot { m m}$ )
Silicon (pure)	2300
Silicon	0.1 – 2300
Insulators	
Amber	$5 imes 10^{14}$
Glass	$10^9-10^{14}$
Lucite	$> 10^{13}$
Mica	$10^{11}-10^{15}$
Quartz (fused)	$75\times10^{16}$
Rubber (hard)	$10^{13}-10^{16}$
Sulfur	$10^{15}$

Material	Resistivity $ ho$ ( $\Omega \cdot { m m}$ )
Teflon	$> 10^{13}$
Wood	$10^8-10^{11}$

Resistivities ho of Various materials at  $20^{\circ}\mathrm{C}$ 

### **Example:**

### Calculating Resistor Diameter: A Headlight Filament

A car headlight filament is made of tungsten and has a cold resistance of  $0.350~\Omega$ . If the filament is a cylinder 4.00 cm long (it may be coiled to save space), what is its diameter?

### Strategy

We can rearrange the equation  $R = \frac{\rho L}{A}$  to find the cross-sectional area A of the filament from the given information. Then its diameter can be found by assuming it has a circular cross-section.

#### Solution

The cross-sectional area, found by rearranging the expression for the resistance of a cylinder given in  $R = \frac{\rho L}{A}$ , is

# **Equation:**

$$A = \frac{\rho L}{R}$$
.

Substituting the given values, and taking  $\rho$  from [link], yields

# **Equation:**

$$A = \frac{(5.6 \times 10^{-8} \ \Omega \cdot m)(4.00 \times 10^{-2} \ m)}{0.350 \ \Omega}$$
  
=  $6.40 \times 10^{-9} \ m^2$ .

The area of a circle is related to its diameter D by

### **Equation:**

$$A=rac{\pi D^2}{4}.$$

Solving for the diameter D, and substituting the value found for A, gives **Equation:** 

$$egin{array}{lcl} D &=& 2 \Big(rac{A}{p}\Big)^{rac{1}{2}} = 2 \Big(rac{6.40 imes 10^{-9} \ \mathrm{m}^2}{3.14}\Big)^{rac{1}{2}} \ &=& 9.0 imes 10^{-5} \ \mathrm{m}. \end{array}$$

#### Discussion

The diameter is just under a tenth of a millimeter. It is quoted to only two digits, because  $\rho$  is known to only two digits.

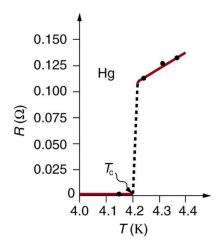
# **Temperature Variation of Resistance**

The resistivity of all materials depends on temperature. Some even become superconductors (zero resistivity) at very low temperatures. (See [link].) Conversely, the resistivity of conductors increases with increasing temperature. Since the atoms vibrate more rapidly and over larger distances at higher temperatures, the electrons moving through a metal make more collisions, effectively making the resistivity higher. Over relatively small temperature changes (about  $100^{\circ}\text{C}$  or less), resistivity  $\rho$  varies with temperature change  $\Delta T$  as expressed in the following equation **Equation:** 

$$\rho = \rho_0 (1 + \alpha \Delta T),$$

where  $\rho_0$  is the original resistivity and  $\alpha$  is the **temperature coefficient of resistivity**. (See the values of  $\alpha$  in [link] below.) For larger temperature changes,  $\alpha$  may vary or a nonlinear equation may be needed to find  $\rho$ . Note

that  $\alpha$  is positive for metals, meaning their resistivity increases with temperature. Some alloys have been developed specifically to have a small temperature dependence. Manganin (which is made of copper, manganese and nickel), for example, has  $\alpha$  close to zero (to three digits on the scale in [link]), and so its resistivity varies only slightly with temperature. This is useful for making a temperature-independent resistance standard, for example.



The resistance of a sample of mercury is zero at very low temperatures—it is a superconductor up to about 4.2 K. Above that critical temperature, its resistance makes a sudden jump and then increases nearly linearly with temperature.

Material	Coefficient $\alpha(1/^{\circ}C)$ [footnote] Values at 20°C.
Conductors	
Silver	$3.8 imes10^{-3}$
Copper	$3.9  imes 10^{-3}$
Gold	$3.4 imes10^{-3}$
Aluminum	$3.9 imes10^{-3}$
Tungsten	$4.5 imes10^{-3}$
Iron	$5.0 imes10^{-3}$
Platinum	$3.93 imes10^{-3}$
Lead	$3.9  imes 10^{-3}$
Manganin (Cu, Mn, Ni alloy)	$0.000 imes10^{-3}$

Material	Coefficient $\alpha$ (1/°C)[footnote] Values at 20°C.
Constantan (Cu, Ni alloy)	$0.002 imes10^{-3}$
Mercury	$0.89  imes 10^{-3}$
Nichrome (Ni, Fe, Cr alloy)	$0.4 imes10^{-3}$
Semiconductors	
Carbon (pure)	$-0.5 imes10^{-3}$
Germanium (pure)	$-50 imes10^{-3}$
Silicon (pure)	$-70 imes10^{-3}$

# Tempature Coefficients of Resistivity $\alpha$

Note also that  $\alpha$  is negative for the semiconductors listed in [link], meaning that their resistivity decreases with increasing temperature. They become better conductors at higher temperature, because increased thermal agitation increases the number of free charges available to carry current. This property of decreasing  $\rho$  with temperature is also related to the type and amount of impurities present in the semiconductors.

The resistance of an object also depends on temperature, since  $R_0$  is directly proportional to  $\rho$ . For a cylinder we know  $R = \rho L/A$ , and so, if L and A do not change greatly with temperature, R will have the same temperature dependence as  $\rho$ . (Examination of the coefficients of linear expansion shows them to be about two orders of magnitude less than typical temperature coefficients of resistivity, and so the effect of temperature on L and A is about two orders of magnitude less than on  $\rho$ .) Thus,

### **Equation:**

$$R = R_0(1 + \alpha \Delta T)$$

is the temperature dependence of the resistance of an object, where  $R_0$  is the original resistance and R is the resistance after a temperature change  $\Delta T$ . Numerous thermometers are based on the effect of temperature on resistance. (See [link].) One of the most common is the thermistor, a semiconductor crystal with a strong temperature dependence, the resistance of which is measured to obtain its temperature. The device is small, so that it quickly comes into thermal equilibrium with the part of a person it touches.



These familiar
thermometers are based
on the automated
measurement of a
thermistor's temperaturedependent resistance.
(credit: Biol, Wikimedia
Commons)

# **Example:**

### Calculating Resistance: Hot-Filament Resistance

Although caution must be used in applying  $\rho = \rho_0(1 + \alpha \Delta T)$  and  $R = R_0(1 + \alpha \Delta T)$  for temperature changes greater than  $100^{\circ}\mathrm{C}$ , for tungsten the equations work reasonably well for very large temperature changes. What, then, is the resistance of the tungsten filament in the previous example if its temperature is increased from room temperature ( $20^{\circ}\mathrm{C}$ ) to a typical operating temperature of  $2850^{\circ}\mathrm{C}$ ?

### Strategy

This is a straightforward application of  $R=R_0(1+\alpha\Delta T)$ , since the original resistance of the filament was given to be  $R_0=0.350~\Omega$ , and the temperature change is  $\Delta T=2830^{\circ}\mathrm{C}$ .

#### Solution

The hot resistance R is obtained by entering known values into the above equation:

### **Equation:**

$$egin{array}{lll} R &=& R_0(1+lpha\Delta T) \ &=& (0.350~\Omega)[1+(4.5 imes10^{-3}/^{
m o}{
m C})(2830^{
m o}{
m C})] \ &=& 4.8~\Omega. \end{array}$$

#### Discussion

This value is consistent with the headlight resistance example in <a href="Ohm's Law: Resistance and Simple Circuits">Ohm's Law: Resistance and Simple Circuits</a>.

#### Note:

### PhET Explorations: Resistance in a Wire

Learn about the physics of resistance in a wire. Change its resistivity, length, and area to see how they affect the wire's resistance. The sizes of the symbols in the equation change along with the diagram of a wire.

https://phet.colorado.edu/sims/html/resistance-in-a-wire/latest/resistance-in-a-wire en.html

# **Section Summary**

- The resistance R of a cylinder of length L and cross-sectional area A is  $R=\frac{\rho L}{A}$ , where  $\rho$  is the resistivity of the material.
- Values of  $\rho$  in [link] show that materials fall into three groups—conductors, semiconductors, and insulators.
- Temperature affects resistivity; for relatively small temperature changes  $\Delta T$ , resistivity is  $\rho = \rho_0 (1 + \alpha \Delta T)$ , where  $\rho_0$  is the original resistivity and  $\alpha$  is the temperature coefficient of resistivity.
- [link] gives values for  $\alpha$ , the temperature coefficient of resistivity.
- The resistance R of an object also varies with temperature:  $R = R_0(1 + \alpha \Delta T)$ , where  $R_0$  is the original resistance, and R is the resistance after the temperature change.

# **Conceptual Questions**

#### **Exercise:**

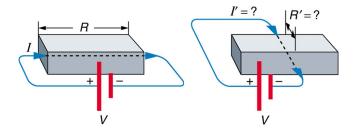
#### **Problem:**

In which of the three semiconducting materials listed in [link] do impurities supply free charges? (Hint: Examine the range of resistivity for each and determine whether the pure semiconductor has the higher or lower conductivity.)

#### **Exercise:**

#### **Problem:**

Does the resistance of an object depend on the path current takes through it? Consider, for example, a rectangular bar—is its resistance the same along its length as across its width? (See [link].)



Does current taking two different paths through the same object encounter different resistance?

#### **Exercise:**

#### **Problem:**

If aluminum and copper wires of the same length have the same resistance, which has the larger diameter? Why?

### **Exercise:**

#### **Problem:**

Explain why  $R = R_0(1 + \alpha \Delta T)$  for the temperature variation of the resistance R of an object is not as accurate as  $\rho = \rho_0(1 + \alpha \Delta T)$ , which gives the temperature variation of resistivity  $\rho$ .

### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

What is the resistance of a 20.0-m-long piece of 12-gauge copper wire having a 2.053-mm diameter?

#### **Solution:**

 $0.104~\Omega$ 

#### **Exercise:**

#### **Problem:**

The diameter of 0-gauge copper wire is 8.252 mm. Find the resistance of a 1.00-km length of such wire used for power transmission.

#### **Exercise:**

#### **Problem:**

If the 0.100-mm diameter tungsten filament in a light bulb is to have a resistance of  $0.200~\Omega$  at  $20.0^{\circ}$ C, how long should it be?

#### **Solution:**

$$2.8 \times 10^{-2} \text{ m}$$

#### **Exercise:**

### **Problem:**

Find the ratio of the diameter of aluminum to copper wire, if they have the same resistance per unit length (as they might in household wiring).

### **Exercise:**

#### **Problem:**

What current flows through a 2.54-cm-diameter rod of pure silicon that is 20.0 cm long, when  $1.00 \times 10^3~V$  is applied to it? (Such a rod may be used to make nuclear-particle detectors, for example.)

### **Solution:**

$$1.10 \times 10^{-3} \text{ A}$$

(a) To what temperature must you raise a copper wire, originally at 20.0°C, to double its resistance, neglecting any changes in dimensions? (b) Does this happen in household wiring under ordinary circumstances?

#### **Exercise:**

#### **Problem:**

A resistor made of Nichrome wire is used in an application where its resistance cannot change more than 1.00% from its value at 20.0°C. Over what temperature range can it be used?

#### **Solution:**

 $-5^{\circ}\mathrm{C}$  to  $45^{\circ}\mathrm{C}$ 

#### **Exercise:**

#### **Problem:**

Of what material is a resistor made if its resistance is 40.0% greater at 100°C than at 20.0°C?

#### **Exercise:**

#### **Problem:**

An electronic device designed to operate at any temperature in the range from  $-10.0^{\circ}\text{C}$  to  $55.0^{\circ}\text{C}$  contains pure carbon resistors. By what factor does their resistance increase over this range?

#### **Solution:**

1.03

(a) Of what material is a wire made, if it is 25.0 m long with a 0.100 mm diameter and has a resistance of  $77.7~\Omega$  at  $20.0^{\circ}$ C? (b) What is its resistance at  $150^{\circ}$ C?

#### **Exercise:**

#### **Problem:**

Assuming a constant temperature coefficient of resistivity, what is the maximum percent decrease in the resistance of a constantan wire starting at  $20.0^{\circ}$ C?

#### **Solution:**

0.06%

#### **Exercise:**

#### **Problem:**

A wire is drawn through a die, stretching it to four times its original length. By what factor does its resistance increase?

#### **Exercise:**

#### **Problem:**

A copper wire has a resistance of  $0.500~\Omega$  at  $20.0^{\circ}\mathrm{C}$ , and an iron wire has a resistance of  $0.525~\Omega$  at the same temperature. At what temperature are their resistances equal?

#### **Solution:**

 $-17^{\circ}\mathrm{C}$ 

(a) Digital medical thermometers determine temperature by measuring the resistance of a semiconductor device called a thermistor (which has  $\alpha=-0.0600/^{\circ}\mathrm{C}$ ) when it is at the same temperature as the patient. What is a patient's temperature if the thermistor's resistance at that temperature is 82.0% of its value at 37.0°C (normal body temperature)? (b) The negative value for  $\alpha$  may not be maintained for very low temperatures. Discuss why and whether this is the case here. (Hint: Resistance can't become negative.)

#### **Exercise:**

### **Problem:** Integrated Concepts

(a) Redo [link] taking into account the thermal expansion of the tungsten filament. You may assume a thermal expansion coefficient of  $12\times10^{-6}$ /°C. (b) By what percentage does your answer differ from that in the example?

### **Solution:**

- (a)  $4.7 \Omega$  (total)
- (b) 3.0% decrease

#### **Exercise:**

#### **Problem:** Unreasonable Results

(a) To what temperature must you raise a resistor made of constantan to double its resistance, assuming a constant temperature coefficient of resistivity? (b) To cut it in half? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable, or which premises are inconsistent?

# **Glossary**

# resistivity

an intrinsic property of a material, independent of its shape or size, directly proportional to the resistance, denoted by  $\rho$ 

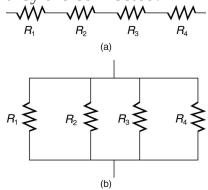
# temperature coefficient of resistivity

an empirical quantity, denoted by  $\alpha$ , which describes the change in resistance or resistivity of a material with temperature

#### Resistors in Series and Parallel

- Draw a circuit with resistors in parallel and in series.
- Calculate the voltage drop of a current across a resistor using Ohm's law.
- Contrast the way total resistance is calculated for resistors in series and in parallel.
- Explain why total resistance of a parallel circuit is less than the smallest resistance of any of the resistors in that circuit.
- Calculate total resistance of a circuit that contains a mixture of resistors connected in series and in parallel.

Most circuits have more than one component, called a **resistor** that limits the flow of charge in the circuit. A measure of this limit on charge flow is called **resistance**. The simplest combinations of resistors are the series and parallel connections illustrated in [link]. The total resistance of a combination of resistors depends on both their individual values and how they are connected.

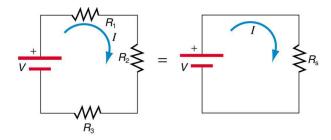


(a) A series connection of resistors. (b) A parallel connection of resistors.

# **Resistors in Series**

When are resistors in **series**? Resistors are in series whenever the flow of charge, called the **current**, must flow through devices sequentially. For example, if current flows through a person holding a screwdriver and into the Earth, then  $R_1$  in [link](a) could be the resistance of the screwdriver's shaft,  $R_2$  the resistance of its handle,  $R_3$  the person's body resistance, and  $R_4$  the resistance of her shoes.

[link] shows resistors in series connected to a **voltage** source. It seems reasonable that the total resistance is the sum of the individual resistances, considering that the current has to pass through each resistor in sequence. (This fact would be an advantage to a person wishing to avoid an electrical shock, who could reduce the current by wearing high-resistance rubbersoled shoes. It could be a disadvantage if one of the resistances were a faulty high-resistance cord to an appliance that would reduce the operating current.)



Three resistors connected in series to a battery (left) and the equivalent single or series resistance (right).

To verify that resistances in series do indeed add, let us consider the loss of electrical power, called a **voltage drop**, in each resistor in [link].

According to **Ohm's law**, the voltage drop, V, across a resistor when a current flows through it is calculated using the equation V = IR, where I equals the current in amps (A) and R is the resistance in ohms ( $\Omega$ ). Another

way to think of this is that V is the voltage necessary to make a current I flow through a resistance R.

So the voltage drop across  $R_1$  is  $V_1 = IR_1$ , that across  $R_2$  is  $V_2 = IR_2$ , and that across  $R_3$  is  $V_3 = IR_3$ . The sum of these voltages equals the voltage output of the source; that is,

### **Equation:**

$$V = V_1 + V_2 + V_3$$
.

This equation is based on the conservation of energy and conservation of charge. Electrical potential energy can be described by the equation PE = qV, where q is the electric charge and V is the voltage. Thus the energy supplied by the source is qV, while that dissipated by the resistors is **Equation:** 

$$qV_1 + qV_2 + qV_3$$
.

#### Note:

#### Connections: Conservation Laws

The derivations of the expressions for series and parallel resistance are based on the laws of conservation of energy and conservation of charge, which state that total charge and total energy are constant in any process. These two laws are directly involved in all electrical phenomena and will be invoked repeatedly to explain both specific effects and the general behavior of electricity.

These energies must be equal, because there is no other source and no other destination for energy in the circuit. Thus,  $qV=qV_1+qV_2+qV_3$ . The charge q cancels, yielding  $V=V_1+V_2+V_3$ , as stated. (Note that the same amount of charge passes through the battery and each resistor in a given amount of time, since there is no capacitance to store charge, there is no place for charge to leak, and charge is conserved.)

Now substituting the values for the individual voltages gives **Equation:** 

$$V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3).$$

Note that for the equivalent single series resistance  $R_s$ , we have **Equation:** 

$$V = IR_{s}$$
.

This implies that the total or equivalent series resistance  $R_{\rm s}$  of three resistors is  $R_{\rm s}=R_1+R_2+R_3$ .

This logic is valid in general for any number of resistors in series; thus, the total resistance  $R_{\rm s}$  of a series connection is

### **Equation:**

$$R_{\rm s} = R_1 + R_2 + R_3 + ...,$$

as proposed. Since all of the current must pass through each resistor, it experiences the resistance of each, and resistances in series simply add up.

### **Example:**

# Calculating Resistance, Current, Voltage Drop, and Power Dissipation: Analysis of a Series Circuit

Suppose the voltage output of the battery in [link] is 12.0 V, and the resistances are  $R_1 = 1.00 \Omega$ ,  $R_2 = 6.00 \Omega$ , and  $R_3 = 13.0 \Omega$ . (a) What is the total resistance? (b) Find the current. (c) Calculate the voltage drop in each resistor, and show these add to equal the voltage output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

### Strategy and Solution for (a)

The total resistance is simply the sum of the individual resistances, as given by this equation:

### **Equation:**

$$egin{array}{lll} R_{
m s} &=& R_1 + R_2 + R_3 \ &=& 1.00~\Omega + 6.00~\Omega + 13.0~\Omega \ &=& 20.0~\Omega. \end{array}$$

### Strategy and Solution for (b)

The current is found using Ohm's law, V = IR. Entering the value of the applied voltage and the total resistance yields the current for the circuit:

### **Equation:**

$$I = rac{V}{R_{
m s}} = rac{12.0 \ {
m V}}{20.0 \ \Omega} = 0.600 \ {
m A}.$$

### Strategy and Solution for (c)

The voltage—or IR drop—in a resistor is given by Ohm's law. Entering the current and the value of the first resistance yields

### **Equation:**

$$V_1 = IR_1 = (0.600 \text{ A})(1.0 \Omega) = 0.600 \text{ V}.$$

Similarly,

# **Equation:**

$$V_2 = IR_2 = (0.600 \text{ A})(6.0 \Omega) = 3.60 \text{ V}$$

and

# **Equation:**

$$V_3 = IR_3 = (0.600 \text{ A})(13.0 \Omega) = 7.80 \text{ V}.$$

# Discussion for (c)

The three IR drops add to 12.0 V, as predicted:

### **Equation:**

$$V_1 + V_2 + V_3 = (0.600 + 3.60 + 7.80) \text{ V} = 12.0 \text{ V}.$$

# Strategy and Solution for (d)

The easiest way to calculate power in watts (W) dissipated by a resistor in a DC circuit is to use **Joule's law**, P = IV, where P is electric power. In this case, each resistor has the same full current flowing through it. By substituting Ohm's law V = IR into Joule's law, we get the power dissipated by the first resistor as

## **Equation:**

$$P_1 = I^2 R_1 = (0.600 \text{ A})^2 (1.00 \Omega) = 0.360 \text{ W}.$$

Similarly,

## **Equation:**

$$P_2 = I^2 R_2 = (0.600 \text{ A})^2 (6.00 \Omega) = 2.16 \text{ W}$$

and

## **Equation:**

$$P_3 = I^2 R_3 = (0.600 \text{ A})^2 (13.0 \Omega) = 4.68 \text{ W}.$$

## Discussion for (d)

Power can also be calculated using either P = IV or  $P = \frac{V^2}{R}$ , where V is the voltage drop across the resistor (not the full voltage of the source). The same values will be obtained.

## Strategy and Solution for (e)

The easiest way to calculate power output of the source is to use  $P=\mathrm{IV}$ , where V is the source voltage. This gives

# **Equation:**

$$P = (0.600 \text{ A})(12.0 \text{ V}) = 7.20 \text{ W}.$$

# Discussion for (e)

Note, coincidentally, that the total power dissipated by the resistors is also 7.20 W, the same as the power put out by the source. That is,

# **Equation:**

$$P_1 + P_2 + P_3 = (0.360 + 2.16 + 4.68) \text{ W} = 7.20 \text{ W}.$$

Power is energy per unit time (watts), and so conservation of energy requires the power output of the source to be equal to the total power dissipated by the resistors.

## Note:

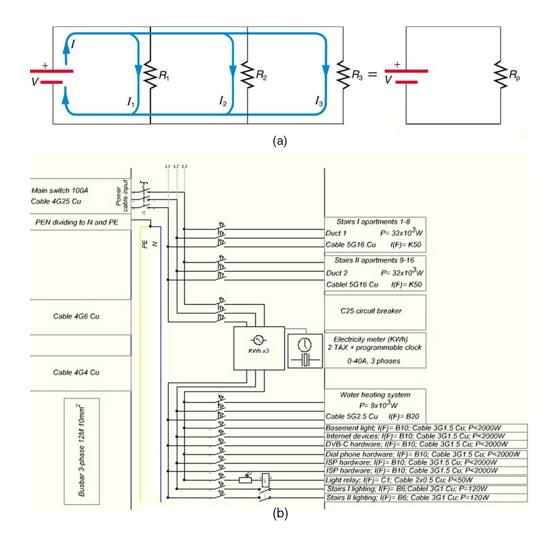
Major Features of Resistors in Series

- 1. Series resistances add:  $R_s = R_1 + R_2 + R_3 + \dots$
- 2. The same current flows through each resistor in series.
- 3. Individual resistors in series do not get the total source voltage, but divide it.

## **Resistors in Parallel**

[link] shows resistors in **parallel**, wired to a voltage source. Resistors are in parallel when each resistor is connected directly to the voltage source by connecting wires having negligible resistance. Each resistor thus has the full voltage of the source applied to it.

Each resistor draws the same current it would if it alone were connected to the voltage source (provided the voltage source is not overloaded). For example, an automobile's headlights, radio, and so on, are wired in parallel, so that they utilize the full voltage of the source and can operate completely independently. The same is true in your house, or any building. (See [link] (b).)



(a) Three resistors connected in parallel to a battery and the equivalent single or parallel resistance. (b) Electrical power setup in a house. (credit: Dmitry G, Wikimedia Commons)

To find an expression for the equivalent parallel resistance  $R_{\rm p}$ , let us consider the currents that flow and how they are related to resistance. Since each resistor in the circuit has the full voltage, the currents flowing through the individual resistors are  $I_1=\frac{V}{R_1}$ ,  $I_2=\frac{V}{R_2}$ , and  $I_3=\frac{V}{R_3}$ . Conservation of charge implies that the total current I produced by the source is the sum of these currents:

# **Equation:**

$$I = I_1 + I_2 + I_3$$
.

Substituting the expressions for the individual currents gives **Equation:** 

$$I = rac{V}{R_1} + rac{V}{R_2} + rac{V}{R_3} = V igg(rac{1}{R_1} + rac{1}{R_2} + rac{1}{R_3}igg).$$

Note that Ohm's law for the equivalent single resistance gives **Equation:** 

$$I = rac{V}{R_{
m p}} = V igg(rac{1}{R_{
m p}}igg).$$

The terms inside the parentheses in the last two equations must be equal. Generalizing to any number of resistors, the total resistance  $R_{\rm p}$  of a parallel connection is related to the individual resistances by

# **Equation:**

$$\frac{1}{R_{\rm p}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{.3}} + \dots$$

This relationship results in a total resistance  $R_{\rm p}$  that is less than the smallest of the individual resistances. (This is seen in the next example.) When resistors are connected in parallel, more current flows from the source than would flow for any of them individually, and so the total resistance is lower.

# **Example:**

# Calculating Resistance, Current, Power Dissipation, and Power Output: Analysis of a Parallel Circuit

Let the voltage output of the battery and resistances in the parallel connection in [link] be the same as the previously considered series

connection: V=12.0 V,  $R_1=1.00 \Omega$ ,  $R_2=6.00 \Omega$ , and  $R_3=13.0 \Omega$ . (a) What is the total resistance? (b) Find the total current. (c) Calculate the currents in each resistor, and show these add to equal the total current output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

## Strategy and Solution for (a)

The total resistance for a parallel combination of resistors is found using the equation below. Entering known values gives

## **Equation:**

$$rac{1}{R_{
m p}} = rac{1}{R_1} + rac{1}{R_2} + rac{1}{R_3} = rac{1}{1.00\,\Omega} + rac{1}{6.00\,\Omega} + rac{1}{13.0\,\Omega}.$$

Thus,

## **Equation:**

$$rac{1}{R_{
m p}} = rac{1.00}{\Omega} + rac{0.1667}{\Omega} + rac{0.07692}{\Omega} = rac{1.2436}{\Omega}.$$

(Note that in these calculations, each intermediate answer is shown with an extra digit.)

We must invert this to find the total resistance  $R_{\rm p}$ . This yields

# **Equation:**

$$R_{
m p} = rac{1}{1.2436} \Omega = 0.8041 \ \Omega.$$

The total resistance with the correct number of significant digits is  $R_{\rm p} = 0.804~\Omega.$ 

# Discussion for (a)

 $R_{
m p}$  is, as predicted, less than the smallest individual resistance.

# Strategy and Solution for (b)

The total current can be found from Ohm's law, substituting  $R_{\rm p}$  for the total resistance. This gives

# **Equation:**

$$I = rac{V}{R_{
m p}} = rac{12.0 \ {
m V}}{0.8041 \ \Omega} = 14.92 \ {
m A}.$$

## Discussion for (b)

Current I for each device is much larger than for the same devices connected in series (see the previous example). A circuit with parallel connections has a smaller total resistance than the resistors connected in series.

## Strategy and Solution for (c)

The individual currents are easily calculated from Ohm's law, since each resistor gets the full voltage. Thus,

## **Equation:**

$$I_1 = rac{V}{R_1} = rac{12.0 \text{ V}}{1.00 \Omega} = 12.0 \text{ A}.$$

Similarly,

## **Equation:**

$$I_2 = rac{V}{R_2} = rac{12.0 \ {
m V}}{6.00 \, \Omega} = 2.00 \ {
m A}$$

and

# **Equation:**

$$I_3 = rac{V}{R_3} = rac{12.0 ext{ V}}{13.0 \, \Omega} = 0.92 ext{ A}.$$

## Discussion for (c)

The total current is the sum of the individual currents:

# **Equation:**

$$I_1 + I_2 + I_3 = 14.92 \text{ A}.$$

This is consistent with conservation of charge.

# Strategy and Solution for (d)

The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three

are known. Let us use  $P=rac{V^2}{R}$ , since each resistor gets full voltage. Thus,

## **Equation:**

$$P_1 = rac{V^2}{R_1} = rac{(12.0 \ {
m V})^2}{1.00 \ \Omega} = 144 \ {
m W}.$$

Similarly,

## **Equation:**

$$P_2 = rac{V^2}{R_2} = rac{(12.0 \ {
m V})^2}{6.00 \ \Omega} = 24.0 \ {
m W}$$

and

## **Equation:**

$$P_3 = rac{V^2}{R_3} = rac{(12.0 \ {
m V})^2}{13.0 \ \Omega} = 11.1 \ {
m W}.$$

## Discussion for (d)

The power dissipated by each resistor is considerably higher in parallel than when connected in series to the same voltage source.

## Strategy and Solution for (e)

The total power can also be calculated in several ways. Choosing P = IV, and entering the total current, yields

## **Equation:**

$$P = IV = (14.92 \text{ A})(12.0 \text{ V}) = 179 \text{ W}.$$

## Discussion for (e)

Total power dissipated by the resistors is also 179 W:

# **Equation:**

$$P_1 + P_2 + P_3 = 144 \text{ W} + 24.0 \text{ W} + 11.1 \text{ W} = 179 \text{ W}.$$

This is consistent with the law of conservation of energy.

#### **Overall Discussion**

Note that both the currents and powers in parallel connections are greater than for the same devices in series.

#### Note:

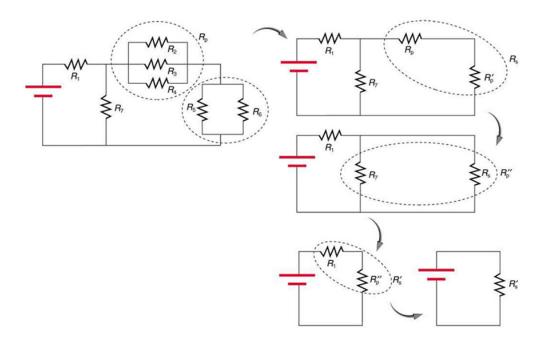
## Major Features of Resistors in Parallel

- 1. Parallel resistance is found from  $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$ , and it is smaller than any individual resistance in the combination.
- 2. Each resistor in parallel has the same full voltage of the source applied to it. (Power distribution systems most often use parallel connections to supply the myriad devices served with the same voltage and to allow them to operate independently.)
- 3. Parallel resistors do not each get the total current; they divide it.

## **Combinations of Series and Parallel**

More complex connections of resistors are sometimes just combinations of series and parallel. These are commonly encountered, especially when wire resistance is considered. In that case, wire resistance is in series with other resistances that are in parallel.

Combinations of series and parallel can be reduced to a single equivalent resistance using the technique illustrated in [link]. Various parts are identified as either series or parallel, reduced to their equivalents, and further reduced until a single resistance is left. The process is more time consuming than difficult.



This combination of seven resistors has both series and parallel parts. Each is identified and reduced to an equivalent resistance, and these are further reduced until a single equivalent resistance is reached.

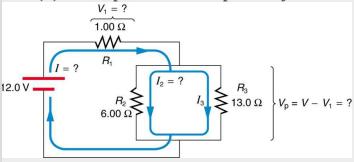
The simplest combination of series and parallel resistance, shown in [link], is also the most instructive, since it is found in many applications. For example,  $R_1$  could be the resistance of wires from a car battery to its electrical devices, which are in parallel.  $R_2$  and  $R_3$  could be the starter motor and a passenger compartment light. We have previously assumed that wire resistance is negligible, but, when it is not, it has important effects, as the next example indicates.

# Example:

# Calculating Resistance, $\operatorname{IR}$ Drop, Current, and Power Dissipation: Combining Series and Parallel Circuits

[link] shows the resistors from the previous two examples wired in a different way—a combination of series and parallel. We can consider  $R_1$  to be the resistance of wires leading to  $R_2$  and  $R_3$ . (a) Find the total

resistance. (b) What is the IR drop in  $R_1$ ? (c) Find the current  $I_2$  through  $R_2$ . (d) What power is dissipated by  $R_2$ ?



These three resistors are connected to a voltage source so that  $R_2$  and  $R_3$  are in parallel with one another and that combination is in series with  $R_1$ .

## Strategy and Solution for (a)

To find the total resistance, we note that  $R_2$  and  $R_3$  are in parallel and their combination  $R_p$  is in series with  $R_1$ . Thus the total (equivalent) resistance of this combination is

## **Equation:**

$$R_{\mathrm{tot}} = R_1 + R_{\mathrm{p}}.$$

First, we find  $R_{\rm p}$  using the equation for resistors in parallel and entering known values:

# **Equation:**

$$rac{1}{R_{
m p}} = rac{1}{R_2} + rac{1}{R_3} = rac{1}{6.00\,\Omega} + rac{1}{13.0\,\Omega} = rac{0.2436}{\Omega}.$$

Inverting gives

# **Equation:**

$$R_{
m p}=rac{1}{0.2436}\Omega=4.11~\Omega.$$

So the total resistance is

## **Equation:**

$$R_{
m tot} = R_1 + R_{
m p} = 1.00~\Omega + 4.11~\Omega = 5.11~\Omega.$$

## Discussion for (a)

The total resistance of this combination is intermediate between the pure series and pure parallel values (20.0  $\Omega$  and 0.804  $\Omega$ , respectively) found for the same resistors in the two previous examples.

## Strategy and Solution for (b)

To find the IR drop in  $R_1$ , we note that the full current I flows through  $R_1$ . Thus its IR drop is

## **Equation:**

$$V_1 = IR_1$$
.

We must find I before we can calculate  $V_1$ . The total current I is found using Ohm's law for the circuit. That is,

## **Equation:**

$$I = rac{V}{R_{
m tot}} = rac{12.0 \ {
m V}}{5.11 \ \Omega} = 2.35 \ {
m A}.$$

Entering this into the expression above, we get

# **Equation:**

$$V_1 = IR_1 = (2.35 \text{ A})(1.00 \Omega) = 2.35 \text{ V}.$$

# Discussion for (b)

The voltage applied to  $R_2$  and  $R_3$  is less than the total voltage by an amount  $V_1$ . When wire resistance is large, it can significantly affect the operation of the devices represented by  $R_2$  and  $R_3$ .

## Strategy and Solution for (c)

To find the current through  $R_2$ , we must first find the voltage applied to it. We call this voltage  $V_p$ , because it is applied to a parallel combination of resistors. The voltage applied to both  $R_2$  and  $R_3$  is reduced by the amount  $V_1$ , and so it is

# **Equation:**

$$V_{\rm p} = V - V_1 = 12.0 \text{ V} - 2.35 \text{ V} = 9.65 \text{ V}.$$

Now the current  $I_2$  through resistance  $R_2$  is found using Ohm's law:

## **Equation:**

$$I_2 = rac{V_{
m p}}{R_2} = rac{9.65 \ {
m V}}{6.00 \ \Omega} = 1.61 \ {
m A}.$$

## Discussion for (c)

The current is less than the 2.00 A that flowed through  $R_2$  when it was connected in parallel to the battery in the previous parallel circuit example.

## Strategy and Solution for (d)

The power dissipated by  $R_2$  is given by

## **Equation:**

$$P_2 = (I_2)^2 R_2 = (1.61 \text{ A})^2 (6.00 \Omega) = 15.5 \text{ W}.$$

## Discussion for (d)

The power is less than the 24.0 W this resistor dissipated when connected in parallel to the 12.0-V source.

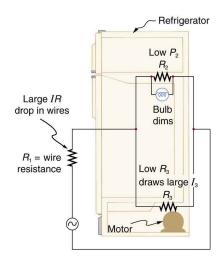
# **Practical Implications**

One implication of this last example is that resistance in wires reduces the current and power delivered to a resistor. If wire resistance is relatively large, as in a worn (or a very long) extension cord, then this loss can be significant. If a large current is drawn, the IR drop in the wires can also be significant.

For example, when you are rummaging in the refrigerator and the motor comes on, the refrigerator light dims momentarily. Similarly, you can see the passenger compartment light dim when you start the engine of your car (although this may be due to resistance inside the battery itself).

What is happening in these high-current situations is illustrated in [link]. The device represented by  $R_3$  has a very low resistance, and so when it is

switched on, a large current flows. This increased current causes a larger IR drop in the wires represented by  $R_1$ , reducing the voltage across the light bulb (which is  $R_2$ ), which then dims noticeably.



Why do lights dim
when a large
appliance is
switched on? The
answer is that the
large current the
appliance motor
draws causes a
significant IR drop
in the wires and
reduces the voltage
across the light.

# **Exercise:** Check Your Understanding

Can any arbitrary combination of resistors be broken down into series and parallel combinations? See if you can draw a circuit diagram of resistors that cannot be broken down into combinations of series and parallel.

#### **Solution:**

No, there are many ways to connect resistors that are not combinations of series and parallel, including loops and junctions. In such cases Kirchhoff's rules, to be introduced in <u>Kirchhoff's Rules</u>, will allow you to analyze the circuit.

#### Note:

## Problem-Solving Strategies for Series and Parallel Resistors

- 1. Draw a clear circuit diagram, labeling all resistors and voltage sources. This step includes a list of the knowns for the problem, since they are labeled in your circuit diagram.
- 2. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
- 3. Determine whether resistors are in series, parallel, or a combination of both series and parallel. Examine the circuit diagram to make this assessment. Resistors are in series if the same current must pass sequentially through them.
- 4. Use the appropriate list of major features for series or parallel connections to solve for the unknowns. There is one list for series and another for parallel. If your problem has a combination of series and parallel, reduce it in steps by considering individual groups of series or parallel connections, as done in this module and the examples. Special note: When finding  $R_{\rm p}$ , the reciprocal must be taken with care.
- 5. Check to see whether the answers are reasonable and consistent. Units and numerical results must be reasonable. Total series resistance

should be greater, whereas total parallel resistance should be smaller, for example. Power should be greater for the same devices in parallel compared with series, and so on.

# **Section Summary**

• The total resistance of an electrical circuit with resistors wired in a series is the sum of the individual resistances:

$$R_{\rm s} = R_1 + R_2 + R_3 + \dots$$

- Each resistor in a series circuit has the same amount of current flowing through it.
- The voltage drop, or power dissipation, across each individual resistor in a series is different, and their combined total adds up to the power source input.
- The total resistance of an electrical circuit with resistors wired in parallel is less than the lowest resistance of any of the components and can be determined using the formula:

## **Equation:**

$$\frac{1}{R_{\rm p}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

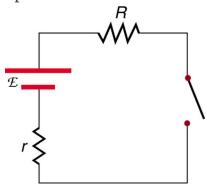
- Each resistor in a parallel circuit has the same full voltage of the source applied to it.
- The current flowing through each resistor in a parallel circuit is different, depending on the resistance.
- If a more complex connection of resistors is a combination of series and parallel, it can be reduced to a single equivalent resistance by identifying its various parts as series or parallel, reducing each to its equivalent, and continuing until a single resistance is eventually reached.

# **Conceptual Questions**

## **Exercise:**

## **Problem:**

A switch has a variable resistance that is nearly zero when closed and extremely large when open, and it is placed in series with the device it controls. Explain the effect the switch in [link] has on current when open and when closed.



A switch is ordinarily in series with a resistance and voltage source. Ideally, the switch has nearly zero resistance when closed but has an extremely large resistance when open. (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)

**Problem:** What is the voltage across the open switch in [link]?

## **Exercise:**

## **Problem:**

There is a voltage across an open switch, such as in [link]. Why, then, is the power dissipated by the open switch small?

#### **Exercise:**

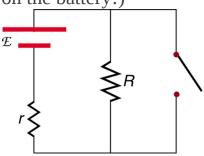
#### **Problem:**

Why is the power dissipated by a closed switch, such as in [<u>link</u>], small?

#### **Exercise:**

#### **Problem:**

A student in a physics lab mistakenly wired a light bulb, battery, and switch as shown in [link]. Explain why the bulb is on when the switch is open, and off when the switch is closed. (Do not try this—it is hard on the battery!)



A wiring mistake put this switch in parallel with the device represented by R. (Note that in this diagram, the script E represents the voltage (or

electromotive force) of the battery.)

#### **Exercise:**

#### **Problem:**

Knowing that the severity of a shock depends on the magnitude of the current through your body, would you prefer to be in series or parallel with a resistance, such as the heating element of a toaster, if shocked by it? Explain.

## **Exercise:**

#### **Problem:**

Would your headlights dim when you start your car's engine if the wires in your automobile were superconductors? (Do not neglect the battery's internal resistance.) Explain.

#### **Exercise:**

#### **Problem:**

Some strings of holiday lights are wired in series to save wiring costs. An old version utilized bulbs that break the electrical connection, like an open switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 40 identical bulbs, what is the normal operating voltage of each? Newer versions use bulbs that short circuit, like a closed switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 39 remaining identical bulbs, what is then the operating voltage of each?

If two household lightbulbs rated 60 W and 100 W are connected in series to household power, which will be brighter? Explain.

#### **Exercise:**

#### **Problem:**

Suppose you are doing a physics lab that asks you to put a resistor into a circuit, but all the resistors supplied have a larger resistance than the requested value. How would you connect the available resistances to attempt to get the smaller value asked for?

#### **Exercise:**

#### **Problem:**

Before World War II, some radios got power through a "resistance cord" that had a significant resistance. Such a resistance cord reduces the voltage to a desired level for the radio's tubes and the like, and it saves the expense of a transformer. Explain why resistance cords become warm and waste energy when the radio is on.

#### **Exercise:**

#### **Problem:**

Some light bulbs have three power settings (not including zero), obtained from multiple filaments that are individually switched and wired in parallel. What is the minimum number of filaments needed for three power settings?

## **Problem Exercises**

Note: Data taken from figures can be assumed to be accurate to three significant digits.

- (a) What is the resistance of ten  $275-\Omega$  resistors connected in series?
- (b) In parallel?

#### **Solution:**

- (a)  $2.75 \text{ k}\Omega$
- (b)  $27.5 \Omega$

## **Exercise:**

#### **Problem:**

(a) What is the resistance of a  $1.00 \times 10^2$  - $\Omega$ , a 2.50-k $\Omega$ , and a 4.00-k $\Omega$  resistor connected in series? (b) In parallel?

#### **Exercise:**

## **Problem:**

What are the largest and smallest resistances you can obtain by connecting a  $36.0-\Omega$ , a  $50.0-\Omega$ , and a  $700-\Omega$  resistor together?

## **Solution:**

- (a)  $786 \Omega$
- (b)  $20.3 \Omega$

## **Exercise:**

#### **Problem:**

An 1800-W toaster, a 1400-W electric frying pan, and a 75-W lamp are plugged into the same outlet in a 15-A, 120-V circuit. (The three devices are in parallel when plugged into the same socket.). (a) What current is drawn by each device? (b) Will this combination blow the 15-A fuse?

Your car's 30.0-W headlight and 2.40-kW starter are ordinarily connected in parallel in a 12.0-V system. What power would one headlight and the starter consume if connected in series to a 12.0-V battery? (Neglect any other resistance in the circuit and any change in resistance in the two devices.)

#### **Solution:**

 $29.6 \, W$ 

## **Exercise:**

#### **Problem:**

(a) Given a 48.0-V battery and  $24.0-\Omega$  and  $96.0-\Omega$  resistors, find the current and power for each when connected in series. (b) Repeat when the resistances are in parallel.

#### **Exercise:**

#### **Problem:**

Referring to the example combining series and parallel circuits and  $[\underline{link}]$ , calculate  $I_3$  in the following two different ways: (a) from the known values of I and  $I_2$ ; (b) using Ohm's law for  $R_3$ . In both parts explicitly show how you follow the steps in the  $\underline{Problem-Solving}$  Strategies for Series and Parallel Resistors.

#### **Solution:**

- (a) 0.74 A
- (b) 0.742 A

Referring to [link]: (a) Calculate  $P_3$  and note how it compares with  $P_3$  found in the first two example problems in this module. (b) Find the total power supplied by the source and compare it with the sum of the powers dissipated by the resistors.

#### **Exercise:**

## **Problem:**

Refer to [link] and the discussion of lights dimming when a heavy appliance comes on. (a) Given the voltage source is 120 V, the wire resistance is  $0.400~\Omega$ , and the bulb is nominally 75.0 W, what power will the bulb dissipate if a total of 15.0 A passes through the wires when the motor comes on? Assume negligible change in bulb resistance. (b) What power is consumed by the motor?

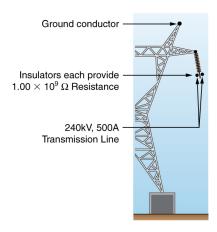
## **Solution:**

- (a) 60.8 W
- (b) 3.18 kW

#### **Exercise:**

#### **Problem:**

A 240-kV power transmission line carrying  $5.00 \times 10^2$  A is hung from grounded metal towers by ceramic insulators, each having a  $1.00 \times 10^9$  - $\Omega$  resistance. [link]. (a) What is the resistance to ground of 100 of these insulators? (b) Calculate the power dissipated by 100 of them. (c) What fraction of the power carried by the line is this? Explicitly show how you follow the steps in the Problem-Solving Strategies for Series and Parallel Resistors.



High-voltage (240-kV) transmission line carrying  $5.00 \times 10^2$  A is hung from a grounded metal transmission tower. The row of ceramic insulators provide  $1.00 \times 10^9 \Omega$  of resistance each.

## **Exercise:**

#### **Problem:**

Show that if two resistors  $R_1$  and  $R_2$  are combined and one is much greater than the other  $(R_1 >> R_2)$ : (a) Their series resistance is very nearly equal to the greater resistance  $R_1$ . (b) Their parallel resistance is very nearly equal to smaller resistance  $R_2$ .

#### **Solution:**

$$egin{aligned} R_{
m s} &= R_1 + R_2 \ \Rightarrow R_{
m s} &pprox R_1 (R_1 >> R_2) \end{aligned}$$

(b) 
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$
,

so that

$$R_{
m p}=rac{R_1R_2}{R_1+R_2}{pprox}rac{R_1R_2}{R_1}=R_2(R_1{>>}R_2).$$

## **Exercise:**

## **Problem:** Unreasonable Results

Two resistors, one having a resistance of  $145~\Omega$ , are connected in parallel to produce a total resistance of  $150~\Omega$ . (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

#### **Exercise:**

## **Problem: Unreasonable Results**

Two resistors, one having a resistance of  $900 \text{ k}\Omega$ , are connected in series to produce a total resistance of  $0.500 \text{ M}\Omega$ . (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

## **Solution:**

- (a)  $-400 \text{ k}\Omega$
- (b) Resistance cannot be negative.
- (c) Series resistance is said to be less than one of the resistors, but it must be greater than any of the resistors.

# **Glossary**

series

a sequence of resistors or other components wired into a circuit one after the other

#### resistor

a component that provides resistance to the current flowing through an electrical circuit

#### resistance

causing a loss of electrical power in a circuit

#### Ohm's law

the relationship between current, voltage, and resistance within an electrical circuit: V = IR

## voltage

the electrical potential energy per unit charge; electric pressure created by a power source, such as a battery

## voltage drop

the loss of electrical power as a current travels through a resistor, wire or other component

#### current

the flow of charge through an electric circuit past a given point of measurement

## Joule's law

the relationship between potential electrical power, voltage, and resistance in an electrical circuit, given by:  $P_e = IV$ 

# parallel

the wiring of resistors or other components in an electrical circuit such that each component receives an equal voltage from the power source; often pictured in a ladder-shaped diagram, with each component on a rung of the ladder

## Electromotive Force: Terminal Voltage

- Compare and contrast the voltage and the electromagnetic force of an electric power source.
- Describe what happens to the terminal voltage, current, and power delivered to a load as internal resistance of the voltage source increases (due to aging of batteries, for example).
- Explain why it is beneficial to use more than one voltage source connected in parallel.

When you forget to turn off your car lights, they slowly dim as the battery runs down. Why don't they simply blink off when the battery's energy is gone? Their gradual dimming implies that battery output voltage decreases as the battery is depleted.

Furthermore, if you connect an excessive number of 12-V lights in parallel to a car battery, they will be dim even when the battery is fresh and even if the wires to the lights have very low resistance. This implies that the battery's output voltage is reduced by the overload.

The reason for the decrease in output voltage for depleted or overloaded batteries is that all voltage sources have two fundamental parts—a source of electrical energy and an **internal resistance**. Let us examine both.

## **Electromotive Force**

You can think of many different types of voltage sources. Batteries themselves come in many varieties. There are many types of mechanical/electrical generators, driven by many different energy sources, ranging from nuclear to wind. Solar cells create voltages directly from light, while thermoelectric devices create voltage from temperature differences.

A few voltage sources are shown in [link]. All such devices create a **potential difference** and can supply current if connected to a resistance. On the small scale, the potential difference creates an electric field that exerts force on charges, causing current. We thus use the name **electromotive force**, abbreviated emf.

Emf is not a force at all; it is a special type of potential difference. To be precise, the electromotive force (emf) is the potential difference of a source when no current is flowing. Units of emf are volts.



A variety of voltage sources (clockwise from top left): the Brazos Wind Farm in Fluvanna, Texas (credit: Leaflet, Wikimedia Commons); the Krasnoyarsk Dam in Russia (credit: Alex Polezhaev); a solar farm (credit: U.S. Department of Energy); and a group of nickel metal hydride batteries (credit: Tiaa Monto). The voltage output of each depends on its construction and load, and equals emf only if there is no load.

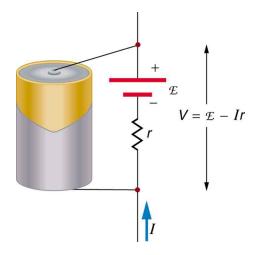
Electromotive force is directly related to the source of potential difference, such as the particular combination of chemicals in a battery. However, emf

differs from the voltage output of the device when current flows. The voltage across the terminals of a battery, for example, is less than the emf when the battery supplies current, and it declines further as the battery is depleted or loaded down. However, if the device's output voltage can be measured without drawing current, then output voltage will equal emf (even for a very depleted battery).

## **Internal Resistance**

As noted before, a 12-V truck battery is physically larger, contains more charge and energy, and can deliver a larger current than a 12-V motorcycle battery. Both are lead-acid batteries with identical emf, but, because of its size, the truck battery has a smaller internal resistance r. Internal resistance is the inherent resistance to the flow of current within the source itself.

[link] is a schematic representation of the two fundamental parts of any voltage source. The emf (represented by a script E in the figure) and internal resistance r are in series. The smaller the internal resistance for a given emf, the more current and the more power the source can supply.



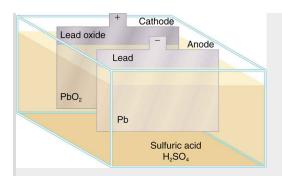
Any voltage source (in this case, a carbon-zinc dry cell) has an emf related to its source of

potential difference, and an internal resistance r related to its construction. (Note that the script E stands for emf.). Also shown are the output terminals across which the terminal voltage V is measured. Since  $V=\mathrm{emf}-\mathrm{Ir},$  terminal voltage equals emf only if there is no current flowing.

The internal resistance r can behave in complex ways. As noted, r increases as a battery is depleted. But internal resistance may also depend on the magnitude and direction of the current through a voltage source, its temperature, and even its history. The internal resistance of rechargeable nickel-cadmium cells, for example, depends on how many times and how deeply they have been depleted.

#### Note:

Things Great and Small: The Submicroscopic Origin of Battery Potential Various types of batteries are available, with emfs determined by the combination of chemicals involved. We can view this as a molecular reaction (what much of chemistry is about) that separates charge. The lead-acid battery used in cars and other vehicles is one of the most common types. A single cell (one of six) of this battery is seen in [link]. The cathode (positive) terminal of the cell is connected to a lead oxide plate, while the anode (negative) terminal is connected to a lead plate. Both plates are immersed in sulfuric acid, the electrolyte for the system.

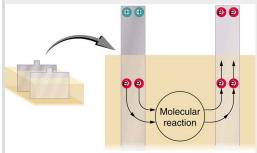


Artist's conception of a lead-acid cell. Chemical reactions in a lead-acid cell separate charge, sending negative charge to the anode, which is connected to the lead plates. The lead oxide plates are connected to the positive or cathode terminal of the cell. Sulfuric acid conducts the charge as well as participating in the chemical reaction.

The details of the chemical reaction are left to the reader to pursue in a chemistry text, but their results at the molecular level help explain the potential created by the battery. [link] shows the result of a single chemical reaction. Two electrons are placed on the anode, making it negative, provided that the cathode supplied two electrons. This leaves the cathode positively charged, because it has lost two electrons. In short, a separation of charge has been driven by a chemical reaction.

Note that the reaction will not take place unless there is a complete circuit to allow two electrons to be supplied to the cathode. Under many circumstances, these electrons come from the anode, flow through a resistance, and return to the cathode. Note also that since the chemical

reactions involve substances with resistance, it is not possible to create the emf without an internal resistance.



Artist's conception of two electrons being forced onto the anode of a cell and two electrons being removed from the cathode of the cell. The chemical reaction in a lead-acid battery places two electrons on the anode and removes two from the cathode. It requires a closed circuit to proceed, since the two electrons must be supplied to the cathode.

Why are the chemicals able to produce a unique potential difference? Quantum mechanical descriptions of molecules, which take into account the types of atoms and numbers of electrons in them, are able to predict the energy states they can have and the energies of reactions between them.

In the case of a lead-acid battery, an energy of 2 eV is given to each electron sent to the anode. Voltage is defined as the electrical potential

energy divided by charge:  $V=\frac{P_{\rm E}}{q}$ . An electron volt is the energy given to a single electron by a voltage of 1 V. So the voltage here is 2 V, since 2 eV is given to each electron. It is the energy produced in each molecular reaction that produces the voltage. A different reaction produces a different energy and, hence, a different voltage.

# **Terminal Voltage**

The voltage output of a device is measured across its terminals and, thus, is called its **terminal voltage** V. Terminal voltage is given by **Equation:** 

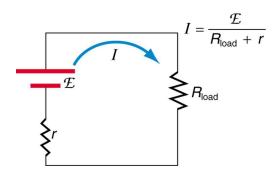
$$V = \text{emf} - \text{Ir},$$

where r is the internal resistance and I is the current flowing at the time of the measurement.

*I* is positive if current flows away from the positive terminal, as shown in [link]. You can see that the larger the current, the smaller the terminal voltage. And it is likewise true that the larger the internal resistance, the smaller the terminal voltage.

Suppose a load resistance  $R_{\rm load}$  is connected to a voltage source, as in [link]. Since the resistances are in series, the total resistance in the circuit is  $R_{\rm load} + r$ . Thus the current is given by Ohm's law to be **Equation:** 

$$I = rac{\mathrm{emf}}{R_{\mathrm{load}} + r}.$$



Schematic of a voltage source and its load  $R_{\rm load}$ . Since the internal resistance r is in series with the load, it can significantly affect the terminal voltage and current delivered to the load. (Note that the script E stands for emf.)

We see from this expression that the smaller the internal resistance r, the greater the current the voltage source supplies to its load  $R_{\rm load}$ . As batteries are depleted, r increases. If r becomes a significant fraction of the load resistance, then the current is significantly reduced, as the following example illustrates.

# **Example:**

# Calculating Terminal Voltage, Power Dissipation, Current, and Resistance: Terminal Voltage and Load

A certain battery has a 12.0-V emf and an internal resistance of  $0.100~\Omega$ . (a) Calculate its terminal voltage when connected to a  $10.0-\Omega$  load. (b) What is the terminal voltage when connected to a  $0.500-\Omega$  load? (c) What power does the  $0.500-\Omega$  load dissipate? (d) If the internal resistance grows

to  $0.500~\Omega$ , find the current, terminal voltage, and power dissipated by a  $0.500-\Omega$  load.

## Strategy

The analysis above gave an expression for current when internal resistance is taken into account. Once the current is found, the terminal voltage can be calculated using the equation V = emf - Ir. Once current is found, the power dissipated by a resistor can also be found.

## Solution for (a)

Entering the given values for the emf, load resistance, and internal resistance into the expression above yields

## **Equation:**

$$I = rac{{
m emf}}{R_{
m load} + r} = rac{12.0 \ {
m V}}{10.1 \, \Omega} = 1.188 \ {
m A}.$$

Enter the known values into the equation V = emf - Ir to get the terminal voltage:

# **Equation:**

$$V = \text{emf} - Ir = 12.0 \text{ V} - (1.188 \text{ A})(0.100 \Omega)$$
  
= 11.9 V.

# Discussion for (a)

The terminal voltage here is only slightly lower than the emf, implying that  $10.0~\Omega$  is a light load for this particular battery.

## Solution for (b)

Similarly, with  $R_{\rm load} = 0.500 \,\Omega$ , the current is

# **Equation:**

$$I = rac{{
m emf}}{R_{
m load} + r} = rac{12.0 \ {
m V}}{0.600 \, \Omega} = 20.0 \ {
m A}.$$

The terminal voltage is now

# **Equation:**

$$V = \text{emf} - Ir = 12.0 \text{ V} - (20.0 \text{ A})(0.100 \Omega)$$
  
= 10.0 V.

## Discussion for (b)

This terminal voltage exhibits a more significant reduction compared with emf, implying  $0.500\,\Omega$  is a heavy load for this battery.

# Solution for (c)

The power dissipated by the 0.500 -  $\Omega$  load can be found using the formula  $P=I^2R$ . Entering the known values gives

# **Equation:**

$$P_{
m load} = I^2 R_{
m load} = (20.0 \ {
m A})^2 (0.500 \ \Omega) = 2.00 imes 10^2 \ {
m W}.$$

## Discussion for (c)

Note that this power can also be obtained using the expressions  $\frac{V^2}{R}$  or IV, where V is the terminal voltage (10.0 V in this case).

## Solution for (d)

Here the internal resistance has increased, perhaps due to the depletion of the battery, to the point where it is as great as the load resistance. As before, we first find the current by entering the known values into the expression, yielding

## **Equation:**

$$I = rac{{
m emf}}{R_{
m load} + r} = rac{12.0 \ {
m V}}{1.00 \ \Omega} = 12.0 \ {
m A}.$$

Now the terminal voltage is

# **Equation:**

$$V = \text{emf} - Ir = 12.0 \text{ V} - (12.0 \text{ A})(0.500 \Omega)$$
  
= 6.00 V,

and the power dissipated by the load is

# **Equation:**

$$P_{\rm load} = I^2 R_{\rm load} = (12.0 \text{ A})^2 (0.500 \Omega) = 72.0 \text{ W}.$$

# Discussion for (d)

We see that the increased internal resistance has significantly decreased terminal voltage, current, and power delivered to a load.

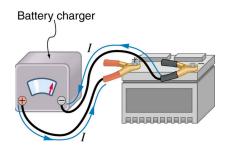
Battery testers, such as those in [link], use small load resistors to intentionally draw current to determine whether the terminal voltage drops below an acceptable level. They really test the internal resistance of the battery. If internal resistance is high, the battery is weak, as evidenced by its low terminal voltage.



These two battery testers measure terminal voltage under a load to determine the condition of a battery. The large device is being used by a U.S. Navy electronics technician to test large batteries aboard the aircraft carrier USS *Nimitz* and has a small resistance that can dissipate large amounts of power. (credit: U.S. Navy photo by Photographer's Mate Airman Jason A. Johnston) The small device is used on small batteries and has a digital display to indicate the acceptability of their terminal voltage. (credit: Keith Williamson)

Some batteries can be recharged by passing a current through them in the direction opposite to the current they supply to a resistance. This is done routinely in cars and batteries for small electrical appliances and electronic devices, and is represented pictorially in [link]. The voltage output of the battery charger must be greater than the emf of the battery to reverse current

through it. This will cause the terminal voltage of the battery to be greater than the emf, since V = emf - Ir, and I is now negative.



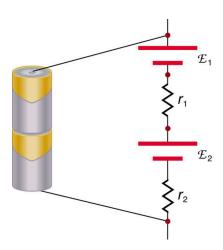
A car battery charger reverses the normal direction of current through a battery, reversing its chemical reaction and replenishing its chemical potential.

# **Multiple Voltage Sources**

There are two voltage sources when a battery charger is used. Voltage sources connected in series are relatively simple. When voltage sources are in series, their internal resistances add and their emfs add algebraically. (See [link].) Series connections of voltage sources are common—for example, in flashlights, toys, and other appliances. Usually, the cells are in series in order to produce a larger total emf.

But if the cells oppose one another, such as when one is put into an appliance backward, the total emf is less, since it is the algebraic sum of the individual emfs.

A battery is a multiple connection of voltaic cells, as shown in [link]. The disadvantage of series connections of cells is that their internal resistances add. One of the authors once owned a 1957 MGA that had two 6-V batteries in series, rather than a single 12-V battery. This arrangement produced a large internal resistance that caused him many problems in starting the engine.

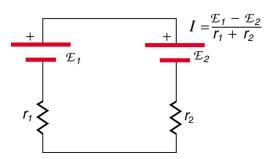


A series connection of two voltage sources. The emfs (each labeled with a script E) and internal resistances add, giving a total emf of emf $_1 + \text{emf}_2$  and a total internal resistance of  $r_1 + r_2$ .



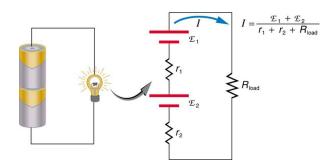
Batteries are multiple connections of individual cells, as shown in this modern rendition of an old print. Single cells, such as AA or C cells, are commonly called batteries, although this is technically incorrect.

If the *series* connection of two voltage sources is made into a complete circuit with the emfs in opposition, then a current of magnitude  $I = \frac{(\text{emf}_1 - \text{emf}_2)}{r_1 + r_2} \text{ flows. See } [\underline{\text{link}}] \text{, for example, which shows a circuit exactly analogous to the battery charger discussed above. If two voltage sources in series with emfs in the same sense are connected to a load <math>R_{\text{load}}$ , as in  $[\underline{\text{link}}]$ , then  $I = \frac{(\text{emf}_1 + \text{emf}_2)}{r_1 + r_2 + R_{\text{load}}}$  flows.



These two voltage sources are connected in series with their emfs in opposition. Current flows in the direction of the greater emf and is limited

to  $I=\frac{(\mathrm{emf_1-emf_2})}{r_1+r_2}$  by the sum of the internal resistances. (Note that each emf is represented by script E in the figure.) A battery charger connected to a battery is an example of such a connection. The charger must have a larger emf than the battery to reverse current through it.



This schematic represents a flashlight with two cells (voltage sources) and a single bulb (load resistance) in series. The current that flows is  $I = \frac{(\text{emf}_1 + \text{emf}_2)}{r_1 + r_2 + R_{\text{load}}}.$  (Note that each emf is represented by script E in the figure.)

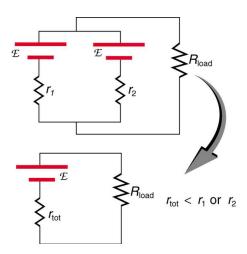
### Note:

# Take-Home Experiment: Flashlight Batteries

Find a flashlight that uses several batteries and find new and old batteries. Based on the discussions in this module, predict the brightness of the flashlight when different combinations of batteries are used. Do your predictions match what you observe? Now place new batteries in the flashlight and leave the flashlight switched on for several hours. Is the flashlight still quite bright? Do the same with the old batteries. Is the flashlight as bright when left on for the same length of time with old and new batteries? What does this say for the case when you are limited in the number of available new batteries?

[link] shows two voltage sources with identical emfs in parallel and connected to a load resistance. In this simple case, the total emf is the same as the individual emfs. But the total internal resistance is reduced, since the internal resistances are in parallel. The parallel connection thus can produce a larger current.

Here,  $I=\frac{\mathrm{emf}}{(r_{\mathrm{tot}}+R_{\mathrm{load}})}$  flows through the load, and  $r_{\mathrm{tot}}$  is less than those of the individual batteries. For example, some diesel-powered cars use two 12-V batteries in parallel; they produce a total emf of 12 V but can deliver the larger current needed to start a diesel engine.



Two voltage sources with identical emfs (each labeled by script E) connected in parallel produce the same emf but have a smaller total internal resistance than the individual sources. Parallel combinations are often used to deliver more current. Here  $I = \frac{\text{emf}}{(r_{\text{tot}} + R_{\text{load}})}$  flows through the load.

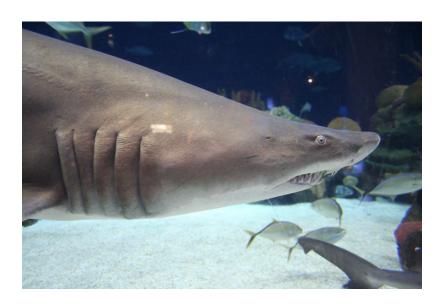
## **Animals as Electrical Detectors**

A number of animals both produce and detect electrical signals. Fish, sharks, platypuses, and echidnas (spiny anteaters) all detect electric fields generated by nerve activity in prey. Electric eels produce their own emf through biological cells (electric organs) called electroplaques, which are arranged in both series and parallel as a set of batteries.

Electroplaques are flat, disk-like cells; those of the electric eel have a voltage of 0.15 V across each one. These cells are usually located toward the head or tail of the animal, although in the case of the electric eel, they are found along the entire body. The electroplaques in the South American eel are arranged in 140 rows, with each row stretching horizontally along the body and containing 5,000 electroplaques. This can yield an emf of approximately 600 V, and a current of 1 A—deadly.

The mechanism for detection of external electric fields is similar to that for producing nerve signals in the cell through depolarization and

repolarization—the movement of ions across the cell membrane. Within the fish, weak electric fields in the water produce a current in a gel-filled canal that runs from the skin to sensing cells, producing a nerve signal. The Australian platypus, one of the very few mammals that lay eggs, can detect fields of 30  $\frac{\text{mV}}{\text{m}}$ , while sharks have been found to be able to sense a field in their snouts as small as 100  $\frac{\text{mV}}{\text{m}}$  ([link]). Electric eels use their own electric fields produced by the electroplaques to stun their prey or enemies.



Sand tiger sharks (*Carcharias taurus*), like this one at the Minnesota Zoo, use electroreceptors in their snouts to locate prey. (credit: Jim Winstead, Flickr)

# **Solar Cell Arrays**

Another example dealing with multiple voltage sources is that of combinations of solar cells—wired in both series and parallel combinations to yield a desired voltage and current. Photovoltaic generation (PV), the conversion of sunlight directly into electricity, is based upon the

photoelectric effect, in which photons hitting the surface of a solar cell create an electric current in the cell.

Most solar cells are made from pure silicon—either as single-crystal silicon, or as a thin film of silicon deposited upon a glass or metal backing. Most single cells have a voltage output of about 0.5 V, while the current output is a function of the amount of sunlight upon the cell (the incident solar radiation—the insolation). Under bright noon sunlight, a current of about  $100~\mathrm{mA/cm^2}$  of cell surface area is produced by typical single-crystal cells.

Individual solar cells are connected electrically in modules to meet electrical-energy needs. They can be wired together in series or in parallel —connected like the batteries discussed earlier. A solar-cell array or module usually consists of between 36 and 72 cells, with a power output of 50 W to 140 W.

The output of the solar cells is direct current. For most uses in a home, AC is required, so a device called an inverter must be used to convert the DC to AC. Any extra output can then be passed on to the outside electrical grid for sale to the utility.

#### Note:

# Take-Home Experiment: Virtual Solar Cells

One can assemble a "virtual" solar cell array by using playing cards, or business or index cards, to represent a solar cell. Combinations of these cards in series and/or parallel can model the required array output. Assume each card has an output of 0.5 V and a current (under bright light) of 2 A. Using your cards, how would you arrange them to produce an output of 6 A at 3 V (18 W)?

Suppose you were told that you needed only 18 W (but no required voltage). Would you need more cards to make this arrangement?

# **Section Summary**

- All voltage sources have two fundamental parts—a source of electrical energy that has a characteristic electromotive force (emf), and an internal resistance r.
- The emf is the potential difference of a source when no current is flowing.
- The numerical value of the emf depends on the source of potential difference.
- The internal resistance r of a voltage source affects the output voltage when a current flows.
- The voltage output of a device is called its terminal voltage V and is given by V = emf Ir, where I is the electric current and is positive when flowing away from the positive terminal of the voltage source.
- When multiple voltage sources are in series, their internal resistances add and their emfs add algebraically.
- Solar cells can be wired in series or parallel to provide increased voltage or current, respectively.

# **Conceptual Questions**

### **Exercise:**

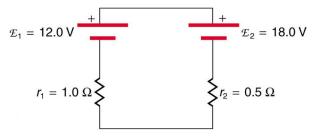
#### **Problem:**

Is every emf a potential difference? Is every potential difference an emf? Explain.

#### **Exercise:**

#### **Problem:**

Explain which battery is doing the charging and which is being charged in [link].



#### **Exercise:**

#### **Problem:**

Given a battery, an assortment of resistors, and a variety of voltage and current measuring devices, describe how you would determine the internal resistance of the battery.

#### **Exercise:**

## **Problem:**

Two different 12-V automobile batteries on a store shelf are rated at 600 and 850 "cold cranking amps." Which has the smallest internal resistance?

#### **Exercise:**

#### **Problem:**

What are the advantages and disadvantages of connecting batteries in series? In parallel?

#### **Exercise:**

#### Problem:

Semitractor trucks use four large 12-V batteries. The starter system requires 24 V, while normal operation of the truck's other electrical components utilizes 12 V. How could the four batteries be connected to produce 24 V? To produce 12 V? Why is 24 V better than 12 V for starting the truck's engine (a very heavy load)?

## **Problem Exercises**

### **Exercise:**

### **Problem:**

Standard automobile batteries have six lead-acid cells in series, creating a total emf of 12.0 V. What is the emf of an individual lead-acid cell?

# **Solution:**

2.00 V

#### Exercise:

### **Problem:**

Carbon-zinc dry cells (sometimes referred to as non-alkaline cells) have an emf of 1.54 V, and they are produced as single cells or in various combinations to form other voltages. (a) How many 1.54-V cells are needed to make the common 9-V battery used in many small electronic devices? (b) What is the actual emf of the approximately 9-V battery? (c) Discuss how internal resistance in the series connection of cells will affect the terminal voltage of this approximately 9-V battery.

## **Exercise:**

#### **Problem:**

What is the output voltage of a 3.0000-V lithium cell in a digital wristwatch that draws 0.300 mA, if the cell's internal resistance is  $2.00 \Omega$ ?

### **Solution:**

2.9994 V

(a) What is the terminal voltage of a large 1.54-V carbon-zinc dry cell used in a physics lab to supply 2.00 A to a circuit, if the cell's internal resistance is  $0.100~\Omega$ ? (b) How much electrical power does the cell produce? (c) What power goes to its load?

#### **Exercise:**

### **Problem:**

What is the internal resistance of an automobile battery that has an emf of 12.0 V and a terminal voltage of 15.0 V while a current of 8.00 A is charging it?

## **Solution:**

 $0.375 \Omega$ 

#### **Exercise:**

### **Problem:**

(a) Find the terminal voltage of a 12.0-V motorcycle battery having a  $0.600-\Omega$  internal resistance, if it is being charged by a current of 10.0 A. (b) What is the output voltage of the battery charger?

#### **Exercise:**

#### Problem:

A car battery with a 12-V emf and an internal resistance of  $0.050\,\Omega$  is being charged with a current of 60 A. Note that in this process the battery is being charged. (a) What is the potential difference across its terminals? (b) At what rate is thermal energy being dissipated in the battery? (c) At what rate is electric energy being converted to chemical energy? (d) What are the answers to (a) and (b) when the battery is used to supply 60 A to the starter motor?

The hot resistance of a flashlight bulb is  $2.30\,\Omega$ , and it is run by a 1.58-V alkaline cell having a 0.100- $\Omega$  internal resistance. (a) What current flows? (b) Calculate the power supplied to the bulb using  $I^2R_{\rm bulb}$ . (c) Is this power the same as calculated using  $\frac{V^2}{R_{\rm bulb}}$ ?

### **Solution:**

- (a) 0.658 A
- (b) 0.997 W
- (c) 0.997 W; yes

#### **Exercise:**

### **Problem:**

The label on a portable radio recommends the use of rechargeable nickel-cadmium cells (nicads), although they have a 1.25-V emf while alkaline cells have a 1.58-V emf. The radio has a  $3.20-\Omega$  resistance. (a) Draw a circuit diagram of the radio and its batteries. Now, calculate the power delivered to the radio. (b) When using Nicad cells each having an internal resistance of  $0.0400~\Omega$ . (c) When using alkaline cells each having an internal resistance of  $0.200~\Omega$ . (d) Does this difference seem significant, considering that the radio's effective resistance is lowered when its volume is turned up?

An automobile starter motor has an equivalent resistance of  $0.0500\,\Omega$  and is supplied by a 12.0-V battery with a  $0.0100\text{-}\Omega$  internal resistance. (a) What is the current to the motor? (b) What voltage is applied to it? (c) What power is supplied to the motor? (d) Repeat these calculations for when the battery connections are corroded and add  $0.0900\,\Omega$  to the circuit. (Significant problems are caused by even small amounts of unwanted resistance in low-voltage, high-current applications.)

### **Solution:**

- (a) 200 A
- (b) 10.0 V
- (c) 2.00 kW
- (d)  $0.1000 \Omega$ ; 80.0 A, 4.0 V, 320 W

#### **Exercise:**

#### **Problem:**

A child's electronic toy is supplied by three 1.58-V alkaline cells having internal resistances of  $0.0200\,\Omega$  in series with a 1.53-V carbonzinc dry cell having a  $0.100\text{-}\Omega$  internal resistance. The load resistance is  $10.0\,\Omega$ . (a) Draw a circuit diagram of the toy and its batteries. (b) What current flows? (c) How much power is supplied to the load? (d) What is the internal resistance of the dry cell if it goes bad, resulting in only 0.500 W being supplied to the load?

(a) What is the internal resistance of a voltage source if its terminal voltage drops by 2.00 V when the current supplied increases by 5.00 A? (b) Can the emf of the voltage source be found with the information supplied?

#### **Solution:**

- (a)  $0.400 \Omega$
- (b) No, there is only one independent equation, so only r can be found.

#### **Exercise:**

### **Problem:**

A person with body resistance between his hands of  $10.0~\mathrm{k}\Omega$  accidentally grasps the terminals of a 20.0-kV power supply. (Do NOT do this!) (a) Draw a circuit diagram to represent the situation. (b) If the internal resistance of the power supply is  $2000~\Omega$ , what is the current through his body? (c) What is the power dissipated in his body? (d) If the power supply is to be made safe by increasing its internal resistance, what should the internal resistance be for the maximum current in this situation to be  $1.00~\mathrm{mA}$  or less? (e) Will this modification compromise the effectiveness of the power supply for driving low-resistance devices? Explain your reasoning.

#### **Exercise:**

#### **Problem:**

Electric fish generate current with biological cells called electroplaques, which are physiological emf devices. The electroplaques in the South American eel are arranged in 140 rows, each row stretching horizontally along the body and each containing 5000 electroplaques. Each electroplaque has an emf of 0.15 V and internal resistance of  $0.25\,\Omega$ . If the water surrounding the fish has resistance of  $800\,\Omega$ , how much current can the eel produce in water from near its head to near its tail?

#### **Exercise:**

# **Problem:** Integrated Concepts

A 12.0-V emf automobile battery has a terminal voltage of 16.0 V when being charged by a current of 10.0 A. (a) What is the battery's internal resistance? (b) What power is dissipated inside the battery? (c) At what rate (in  ${}^{\circ}\text{C/min}$ ) will its temperature increase if its mass is 20.0 kg and it has a specific heat of 0.300 kcal/kg  ${}^{\circ}\text{C}$ , assuming no heat escapes?

## **Exercise:**

#### **Problem:** Unreasonable Results

A 1.58-V alkaline cell with a  $0.200-\Omega$  internal resistance is supplying 8.50 A to a load. (a) What is its terminal voltage? (b) What is the value of the load resistance? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable or inconsistent?

### **Solution:**

- (a) -0.120 V
- (b)  $-1.41 \times 10^{-2} \Omega$
- (c) Negative terminal voltage; negative load resistance.
- (d) The assumption that such a cell could provide 8.50 A is inconsistent with its internal resistance.

#### Exercise:

#### **Problem:** Unreasonable Results

(a) What is the internal resistance of a 1.54-V dry cell that supplies 1.00 W of power to a 15.0- $\Omega$  bulb? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

# **Glossary**

# electromotive force (emf)

the potential difference of a source of electricity when no current is flowing; measured in volts

## internal resistance

the amount of resistance within the voltage source

# potential difference

the difference in electric potential between two points in an electric circuit, measured in volts

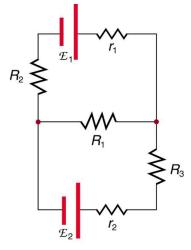
# terminal voltage

the voltage measured across the terminals of a source of potential difference

### Kirchhoff's Rules

• Analyze a complex circuit using Kirchhoff's rules, using the conventions for determining the correct signs of various terms.

Many complex circuits, such as the one in [link], cannot be analyzed with the series-parallel techniques developed in Resistors in Series and Parallel and Electromotive Force: Terminal Voltage. There are, however, two circuit analysis rules that can be used to analyze any circuit, simple or complex. These rules are special cases of the laws of conservation of charge and conservation of energy. The rules are known as **Kirchhoff's rules**, after their inventor Gustav Kirchhoff (1824–1887).



This circuit cannot be reduced to a combination of series and parallel connections. Kirchhoff's rules, special applications of the laws of conservation of charge and energy, can be

used to analyze it. (Note: The script E in the figure represents electromotive force, emf.)

#### Note:

## Kirchhoff's Rules

- Kirchhoff's first rule—the junction rule. The sum of all currents entering a junction must equal the sum of all currents leaving the junction.
- Kirchhoff's second rule—the loop rule. The algebraic sum of changes in potential around any closed circuit path (loop) must be zero.

Explanations of the two rules will now be given, followed by problemsolving hints for applying Kirchhoff's rules, and a worked example that uses them.

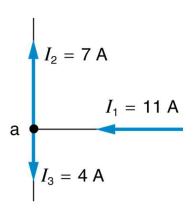
## Kirchhoff's First Rule

Kirchhoff's first rule (the **junction rule**) is an application of the conservation of charge to a junction; it is illustrated in [link]. Current is the flow of charge, and charge is conserved; thus, whatever charge flows into the junction must flow out. Kirchhoff's first rule requires that  $I_1 = I_2 + I_3$  (see figure). Equations like this can and will be used to analyze circuits and to solve circuit problems.

### Note:

# Making Connections: Conservation Laws

Kirchhoff's rules for circuit analysis are applications of **conservation laws** to circuits. The first rule is the application of conservation of charge, while the second rule is the application of conservation of energy. Conservation laws, even used in a specific application, such as circuit analysis, are so basic as to form the foundation of that application.



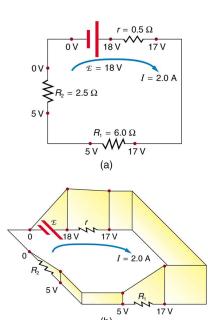
$$I_1 = I_2 + I_3$$

The junction rule. The diagram shows an example of Kirchhoff's first rule where the sum of the currents into a junction equals the sum of the currents out of a junction. In this case, the current going into the junction splits and comes out as two currents, so that  $I_1 = I_2 + I_3$ . Here  $I_1$  must be 11 A, since  $I_2$  is 7 A and  $I_3$  is 4 A.

## Kirchhoff's Second Rule

Kirchhoff's second rule (the **loop rule**) is an application of conservation of energy. The loop rule is stated in terms of potential, V, rather than potential energy, but the two are related since  $PE_{elec} = qV$ . Recall that **emf** is the potential difference of a source when no current is flowing. In a closed loop, whatever energy is supplied by emf must be transferred into other forms by devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit. [link] illustrates the changes in potential in a simple series circuit loop.

Kirchhoff's second rule requires emf  $-\operatorname{Ir} - \operatorname{IR}_1 - \operatorname{IR}_2 = 0$ . Rearranged, this is emf  $=\operatorname{Ir} + IR_1 + IR_2$ , which means the emf equals the sum of the IR (voltage) drops in the loop.



The loop rule. An example of Kirchhoff's second rule where the sum of the changes in potential around a closed loop must be zero. (a) In this standard schematic of a simple series circuit, the emf supplies 18 V, which is reduced to zero by the resistances, with 1 V across the internal resistance, and 12 V and 5 V across the two load resistances, for a total of 18 V. (b) This perspective view represents the

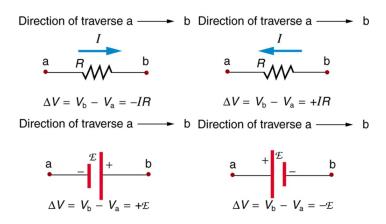
potential as something like a roller coaster, where charge is raised in potential by the emf and lowered by the resistances. (Note that the script E stands for emf.)

# **Applying Kirchhoff's Rules**

By applying Kirchhoff's rules, we generate equations that allow us to find the unknowns in circuits. The unknowns may be currents, emfs, or resistances. Each time a rule is applied, an equation is produced. If there are as many independent equations as unknowns, then the problem can be solved. There are two decisions you must make when applying Kirchhoff's rules. These decisions determine the signs of various quantities in the equations you obtain from applying the rules.

- 1. When applying Kirchhoff's first rule, the junction rule, you must label the current in each branch and decide in what direction it is going. For example, in [link], [link], and [link], currents are labeled  $I_1$ ,  $I_2$ ,  $I_3$ , and I, and arrows indicate their directions. There is no risk here, for if you choose the wrong direction, the current will be of the correct magnitude but negative.
- 2. When applying Kirchhoff's second rule, the loop rule, you must identify a closed loop and decide in which direction to go around it, clockwise or counterclockwise. For example, in [link] the loop was traversed in the same direction as the current (clockwise). Again, there is no risk; going around the circuit in the opposite direction reverses the sign of every term in the equation, which is like multiplying both sides of the equation by −1.

[link] and the following points will help you get the plus or minus signs right when applying the loop rule. Note that the resistors and emfs are traversed by going from a to b. In many circuits, it will be necessary to construct more than one loop. In traversing each loop, one needs to be consistent for the sign of the change in potential. (See [link].)



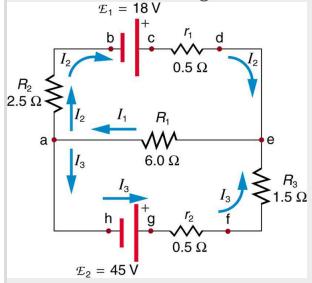
Each of these resistors and voltage sources is traversed from a to b. The potential changes are shown beneath each element and are explained in the text. (Note that the script E stands for emf.)

- When a resistor is traversed in the same direction as the current, the change in potential is —IR. (See [link].)
- When a resistor is traversed in the direction opposite to the current, the change in potential is +IR. (See [link].)
- When an emf is traversed from to + (the same direction it moves positive charge), the change in potential is +emf. (See [link].)
- When an emf is traversed from + to (opposite to the direction it moves positive charge), the change in potential is –emf. (See [link].)

## **Example:**

# **Calculating Current: Using Kirchhoff's Rules**

Find the currents flowing in the circuit in [link].



This circuit is similar to that in [link], but the resistances and emfs are specified. (Each emf is denoted by script E.) The currents in each branch are labeled and assumed to move in the directions shown. This example uses Kirchhoff's rules to find the currents.

# **Strategy**

This circuit is sufficiently complex that the currents cannot be found using Ohm's law and the series-parallel techniques—it is necessary to use Kirchhoff's rules. Currents have been labeled  $I_1$ ,  $I_2$ , and  $I_3$  in the figure and assumptions have been made about their directions. Locations on the diagram have been labeled with letters a through h. In the solution we will apply the junction and loop rules, seeking three independent equations to allow us to solve for the three unknown currents.

#### Solution

We begin by applying Kirchhoff's first or junction rule at point a. This gives

## **Equation:**

$$I_1=I_2+I_3,$$

since  $I_1$  flows into the junction, while  $I_2$  and  $I_3$  flow out. Applying the junction rule at e produces exactly the same equation, so that no new information is obtained. This is a single equation with three unknowns—three independent equations are needed, and so the loop rule must be applied.

Now we consider the loop abcdea. Going from a to b, we traverse  $R_2$  in the same (assumed) direction of the current  $I_2$ , and so the change in potential is  $-I_2R_2$ . Then going from b to c, we go from – to +, so that the change in potential is  $+\mathrm{emf}_1$ . Traversing the internal resistance  $r_1$  from c to d gives  $-I_2r_1$ . Completing the loop by going from d to a again traverses a resistor in the same direction as its current, giving a change in potential of  $-I_1R_1$ .

The loop rule states that the changes in potential sum to zero. Thus,

# **Equation:**

$$-I_2R_2+\mathrm{emf}_1-I_2r_1-I_1R_1=-I_2(R_2+r_1)+\mathrm{emf}_1-I_1R_1=0.$$

Substituting values from the circuit diagram for the resistances and emf, and canceling the ampere unit gives

# **Equation:**

$$-3I_2 + 18 - 6I_1 = 0.$$

Now applying the loop rule to aefgha (we could have chosen abcdefgha as well) similarly gives

# **Equation:**

$$+I_1R_1+I_3R_3+I_3r_2-\mathrm{emf}_2=+I_1R_1+I_3(R_3+r_2)-\mathrm{emf}_2=0.$$

Note that the signs are reversed compared with the other loop, because elements are traversed in the opposite direction. With values entered, this becomes

# **Equation:**

$$+6I_1+2I_3-45=0.$$

These three equations are sufficient to solve for the three unknown currents. First, solve the second equation for  $I_2$ :

## **Equation:**

$$I_2 = 6 - 2I_1$$
.

Now solve the third equation for  $I_3$ :

## **Equation:**

$$I_3 = 22.5 - 3I_1$$
.

Substituting these two new equations into the first one allows us to find a value for  $I_1$ :

## **Equation:**

$$I_1 = I_2 + I_3 = (6 - 2I_1) + (22.5 - 3I_1) = 28.5 - 5I_1.$$

Combining terms gives

# **Equation:**

$$6I_1 = 28.5$$
, and

# **Equation:**

$$I_1 = 4.75 \text{ A}.$$

Substituting this value for  $I_1$  back into the fourth equation gives

# **Equation:**

$$I_2 = 6 - 2I_1 = 6 - 9.50$$

# **Equation:**

$$I_2 = -3.50 \text{ A}.$$

The minus sign means  $I_2$  flows in the direction opposite to that assumed in  $[\underline{link}]$ .

Finally, substituting the value for  $I_1$  into the fifth equation gives

# **Equation:**

$$I_3 = 22.5 - 3I_1 = 22.5 - 14.25$$

# **Equation:**

$$I_3 = 8.25 \text{ A}.$$

### Discussion

Just as a check, we note that indeed  $I_1 = I_2 + I_3$ . The results could also have been checked by entering all of the values into the equation for the abcdefgha loop.

#### Note:

# Problem-Solving Strategies for Kirchhoff's Rules

- 1. Make certain there is a clear circuit diagram on which you can label all known and unknown resistances, emfs, and currents. If a current is unknown, you must assign it a direction. This is necessary for determining the signs of potential changes. If you assign the direction incorrectly, the current will be found to have a negative value—no harm done.
- 2. Apply the junction rule to any junction in the circuit. Each time the junction rule is applied, you should get an equation with a current that does not appear in a previous application—if not, then the equation is redundant.
- 3. Apply the loop rule to as many loops as needed to solve for the unknowns in the problem. (There must be as many independent equations as unknowns.) To apply the loop rule, you must choose a direction to go around the loop. Then carefully and consistently determine the signs of the potential changes for each element using the four bulleted points discussed above in conjunction with [link].
- 4. Solve the simultaneous equations for the unknowns. This may involve many algebraic steps, requiring careful checking and rechecking.
- 5. Check to see whether the answers are reasonable and consistent. The numbers should be of the correct order of magnitude, neither

exceedingly large nor vanishingly small. The signs should be reasonable—for example, no resistance should be negative. Check to see that the values obtained satisfy the various equations obtained from applying the rules. The currents should satisfy the junction rule, for example.

The material in this section is correct in theory. We should be able to verify it by making measurements of current and voltage. In fact, some of the devices used to make such measurements are straightforward applications of the principles covered so far and are explored in the next modules. As we shall see, a very basic, even profound, fact results—making a measurement alters the quantity being measured.

#### **Exercise:**

# **Check Your Understanding**

### **Problem:**

Can Kirchhoff's rules be applied to simple series and parallel circuits or are they restricted for use in more complicated circuits that are not combinations of series and parallel?

### **Solution:**

Kirchhoff's rules can be applied to any circuit since they are applications to circuits of two conservation laws. Conservation laws are the most broadly applicable principles in physics. It is usually mathematically simpler to use the rules for series and parallel in simpler circuits so we emphasize Kirchhoff's rules for use in more complicated situations. But the rules for series and parallel can be derived from Kirchhoff's rules. Moreover, Kirchhoff's rules can be expanded to devices other than resistors and emfs, such as capacitors, and are one of the basic analysis devices in circuit analysis.

# **Section Summary**

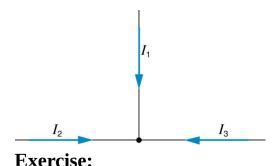
- Kirchhoff's rules can be used to analyze any circuit, simple or complex.
- Kirchhoff's first rule—the junction rule: The sum of all currents entering a junction must equal the sum of all currents leaving the junction.
- Kirchhoff's second rule—the loop rule: The algebraic sum of changes in potential around any closed circuit path (loop) must be zero.
- The two rules are based, respectively, on the laws of conservation of charge and energy.
- When calculating potential and current using Kirchhoff's rules, a set of conventions must be followed for determining the correct signs of various terms.
- The simpler series and parallel rules are special cases of Kirchhoff's rules.

# **Conceptual Questions**

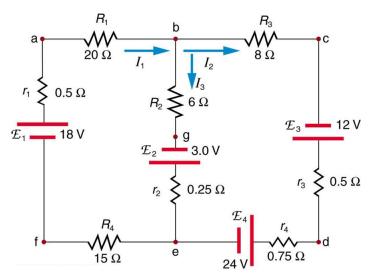
### **Exercise:**

## **Problem:**

Can all of the currents going into the junction in [link] be positive? Explain.



Apply the junction rule to junction b in [link]. Is any new information gained by applying the junction rule at e? (In the figure, each emf is represented by script E.)



### **Exercise:**

### **Problem:**

(a) What is the potential difference going from point a to point b in [link]? (b) What is the potential difference going from c to b? (c) From e to g? (d) From e to d?

### **Exercise:**

**Problem:** Apply the loop rule to loop afedcba in [link].

## **Exercise:**

**Problem:** Apply the loop rule to loops abgefa and cbgedc in [link].

# **Problem Exercises**

# **Exercise:**

**Problem:** Apply the loop rule to loop abcdefgha in [link].

# **Solution:**

# **Equation:**

$$-I_2R_2 + \mathrm{emf}_1 - \mathrm{I}_2r_1 + \mathrm{I}_3R_3 + \mathrm{I}_3r_2 - \mathrm{emf}_2 = 0$$

## **Exercise:**

**Problem:** Apply the loop rule to loop aedcba in [link].

# **Exercise:**

## **Problem:**

Verify the second equation in  $[\underline{link}]$  by substituting the values found for the currents  $I_1$  and  $I_2$ .

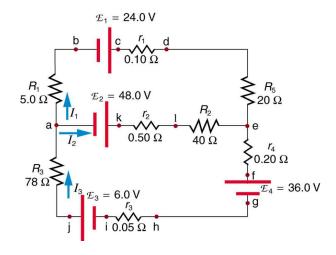
### **Exercise:**

### **Problem:**

Verify the third equation in  $[\underline{link}]$  by substituting the values found for the currents  $I_1$  and  $I_3$ .

# **Exercise:**

**Problem:** Apply the junction rule at point a in [link].



**Solution: Equation:** 

$$I_3 = I_1 + I_2$$

**Exercise:** 

**Problem:** Apply the loop rule to loop abcdefghija in [link].

**Exercise:** 

**Problem:** Apply the loop rule to loop akledcba in [<u>link</u>].

Solution: Equation:

$$\mathrm{emf}_2 - I_2 r_2 - I_2 R_2 + I_1 R_5 + I_1 r_1 - \mathrm{emf}_1 + I_1 R_1 = 0$$

**Exercise:** 

**Problem:** 

Find the currents flowing in the circuit in [<u>link</u>]. Explicitly show how you follow the steps in the <u>Problem-Solving Strategies for Series and Parallel Resistors</u>.

Solve [link], but use loop abcdefgha instead of loop akledcba. Explicitly show how you follow the steps in the Problem-Solving Strategies for Series and Parallel Resistors.

## **Solution:**

- (a)  $I_1 = 4.75 \text{ A}$
- (b)  $I_2 = -3.5 \text{ A}$
- (c)  $I_3 = 8.25 \text{ A}$

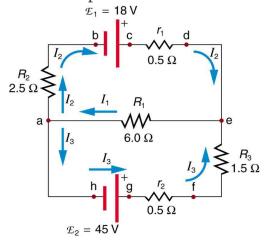
## **Exercise:**

**Problem:** Find the currents flowing in the circuit in [link].

## **Exercise:**

### **Problem:** Unreasonable Results

Consider the circuit in [link], and suppose that the emfs are unknown and the currents are given to be  $I_1 = 5.00 \text{ A}$ ,  $I_2 = 3.0 \text{ A}$ , and  $I_3 = -2.00 \text{ A}$ . (a) Could you find the emfs? (b) What is wrong with the assumptions?



## **Solution:**

- (a) No, you would get inconsistent equations to solve.
- (b)  $I_1 \neq I_2 + I_3$ . The assumed currents violate the junction rule.

# **Glossary**

### Kirchhoff's rules

a set of two rules, based on conservation of charge and energy, governing current and changes in potential in an electric circuit

# junction rule

Kirchhoff's first rule, which applies the conservation of charge to a junction; current is the flow of charge; thus, whatever charge flows into the junction must flow out; the rule can be stated  $I_1 = I_2 + I_3$ 

# loop rule

Kirchhoff's second rule, which states that in a closed loop, whatever energy is supplied by emf must be transferred into other forms by devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit. Thus, the emf equals the sum of the IR (voltage) drops in the loop and can be stated:

$$\mathrm{emf} = Ir + IR_1 + IR_2$$

#### conservation laws

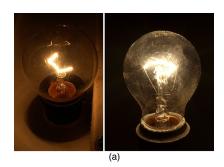
require that energy and charge be conserved in a system

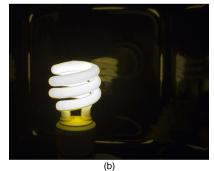
## Electric Power and Energy

- Calculate the power dissipated by a resistor and power supplied by a power supply.
- Calculate the cost of electricity under various circumstances.

### **Power in Electric Circuits**

Power is associated by many people with electricity. Knowing that power is the rate of energy use or energy conversion, what is the expression for **electric power**? Power transmission lines might come to mind. We also think of lightbulbs in terms of their power ratings in watts. Let us compare a 25-W bulb with a 60-W bulb. (See [link](a).) Since both operate on the same voltage, the 60-W bulb must draw more current to have a greater power rating. Thus the 60-W bulb's resistance must be lower than that of a 25-W bulb. If we increase voltage, we also increase power. For example, when a 25-W bulb that is designed to operate on 120 V is connected to 240 V, it briefly glows very brightly and then burns out. Precisely how are voltage, current, and resistance related to electric power?





(a) Which of these lightbulbs, the 25-W bulb (upper left) or the 60-W bulb (upper right), has the higher resistance? Which draws more current? Which uses the most energy? Can you tell from the color that the 25-W filament is cooler? Is the brighter bulb a different color and if so why? (credits: Dickbauch. Wikimedia Commons; Greg Westfall, Flickr) (b) This compact fluorescent light (CFL) puts out the same intensity of light as the 60-W bulb, but at 1/4 to 1/10 the input power. (credit: dbgg1979, Flickr)

Electric energy depends on both the voltage involved and the charge moved. This is expressed most simply as PE = qV, where q is the charge moved and V is the voltage (or more precisely, the potential difference the

charge moves through). Power is the rate at which energy is moved, and so electric power is

### **Equation:**

$$P = \frac{\mathrm{PE}}{t} = \frac{\mathrm{qV}}{t}.$$

Recognizing that current is I=q/t (note that  $\Delta t=t$  here), the expression for power becomes

# **Equation:**

$$P = IV$$
.

Electric power (P) is simply the product of current times voltage. Power has familiar units of watts. Since the SI unit for potential energy (PE) is the joule, power has units of joules per second, or watts. Thus,  $1 \text{ A} \cdot \text{V} = 1 \text{ W}$ . For example, cars often have one or more auxiliary power outlets with which you can charge a cell phone or other electronic devices. These outlets may be rated at 20 A, so that the circuit can deliver a maximum power P = IV = (20 A)(12 V) = 240 W. In some applications, electric power may be expressed as volt-amperes or even kilovolt-amperes ( $1 \text{ kA} \cdot \text{V} = 1 \text{ kW}$ ).

To see the relationship of power to resistance, we combine Ohm's law with P = IV. Substituting I = V/R gives  $P = (V/R)V = V^2/R$ . Similarly, substituting V = IR gives  $P = I(IR) = I^2R$ . Three expressions for electric power are listed together here for convenience:

# **Equation:**

$$P = IV$$

**Equation:** 

$$P = \frac{V^2}{R}$$

$$P = I^2 R$$
.

Note that the first equation is always valid, whereas the other two can be used only for resistors. In a simple circuit, with one voltage source and a single resistor, the power supplied by the voltage source and that dissipated by the resistor are identical. (In more complicated circuits, P can be the power dissipated by a single device and not the total power in the circuit.)

Different insights can be gained from the three different expressions for electric power. For example,  $P=V^2/R$  implies that the lower the resistance connected to a given voltage source, the greater the power delivered. Furthermore, since voltage is squared in  $P=V^2/R$ , the effect of applying a higher voltage is perhaps greater than expected. Thus, when the voltage is doubled to a 25-W bulb, its power nearly quadruples to about 100 W, burning it out. If the bulb's resistance remained constant, its power would be exactly 100 W, but at the higher temperature its resistance is higher, too.

# **Example:**

# Calculating Power Dissipation and Current: Hot and Cold Power

(a) Consider the examples given in <u>Ohm's Law: Resistance and Simple Circuits</u> and <u>Resistance and Resistivity</u>. Then find the power dissipated by the car headlight in these examples, both when it is hot and when it is cold.

(b) What current does it draw when cold?

# Strategy for (a)

For the hot headlight, we know voltage and current, so we can use  $P=\mathrm{IV}$  to find the power. For the cold headlight, we know the voltage and resistance, so we can use  $P=V^2/R$  to find the power.

# Solution for (a)

Entering the known values of current and voltage for the hot headlight, we obtain

$$P = IV = (2.50 \text{ A})(12.0 \text{ V}) = 30.0 \text{ W}.$$

The cold resistance was  $0.350~\Omega$ , and so the power it uses when first switched on is

# **Equation:**

$$P = rac{V^2}{R} = rac{(12.0 \text{ V})^2}{0.350 \Omega} = 411 \text{ W}.$$

# Discussion for (a)

The 30 W dissipated by the hot headlight is typical. But the 411 W when cold is surprisingly higher. The initial power quickly decreases as the bulb's temperature increases and its resistance increases.

# Strategy and Solution for (b)

The current when the bulb is cold can be found several different ways. We rearrange one of the power equations,  $P = I^2R$ , and enter known values, obtaining

## **Equation:**

$$I = \sqrt{rac{P}{R}} = \sqrt{rac{411 \ \mathrm{W}}{0.350 \ \Omega}} = 34.3 \ \mathrm{A}.$$

# Discussion for (b)

The cold current is remarkably higher than the steady-state value of 2.50 A, but the current will quickly decline to that value as the bulb's temperature increases. Most fuses and circuit breakers (used to limit the current in a circuit) are designed to tolerate very high currents briefly as a device comes on. In some cases, such as with electric motors, the current remains high for several seconds, necessitating special "slow blow" fuses.

# The Cost of Electricity

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. Since P=E/t, we see that

is the energy used by a device using power P for a time interval t. For example, the more lightbulbs burning, the greater P used; the longer they are on, the greater t is. The energy unit on electric bills is the kilowatt-hour  $(kW \cdot h)$ , consistent with the relationship E = Pt. It is easy to estimate the cost of operating electric appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized energy units such as food calories, can be converted to joules. You can prove to yourself that  $1 \ kW \cdot h = 3.6 \times 10^6 \ J$ .

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This will not only reduce the cost, but it will also result in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home's use of energy goes to lighting, while the number for commercial establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFL). (See [link](b).) Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiralshaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.) The heat transfer from these CFLs is less, and they last up to 10 times longer. The significance of an investment in such bulbs is addressed in the next example. New white LED lights (which are clusters of small LED bulbs) are even more efficient (twice that of CFLs) and last 5 times longer than CFLs. However, their cost is still high.

#### Note:

Making Connections: Energy, Power, and Time

The relationship  $E=\mathrm{Pt}$  is one that you will find useful in many different contexts. The energy your body uses in exercise is related to the power level and duration of your activity, for example. The amount of heating by a power source is related to the power level and time it is applied. Even the radiation dose of an X-ray image is related to the power and time of exposure.

# **Example:**

# Calculating the Cost Effectiveness of Compact Fluorescent Lights (CFL)

If the cost of electricity in your area is 12 cents per kWh, what is the total cost (capital plus operation) of using a 60-W incandescent bulb for 1000 hours (the lifetime of that bulb) if the bulb cost 25 cents? (b) If we replace this bulb with a compact fluorescent light that provides the same light output, but at one-quarter the wattage, and which costs \$1.50 but lasts 10 times longer (10,000 hours), what will that total cost be?

# Strategy

To find the operating cost, we first find the energy used in kilowatt-hours and then multiply by the cost per kilowatt-hour.

# Solution for (a)

The energy used in kilowatt-hours is found by entering the power and time into the expression for energy:

# **Equation:**

$$E = Pt = (60 \text{ W})(1000 \text{ h}) = 60,000 \text{ W} \cdot \text{h}.$$

In kilowatt-hours, this is

# **Equation:**

$$E = 60.0 \text{ kW} \cdot \text{h}.$$

Now the electricity cost is

$$cost = (60.0 \text{ kW} \cdot \text{h})(\$0.12/\text{kW} \cdot \text{h}) = \$7.20.$$

The total cost will be \$7.20 for 1000 hours (about one-half year at 5 hours per day).

## **Solution for (b)**

Since the CFL uses only 15 W and not 60 W, the electricity cost will be \$7.20/4 = \$1.80. The CFL will last 10 times longer than the incandescent, so that the investment cost will be 1/10 of the bulb cost for that time period of use, or 0.1(\$1.50) = \$0.15. Therefore, the total cost will be \$1.95 for 1000 hours.

### Discussion

Therefore, it is much cheaper to use the CFLs, even though the initial investment is higher. The increased cost of labor that a business must include for replacing the incandescent bulbs more often has not been figured in here.

#### Note:

# Making Connections: Take-Home Experiment—Electrical Energy Use Inventory

1) Make a list of the power ratings on a range of appliances in your home or room. Explain why something like a toaster has a higher rating than a digital clock. Estimate the energy consumed by these appliances in an average day (by estimating their time of use). Some appliances might only state the operating current. If the household voltage is 120 V, then use P = IV. 2) Check out the total wattage used in the rest rooms of your school's floor or building. (You might need to assume the long fluorescent lights in use are rated at 32 W.) Suppose that the building was closed all weekend and that these lights were left on from 6 p.m. Friday until 8 a.m. Monday. What would this oversight cost? How about for an entire year of weekends?

# **Section Summary**

• Electric power *P* is the rate (in watts) that energy is supplied by a source or dissipated by a device.

•	Three expressions for electrical power are
	<b>Equation:</b>

$$P = IV$$
,

**Equation:** 

$$P = \frac{V^2}{R},$$

and

**Equation:** 

$$P = I^2 R$$
.

• The energy used by a device with a power P over a time t is  $E=\operatorname{Pt}$ .

# **Conceptual Questions**

### **Exercise:**

### **Problem:**

Why do incandescent lightbulbs grow dim late in their lives, particularly just before their filaments break?

### **Exercise:**

### **Problem:**

The power dissipated in a resistor is given by  $P=V^2/R$ , which means power decreases if resistance increases. Yet this power is also given by  $P=I^2R$ , which means power increases if resistance increases. Explain why there is no contradiction here.

# **Problem Exercises**

What is the power of a  $1.00 \times 10^2$  MV lightning bolt having a current of  $2.00 \times 10^4$  A?

### **Solution:**

 $2.00 \times 10^{12} \text{ W}$ 

### **Exercise:**

#### **Problem:**

What power is supplied to the starter motor of a large truck that draws 250 A of current from a 24.0-V battery hookup?

### **Exercise:**

### **Problem:**

A charge of 4.00 C of charge passes through a pocket calculator's solar cells in 4.00 h. What is the power output, given the calculator's voltage output is 3.00 V? (See [link].)



The strip of solar cells just above the keys of this calculator convert

```
light to electricity
to supply its energy
needs. (credit:
Evan-Amos,
Wikimedia
Commons)
```

### **Problem:**

How many watts does a flashlight that has  $6.00 \times 10^2$  C pass through it in 0.500 h use if its voltage is 3.00 V?

### **Exercise:**

#### **Problem:**

Find the power dissipated in each of these extension cords: (a) an extension cord having a 0.0600 -  $\Omega$  resistance and through which 5.00 A is flowing; (b) a cheaper cord utilizing thinner wire and with a resistance of  $0.300~\Omega$ .

### **Solution:**

- (a) 1.50 W
- (b) 7.50 W

#### **Exercise:**

#### **Problem:**

Verify that the units of a volt-ampere are watts, as implied by the equation P = IV.

Show that the units  $1~{
m V}^2/\Omega=1{
m W}$ , as implied by the equation  $P=V^2/R$ .

### **Solution:**

$$\frac{V^2}{\Omega} = \frac{V^2}{V/A} = AV = \left(\frac{C}{s}\right)\left(\frac{J}{C}\right) = \frac{J}{s} = 1 \text{ W}$$

### **Exercise:**

### **Problem:**

Show that the units  $1 A^2 \cdot \Omega = 1 W$ , as implied by the equation  $P = I^2 R$ .

### **Exercise:**

### **Problem:**

Verify the energy unit equivalence that  $1 \text{ kW} \cdot \text{h} = 3.60 \times 10^6 \text{ J}$ .

### **Solution:**

$$1~{
m kW}\cdot{
m h}{
m =}{\left(rac{1 imes10^3~{
m J}}{1~{
m s}}
ight)}(1~{
m h}){\left(rac{3600~{
m s}}{1~{
m h}}
ight)}=3.60 imes10^6~{
m J}$$

### **Exercise:**

## **Problem:**

Electrons in an X-ray tube are accelerated through  $1.00 \times 10^2 \ kV$  and directed toward a target to produce X-rays. Calculate the power of the electron beam in this tube if it has a current of 15.0 mA.

An electric water heater consumes 5.00 kW for 2.00 h per day. What is the cost of running it for one year if electricity costs  $12.0 \text{ cents/kW} \cdot \text{h}$ ? See [link].



On-demand electric hot water heater. Heat is supplied to water only when needed. (credit: aviddavid, Flickr)

### **Solution:**

\$438/y

### **Exercise:**

### **Problem:**

With a 1200-W toaster, how much electrical energy is needed to make a slice of toast (cooking time = 1 minute)? At  $9.0 \text{ cents/kW} \cdot h$ , how much does this cost?

What would be the maximum cost of a CFL such that the total cost (investment plus operating) would be the same for both CFL and incandescent 60-W bulbs? Assume the cost of the incandescent bulb is 25 cents and that electricity costs 10 cents/kWh. Calculate the cost for 1000 hours, as in the cost effectiveness of CFL example.

### **Solution:**

\$6.25

#### **Exercise:**

### **Problem:**

Some makes of older cars have 6.00-V electrical systems. (a) What is the hot resistance of a 30.0-W headlight in such a car? (b) What current flows through it?

#### **Exercise:**

### **Problem:**

Alkaline batteries have the advantage of putting out constant voltage until very nearly the end of their life. How long will an alkaline battery rated at  $1.00~{\rm A}\cdot{\rm h}$  and  $1.58~{\rm V}$  keep a  $1.00-{\rm W}$  flashlight bulb burning?

#### **Solution:**

1.58 h

### **Exercise:**

### **Problem:**

A cauterizer, used to stop bleeding in surgery, puts out 2.00 mA at 15.0 kV. (a) What is its power output? (b) What is the resistance of the path?

The average television is said to be on 6 hours per day. Estimate the yearly cost of electricity to operate 100 million TVs, assuming their power consumption averages 150 W and the cost of electricity averages  $12.0 \; \text{cents/kW} \cdot \text{h}$ .

### **Solution:**

\$3.94 billion/year

### **Exercise:**

#### **Problem:**

An old lightbulb draws only 50.0 W, rather than its original 60.0 W, due to evaporative thinning of its filament. By what factor is its diameter reduced, assuming uniform thinning along its length? Neglect any effects caused by temperature differences.

#### **Exercise:**

### **Problem:**

00-gauge copper wire has a diameter of 9.266 mm. Calculate the power loss in a kilometer of such wire when it carries  $1.00\times10^2~A$ .

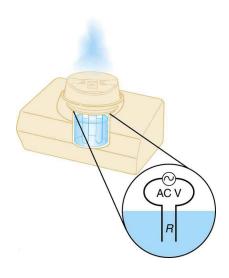
#### **Solution:**

25.5 W

#### **Exercise:**

# **Problem:** Integrated Concepts

Cold vaporizers pass a current through water, evaporating it with only a small increase in temperature. One such home device is rated at 3.50 A and utilizes 120 V AC with 95.0% efficiency. (a) What is the vaporization rate in grams per minute? (b) How much water must you put into the vaporizer for 8.00 h of overnight operation? (See [link].)



This cold vaporizer passes current directly through water, vaporizing it directly with relatively little temperature increase.

# **Problem:** Integrated Concepts

(a) What energy is dissipated by a lightning bolt having a 20,000-A current, a voltage of  $1.00 \times 10^2$  MV, and a length of 1.00 ms? (b) What mass of tree sap could be raised from  $18.0^{\circ}$ C to its boiling point and then evaporated by this energy, assuming sap has the same thermal characteristics as water?

# **Solution:**

- (a)  $2.00 \times 10^9 \text{ J}$
- (b) 769 kg

# **Problem:** Integrated Concepts

What current must be produced by a 12.0-V battery-operated bottle warmer in order to heat 75.0 g of glass, 250 g of baby formula, and  $3.00 \times 10^2$  g of aluminum from  $20.0^{\circ}$ C to  $90.0^{\circ}$ C in 5.00 min?

### **Exercise:**

### **Problem:** Integrated Concepts

How much time is needed for a surgical cauterizer to raise the temperature of 1.00 g of tissue from 37.0°C to 100°C and then boil away 0.500 g of water, if it puts out 2.00 mA at 15.0 kV? Ignore heat transfer to the surroundings.

### **Solution:**

45.0 s

### **Exercise:**

# **Problem:** Integrated Concepts

Hydroelectric generators (see [link]) at Hoover Dam produce a maximum current of  $8.00 \times 10^3$  A at 250 kV. (a) What is the power output? (b) The water that powers the generators enters and leaves the system at low speed (thus its kinetic energy does not change) but loses 160 m in altitude. How many cubic meters per second are needed, assuming 85.0% efficiency?



Hydroelectric generators at the Hoover dam. (credit: Jon Sullivan)

# **Problem:** Integrated Concepts

(a) Assuming 95.0% efficiency for the conversion of electrical power by the motor, what current must the 12.0-V batteries of a 750-kg electric car be able to supply: (a) To accelerate from rest to 25.0 m/s in 1.00 min? (b) To climb a  $2.00 \times 10^2$ -m-high hill in 2.00 min at a constant 25.0-m/s speed while exerting  $5.00 \times 10^2$  N of force to overcome air resistance and friction? (c) To travel at a constant 25.0-m/s speed, exerting a  $5.00 \times 10^2$  N force to overcome air resistance and friction? See [link].



This REVAi, an electric

car, gets recharged on a street in London. (credit: Frank Hebbert)

### **Solution:**

- (a) 343 A
- (b)  $2.17 \times 10^3$  A
- (c)  $1.10 \times 10^3$  A

#### **Exercise:**

### **Problem:** Integrated Concepts

A light-rail commuter train draws 630 A of 650-V DC electricity when accelerating. (a) What is its power consumption rate in kilowatts? (b) How long does it take to reach 20.0 m/s starting from rest if its loaded mass is  $5.30\times10^4$  kg, assuming 95.0% efficiency and constant power? (c) Find its average acceleration. (d) Discuss how the acceleration you found for the light-rail train compares to what might be typical for an automobile.

#### **Exercise:**

# **Problem:** Integrated Concepts

(a) An aluminum power transmission line has a resistance of  $0.0580~\Omega/\mathrm{km}$ . What is its mass per kilometer? (b) What is the mass per kilometer of a copper line having the same resistance? A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

#### **Solution:**

(a) 
$$1.23 \times 10^3 \text{ kg}$$

(b) 
$$2.64 \times 10^3 \text{ kg}$$

# **Problem:** Integrated Concepts

(a) An immersion heater utilizing 120 V can raise the temperature of a  $1.00 \times 10^2$ -g aluminum cup containing 350 g of water from 20.0°C to 95.0°C in 2.00 min. Find its resistance, assuming it is constant during the process. (b) A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

### **Exercise:**

# **Problem:** Integrated Concepts

(a) What is the cost of heating a hot tub containing 1500 kg of water from  $10.0^{\circ}$ C to  $40.0^{\circ}$ C, assuming 75.0% efficiency to account for heat transfer to the surroundings? The cost of electricity is  $9 \text{ cents/kW} \cdot h$ . (b) What current was used by the 220-V AC electric heater, if this took 4.00 h?

### **Exercise:**

### **Problem:** Unreasonable Results

(a) What current is needed to transmit  $1.00 \times 10^2$  MW of power at 480 V? (b) What power is dissipated by the transmission lines if they have a 1.00 -  $\Omega$  resistance? (c) What is unreasonable about this result? (d) Which assumptions are unreasonable, or which premises are inconsistent?

### **Solution:**

(a) 
$$2.08 \times 10^5 \text{ A}$$

- (b)  $4.33 \times 10^4 \text{ MW}$
- (c) The transmission lines dissipate more power than they are supposed to transmit.
- (d) A voltage of 480 V is unreasonably low for a transmission voltage. Long-distance transmission lines are kept at much higher voltages (often hundreds of kilovolts) to reduce power losses.

#### **Problem:** Unreasonable Results

(a) What current is needed to transmit  $1.00 \times 10^2$  MW of power at 10.0 kV? (b) Find the resistance of 1.00 km of wire that would cause a 0.0100% power loss. (c) What is the diameter of a 1.00-km-long copper wire having this resistance? (d) What is unreasonable about these results? (e) Which assumptions are unreasonable, or which premises are inconsistent?

#### **Exercise:**

### **Problem:** Construct Your Own Problem

Consider an electric immersion heater used to heat a cup of water to make tea. Construct a problem in which you calculate the needed resistance of the heater so that it increases the temperature of the water and cup in a reasonable amount of time. Also calculate the cost of the electrical energy used in your process. Among the things to be considered are the voltage used, the masses and heat capacities involved, heat losses, and the time over which the heating takes place. Your instructor may wish for you to consider a thermal safety switch (perhaps bimetallic) that will halt the process before damaging temperatures are reached in the immersion unit.

# Glossary

# electric power

the rate at which electrical energy is supplied by a source or dissipated by a device; it is the product of current times voltage

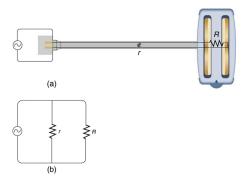
### Electric Hazards and the Human Body

- Define thermal hazard, shock hazard, and short circuit.
- Explain what effects various levels of current have on the human body.

There are two known hazards of electricity—thermal and shock. A **thermal hazard** is one where excessive electric power causes undesired thermal effects, such as starting a fire in the wall of a house. A **shock hazard** occurs when electric current passes through a person. Shocks range in severity from painful, but otherwise harmless, to heart-stopping lethality. This section considers these hazards and the various factors affecting them in a quantitative manner. <u>Electrical Safety: Systems and Devices</u> will consider systems and devices for preventing electrical hazards.

# **Thermal Hazards**

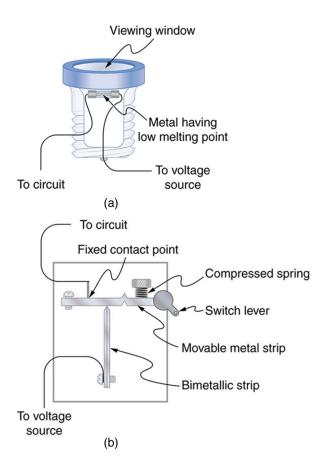
Electric power causes undesired heating effects whenever electric energy is converted to thermal energy at a rate faster than it can be safely dissipated. A classic example of this is the **short circuit**, a low-resistance path between terminals of a voltage source. An example of a short circuit is shown in [link]. Insulation on wires leading to an appliance has worn through, allowing the two wires to come into contact. Such an undesired contact with a high voltage is called a *short*. Since the resistance of the short, r, is very small, the power dissipated in the short,  $P = V^2/r$ , is very large. For example, if V is 120 V and r is 0.100  $\Omega$ , then the power is 144 kW, *much* greater than that used by a typical household appliance. Thermal energy delivered at this rate will very quickly raise the temperature of surrounding materials, melting or perhaps igniting them.



A short circuit is an undesired low-resistance path across a voltage source. (a) Worn insulation on the wires of a toaster allow them to come into contact with a low resistance r. Since  $P = V^2/r$ , thermal power is created so rapidly that the cord melts or burns. (b) A schematic of the short circuit.

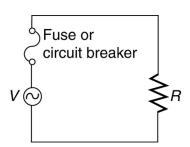
One particularly insidious aspect of a short circuit is that its resistance may actually be decreased due to the increase in temperature. This can happen if the short creates ionization. These charged atoms and molecules are free to move and, thus, lower the resistance r. Since  $P = V^2/r$ , the power dissipated in the short rises, possibly causing more ionization, more power, and so on. High voltages, such as the 480-V AC used in some industrial applications, lend themselves to this hazard, because higher voltages create higher initial power production in a short.

Another serious, but less dramatic, thermal hazard occurs when wires supplying power to a user are overloaded with too great a current. As discussed in the previous section, the power dissipated in the supply wires is  $P=I^2R_{\rm w}$ , where  $R_{\rm w}$  is the resistance of the wires and I the current flowing through them. If either I or  $R_{\rm w}$  is too large, the wires overheat. For example, a worn appliance cord (with some of its braided wires broken) may have  $R_{\rm w}=2.00~\Omega$  rather than the  $0.100~\Omega$  it should be. If  $10.0~\Lambda$  of current passes through the cord, then  $P=I^2R_{\rm w}=200~{\rm W}$  is dissipated in the cord—much more than is safe. Similarly, if a wire with a  $0.100~\Omega$  resistance is meant to carry a few amps, but is instead carrying  $100~\Lambda$ , it will severely overheat. The power dissipated in the wire will in that case be  $P=1000~{\rm W}$ . Fuses and circuit breakers are used to limit excessive currents. (See [link] and [link].) Each device opens the circuit automatically when a sustained current exceeds safe limits.



(a) A fuse has a metal strip with a low melting point that, when overheated by an excessive

current, permanently breaks the connection of a circuit to a voltage source. (b) A circuit breaker is an automatic but restorable electric switch. The one shown here has a bimetallic strip that bends to the right and into the notch if overheated. The spring then forces the metal strip downward, breaking the electrical connection at the points.



Schematic of a circuit with a fuse or circuit breaker in it.
Fuses and circuit breakers act like automatic switches that open when sustained current exceeds desired limits.

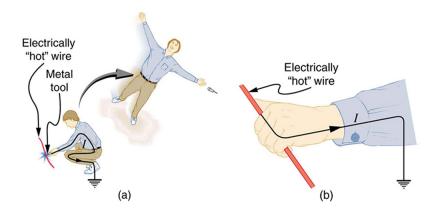
Fuses and circuit breakers for typical household voltages and currents are relatively simple to produce, but those for large voltages and currents experience special problems. For example, when a circuit breaker tries to interrupt the flow of high-voltage electricity, a spark can jump across its points that ionizes the air in the gap and allows the current to continue flowing. Large circuit breakers found in power-distribution systems employ insulating gas and even use jets of gas to blow out such sparks. Here AC is safer than DC, since AC current goes through zero 120 times per second, giving a quick opportunity to extinguish these arcs.

### **Shock Hazards**

Electrical currents through people produce tremendously varied effects. An electrical current can be used to block back pain. The possibility of using electrical current to stimulate muscle action in paralyzed limbs, perhaps allowing paraplegics to walk, is under study. TV dramatizations in which electrical shocks are used to bring a heart attack victim out of ventricular fibrillation (a massively irregular, often fatal, beating of the heart) are more than common. Yet most electrical shock fatalities occur because a current put the heart into fibrillation. A pacemaker uses electrical shocks to stimulate the heart to beat properly. Some fatal shocks do not produce burns, but warts can be safely burned off with electric current (though freezing using liquid nitrogen is now more common). Of course, there are consistent explanations for these disparate effects. The major factors upon which the effects of electrical shock depend are

- 1. The amount of current I
- 2. The path taken by the current
- 3. The duration of the shock
- 4. The frequency f of the current (f = 0 for DC)

[link] gives the effects of electrical shocks as a function of current for a typical accidental shock. The effects are for a shock that passes through the trunk of the body, has a duration of 1 s, and is caused by 60-Hz power.



An electric current can cause muscular contractions with varying effects. (a) The victim is "thrown" backward by involuntary muscle contractions that extend the legs and torso. (b) The victim can't let go of the wire that is stimulating all the muscles in the hand. Those that close the fingers are stronger than those that open them.

Current (mA)	Effect
1	Threshold of sensation
5	Maximum harmless current
10–20	Onset of sustained muscular contraction; cannot let go for duration of shock; contraction of chest muscles may stop breathing during shock

Current (mA)	Effect
50	Onset of pain
100– 300+	Ventricular fibrillation possible; often fatal
300	Onset of burns depending on concentration of current
6000 (6 A)	Onset of sustained ventricular contraction and respiratory paralysis; both cease when shock ends; heartbeat may return to normal; used to defibrillate the heart

Effects of Electrical Shock as a Function of Current[footnote] For an average male shocked through trunk of body for 1 s by 60-Hz AC. Values for females are 60–80% of those listed.

Our bodies are relatively good conductors due to the water in our bodies. Given that larger currents will flow through sections with lower resistance (to be further discussed in the next chapter), electric currents preferentially flow through paths in the human body that have a minimum resistance in a direct path to earth. The earth is a natural electron sink. Wearing insulating shoes, a requirement in many professions, prohibits a pathway for electrons by providing a large resistance in that path. Whenever working with high-power tools (drills), or in risky situations, ensure that you do not provide a pathway for current flow (especially through the heart).

Very small currents pass harmlessly and unfelt through the body. This happens to you regularly without your knowledge. The threshold of sensation is only 1 mA and, although unpleasant, shocks are apparently harmless for currents less than 5 mA. A great number of safety rules take the 5-mA value for the maximum allowed shock. At 10 to 20 mA and above, the current can stimulate sustained muscular contractions much as regular nerve impulses do. People sometimes say they were knocked across the room by a shock, but what really happened was that certain muscles

contracted, propelling them in a manner not of their own choosing. (See [link](a).) More frightening, and potentially more dangerous, is the "can't let go" effect illustrated in [link](b). The muscles that close the fingers are stronger than those that open them, so the hand closes involuntarily on the wire shocking it. This can prolong the shock indefinitely. It can also be a danger to a person trying to rescue the victim, because the rescuer's hand may close about the victim's wrist. Usually the best way to help the victim is to give the fist a hard knock/blow/jar with an insulator or to throw an insulator at the fist. Modern electric fences, used in animal enclosures, are now pulsed on and off to allow people who touch them to get free, rendering them less lethal than in the past.

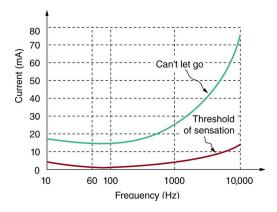
Greater currents may affect the heart. Its electrical patterns can be disrupted, so that it beats irregularly and ineffectively in a condition called "ventricular fibrillation." This condition often lingers after the shock and is fatal due to a lack of blood circulation. The threshold for ventricular fibrillation is between 100 and 300 mA. At about 300 mA and above, the shock can cause burns, depending on the concentration of current—the more concentrated, the greater the likelihood of burns.

Very large currents cause the heart and diaphragm to contract for the duration of the shock. Both the heart and breathing stop. Interestingly, both often return to normal following the shock. The electrical patterns on the heart are completely erased in a manner that the heart can start afresh with normal beating, as opposed to the permanent disruption caused by smaller currents that can put the heart into ventricular fibrillation. The latter is something like scribbling on a blackboard, whereas the former completely erases it. TV dramatizations of electric shock used to bring a heart attack victim out of ventricular fibrillation also show large paddles. These are used to spread out current passed through the victim to reduce the likelihood of burns.

Current is the major factor determining shock severity (given that other conditions such as path, duration, and frequency are fixed, such as in the table and preceding discussion). A larger voltage is more hazardous, but since I=V/R, the severity of the shock depends on the combination of voltage and resistance. For example, a person with dry skin has a resistance

of about  $200~\mathrm{k}\Omega$ . If he comes into contact with 120-V AC, a current  $I=(120~\mathrm{V})/(200~\mathrm{k}\Omega)=0.6~\mathrm{mA}$  passes harmlessly through him. The same person soaking wet may have a resistance of  $10.0~\mathrm{k}\Omega$  and the same 120 V will produce a current of 12 mA—above the "can't let go" threshold and potentially dangerous.

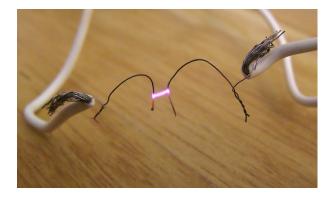
Most of the body's resistance is in its dry skin. When wet, salts go into ion form, lowering the resistance significantly. The interior of the body has a much lower resistance than dry skin because of all the ionic solutions and fluids it contains. If skin resistance is bypassed, such as by an intravenous infusion, a catheter, or exposed pacemaker leads, a person is rendered **microshock sensitive**. In this condition, currents about 1/1000 those listed in [link] produce similar effects. During open-heart surgery, currents as small as  $20~\mu\text{A}$  can be used to still the heart. Stringent electrical safety requirements in hospitals, particularly in surgery and intensive care, are related to the doubly disadvantaged microshock-sensitive patient. The break in the skin has reduced his resistance, and so the same voltage causes a greater current, and a much smaller current has a greater effect.



Graph of average values for the threshold of sensation and the "can't let go" current as a function of frequency. The lower the value, the

# more sensitive the body is at that frequency.

Factors other than current that affect the severity of a shock are its path, duration, and AC frequency. Path has obvious consequences. For example, the heart is unaffected by an electric shock through the brain, such as may be used to treat manic depression. And it is a general truth that the longer the duration of a shock, the greater its effects. [link] presents a graph that illustrates the effects of frequency on a shock. The curves show the minimum current for two different effects, as a function of frequency. The lower the current needed, the more sensitive the body is at that frequency. Ironically, the body is most sensitive to frequencies near the 50- or 60-Hz frequencies in common use. The body is slightly less sensitive for DC ( f=0), mildly confirming Edison's claims that AC presents a greater hazard. At higher and higher frequencies, the body becomes progressively less sensitive to any effects that involve nerves. This is related to the maximum rates at which nerves can fire or be stimulated. At very high frequencies, electrical current travels only on the surface of a person. Thus a wart can be burned off with very high frequency current without causing the heart to stop. (Do not try this at home with 60-Hz AC!) Some of the spectacular demonstrations of electricity, in which high-voltage arcs are passed through the air and over people's bodies, employ high frequencies and low currents. (See [link].) Electrical safety devices and techniques are discussed in detail in Electrical Safety: Systems and Devices.



Is this electric arc dangerous?

The answer depends on the AC frequency and the power involved. (credit: Khimich Alex, Wikimedia Commons)

# **Section Summary**

- The two types of electric hazards are thermal (excessive power) and shock (current through a person).
- Shock severity is determined by current, path, duration, and AC frequency.
- [link] lists shock hazards as a function of current.
- [link] graphs the threshold current for two hazards as a function of frequency.

# **Conceptual Questions**

#### **Exercise:**

### **Problem:**

Using an ohmmeter, a student measures the resistance between various points on his body. He finds that the resistance between two points on the same finger is about the same as the resistance between two points on opposite hands—both are several hundred thousand ohms. Furthermore, the resistance decreases when more skin is brought into contact with the probes of the ohmmeter. Finally, there is a dramatic drop in resistance (to a few thousand ohms) when the skin is wet. Explain these observations and their implications regarding skin and internal resistance of the human body.

#### **Exercise:**

**Problem:** What are the two major hazards of electricity?

**Problem:** Why isn't a short circuit a shock hazard?

**Exercise:** 

### **Problem:**

What determines the severity of a shock? Can you say that a certain voltage is hazardous without further information?

#### **Exercise:**

#### **Problem:**

An electrified needle is used to burn off warts, with the circuit being completed by having the patient sit on a large butt plate. Why is this plate large?

#### **Exercise:**

### **Problem:**

Some surgery is performed with high-voltage electricity passing from a metal scalpel through the tissue being cut. Considering the nature of electric fields at the surface of conductors, why would you expect most of the current to flow from the sharp edge of the scalpel? Do you think high- or low-frequency AC is used?

### **Exercise:**

#### Problem:

Some devices often used in bathrooms, such as hairdryers, often have safety messages saying "Do not use when the bathtub or basin is full of water." Why is this so?

#### **Exercise:**

#### **Problem:**

We are often advised to not flick electric switches with wet hands, dry your hand first. We are also advised to never throw water on an electric fire. Why is this so?

#### **Problem:**

Before working on a power transmission line, linemen will touch the line with the back of the hand as a final check that the voltage is zero. Why the back of the hand?

#### **Exercise:**

### **Problem:**

Why is the resistance of wet skin so much smaller than dry, and why do blood and other bodily fluids have low resistances?

### **Exercise:**

### **Problem:**

Could a person on intravenous infusion (an IV) be microshock sensitive?

#### **Exercise:**

#### **Problem:**

In view of the small currents that cause shock hazards and the larger currents that circuit breakers and fuses interrupt, how do they play a role in preventing shock hazards?

### **Problem Exercises**

#### **Exercise:**

#### **Problem:**

(a) How much power is dissipated in a short circuit of 240-V AC through a resistance of  $0.250~\Omega$ ? (b) What current flows?

#### **Solution:**

(a) 230 kW

(b) 960 A

### **Exercise:**

### **Problem:**

What voltage is involved in a 1.44-kW short circuit through a 0.100 -  $\Omega$  resistance?

#### **Exercise:**

### **Problem:**

Find the current through a person and identify the likely effect on her if she touches a 120-V AC source: (a) if she is standing on a rubber mat and offers a total resistance of 300 k $\Omega$ ; (b) if she is standing barefoot on wet grass and has a resistance of only 4000 k $\Omega$ .

#### **Solution:**

- (a) 0.400 mA, no effect
- (b) 26.7 mA, muscular contraction for duration of the shock (can't let go)

### **Exercise:**

### **Problem:**

While taking a bath, a person touches the metal case of a radio. The path through the person to the drainpipe and ground has a resistance of  $4000~\Omega$ . What is the smallest voltage on the case of the radio that could cause ventricular fibrillation?

Foolishly trying to fish a burning piece of bread from a toaster with a metal butter knife, a man comes into contact with 120-V AC. He does not even feel it since, luckily, he is wearing rubber-soled shoes. What is the minimum resistance of the path the current follows through the person?

#### **Solution:**

 $1.20 \times 10^{5} \Omega$ 

### **Exercise:**

### **Problem:**

(a) During surgery, a current as small as  $20.0~\mu A$  applied directly to the heart may cause ventricular fibrillation. If the resistance of the exposed heart is  $300~\Omega$ , what is the smallest voltage that poses this danger? (b) Does your answer imply that special electrical safety precautions are needed?

### **Exercise:**

### **Problem:**

(a) What is the resistance of a 220-V AC short circuit that generates a peak power of 96.8 kW? (b) What would the average power be if the voltage was 120 V AC?

#### **Solution:**

- (a)  $1.00 \Omega$
- (b) 14.4 kW

## **Problem:**

A heart defibrillator passes 10.0 A through a patient's torso for 5.00 ms in an attempt to restore normal beating. (a) How much charge passed? (b) What voltage was applied if 500 J of energy was dissipated? (c) What was the path's resistance? (d) Find the temperature increase caused in the 8.00 kg of affected tissue.

#### **Exercise:**

## **Problem:** Integrated Concepts

A short circuit in a 120-V appliance cord has a 0.500- $\Omega$  resistance. Calculate the temperature rise of the 2.00 g of surrounding materials, assuming their specific heat capacity is  $0.200 \text{ cal/g} \cdot ^{\circ} \text{C}$  and that it takes 0.0500 s for a circuit breaker to interrupt the current. Is this likely to be damaging?

#### **Solution:**

Temperature increases 860° C. It is very likely to be damaging.

#### **Exercise:**

#### **Problem:** Construct Your Own Problem

Consider a person working in an environment where electric currents might pass through her body. Construct a problem in which you calculate the resistance of insulation needed to protect the person from harm. Among the things to be considered are the voltage to which the person might be exposed, likely body resistance (dry, wet, ...), and acceptable currents (safe but sensed, safe and unfelt, ...).

# Glossary

#### thermal hazard

a hazard in which electric current causes undesired thermal effects

## shock hazard

when electric current passes through a person

## short circuit

also known as a "short," a low-resistance path between terminals of a voltage source

## microshock sensitive

a condition in which a person's skin resistance is bypassed, possibly by a medical procedure, rendering the person vulnerable to electrical shock at currents about 1/1000 the normally required level

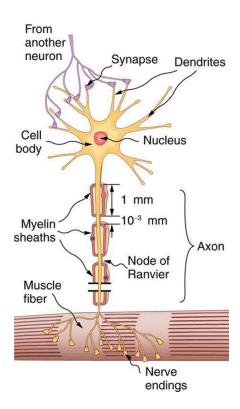
# Nerve Conduction–Electrocardiograms

- Explain the process by which electric signals are transmitted along a neuron.
- Explain the effects myelin sheaths have on signal propagation.
- Explain what the features of an ECG signal indicate.

## **Nerve Conduction**

Electric currents in the vastly complex system of billions of nerves in our body allow us to sense the world, control parts of our body, and think. These are representative of the three major functions of nerves. First, nerves carry messages from our sensory organs and others to the central nervous system, consisting of the brain and spinal cord. Second, nerves carry messages from the central nervous system to muscles and other organs. Third, nerves transmit and process signals within the central nervous system. The sheer number of nerve cells and the incredibly greater number of connections between them makes this system the subtle wonder that it is. **Nerve conduction** is a general term for electrical signals carried by nerve cells. It is one aspect of **bioelectricity**, or electrical effects in and created by biological systems.

Nerve cells, properly called *neurons*, look different from other cells—they have tendrils, some of them many centimeters long, connecting them with other cells. (See [link].) Signals arrive at the cell body across *synapses* or through *dendrites*, stimulating the neuron to generate its own signal, sent along its long *axon* to other nerve or muscle cells. Signals may arrive from many other locations and be transmitted to yet others, conditioning the synapses by use, giving the system its complexity and its ability to learn.

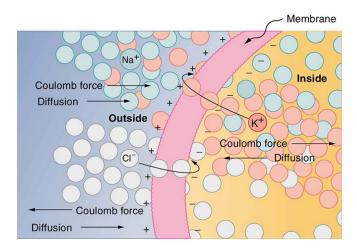


A neuron with its dendrites and long axon. Signals in the form of electric currents reach the cell body through dendrites and across synapses, stimulating the neuron to generate its own signal sent down the axon. The number of interconnections can be far greater than shown here.

The method by which these electric currents are generated and transmitted is more complex than the simple movement of free charges in a conductor,

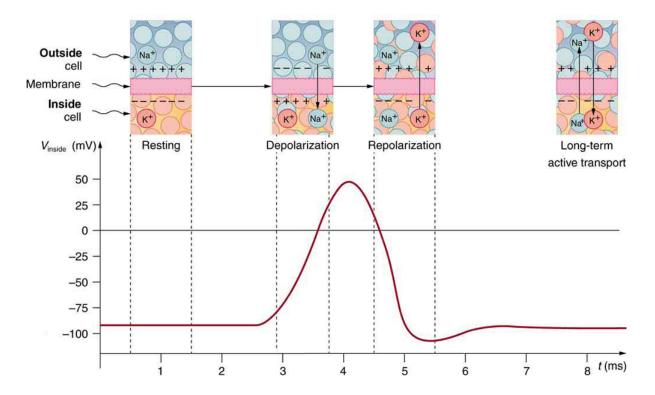
but it can be understood with principles already discussed in this text. The most important of these are the Coulomb force and diffusion.

[link] illustrates how a voltage (potential difference) is created across the cell membrane of a neuron in its resting state. This thin membrane separates electrically neutral fluids having differing concentrations of ions, the most important varieties being Na<sup>+</sup>, K<sup>+</sup>, and Cl<sup>-</sup> (these are sodium, potassium, and chlorine ions with single plus or minus charges as indicated). As discussed in Molecular Transport Phenomena: Diffusion, Osmosis, and Related Processes, free ions will diffuse from a region of high concentration to one of low concentration. But the cell membrane is **semipermeable**, meaning that some ions may cross it while others cannot. In its resting state, the cell membrane is permeable to  $K^+$  and  $Cl^-$ , and impermeable to  $Na^+$ . Diffusion of K<sup>+</sup> and Cl<sup>-</sup> thus creates the layers of positive and negative charge on the outside and inside of the membrane. The Coulomb force prevents the ions from diffusing across in their entirety. Once the charge layer has built up, the repulsion of like charges prevents more from moving across, and the attraction of unlike charges prevents more from leaving either side. The result is two layers of charge right on the membrane, with diffusion being balanced by the Coulomb force. A tiny fraction of the charges move across and the fluids remain neutral (other ions are present), while a separation of charge and a voltage have been created across the membrane.



The semipermeable membrane of a

cell has different concentrations of ions inside and out. Diffusion moves the  $K^+$  and  $Cl^-$  ions in the direction shown, until the Coulomb force halts further transfer. This results in a layer of positive charge on the outside, a layer of negative charge on the inside, and thus a voltage across the cell membrane. The membrane is normally impermeable to  $Na^+$ .



An action potential is the pulse of voltage inside a nerve cell graphed here. It is caused by movements of ions across the cell membrane as shown. Depolarization occurs when a stimulus makes the membrane permeable to  $\mathrm{Na}^+$  ions. Repolarization follows as the membrane

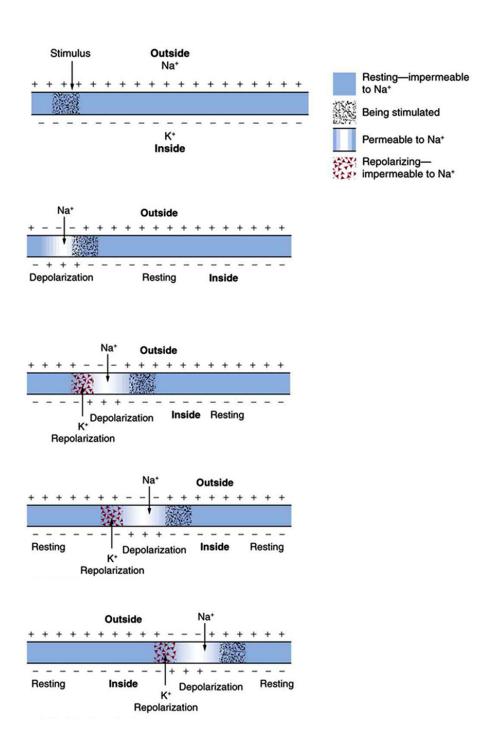
again becomes impermeable to  $\mathrm{Na}^+,$  and  $\mathrm{K}^+$  moves from high to low concentration. In the long term, active transport slowly maintains the concentration differences, but the cell may fire hundreds of times in rapid succession without seriously depleting them.

The separation of charge creates a potential difference of 70 to 90 mV across the cell membrane. While this is a small voltage, the resulting electric field (E=V/d) across the only 8-nm-thick membrane is immense (on the order of 11 MV/m!) and has fundamental effects on its structure and permeability. Now, if the exterior of a neuron is taken to be at 0 V, then the interior has a *resting potential* of about -90 mV. Such voltages are created across the membranes of almost all types of animal cells but are largest in nerve and muscle cells. In fact, fully 25% of the energy used by cells goes toward creating and maintaining these potentials.

Electric currents along the cell membrane are created by any stimulus that changes the membrane's permeability. The membrane thus temporarily becomes permeable to  $\mathrm{Na}^+$ , which then rushes in, driven both by diffusion and the Coulomb force. This inrush of  $\mathrm{Na}^+$  first neutralizes the inside membrane, or *depolarizes* it, and then makes it slightly positive. The depolarization causes the membrane to again become impermeable to  $\mathrm{Na}^+$ , and the movement of  $\mathrm{K}^+$  quickly returns the cell to its resting potential, or *repolarizes* it. This sequence of events results in a voltage pulse, called the *action potential*. (See [link].) Only small fractions of the ions move, so that the cell can fire many hundreds of times without depleting the excess concentrations of  $\mathrm{Na}^+$  and  $\mathrm{K}^+$ . Eventually, the cell must replenish these ions to maintain the concentration differences that create bioelectricity. This sodium-potassium pump is an example of *active transport*, wherein cell energy is used to move ions across membranes against diffusion gradients and the Coulomb force.

The action potential is a voltage pulse at one location on a cell membrane. How does it get transmitted along the cell membrane, and in particular down an axon, as a nerve impulse? The answer is that the changing voltage and electric fields affect the permeability of the adjacent cell membrane, so

that the same process takes place there. The adjacent membrane depolarizes, affecting the membrane further down, and so on, as illustrated in [link]. Thus the action potential stimulated at one location triggers a *nerve impulse* that moves slowly (about 1 m/s) along the cell membrane.



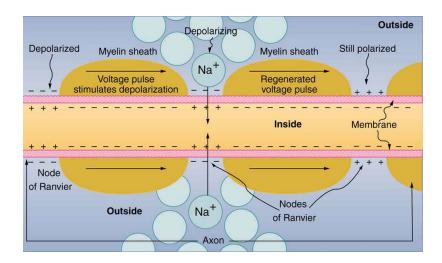
A nerve impulse is the propagation of an action potential along a cell membrane. A stimulus causes an action potential at one location, which changes the permeability of the adjacent membrane, causing an action potential there. This in turn affects the membrane further down, so that the action potential moves slowly (in electrical terms) along the cell membrane. Although the impulse is due to Na<sup>+</sup> and K<sup>+</sup> going across the membrane, it is equivalent to a wave of charge moving along the outside and inside of the membrane.

Some axons, like that in [link], are sheathed with *myelin*, consisting of fatcontaining cells. [link] shows an enlarged view of an axon having myelin sheaths characteristically separated by unmyelinated gaps (called nodes of Ranvier). This arrangement gives the axon a number of interesting properties. Since myelin is an insulator, it prevents signals from jumping between adjacent nerves (cross talk). Additionally, the myelinated regions transmit electrical signals at a very high speed, as an ordinary conductor or resistor would. There is no action potential in the myelinated regions, so that no cell energy is used in them. There is an IR signal loss in the myelin, but the signal is regenerated in the gaps, where the voltage pulse triggers the action potential at full voltage. So a myelinated axon transmits a nerve impulse faster, with less energy consumption, and is better protected from cross talk than an unmyelinated one. Not all axons are myelinated, so that cross talk and slow signal transmission are a characteristic of the normal operation of these axons, another variable in the nervous system.

The degeneration or destruction of the myelin sheaths that surround the nerve fibers impairs signal transmission and can lead to numerous neurological effects. One of the most prominent of these diseases comes from the body's own immune system attacking the myelin in the central nervous system—multiple sclerosis. MS symptoms include fatigue, vision problems, weakness of arms and legs, loss of balance, and tingling or

numbness in one's extremities (neuropathy). It is more apt to strike younger adults, especially females. Causes might come from infection, environmental or geographic affects, or genetics. At the moment there is no known cure for MS.

Most animal cells can fire or create their own action potential. Muscle cells contract when they fire and are often induced to do so by a nerve impulse. In fact, nerve and muscle cells are physiologically similar, and there are even hybrid cells, such as in the heart, that have characteristics of both nerves and muscles. Some animals, like the infamous electric eel (see [link]), use muscles ganged so that their voltages add in order to create a shock great enough to stun prey.



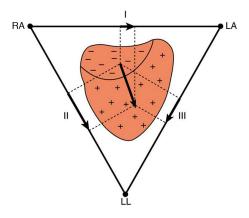
Propagation of a nerve impulse down a myelinated axon, from left to right. The signal travels very fast and without energy input in the myelinated regions, but it loses voltage. It is regenerated in the gaps. The signal moves faster than in unmyelinated axons and is insulated from signals in other nerves, limiting cross talk.



An electric eel flexes its muscles to create a voltage that stuns prey. (credit: chrisbb, Flickr)

# **Electrocardiograms**

Just as nerve impulses are transmitted by depolarization and repolarization of adjacent membrane, the depolarization that causes muscle contraction can also stimulate adjacent muscle cells to depolarize (fire) and contract. Thus, a depolarization wave can be sent across the heart, coordinating its rhythmic contractions and enabling it to perform its vital function of propelling blood through the circulatory system. [link] is a simplified graphic of a depolarization wave spreading across the heart from the *sinoarterial (SA) node*, the heart's natural pacemaker.

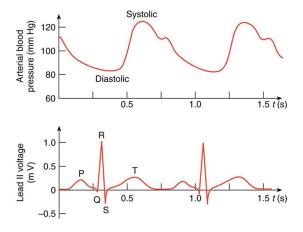


The outer surface of the heart changes from positive to negative during depolarization. This wave of depolarization is spreading from the top of the heart and is represented by a vector pointing in the direction of the wave. This vector is a voltage (potential difference) vector. Three electrodes, labeled RA, LA, and LL, are placed on the patient. Each pair (called leads I, II, and III) measures a component of the depolarization vector and is graphed in an ECG.

An **electrocardiogram** (**ECG**) is a record of the voltages created by the wave of depolarization and subsequent repolarization in the heart. Voltages between pairs of electrodes placed on the chest are vector components of the voltage wave on the heart. Standard ECGs have 12 or more electrodes, but only three are shown in [link] for clarity. Decades ago, three-electrode ECGs were performed by placing electrodes on the left and right arms and the left leg. The voltage between the right arm and the left leg is called the *lead II potential* and is the most often graphed. We shall examine the lead II potential as an indicator of heart-muscle function and see that it is coordinated with arterial blood pressure as well.

Heart function and its four-chamber action are explored in <u>Viscosity and Laminar Flow; Poiseuille's Law</u>. Basically, the right and left atria receive blood from the body and lungs, respectively, and pump the blood into the ventricles. The right and left ventricles, in turn, pump blood through the lungs and the rest of the body, respectively. Depolarization of the heart muscle causes it to contract. After contraction it is repolarized to ready it for the next beat. The ECG measures components of depolarization and repolarization of the heart muscle and can yield significant information on the functioning and malfunctioning of the heart.

[link] shows an ECG of the lead II potential and a graph of the corresponding arterial blood pressure. The major features are labeled P, Q, R, S, and T. The *P wave* is generated by the depolarization and contraction of the atria as they pump blood into the ventricles. The *QRS complex* is created by the depolarization of the ventricles as they pump blood to the lungs and body. Since the shape of the heart and the path of the depolarization wave are not simple, the QRS complex has this typical shape and time span. The lead II QRS signal also masks the repolarization of the atria, which occur at the same time. Finally, the *T wave* is generated by the repolarization of the ventricles and is followed by the next P wave in the next heartbeat. Arterial blood pressure varies with each part of the heartbeat, with systolic (maximum) pressure occurring closely after the QRS complex, which signals contraction of the ventricles.



A lead II ECG with

corresponding arterial blood pressure. The QRS complex is created by the depolarization and contraction of the ventricles and is followed shortly by the maximum or systolic blood pressure. See text for further description.

Taken together, the 12 leads of a state-of-the-art ECG can yield a wealth of information about the heart. For example, regions of damaged heart tissue, called infarcts, reflect electrical waves and are apparent in one or more lead potentials. Subtle changes due to slight or gradual damage to the heart are most readily detected by comparing a recent ECG to an older one. This is particularly the case since individual heart shape, size, and orientation can cause variations in ECGs from one individual to another. ECG technology has advanced to the point where a portable ECG monitor with a liquid crystal instant display and a printer can be carried to patients' homes or used in emergency vehicles. See [link].



This NASA scientist and NEEMO 5 aquanaut's heart rate and other vital signs

are being recorded by
a portable device
while living in an
underwater habitat.
(credit: NASA, Life
Sciences Data Archive
at Johnson Space
Center, Houston,
Texas)

#### Note:

PhET Explorations: Neuron

Stimulate a neuron and monitor what happens. Pause, rewind, and move forward in time in order to observe the ions as they move across the neuron membrane.

https://phet.colorado.edu/sims/html/neuron/latest/neuron\_en.html

# **Section Summary**

- Electric potentials in neurons and other cells are created by ionic concentration differences across semipermeable membranes.
- Stimuli change the permeability and create action potentials that propagate along neurons.
- Myelin sheaths speed this process and reduce the needed energy input.
- This process in the heart can be measured with an electrocardiogram (ECG).

# **Conceptual Questions**

#### **Exercise:**

## **Problem:**

Note that in [link], both the concentration gradient and the Coulomb force tend to move Na<sup>+</sup> ions into the cell. What prevents this?

## **Exercise:**

#### **Problem:**

Define depolarization, repolarization, and the action potential.

### **Exercise:**

#### **Problem:**

Explain the properties of myelinated nerves in terms of the insulating properties of myelin.

## **Problems & Exercises**

#### **Exercise:**

# **Problem:** Integrated Concepts

Use the ECG in [link] to determine the heart rate in beats per minute assuming a constant time between beats.

#### **Solution:**

80 beats/minute

#### **Exercise:**

## **Problem:** Integrated Concepts

(a) Referring to [link], find the time systolic pressure lags behind the middle of the QRS complex. (b) Discuss the reasons for the time lag.

# Glossary

## nerve conduction

the transport of electrical signals by nerve cells

## bioelectricity

electrical effects in and created by biological systems

# semipermeable

property of a membrane that allows only certain types of ions to cross it

## electrocardiogram (ECG)

usually abbreviated ECG, a record of voltages created by depolarization and repolarization, especially in the heart

# Introduction to Magnetism class="introduction"

The magnificen t spectacle of the Aurora Borealis, or northern lights, glows in the northern sky above Bear Lake near Eielson Air Force Base, Alaska. Shaped by the Earth's magnetic field, this light is produced by radiation spewed from solar storms. (credit: Senior Airman Joshua Strang, via Flickr)



One evening, an Alaskan sticks a note to his refrigerator with a small magnet. Through the kitchen window, the Aurora Borealis glows in the night sky. This grand spectacle is shaped by the same force that holds the note to the refrigerator.

People have been aware of magnets and magnetism for thousands of years. The earliest records date to well before the time of Christ, particularly in a region of Asia Minor called Magnesia (the name of this region is the source of words like *magnetic*). Magnetic rocks found in Magnesia, which is now part of western Turkey, stimulated interest during ancient times. A practical application for magnets was found later, when they were employed as navigational compasses. The use of magnets in compasses resulted not only in improved long-distance sailing, but also in the names of "north" and "south" being given to the two types of magnetic poles.

Today magnetism plays many important roles in our lives. Physicists' understanding of magnetism has enabled the development of technologies that affect our everyday lives. The iPod in your purse or backpack, for example, wouldn't have been possible without the applications of magnetism and electricity on a small scale.

The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology. The 2007 Nobel Prize in Physics went to Albert Fert from France and Peter Grunberg from Germany for this discovery of *giant magnetoresistance* and its applications to computer memory.

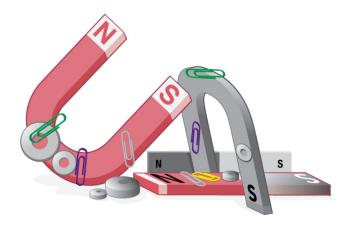
All electric motors, with uses as diverse as powering refrigerators, starting cars, and moving elevators, contain magnets. Generators, whether producing hydroelectric power or running bicycle lights, use magnetic fields. Recycling facilities employ magnets to separate iron from other refuse. Hundreds of millions of dollars are spent annually on magnetic containment of fusion as a future energy source. Magnetic resonance imaging (MRI) has become an important diagnostic tool in the field of medicine, and the use of magnetism to explore brain activity is a subject of contemporary research and development. The list of applications also includes computer hard drives, tape recording, detection of inhaled asbestos, and levitation of high-speed trains. Magnetism is used to explain atomic energy levels, cosmic rays, and charged particles trapped in the Van Allen belts. Once again, we will find all these disparate phenomena are linked by a small number of underlying physical principles.



Engineering of technology like iPods would not be possible without a deep understanding magnetism. (credit: Jesse! S?, Flickr)

## Magnets

- Describe the difference between the north and south poles of a magnet.
- Describe how magnetic poles interact with each other.



Magnets come in various shapes, sizes, and strengths. All have both a north pole and a south pole. There is never an isolated pole (a monopole).

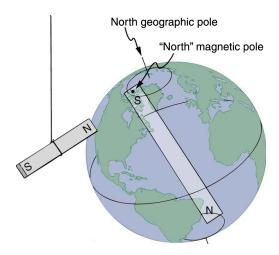
All magnets attract iron, such as that in a refrigerator door. However, magnets may attract or repel other magnets. Experimentation shows that all magnets have two poles. If freely suspended, one pole will point toward the north. The two poles are thus named the **north magnetic pole** and the **south magnetic pole** (or more properly, north-seeking and south-seeking poles, for the attractions in those directions).

#### Note:

# Universal Characteristics of Magnets and Magnetic Poles

It is a universal characteristic of all magnets that *like poles repel and unlike poles attract*. (Note the similarity with electrostatics: unlike charges attract and like charges repel.)

Further experimentation shows that it is *impossible to separate north and south poles* in the manner that + and – charges can be separated.



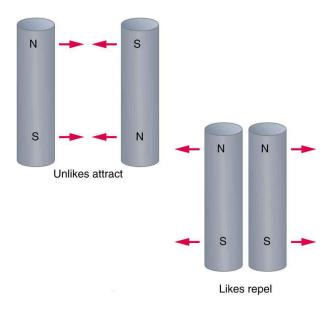
One end of a bar magnet is suspended from a thread that points toward north. The magnet's two poles are labeled N and S for north-seeking and south-seeking poles, respectively.

## Note:

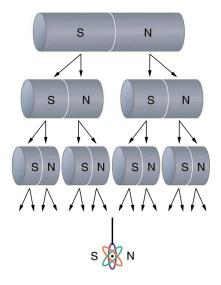
# Misconception Alert: Earth's Geographic North Pole Hides an S

The Earth acts like a very large bar magnet with its south-seeking pole near the geographic North Pole. That is why the north pole of your compass is attracted toward the geographic north pole of the Earth—because the magnetic pole that is near the geographic North Pole is actually a south magnetic pole! Confusion arises because the geographic term "North Pole" has come to be used (incorrectly) for the magnetic pole that is near the

North Pole. Thus, "North magnetic pole" is actually a misnomer—it should be called the South magnetic pole.



Unlike poles attract, whereas like poles repel.



North and south poles always occur in pairs. Attempts

to separate them result in more pairs of poles. If we continue to split the magnet, we will eventually get down to an iron atom with a north pole and a south pole—these, too, cannot be separated.

The fact that magnetic poles always occur in pairs of north and south is true from the very large scale—for example, sunspots always occur in pairs that are north and south magnetic poles—all the way down to the very small scale. Magnetic atoms have both a north pole and a south pole, as do many types of subatomic particles, such as electrons, protons, and neutrons.

### Note:

Making Connections: Take-Home Experiment—Refrigerator Magnets We know that like magnetic poles repel and unlike poles attract. See if you can show this for two refrigerator magnets. Will the magnets stick if you turn them over? Why do they stick to the door anyway? What can you say about the magnetic properties of the door next to the magnet? Do refrigerator magnets stick to metal or plastic spoons? Do they stick to all types of metal?

# **Section Summary**

• Magnetism is a subject that includes the properties of magnets, the effect of the magnetic force on moving charges and currents, and the

- creation of magnetic fields by currents.
- There are two types of magnetic poles, called the north magnetic pole and south magnetic pole.
- North magnetic poles are those that are attracted toward the Earth's geographic north pole.
- Like poles repel and unlike poles attract.
- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.

# **Conceptual Questions**

## **Exercise:**

#### **Problem:**

Volcanic and other such activity at the mid-Atlantic ridge extrudes material to fill the gap between separating tectonic plates associated with continental drift. The magnetization of rocks is found to reverse in a coordinated manner with distance from the ridge. What does this imply about the Earth's magnetic field and how could the knowledge of the spreading rate be used to give its historical record?

# Glossary

# north magnetic pole

the end or the side of a magnet that is attracted toward Earth's geographic north pole

## south magnetic pole

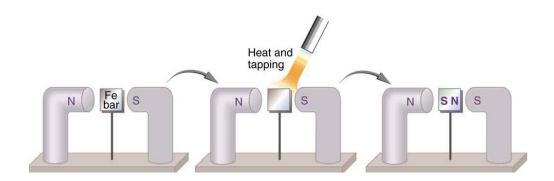
the end or the side of a magnet that is attracted toward Earth's geographic south pole

## Ferromagnets and Electromagnets

- Define ferromagnet.
- Describe the role of magnetic domains in magnetization.
- Explain the significance of the Curie temperature.
- Describe the relationship between electricity and magnetism.

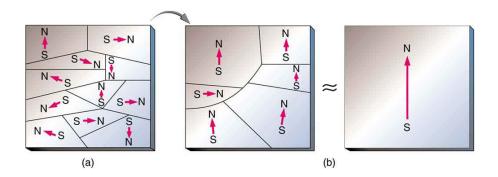
# **Ferromagnets**

Only certain materials, such as iron, cobalt, nickel, and gadolinium, exhibit strong magnetic effects. Such materials are called **ferromagnetic**, after the Latin word for iron, *ferrum*. A group of materials made from the alloys of the rare earth elements are also used as strong and permanent magnets; a popular one is neodymium. Other materials exhibit weak magnetic effects, which are detectable only with sensitive instruments. Not only do ferromagnetic materials respond strongly to magnets (the way iron is attracted to magnets), they can also be **magnetized** themselves—that is, they can be induced to be magnetic or made into permanent magnets.



An unmagnetized piece of iron is placed between two magnets, heated, and then cooled, or simply tapped when cold. The iron becomes a permanent magnet with the poles aligned as shown: its south pole is adjacent to the north pole of the original magnet, and its north pole is adjacent to the south pole of the original magnet. Note that there are attractive forces between the magnets.

When a magnet is brought near a previously unmagnetized ferromagnetic material, it causes local magnetization of the material with unlike poles closest, as in [link]. (This results in the attraction of the previously unmagnetized material to the magnet.) What happens on a microscopic scale is illustrated in [link]. The regions within the material called **domains** act like small bar magnets. Within domains, the poles of individual atoms are aligned. Each atom acts like a tiny bar magnet. Domains are small and randomly oriented in an unmagnetized ferromagnetic object. In response to an external magnetic field, the domains may grow to millimeter size, aligning themselves as shown in [link](b). This induced magnetization can be made permanent if the material is heated and then cooled, or simply tapped in the presence of other magnets.



(a) An unmagnetized piece of iron (or other ferromagnetic material) has randomly oriented domains. (b) When magnetized by an external field, the domains show greater alignment, and some grow at the expense of others. Individual atoms are aligned within domains; each atom acts like a tiny bar magnet.

Conversely, a permanent magnet can be demagnetized by hard blows or by heating it in the absence of another magnet. Increased thermal motion at higher temperature can disrupt and randomize the orientation and the size of

the domains. There is a well-defined temperature for ferromagnetic materials, which is called the **Curie temperature**, above which they cannot be magnetized. The Curie temperature for iron is 1043 K °, which is well above room temperature. There are several elements and alloys that have Curie temperatures much lower than room temperature and are ferromagnetic only below those temperatures.

# **Electromagnets**

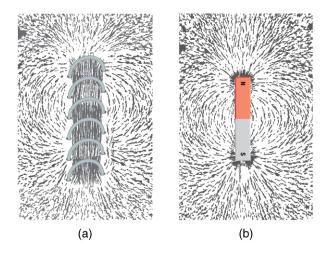
Early in the 19th century, it was discovered that electrical currents cause magnetic effects. The first significant observation was by the Danish scientist Hans Christian Oersted (1777–1851), who found that a compass needle was deflected by a current-carrying wire. This was the first significant evidence that the movement of charges had any connection with magnets. **Electromagnetism** is the use of electric current to make magnets. These temporarily induced magnets are called **electromagnets**. Electromagnets are employed for everything from a wrecking yard crane that lifts scrapped cars to controlling the beam of a 90-km-circumference particle accelerator to the magnets in medical imaging machines (See [link]).



Instrument for magnetic resonance imaging (MRI). The device uses a superconducting

cylindrical coil for the main magnetic field. The patient goes into this "tunnel" on the gurney. (credit: Bill McChesney, Flickr)

[link] shows that the response of iron filings to a current-carrying coil and to a permanent bar magnet. The patterns are similar. In fact, electromagnets and ferromagnets have the same basic characteristics—for example, they have north and south poles that cannot be separated and for which like poles repel and unlike poles attract.



Iron filings near (a) a current-carrying coil and (b) a magnet act like tiny compass needles, showing the shape of their fields. Their response to a current-carrying coil and a permanent magnet is seen to be very similar, especially near the ends of the coil and the magnet.

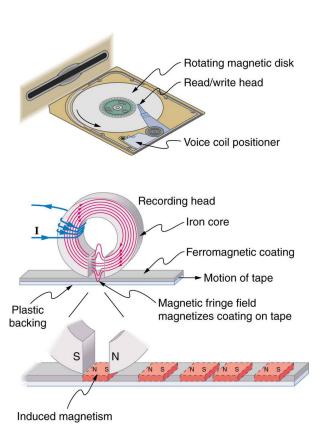
Combining a ferromagnet with an electromagnet can produce particularly strong magnetic effects. (See [link].) Whenever strong magnetic effects are needed, such as lifting scrap metal, or in particle accelerators, electromagnets are enhanced by ferromagnetic materials. Limits to how strong the magnets can be made are imposed by coil resistance (it will overheat and melt at sufficiently high current), and so superconducting magnets may be employed. These are still limited, because superconducting properties are destroyed by too great a magnetic field.



An electromagnet with a ferromagnetic core can produce very strong magnetic effects. Alignment of domains in the core produces a magnet, the poles of which are aligned with the electromagnet

•

[link] shows a few uses of combinations of electromagnets and ferromagnets. Ferromagnetic materials can act as memory devices, because the orientation of the magnetic fields of small domains can be reversed or erased. Magnetic information storage on videotapes and computer hard drives are among the most common applications. This property is vital in our digital world.



An electromagnet induces regions of permanent magnetism on a floppy disk coated with a ferromagnetic material. The information stored here is digital (a region is either magnetic or not); in other applications, it can be analog

(with a varying strength), such as on audiotapes.

# **Current: The Source of All Magnetism**

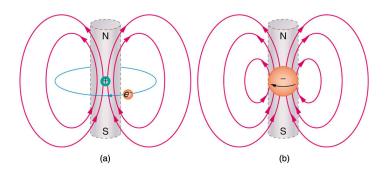
An electromagnet creates magnetism with an electric current. In later sections we explore this more quantitatively, finding the strength and direction of magnetic fields created by various currents. But what about ferromagnets? [link] shows models of how electric currents create magnetism at the submicroscopic level. (Note that we cannot directly observe the paths of individual electrons about atoms, and so a model or visual image, consistent with all direct observations, is made. We can directly observe the electron's orbital angular momentum, its spin momentum, and subsequent magnetic moments, all of which are explained with electric-current-creating subatomic magnetism.) Currents, including those associated with other submicroscopic particles like protons, allow us to explain ferromagnetism and all other magnetic effects. Ferromagnetism, for example, results from an internal cooperative alignment of electron spins, possible in some materials but not in others.

Crucial to the statement that electric current is the source of all magnetism is the fact that it is impossible to separate north and south magnetic poles. (This is far different from the case of positive and negative charges, which are easily separated.) A current loop always produces a magnetic dipole—that is, a magnetic field that acts like a north pole and south pole pair. Since isolated north and south magnetic poles, called **magnetic monopoles**, are not observed, currents are used to explain all magnetic effects. If magnetic monopoles did exist, then we would have to modify this underlying connection that all magnetism is due to electrical current. There is no known reason that magnetic monopoles should not exist—they are simply never observed—and so searches at the subnuclear level continue. If they do not exist, we would like to find out why not. If they do exist, we would like to see evidence of them.

## Note:

## **Electric Currents and Magnetism**

Electric current is the source of all magnetism.



(a) In the planetary model of the atom, an electron orbits a nucleus, forming a closed-current loop and producing a magnetic field with a north pole and a south pole. (b) Electrons have spin and can be crudely pictured as rotating charge, forming a current that produces a magnetic field with a north pole and a south pole. Neither the planetary model nor the image of a spinning electron is completely consistent with modern physics. However, they do provide a useful way of understanding phenomena.

## Note:

# PhET Explorations: Magnets and Electromagnets

Explore the interactions between a compass and bar magnet. Discover how you can use a battery and wire to make a magnet! Can you make it a stronger magnet? Can you make the magnetic field reverse?

# **Section Summary**

- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.
- All magnetism is created by electric current.
- Ferromagnetic materials, such as iron, are those that exhibit strong magnetic effects.
- The atoms in ferromagnetic materials act like small magnets (due to currents within the atoms) and can be aligned, usually in millimeter-sized regions called domains.
- Domains can grow and align on a larger scale, producing permanent magnets. Such a material is magnetized, or induced to be magnetic.
- Above a material's Curie temperature, thermal agitation destroys the alignment of atoms, and ferromagnetism disappears.
- Electromagnets employ electric currents to make magnetic fields, often aided by induced fields in ferromagnetic materials.

# Glossary

# ferromagnetic

materials, such as iron, cobalt, nickel, and gadolinium, that exhibit strong magnetic effects

# magnetized

to be turned into a magnet; to be induced to be magnetic

#### domains

regions within a material that behave like small bar magnets

# Curie temperature

the temperature above which a ferromagnetic material cannot be magnetized

# electromagnetism

the use of electrical currents to induce magnetism

## electromagnet

an object that is temporarily magnetic when an electrical current is passed through it

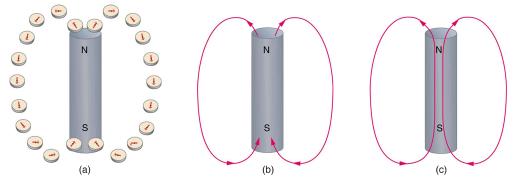
# magnetic monopoles

an isolated magnetic pole; a south pole without a north pole, or vice versa (no magnetic monopole has ever been observed)

## Magnetic Fields and Magnetic Field Lines

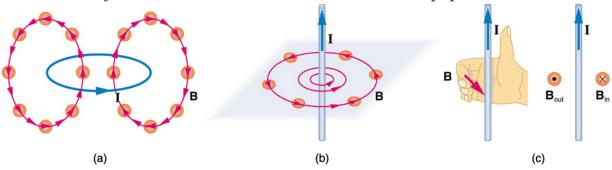
• Define magnetic field and describe the magnetic field lines of various magnetic fields.

Einstein is said to have been fascinated by a compass as a child, perhaps musing on how the needle felt a force without direct physical contact. His ability to think deeply and clearly about action at a distance, particularly for gravitational, electric, and magnetic forces, later enabled him to create his revolutionary theory of relativity. Since magnetic forces act at a distance, we define a **magnetic field** to represent magnetic forces. The pictorial representation of **magnetic field lines** is very useful in visualizing the strength and direction of the magnetic field. As shown in [link], the **direction of magnetic field lines** is defined to be the direction in which the north end of a compass needle points. The magnetic field is traditionally called the **B-field**.



Magnetic field lines are defined to have the direction that a small compass points when placed at a location. (a) If small compasses are used to map the magnetic field around a bar magnet, they will point in the directions shown: away from the north pole of the magnet, toward the south pole of the magnet. (Recall that the Earth's north magnetic pole is really a south pole in terms of definitions of poles on a bar magnet.) (b) Connecting the arrows gives continuous magnetic field lines. The strength of the field is proportional to the closeness (or density) of the lines. (c) If the interior of the magnet could be probed, the field lines would be found to form continuous closed loops.

Small compasses used to test a magnetic field will not disturb it. (This is analogous to the way we tested electric fields with a small test charge. In both cases, the fields represent only the object creating them and not the probe testing them.) [link] shows how the magnetic field appears for a current loop and a long straight wire, as could be explored with small compasses. A small compass placed in these fields will align itself parallel to the field line at its location, with its north pole pointing in the direction of *B*. Note the symbols used for field into and out of the paper.



Small compasses could be used to map the fields shown here. (a) The magnetic field of a circular current loop is similar to that of a bar magnet. (b) A long and straight wire creates a field with magnetic field lines forming circular loops. (c) When the wire is in the plane of the paper, the field is perpendicular to the paper. Note that the symbols used for the field pointing inward (like the tail of an arrow) and the field pointing outward (like the tip of an arrow).

### Note:

## Making Connections: Concept of a Field

A field is a way of mapping forces surrounding any object that can act on another object at a distance without apparent physical connection. The field represents the object generating it. Gravitational fields map gravitational forces, electric fields map electrical forces, and magnetic fields map magnetic forces.

Extensive exploration of magnetic fields has revealed a number of hardand-fast rules. We use magnetic field lines to represent the field (the lines are a pictorial tool, not a physical entity in and of themselves). The properties of magnetic field lines can be summarized by these rules:

- 1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line.
- 2. The strength of the field is proportional to the closeness of the lines. It is exactly proportional to the number of lines per unit area perpendicular to the lines (called the areal density).
- 3. Magnetic field lines can never cross, meaning that the field is unique at any point in space.
- 4. Magnetic field lines are continuous, forming closed loops without beginning or end. They go from the north pole to the south pole.

The last property is related to the fact that the north and south poles cannot be separated. It is a distinct difference from electric field lines, which begin and end on the positive and negative charges. If magnetic monopoles existed, then magnetic field lines would begin and end on them.

# **Section Summary**

- Magnetic fields can be pictorially represented by magnetic field lines, the properties of which are as follows:
- 1. The field is tangent to the magnetic field line.
- 2. Field strength is proportional to the line density.
- 3. Field lines cannot cross.
- 4. Field lines are continuous loops.

# **Conceptual Questions**

### **Exercise:**

## **Problem:**

Explain why the magnetic field would not be unique (that is, not have a single value) at a point in space where magnetic field lines might cross. (Consider the direction of the field at such a point.)

### **Exercise:**

## **Problem:**

List the ways in which magnetic field lines and electric field lines are similar. For example, the field direction is tangent to the line at any point in space. Also list the ways in which they differ. For example, electric force is parallel to electric field lines, whereas magnetic force on moving charges is perpendicular to magnetic field lines.

### **Exercise:**

### **Problem:**

Noting that the magnetic field lines of a bar magnet resemble the electric field lines of a pair of equal and opposite charges, do you expect the magnetic field to rapidly decrease in strength with distance from the magnet? Is this consistent with your experience with magnets?

### **Exercise:**

### **Problem:**

Is the Earth's magnetic field parallel to the ground at all locations? If not, where is it parallel to the surface? Is its strength the same at all locations? If not, where is it greatest?

# Glossary

magnetic field

the representation of magnetic forces

## *B*-field

another term for magnetic field

# magnetic field lines

the pictorial representation of the strength and the direction of a magnetic field

# direction of magnetic field lines

the direction that the north end of a compass needle points

## Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

- Describe the effects of magnetic fields on moving charges.
- Use the right hand rule 1 to determine the velocity of a charge, the direction of the magnetic field, and the direction of the magnetic force on a moving charge.
- Calculate the magnetic force on a moving charge.

What is the mechanism by which one magnet exerts a force on another? The answer is related to the fact that all magnetism is caused by current, the flow of charge. *Magnetic fields exert forces on moving charges*, and so they exert forces on other magnets, all of which have moving charges.

# **Right Hand Rule 1**

The magnetic force on a moving charge is one of the most fundamental known. Magnetic force is as important as the electrostatic or Coulomb force. Yet the magnetic force is more complex, in both the number of factors that affects it and in its direction, than the relatively simple Coulomb force. The magnitude of the **magnetic force** F on a charge q moving at a speed v in a magnetic field of strength B is given by

## **Equation:**

$$F = \text{qvB} \sin \theta$$
,

where  $\theta$  is the angle between the directions of  $\mathbf{v}$  and  $\mathbf{B}$ . This force is often called the **Lorentz force**. In fact, this is how we define the magnetic field strength B—in terms of the force on a charged particle moving in a magnetic field. The SI unit for magnetic field strength B is called the **tesla** (T) after the eccentric but brilliant inventor Nikola Tesla (1856–1943). To determine how the tesla relates to other SI units, we solve  $F = \text{qvB} \sin \theta$  for B.

# **Equation:**

$$B = \frac{F}{\operatorname{qv}\sin\theta}$$

Because  $\sin \theta$  is unitless, the tesla is

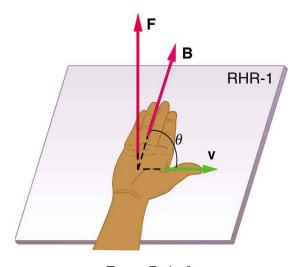
## **Equation:**

$$1~\mathrm{T} = \frac{1~\mathrm{N}}{\mathrm{C}\cdot\mathrm{m/s}} = \frac{1~\mathrm{N}}{\mathrm{A}\cdot\mathrm{m}}$$

(note that C/s = A).

Another smaller unit, called the **gauss** (G), where  $1~\mathrm{G}=10^{-4}~\mathrm{T}$ , is sometimes used. The strongest permanent magnets have fields near 2 T; superconducting electromagnets may attain 10 T or more. The Earth's magnetic field on its surface is only about  $5\times10^{-5}~\mathrm{T}$ , or 0.5 G.

The *direction* of the magnetic force  $\mathbf{F}$  is perpendicular to the plane formed by  $\mathbf{v}$  and  $\mathbf{B}$ , as determined by the **right hand rule 1** (or RHR-1), which is illustrated in [link]. RHR-1 states that, to determine the direction of the magnetic force on a positive moving charge, you point the thumb of the right hand in the direction of  $\mathbf{v}$ , the fingers in the direction of  $\mathbf{B}$ , and a perpendicular to the palm points in the direction of  $\mathbf{F}$ . One way to remember this is that there is one velocity, and so the thumb represents it. There are many field lines, and so the fingers represent them. The force is in the direction you would push with your palm. The force on a negative charge is in exactly the opposite direction to that on a positive charge.



 $F = qvB \sin \theta$ 

 ${f F} \perp {f plane}$  of  ${f v}$  and  ${f B}$ 

Magnetic fields exert forces on moving charges. This force is one of the most basic known. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by **v** and **B** and follows right hand rule—1 (RHR-1) as shown. The magnitude of the force is proportional to *q*, *v*, *B*, and the sine of the angle between **v** and **B**.

## Note:

# Making Connections: Charges and Magnets

There is no magnetic force on static charges. However, there is a magnetic force on moving charges. When charges are stationary, their electric fields do not affect magnets. But, when charges move, they produce magnetic

fields that exert forces on other magnets. When there is relative motion, a connection between electric and magnetic fields emerges—each affects the other.

## **Example:**

# Calculating Magnetic Force: Earth's Magnetic Field on a Charged Glass Rod

With the exception of compasses, you seldom see or personally experience forces due to the Earth's small magnetic field. To illustrate this, suppose that in a physics lab you rub a glass rod with silk, placing a 20-nC positive charge on it. Calculate the force on the rod due to the Earth's magnetic field, if you throw it with a horizontal velocity of 10 m/s due west in a place where the Earth's field is due north parallel to the ground. (The direction of the force is determined with right hand rule 1 as shown in [link].)

North

B

North

V

F down

(a)

(b)

A positively charged object moving due west in a region where the Earth's magnetic field is due north experiences a force that is straight down as shown. A negative charge moving in the same direction would feel a force straight up.

## Strategy

We are given the charge, its velocity, and the magnetic field strength and direction. We can thus use the equation  $F = qvB \sin \theta$  to find the force.

## Solution

The magnetic force is

## **Equation:**

$$F = qvb \sin \theta$$
.

We see that  $\sin \theta = 1$ , since the angle between the velocity and the direction of the field is 90°. Entering the other given quantities yields

## **Equation:**

$$egin{array}{lll} F &=& ig(20 imes10^{-9}\ {
m C}ig) ig(10\ {
m m/s}ig) ig(5 imes10^{-5}\ {
m T}ig) \ &=& 1 imes10^{-11}\ {
m (C\cdot m/s)} igg(rac{{
m N}}{{
m C\cdot m/s}}igg) = 1 imes10^{-11}\ {
m N}. \end{array}$$

## Discussion

This force is completely negligible on any macroscopic object, consistent with experience. (It is calculated to only one digit, since the Earth's field varies with location and is given to only one digit.) The Earth's magnetic field, however, does produce very important effects, particularly on submicroscopic particles. Some of these are explored in <a href="Force on a Moving Charge in a Magnetic Field: Examples and Applications">Force on a Moving Charge in a Magnetic Field: Examples and Applications</a>.

# **Section Summary**

• Magnetic fields exert a force on a moving charge *q*, the magnitude of which is

# **Equation:**

$$F = qvB \sin \theta$$
,

where  $\theta$  is the angle between the directions of v and B.

• The SI unit for magnetic field strength B is the tesla (T), which is related to other units by

## **Equation:**

$$1 T = \frac{1 N}{C \cdot m/s} = \frac{1 N}{A \cdot m}.$$

- The *direction* of the force on a moving charge is given by right hand rule 1 (RHR-1): Point the thumb of the right hand in the direction of v, the fingers in the direction of B, and a perpendicular to the palm points in the direction of F.
- The force is perpendicular to the plane formed by **v** and **B**. Since the force is zero if **v** is parallel to **B**, charged particles often follow magnetic field lines rather than cross them.

# **Conceptual Questions**

## **Exercise:**

### **Problem:**

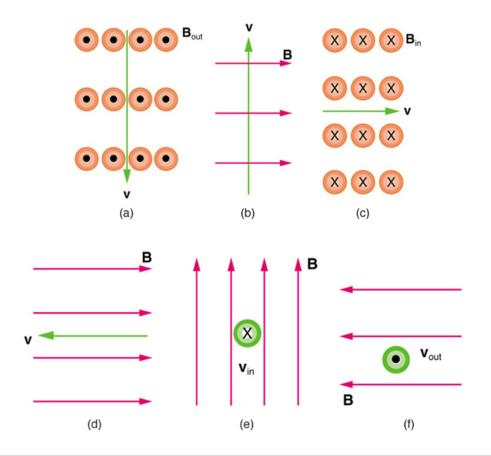
If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in that region is necessarily zero?

## **Problems & Exercises**

## **Exercise:**

### **Problem:**

What is the direction of the magnetic force on a positive charge that moves as shown in each of the six cases shown in [link]?



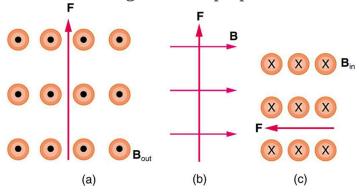
# **Solution:**

- (a) Left (West)
- (b) Into the page
- (c) Up (North)
- (d) No force
- (e) Right (East)
- (f) Down (South)

# **Exercise:**

**Problem:** Repeat [link] for a negative charge.

What is the direction of the velocity of a negative charge that experiences the magnetic force shown in each of the three cases in  $[\underline{link}]$ , assuming it moves perpendicular to  $\mathbf{B}$ ?



## **Solution:**

- (a) East (right)
- (b) Into page
- (c) South (down)

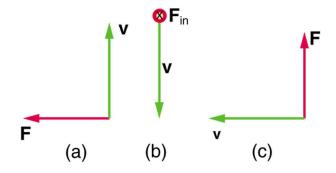
## **Exercise:**

**Problem:** Repeat [link] for a positive charge.

## **Exercise:**

## **Problem:**

What is the direction of the magnetic field that produces the magnetic force on a positive charge as shown in each of the three cases in the figure below, assuming  $\bf B$  is perpendicular to  $\bf v$ ?



## **Solution:**

- (a) Into page
- (b) West (left)
- (c) Out of page

## **Exercise:**

**Problem:** Repeat [link] for a negative charge.

## **Exercise:**

## **Problem:**

What is the maximum magnitude of the force on an aluminum rod with a 0.100- $\mu C$  charge that you pass between the poles of a 1.50-T permanent magnet at a speed of 5.00 m/s? In what direction is the force?

## **Solution:**

 $7.50\times 10^{-7}\ N$  perpendicular to both the magnetic field lines and the velocity

(a) Aircraft sometimes acquire small static charges. Suppose a supersonic jet has a 0.500- $\mu C$  charge and flies due west at a speed of 660 m/s over the Earth's magnetic south pole (near Earth's geographic north pole), where the  $8.00 \times 10^{-5}$ -T magnetic field points straight down. What are the direction and the magnitude of the magnetic force on the plane? (b) Discuss whether the value obtained in part (a) implies this is a significant or negligible effect.

## **Exercise:**

### **Problem:**

(a) A cosmic ray proton moving toward the Earth at  $5.00 \times 10^7$  m/s experiences a magnetic force of  $1.70 \times 10^{-16}$  N. What is the strength of the magnetic field if there is a  $45^\circ$  angle between it and the proton's velocity? (b) Is the value obtained in part (a) consistent with the known strength of the Earth's magnetic field on its surface? Discuss.

## **Solution:**

(a) 
$$3.01 \times 10^{-5} \text{ T}$$

(b) This is slightly less then the magnetic field strength of  $5 \times 10^{-5} \mathrm{~T}$  at the surface of the Earth, so it is consistent.

### **Exercise:**

## **Problem:**

An electron moving at  $4.00 \times 10^3 \ \mathrm{m/s}$  in a 1.25-T magnetic field experiences a magnetic force of  $1.40 \times 10^{-16} \ \mathrm{N}$ . What angle does the velocity of the electron make with the magnetic field? There are two answers.

(a) A physicist performing a sensitive measurement wants to limit the magnetic force on a moving charge in her equipment to less than  $1.00 \times 10^{-12}$  N. What is the greatest the charge can be if it moves at a maximum speed of 30.0 m/s in the Earth's field? (b) Discuss whether it would be difficult to limit the charge to less than the value found in (a) by comparing it with typical static electricity and noting that static is often absent.

## **Solution:**

- (a)  $6.67 \times 10^{-10}~\mathrm{C}$  (taking the Earth's field to be  $5.00 \times 10^{-5}~\mathrm{T}$ )
- (b) Less than typical static, therefore difficult

# Glossary

right hand rule 1 (RHR-1)

the rule to determine the direction of the magnetic force on a positive moving charge: when the thumb of the right hand points in the direction of the charge's velocity  ${\bf v}$  and the fingers point in the direction of the magnetic field  ${\bf B}$ , then the force on the charge is perpendicular and away from the palm; the force on a negative charge is perpendicular and into the palm

### Lorentz force

the force on a charge moving in a magnetic field

tesla

T, the SI unit of the magnetic field strength;  $1~T=\frac{1~\mathrm{N}}{\mathrm{A}\cdot\mathrm{m}}$ 

# magnetic force

the force on a charge produced by its motion through a magnetic field; the Lorentz force

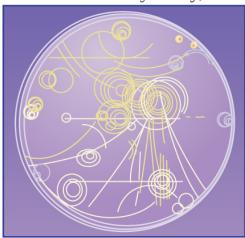
gauss

 $^{\circ}$  G, the unit of the magnetic field strength;  $1~\mathrm{G}=10^{-4}~\mathrm{T}$ 

# Force on a Moving Charge in a Magnetic Field: Examples and Applications

- Describe the effects of a magnetic field on a moving charge.
- Calculate the radius of curvature of the path of a charge that is moving in a magnetic field.

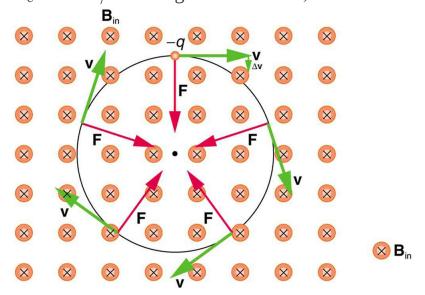
Magnetic force can cause a charged particle to move in a circular or spiral path. Cosmic rays are energetic charged particles in outer space, some of which approach the Earth. They can be forced into spiral paths by the Earth's magnetic field. Protons in giant accelerators are kept in a circular path by magnetic force. The bubble chamber photograph in [link] shows charged particles moving in such curved paths. The curved paths of charged particles in magnetic fields are the basis of a number of phenomena and can even be used analytically, such as in a mass spectrometer.



Trails of bubbles are produced by high-energy charged particles moving through the superheated liquid hydrogen in this artist's rendition of a bubble chamber. There is a strong magnetic field perpendicular to the page that causes the curved paths of the path can be

used to find the mass, charge, and energy of the particle.

So does the magnetic force cause circular motion? Magnetic force is always perpendicular to velocity, so that it does no work on the charged particle. The particle's kinetic energy and speed thus remain constant. The direction of motion is affected, but not the speed. This is typical of uniform circular motion. The simplest case occurs when a charged particle moves perpendicular to a uniform B-field, such as shown in [link]. (If this takes place in a vacuum, the magnetic field is the dominant factor determining the motion.) Here, the magnetic force supplies the centripetal force  $F_c = mv^2/r$ . Noting that  $\sin \theta = 1$ , we see that F = qvB.



A negatively charged particle moves in the plane of the page in a region where the magnetic field is perpendicular into the page (represented by the small circles with x's—like the tails of arrows). The magnetic force is perpendicular to the velocity, and so velocity changes in direction but not magnitude. Uniform circular motion results.

Because the magnetic force F supplies the centripetal force  $F_c$ , we have **Equation:** 

$$ext{qvB} = rac{mv^2}{r}.$$

Solving for r yields

## **Equation:**

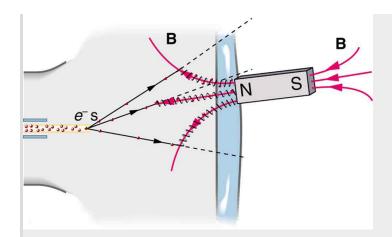
$$r=rac{\mathrm{mv}}{\mathrm{qB}}.$$

Here, r is the radius of curvature of the path of a charged particle with mass m and charge q, moving at a speed v perpendicular to a magnetic field of strength B. If the velocity is not perpendicular to the magnetic field, then v is the component of the velocity perpendicular to the field. The component of the velocity parallel to the field is unaffected, since the magnetic force is zero for motion parallel to the field. This produces a spiral motion rather than a circular one.

# **Example:**

# Calculating the Curvature of the Path of an Electron Moving in a Magnetic Field: A Magnet on a TV Screen

A magnet brought near an old-fashioned TV screen such as in [link] (TV sets with cathode ray tubes instead of LCD screens) severely distorts its picture by altering the path of the electrons that make its phosphors glow. (Don't try this at home, as it will permanently magnetize and ruin the TV.) To illustrate this, calculate the radius of curvature of the path of an electron having a velocity of  $6.00 \times 10^7$  m/s (corresponding to the accelerating voltage of about 10.0 kV used in some TVs) perpendicular to a magnetic field of strength B = 0.500 T (obtainable with permanent magnets).



Side view showing what happens when a magnet comes in contact with a computer monitor or TV screen. Electrons moving toward the screen spiral about magnetic field lines, maintaining the component of their velocity parallel to the field lines. This distorts the image on the screen.

## **Strategy**

We can find the radius of curvature r directly from the equation  $r = \frac{mv}{qB}$ , since all other quantities in it are given or known.

## Solution

Using known values for the mass and charge of an electron, along with the given values of v and B gives us

## **Equation:**

$$egin{array}{lll} r = rac{
m mv}{
m qB} & = & rac{ig(9.11 imes 10^{-31} {
m \, kg}ig)ig(6.00 imes 10^7 {
m \, m/s}ig)}{ig(1.60 imes 10^{-19} {
m \, C}ig)ig(0.500 {
m \, T}ig)} \ & = & 6.83 imes 10^{-4} {
m \, m} \end{array}$$

or

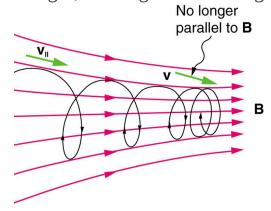
# **Equation:**

$$r = 0.683 \text{ mm}.$$

## Discussion

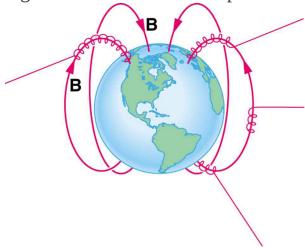
The small radius indicates a large effect. The electrons in the TV picture tube are made to move in very tight circles, greatly altering their paths and distorting the image.

[link] shows how electrons not moving perpendicular to magnetic field lines follow the field lines. The component of velocity parallel to the lines is unaffected, and so the charges spiral along the field lines. If field strength increases in the direction of motion, the field will exert a force to slow the charges, forming a kind of magnetic mirror, as shown below.



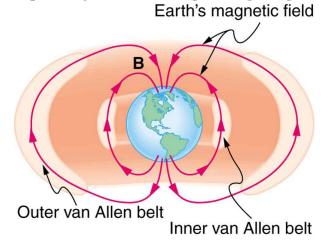
When a charged particle moves along a magnetic field line into a region where the field becomes stronger, the particle experiences a force that reduces the component of velocity parallel to the field. This force slows the motion along the field line and here reverses it, forming a "magnetic mirror."

The properties of charged particles in magnetic fields are related to such different things as the Aurora Australis or Aurora Borealis and particle accelerators. *Charged particles approaching magnetic field lines may get trapped in spiral orbits about the lines rather than crossing them*, as seen above. Some cosmic rays, for example, follow the Earth's magnetic field lines, entering the atmosphere near the magnetic poles and causing the southern or northern lights through their ionization of molecules in the atmosphere. This glow of energized atoms and molecules is seen in [link]. Those particles that approach middle latitudes must cross magnetic field lines, and many are prevented from penetrating the atmosphere. Cosmic rays are a component of background radiation; consequently, they give a higher radiation dose at the poles than at the equator.



Energetic electrons and protons, components of cosmic rays, from the Sun and deep outer space often follow the Earth's magnetic field lines rather than cross them. (Recall that the Earth's north magnetic pole is really a south pole in terms of a bar magnet.)

Some incoming charged particles become trapped in the Earth's magnetic field, forming two belts above the atmosphere known as the Van Allen radiation belts after the discoverer James A. Van Allen, an American astrophysicist. (See [link].) Particles trapped in these belts form radiation fields (similar to nuclear radiation) so intense that manned space flights avoid them and satellites with sensitive electronics are kept out of them. In the few minutes it took lunar missions to cross the Van Allen radiation belts, astronauts received radiation doses more than twice the allowed annual exposure for radiation workers. Other planets have similar belts, especially those having strong magnetic fields like Jupiter.



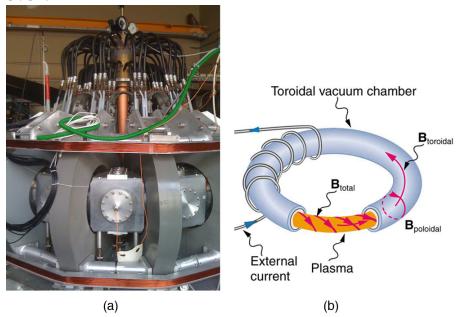
The Van Allen radiation belts are two regions in which energetic charged particles are trapped in the Earth's magnetic field. One belt lies about 300 km above the Earth's surface. the other about 16,000 km. Charged particles in these belts migrate along magnetic field lines and are partially reflected away from the poles by the stronger fields there. The charged particles that enter the atmosphere are replenished by the Sun and sources in deep outer space.

Back on Earth, we have devices that employ magnetic fields to contain charged particles. Among them are the giant particle accelerators that have been used to explore the substructure of matter. (See [link].) Magnetic fields not only control the direction of the charged particles, they also are used to focus particles into beams and overcome the repulsion of like charges in these beams.



The Fermilab facility in Illinois has a large particle accelerator (the most powerful in the world until 2008) that employs magnetic fields (magnets seen here in orange) to contain and direct its beam. This and other accelerators have been in use for several decades and have allowed us to discover some of the laws underlying all matter. (credit: ammcrim, Flickr)

Thermonuclear fusion (like that occurring in the Sun) is a hope for a future clean energy source. One of the most promising devices is the *tokamak*, which uses magnetic fields to contain (or trap) and direct the reactive charged particles. (See [link].) Less exotic, but more immediately practical, amplifiers in microwave ovens use a magnetic field to contain oscillating electrons. These oscillating electrons generate the microwaves sent into the oven.



Tokamaks such as the one shown in the figure are being studied with the goal of economical production of energy by nuclear fusion. Magnetic fields in the doughnut-shaped device contain and direct the reactive charged particles. (credit: David Mellis, Flickr)

Mass spectrometers have a variety of designs, and many use magnetic fields to measure mass. The curvature of a charged particle's path in the field is related to its mass and is measured to obtain mass information. (See <a href="More Applications of Magnetism">More Applications of Magnetism</a>.) Historically, such techniques were employed in the first direct observations of electron charge and mass. Today, mass

spectrometers (sometimes coupled with gas chromatographs) are used to determine the make-up and sequencing of large biological molecules.

# **Section Summary**

 Magnetic force can supply centripetal force and cause a charged particle to move in a circular path of radius
 Equation:

$$r = \frac{\mathrm{mv}}{\mathrm{qB}},$$

where v is the component of the velocity perpendicular to B for a charged particle with mass m and charge q.

# **Conceptual Questions**

### **Exercise:**

### **Problem:**

How can the motion of a charged particle be used to distinguish between a magnetic and an electric field?

### **Exercise:**

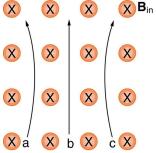
### **Problem:**

High-velocity charged particles can damage biological cells and are a component of radiation exposure in a variety of locations ranging from research facilities to natural background. Describe how you could use a magnetic field to shield yourself.

If a cosmic ray proton approaches the Earth from outer space along a line toward the center of the Earth that lies in the plane of the equator, in what direction will it be deflected by the Earth's magnetic field? What about an electron? A neutron?

## **Exercise:**

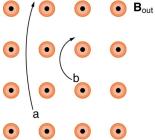
**Problem:** What are the signs of the charges on the particles in [link]?



## **Exercise:**

## **Problem:**

Which of the particles in [link] has the greatest velocity, assuming they have identical charges and masses?



## **Exercise:**

### **Problem:**

Which of the particles in [link] has the greatest mass, assuming all have identical charges and velocities?

While operating, a high-precision TV monitor is placed on its side during maintenance. The image on the monitor changes color and blurs slightly. Discuss the possible relation of these effects to the Earth's magnetic field.

## **Problems & Exercises**

If you need additional support for these problems, see <u>More Applications of Magnetism</u>.

### **Exercise:**

### **Problem:**

A cosmic ray electron moves at  $7.50 \times 10^6$  m/s perpendicular to the Earth's magnetic field at an altitude where field strength is  $1.00 \times 10^{-5}$  T. What is the radius of the circular path the electron follows?

### **Solution:**

4.27 m

## **Exercise:**

### **Problem:**

A proton moves at  $7.50 \times 10^7~\mathrm{m/s}$  perpendicular to a magnetic field. The field causes the proton to travel in a circular path of radius 0.800 m. What is the field strength?

(a) Viewers of *Star Trek* hear of an antimatter drive on the Starship *Enterprise*. One possibility for such a futuristic energy source is to store antimatter charged particles in a vacuum chamber, circulating in a magnetic field, and then extract them as needed. Antimatter annihilates with normal matter, producing pure energy. What strength magnetic field is needed to hold antiprotons, moving at  $5.00 \times 10^7 \, \mathrm{m/s}$  in a circular path 2.00 m in radius? Antiprotons have the same mass as protons but the opposite (negative) charge. (b) Is this field strength obtainable with today's technology or is it a futuristic possibility?

### **Solution:**

- (a) 0.261 T
- (b) This strength is definitely obtainable with today's technology. Magnetic field strengths of 0.500 T are obtainable with permanent magnets.

### **Exercise:**

#### **Problem:**

(a) An oxygen-16 ion with a mass of  $2.66 \times 10^{-26}$  kg travels at  $5.00 \times 10^6$  m/s perpendicular to a 1.20-T magnetic field, which makes it move in a circular arc with a 0.231-m radius. What positive charge is on the ion? (b) What is the ratio of this charge to the charge of an electron? (c) Discuss why the ratio found in (b) should be an integer.

## **Exercise:**

## **Problem:**

What radius circular path does an electron travel if it moves at the same speed and in the same magnetic field as the proton in [link]?

## **Solution:**

$$4.36 \times 10^{-4} \text{ m}$$

## **Exercise:**

## **Problem:**

A velocity selector in a mass spectrometer uses a 0.100-T magnetic field. (a) What electric field strength is needed to select a speed of  $4.00 \times 10^6 \ \mathrm{m/s?}$  (b) What is the voltage between the plates if they are separated by 1.00 cm?

## **Exercise:**

### **Problem:**

An electron in a TV CRT moves with a speed of  $6.00 \times 10^7$  m/s, in a direction perpendicular to the Earth's field, which has a strength of  $5.00 \times 10^{-5}$  T. (a) What strength electric field must be applied perpendicular to the Earth's field to make the electron moves in a straight line? (b) If this is done between plates separated by 1.00 cm, what is the voltage applied? (Note that TVs are usually surrounded by a ferromagnetic material to shield against external magnetic fields and avoid the need for such a correction.)

## **Solution:**

- (a) 3.00 kV/m
- (b) 30.0 V

(a) At what speed will a proton move in a circular path of the same radius as the electron in [link]? (b) What would the radius of the path be if the proton had the same speed as the electron? (c) What would the radius be if the proton had the same kinetic energy as the electron? (d) The same momentum?

### **Exercise:**

#### **Problem:**

A mass spectrometer is being used to separate common oxygen-16 from the much rarer oxygen-18, taken from a sample of old glacial ice. (The relative abundance of these oxygen isotopes is related to climatic temperature at the time the ice was deposited.) The ratio of the masses of these two ions is 16 to 18, the mass of oxygen-16 is  $2.66 \times 10^{-26}$  kg, and they are singly charged and travel at  $5.00 \times 10^6$  m/s in a 1.20-T magnetic field. What is the separation between their paths when they hit a target after traversing a semicircle?

## **Solution:**

 $0.173 \, \mathrm{m}$ 

### **Exercise:**

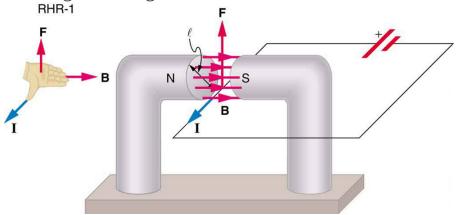
### Problem:

(a) Triply charged uranium-235 and uranium-238 ions are being separated in a mass spectrometer. (The much rarer uranium-235 is used as reactor fuel.) The masses of the ions are  $3.90 \times 10^{-25}$  kg and  $3.95 \times 10^{-25}$  kg, respectively, and they travel at  $3.00 \times 10^{5}$  m/s in a 0.250-T field. What is the separation between their paths when they hit a target after traversing a semicircle? (b) Discuss whether this distance between their paths seems to be big enough to be practical in the separation of uranium-235 from uranium-238.

## Magnetic Force on a Current-Carrying Conductor

- Describe the effects of a magnetic force on a current-carrying conductor.
- Calculate the magnetic force on a current-carrying conductor.

Because charges ordinarily cannot escape a conductor, the magnetic force on charges moving in a conductor is transmitted to the conductor itself.



The magnetic field exerts a force on a current-carrying wire in a direction given by the right hand rule 1 (the same direction as that on the individual moving charges). This force can easily be large enough to move the wire, since typical currents consist of very large numbers of moving charges.

We can derive an expression for the magnetic force on a current by taking a sum of the magnetic forces on individual charges. (The forces add because they are in the same direction.) The force on an individual charge moving at the drift velocity  $v_{\rm d}$  is given by  $F=qv_{\rm d}B\sin\theta$ . Taking B to be uniform over a length of wire l and zero elsewhere, the total magnetic force on the wire is then  $F=(qv_{\rm d}B\sin\theta)(N)$ , where N is the number of charge carriers in the section of wire of length l. Now, N=nV, where n is the number of charge carriers per unit volume and V is the volume of wire in the field. Noting that V=Al, where A is the cross-sectional area of the

wire, then the force on the wire is  $F = (qv_{\rm d}B\sin\theta)({\rm nAl})$ . Gathering terms,

## **Equation:**

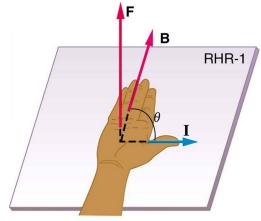
$$F = (nqAv_{\rm d})lB\sin\theta.$$

Because  $nqAv_{\rm d}=I$  (see Current),

## **Equation:**

$$F = \text{IlB sin } \theta$$

is the equation for magnetic force on a length l of wire carrying a current I in a uniform magnetic field B, as shown in  $[\underline{\text{link}}]$ . If we divide both sides of this expression by l, we find that the magnetic force per unit length of wire in a uniform field is  $\frac{F}{l} = IB \sin \theta$ . The direction of this force is given by RHR-1, with the thumb in the direction of the current I. Then, with the fingers in the direction of B, a perpendicular to the palm points in the direction of F, as in  $[\underline{\text{link}}]$ .



 ${f F} \perp {f plane} \ {f of} \ {f I} \ {f and} \ {f B}$ 

The force on a currentcarrying wire in a magnetic field is  $F = \text{IlB sin } \theta$ . Its

 $F = I\ell B \sin \theta$ 

direction is given by RHR-1.

## **Example:**

# Calculating Magnetic Force on a Current-Carrying Wire: A Strong Magnetic Field

Calculate the force on the wire shown in [link], given B=1.50 T, l=5.00 cm, and I=20.0 A.

## Strategy

The force can be found with the given information by using  $F = IlB \sin \theta$  and noting that the angle  $\theta$  between I and B is  $90^{\circ}$ , so that  $\sin \theta = 1$ .

## Solution

Entering the given values into  $F = IlB \sin \theta$  yields

## **Equation:**

$$F = \text{IlB sin } \theta = (20.0 \text{ A})(0.0500 \text{ m})(1.50 \text{ T})(1).$$

The units for tesla are  $1~T=rac{N}{A\cdot m}$ ; thus,

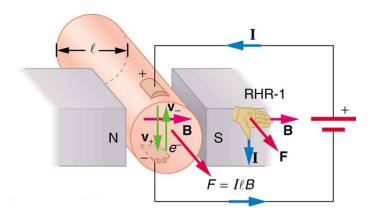
# **Equation:**

$$F = 1.50 \text{ N}.$$

## **Discussion**

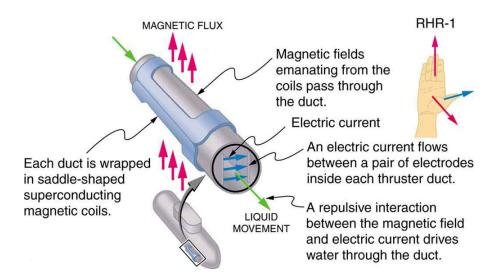
This large magnetic field creates a significant force on a small length of wire.

Magnetic force on current-carrying conductors is used to convert electric energy to work. (Motors are a prime example—they employ loops of wire and are considered in the next section.) Magnetohydrodynamics (MHD) is the technical name given to a clever application where magnetic force pumps fluids without moving mechanical parts. (See [link].)



Magnetohydrodynamics. The magnetic force on the current passed through this fluid can be used as a nonmechanical pump.

A strong magnetic field is applied across a tube and a current is passed through the fluid at right angles to the field, resulting in a force on the fluid parallel to the tube axis as shown. The absence of moving parts makes this attractive for moving a hot, chemically active substance, such as the liquid sodium employed in some nuclear reactors. Experimental artificial hearts are testing with this technique for pumping blood, perhaps circumventing the adverse effects of mechanical pumps. (Cell membranes, however, are affected by the large fields needed in MHD, delaying its practical application in humans.) MHD propulsion for nuclear submarines has been proposed, because it could be considerably quieter than conventional propeller drives. The deterrent value of nuclear submarines is based on their ability to hide and survive a first or second nuclear strike. As we slowly disassemble our nuclear weapons arsenals, the submarine branch will be the last to be decommissioned because of this ability (See [link].) Existing MHD drives are heavy and inefficient—much development work is needed.



An MHD propulsion system in a nuclear submarine could produce significantly less turbulence than propellers and allow it to run more silently. The development of a silent drive submarine was dramatized in the book and the film *The Hunt for Red October*.

## **Section Summary**

• The magnetic force on current-carrying conductors is given by **Equation:** 

$$F = \text{IlB sin } \theta$$
,

where I is the current, l is the length of a straight conductor in a uniform magnetic field B, and  $\theta$  is the angle between I and B. The force follows RHR-1 with the thumb in the direction of I.

## **Conceptual Questions**

## **Exercise:**

## **Problem:**

Draw a sketch of the situation in [link] showing the direction of electrons carrying the current, and use RHR-1 to verify the direction of the force on the wire.

#### **Exercise:**

#### **Problem:**

Verify that the direction of the force in an MHD drive, such as that in [link], does not depend on the sign of the charges carrying the current across the fluid.

## **Exercise:**

## **Problem:**

Why would a magnetohydrodynamic drive work better in ocean water than in fresh water? Also, why would superconducting magnets be desirable?

#### **Exercise:**

#### **Problem:**

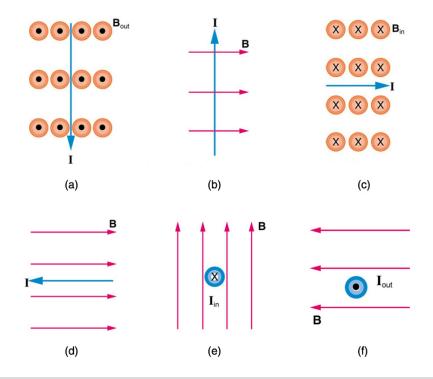
Which is more likely to interfere with compass readings, AC current in your refrigerator or DC current when you start your car? Explain.

## **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

What is the direction of the magnetic force on the current in each of the six cases in [link]?



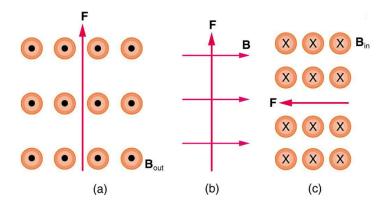
## **Solution:**

- (a) west (left)
- (b) into page
- (c) north (up)
- (d) no force
- (e) east (right)
- (f) south (down)

## **Exercise:**

## **Problem:**

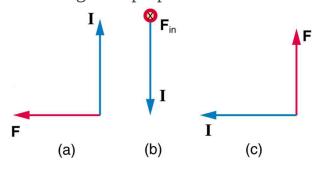
What is the direction of a current that experiences the magnetic force shown in each of the three cases in  $[\underline{link}]$ , assuming the current runs perpendicular to B?



## **Exercise:**

## **Problem:**

What is the direction of the magnetic field that produces the magnetic force shown on the currents in each of the three cases in [link], assuming is perpendicular to ?



## **Solution:**

- (a) into page
- (b) west (left)
- (c) out of page

## **Exercise:**

## **Problem:**

(a) What is the force per meter on a lightning bolt at the equator that carries 20,000 A perpendicular to the Earth's  $3.00 \times 10^{-5}$ -T field? (b) What is the direction of the force if the current is straight up and the Earth's field direction is due north, parallel to the ground?

#### **Exercise:**

#### **Problem:**

(a) A DC power line for a light-rail system carries 1000 A at an angle of  $30.0^{\circ}$  to the Earth's  $5.00 \times 10^{-5}$  -T field. What is the force on a 100-m section of this line? (b) Discuss practical concerns this presents, if any.

#### **Solution:**

- (a) 2.50 N
- (b) This is about half a pound of force per 100 m of wire, which is much less than the weight of the wire itself. Therefore, it does not cause any special concerns.

#### **Exercise:**

#### **Problem:**

What force is exerted on the water in an MHD drive utilizing a 25.0-cm-diameter tube, if 100-A current is passed across the tube that is perpendicular to a 2.00-T magnetic field? (The relatively small size of this force indicates the need for very large currents and magnetic fields to make practical MHD drives.)

## **Exercise:**

## **Problem:**

A wire carrying a 30.0-A current passes between the poles of a strong magnet that is perpendicular to its field and experiences a 2.16-N force on the 4.00 cm of wire in the field. What is the average field strength?

#### **Solution:**

1.80 T

#### **Exercise:**

## **Problem:**

(a) A 0.750-m-long section of cable carrying current to a car starter motor makes an angle of  $60^{\circ}$  with the Earth's  $5.50 \times 10^{-5}$  T field. What is the current when the wire experiences a force of  $7.00 \times 10^{-3}$  N? (b) If you run the wire between the poles of a strong horseshoe magnet, subjecting 5.00 cm of it to a 1.75-T field, what force is exerted on this segment of wire?

#### **Exercise:**

#### **Problem:**

(a) What is the angle between a wire carrying an 8.00-A current and the 1.20-T field it is in if 50.0 cm of the wire experiences a magnetic force of 2.40 N? (b) What is the force on the wire if it is rotated to make an angle of 90° with the field?

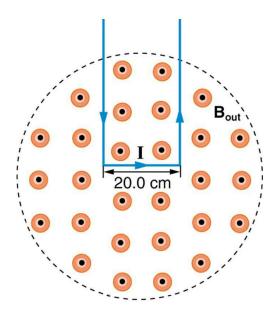
## **Solution:**

- (a)  $30^{\circ}$
- (b) 4.80 N

#### **Exercise:**

## **Problem:**

The force on the rectangular loop of wire in the magnetic field in [link] can be used to measure field strength. The field is uniform, and the plane of the loop is perpendicular to the field. (a) What is the direction of the magnetic force on the loop? Justify the claim that the forces on the sides of the loop are equal and opposite, independent of how much of the loop is in the field and do not affect the net force on the loop. (b) If a current of 5.00 A is used, what is the force per tesla on the 20.0-cm-wide loop?

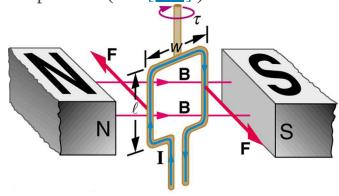


A rectangular loop of wire carrying a current is perpendicular to a magnetic field. The field is uniform in the region shown and is zero outside that region.

## Torque on a Current Loop: Motors and Meters

- Describe how motors and meters work in terms of torque on a current loop.
- Calculate the torque on a current-carrying loop in a magnetic field.

**Motors** are the most common application of magnetic force on current-carrying wires. Motors have loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft. Electrical energy is converted to mechanical work in the process. (See [link].)

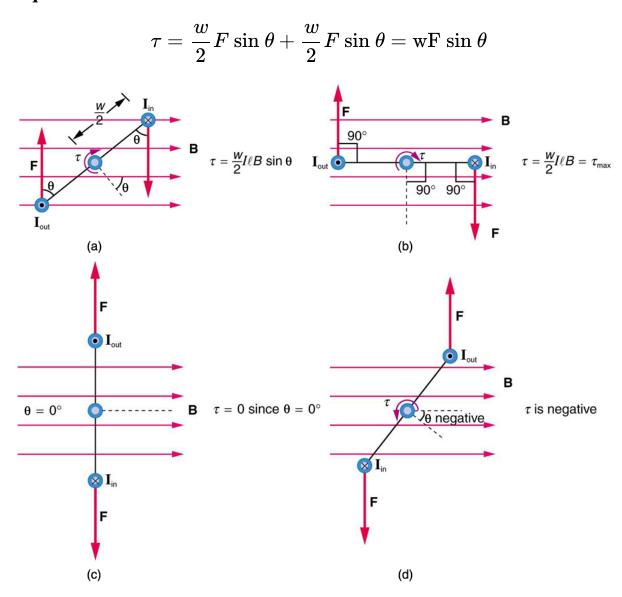


Torque on a current loop. A current-carrying loop of wire attached to a vertically rotating shaft feels magnetic forces that produce a clockwise torque as viewed from above.

Let us examine the force on each segment of the loop in [link] to find the torques produced about the axis of the vertical shaft. (This will lead to a useful equation for the torque on the loop.) We take the magnetic field to be uniform over the rectangular loop, which has width w and height l. First, we note that the forces on the top and bottom segments are vertical and, therefore, parallel to the shaft, producing no torque. Those vertical forces are equal in magnitude and opposite in direction, so that they also produce no net force on the loop. [link] shows views of the loop from above. Torque

is defined as  $\tau=\mathrm{rF}\sin\theta$ , where F is the force, r is the distance from the pivot that the force is applied, and  $\theta$  is the angle between r and F. As seen in [link](a), right hand rule 1 gives the forces on the sides to be equal in magnitude and opposite in direction, so that the net force is again zero. However, each force produces a clockwise torque. Since r=w/2, the torque on each vertical segment is  $(w/2)F\sin\theta$ , and the two add to give a total torque.

## **Equation:**



Top views of a current-carrying loop in a magnetic field. (a) The equation for torque is derived using this view. Note that the perpendicular to the loop makes an angle  $\theta$  with the field that is the

same as the angle between w/2 and  ${\bf F}$ . (b) The maximum torque occurs when  $\theta$  is a right angle and  $\sin\theta=1$ . (c) Zero (minimum) torque occurs when  $\theta$  is zero and  $\sin\theta=0$ . (d) The torque reverses once the loop rotates past  $\theta=0$ .

Now, each vertical segment has a length l that is perpendicular to B, so that the force on each is  $F=\mathrm{IlB}$ . Entering F into the expression for torque yields

## **Equation:**

 $\tau = \text{wIlB sin } \theta$ .

If we have a multiple loop of N turns, we get N times the torque of one loop. Finally, note that the area of the loop is  $A=\mathrm{wl}$ ; the expression for the torque becomes

## **Equation:**

 $\tau = \text{NIAB sin } \theta$ .

This is the torque on a current-carrying loop in a uniform magnetic field. This equation can be shown to be valid for a loop of any shape. The loop carries a current I, has N turns, each of area A, and the perpendicular to the loop makes an angle  $\theta$  with the field B. The net force on the loop is zero.

## **Example:**

## Calculating Torque on a Current-Carrying Loop in a Strong Magnetic Field

Find the maximum torque on a 100-turn square loop of a wire of 10.0 cm on a side that carries 15.0 A of current in a 2.00-T field.

## Strategy

Torque on the loop can be found using  $\tau = \text{NIAB sin } \theta$ . Maximum torque occurs when  $\theta = 90^{\circ}$  and  $\sin \theta = 1$ .

#### Solution

For  $\sin \theta = 1$ , the maximum torque is

## **Equation:**

$$\tau_{\rm max} = {
m NIAB}.$$

Entering known values yields

## **Equation:**

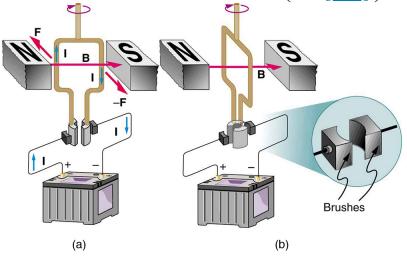
$$\tau_{\text{max}} = (100)(15.0 \text{ A})(0.100 \text{ m}^2)(2.00 \text{ T})$$

$$= 30.0 \text{ N} \cdot \text{m}.$$

## Discussion

This torque is large enough to be useful in a motor.

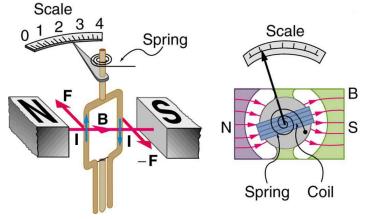
The torque found in the preceding example is the maximum. As the coil rotates, the torque decreases to zero at  $\theta=0$ . The torque then *reverses* its direction once the coil rotates past  $\theta=0$ . (See [link](d).) This means that, unless we do something, the coil will oscillate back and forth about equilibrium at  $\theta=0$ . To get the coil to continue rotating in the same direction, we can reverse the current as it passes through  $\theta=0$  with automatic switches called *brushes*. (See [link].)



(a) As the angular momentum of the coil carries it through  $\theta = 0$ , the brushes reverse

the current to keep the torque clockwise. (b)
The coil will rotate continuously in the
clockwise direction, with the current
reversing each half revolution to maintain
the clockwise torque.

**Meters**, such as those in analog fuel gauges on a car, are another common application of magnetic torque on a current-carrying loop. [link] shows that a meter is very similar in construction to a motor. The meter in the figure has its magnets shaped to limit the effect of  $\theta$  by making B perpendicular to the loop over a large angular range. Thus the torque is proportional to I and not  $\theta$ . A linear spring exerts a counter-torque that balances the current-produced torque. This makes the needle deflection proportional to I. If an exact proportionality cannot be achieved, the gauge reading can be calibrated. To produce a galvanometer for use in analog voltmeters and ammeters that have a low resistance and respond to small currents, we use a large loop area A, high magnetic field B, and low-resistance coils.



Meters are very similar to motors but only rotate through a part of a revolution. The magnetic poles of this meter are shaped to keep the component of B perpendicular to the loop constant, so that the torque does not depend on  $\theta$  and the deflection

against the return spring is proportional only to the current I.

## **Section Summary**

• The torque  $\tau$  on a current-carrying loop of any shape in a uniform magnetic field. is

## **Equation:**

$$\tau = \text{NIAB sin } \theta$$
,

where N is the number of turns, I is the current, A is the area of the loop, B is the magnetic field strength, and  $\theta$  is the angle between the perpendicular to the loop and the magnetic field.

## **Conceptual Questions**

## **Exercise:**

#### **Problem:**

Draw a diagram and use RHR-1 to show that the forces on the top and bottom segments of the motor's current loop in [link] are vertical and produce no torque about the axis of rotation.

## **Problems & Exercises**

#### Exercise:

#### **Problem:**

(a) By how many percent is the torque of a motor decreased if its permanent magnets lose 5.0% of their strength? (b) How many percent would the current need to be increased to return the torque to original values?

#### **Solution:**

- (a)  $\tau$  decreases by 5.00% if B decreases by 5.00%
- (b) 5.26% increase

#### **Exercise:**

## **Problem:**

(a) What is the maximum torque on a 150-turn square loop of wire 18.0 cm on a side that carries a 50.0-A current in a 1.60-T field? (b) What is the torque when  $\theta$  is  $10.9^{\circ}$ ?

#### **Exercise:**

## **Problem:**

Find the current through a loop needed to create a maximum torque of  $9.00~\mathrm{N}\cdot\mathrm{m}$ . The loop has 50 square turns that are 15.0 cm on a side and is in a uniform 0.800-T magnetic field.

## **Solution:**

10.0 A

#### **Exercise:**

#### **Problem:**

Calculate the magnetic field strength needed on a 200-turn square loop 20.0 cm on a side to create a maximum torque of  $300~\rm N\cdot m$  if the loop is carrying 25.0 A.

## **Exercise:**

## **Problem:**

Since the equation for torque on a current-carrying loop is  $\tau = \text{NIAB sin } \theta$ , the units of  $N \cdot m$  must equal units of  $A \cdot m^2 T$ . Verify this.

## **Solution:**

$$A \cdot m^2 \cdot T = A \cdot m^2 \left( \frac{N}{A \cdot m} \right) = N \cdot m.$$

#### **Exercise:**

## **Problem:**

(a) At what angle  $\theta$  is the torque on a current loop 90.0% of maximum? (b) 50.0% of maximum? (c) 10.0% of maximum?

#### **Exercise:**

#### **Problem:**

A proton has a magnetic field due to its spin on its axis. The field is similar to that created by a circular current loop  $0.650 \times 10^{-15}$  m in radius with a current of  $1.05 \times 10^4$  A (no kidding). Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

## **Solution:**

$$3.48 \times 10^{-26} \; \mathrm{N \cdot m}$$

#### **Exercise:**

#### **Problem:**

(a) A 200-turn circular loop of radius 50.0 cm is vertical, with its axis on an east-west line. A current of 100 A circulates clockwise in the loop when viewed from the east. The Earth's field here is due north, parallel to the ground, with a strength of  $3.00 \times 10^{-5}$  T. What are the direction and magnitude of the torque on the loop? (b) Does this device have any practical applications as a motor?

#### **Exercise:**

#### **Problem:**

Repeat [link], but with the loop lying flat on the ground with its current circulating counterclockwise (when viewed from above) in a location where the Earth's field is north, but at an angle  $45.0^{\circ}$  below the horizontal and with a strength of  $6.00 \times 10^{-5}$  T.

## **Solution:**

- (a)  $0.666 \text{ N} \cdot \text{m}$  west
- (b) This is not a very significant torque, so practical use would be limited. Also, the current would need to be alternated to make the loop rotate (otherwise it would oscillate).

## Glossary

#### motor

loop of wire in a magnetic field; when current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft; electrical energy is converted to mechanical work in the process

#### meter

common application of magnetic torque on a current-carrying loop that is very similar in construction to a motor; by design, the torque is proportional to I and not  $\theta$ , so the needle deflection is proportional to the current

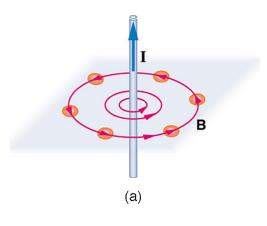
## Magnetic Fields Produced by Currents: Ampere's Law

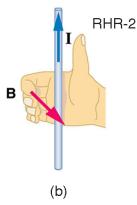
- Calculate current that produces a magnetic field.
- Use the right hand rule 2 to determine the direction of current or the direction of magnetic field loops.

How much current is needed to produce a significant magnetic field, perhaps as strong as the Earth's field? Surveyors will tell you that overhead electric power lines create magnetic fields that interfere with their compass readings. Indeed, when Oersted discovered in 1820 that a current in a wire affected a compass needle, he was not dealing with extremely large currents. How does the shape of wires carrying current affect the shape of the magnetic field created? We noted earlier that a current loop created a magnetic field similar to that of a bar magnet, but what about a straight wire or a toroid (doughnut)? How is the direction of a current-created field related to the direction of the current? Answers to these questions are explored in this section, together with a brief discussion of the law governing the fields created by currents.

## Magnetic Field Created by a Long Straight Current-Carrying Wire: Right Hand Rule 2

Magnetic fields have both direction and magnitude. As noted before, one way to explore the direction of a magnetic field is with compasses, as shown for a long straight current-carrying wire in [link]. Hall probes can determine the magnitude of the field. The field around a long straight wire is found to be in circular loops. The **right hand rule 2** (RHR-2) emerges from this exploration and is valid for any current segment—point the thumb in the direction of the current, and the fingers curl in the direction of the magnetic field loops created by it.





(a) Compasses placed near a long straight current-carrying wire indicate that field lines form circular loops centered on the wire. (b) Right hand rule 2 states that, if the right hand thumb points in the direction of the current, the fingers curl in the direction of the field. This rule is consistent with the field mapped for the long straight wire and is valid for any current segment.

The magnetic field strength (magnitude) produced by a long straight current-carrying wire is found by experiment to be **Equation:** 

$$B = rac{\mu_0 I}{2\pi r} ext{ (long straight wire)},$$

where I is the current, r is the shortest distance to the wire, and the constant  $\mu_0 = 4\pi \times 10^{-7} \, \mathrm{T \cdot m/A}$  is the **permeability of free space**. ( $\mu_0$  is one of the basic constants in nature. We will see later that  $\mu_0$  is related to the speed of light.) Since the wire is very long, the magnitude of the field depends only on distance from the wire r, not on position along the wire.

## **Example:**

## Calculating Current that Produces a Magnetic Field

Find the current in a long straight wire that would produce a magnetic field twice the strength of the Earth's at a distance of 5.0 cm from the wire.

## Strategy

The Earth's field is about  $5.0 \times 10^{-5}$  T, and so here B due to the wire is taken to be  $1.0 \times 10^{-4}$  T. The equation  $B = \frac{\mu_0 I}{2\pi r}$  can be used to find I, since all other quantities are known.

#### Solution

Solving for I and entering known values gives

## **Equation:**

$$egin{array}{lcl} I & = & rac{2\pi rB}{\mu_0} = rac{2\pi (5.0 imes 10^{-2} \ \mathrm{m}) \left(1.0 imes 10^{-4} \ \mathrm{T}
ight)}{4\pi imes 10^{-7} \ \mathrm{T\cdot m/A}} \ & = & 25 \ \mathrm{A.} \end{array}$$

#### Discussion

So a moderately large current produces a significant magnetic field at a distance of 5.0 cm from a long straight wire. Note that the answer is stated to only two digits, since the Earth's field is specified to only two digits in this example.

## **Ampere's Law and Others**

The magnetic field of a long straight wire has more implications than you might at first suspect. Each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment. The formal statement of the direction and magnitude of the field due to each segment is called the **Biot-Savart law**. Integral calculus is needed to sum the field for an arbitrary shape current. This results in a more complete law, called **Ampere's law**, which relates magnetic field and current in a general way. Ampere's law in turn is a part of **Maxwell's equations**, which give a complete theory of all electromagnetic phenomena. Considerations of how Maxwell's equations appear to different observers led to the modern theory of relativity, and the realization that electric and magnetic fields are different manifestations of the same thing. Most of this is beyond the scope of this text in both mathematical level, requiring calculus, and in the amount of space that can be devoted to it. But for the interested student, and particularly for those who continue in physics, engineering, or similar pursuits, delving into these matters further will reveal descriptions of nature that are elegant as well as profound. In this text, we shall keep the general features in mind, such as RHR-2 and the rules for magnetic field lines listed in Magnetic Fields and Magnetic Field Lines, while concentrating on the fields created in certain important situations.

#### Note:

Making Connections: Relativity

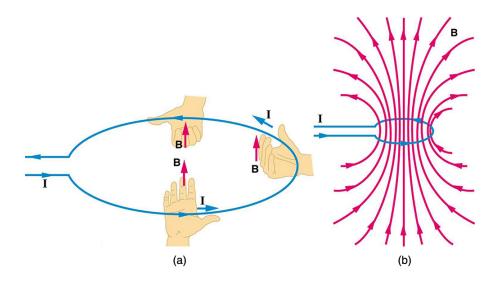
Hearing all we do about Einstein, we sometimes get the impression that he invented relativity out of nothing. On the contrary, one of Einstein's motivations was to solve difficulties in knowing how different observers see magnetic and electric fields.

# Magnetic Field Produced by a Current-Carrying Circular Loop

The magnetic field near a current-carrying loop of wire is shown in [link]. Both the direction and the magnitude of the magnetic field produced by a current-carrying loop are complex. RHR-2 can be used to give the direction of the field near the loop, but mapping with compasses and the rules about field lines given in Magnetic Fields and Magnetic Field Lines are needed for more detail. There is a simple formula for the magnetic field strength at the center of a circular loop. It is Equation:

$$B = \frac{\mu_0 I}{2R}$$
 (at center of loop),

where R is the radius of the loop. This equation is very similar to that for a straight wire, but it is valid *only* at the center of a circular loop of wire. The similarity of the equations does indicate that similar field strength can be obtained at the center of a loop. One way to get a larger field is to have N loops; then, the field is  $B = N\mu_0 I/(2R)$ . Note that the larger the loop, the smaller the field at its center, because the current is farther away.

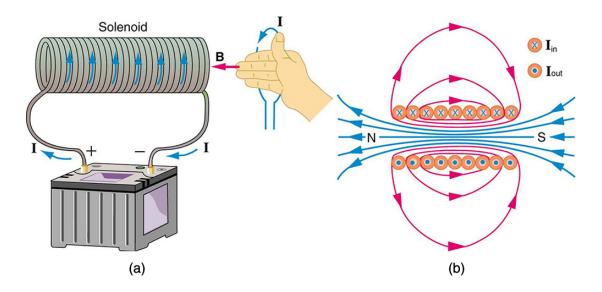


(a) RHR-2 gives the direction of the magnetic field inside and outside a current-carrying loop. (b) More detailed mapping with compasses or with a

Hall probe completes the picture. The field is similar to that of a bar magnet.

## Magnetic Field Produced by a Current-Carrying Solenoid

A **solenoid** is a long coil of wire (with many turns or loops, as opposed to a flat loop). Because of its shape, the field inside a solenoid can be very uniform, and also very strong. The field just outside the coils is nearly zero. [link] shows how the field looks and how its direction is given by RHR-2.



(a) Because of its shape, the field inside a solenoid of length l is remarkably uniform in magnitude and direction, as indicated by the straight and uniformly spaced field lines. The field outside the coils is nearly zero. (b) This cutaway shows the magnetic field generated by the current in the solenoid.

The magnetic field inside of a current-carrying solenoid is very uniform in direction and magnitude. Only near the ends does it begin to weaken and change direction. The field outside has similar complexities to flat loops

and bar magnets, but the **magnetic field strength inside a solenoid** is simply

## **Equation:**

$$B = \mu_0 \text{nI}$$
 (inside a solenoid),

where n is the number of loops per unit length of the solenoid (n = N/l, with N being the number of loops and l the length). Note that B is the field strength anywhere in the uniform region of the interior and not just at the center. Large uniform fields spread over a large volume are possible with solenoids, as  $[\underline{link}]$  implies.

## **Example:**

## Calculating Field Strength inside a Solenoid

What is the field inside a 2.00-m-long solenoid that has 2000 loops and carries a 1600-A current?

## **Strategy**

To find the field strength inside a solenoid, we use  $B = \mu_0 nI$ . First, we note the number of loops per unit length is

## **Equation:**

$$n = rac{N}{l} = rac{2000}{2.00 ext{ m}} = 1000 ext{ m}^{-1} = 10 ext{ cm}^{-1}.$$

#### Solution

Substituting known values gives

## **Equation:**

$$B = \mu_0 \mathrm{nI} = \left(4\pi \times 10^{-7} \; \mathrm{T \cdot m/A}\right) \left(1000 \; \mathrm{m}^{-1}\right) (1600 \; \mathrm{A}) = 2.01 \; \mathrm{T}.$$

#### Discussion

This is a large field strength that could be established over a large-diameter solenoid, such as in medical uses of magnetic resonance imaging (MRI). The very large current is an indication that the fields of this strength are not

easily achieved, however. Such a large current through 1000 loops squeezed into a meter's length would produce significant heating. Higher currents can be achieved by using superconducting wires, although this is expensive. There is an upper limit to the current, since the superconducting state is disrupted by very large magnetic fields.

There are interesting variations of the flat coil and solenoid. For example, the toroidal coil used to confine the reactive particles in tokamaks is much like a solenoid bent into a circle. The field inside a toroid is very strong but circular. Charged particles travel in circles, following the field lines, and collide with one another, perhaps inducing fusion. But the charged particles do not cross field lines and escape the toroid. A whole range of coil shapes are used to produce all sorts of magnetic field shapes. Adding ferromagnetic materials produces greater field strengths and can have a significant effect on the shape of the field. Ferromagnetic materials tend to trap magnetic fields (the field lines bend into the ferromagnetic material, leaving weaker fields outside it) and are used as shields for devices that are adversely affected by magnetic fields, including the Earth's magnetic field.

#### Note:

## PhET Explorations: Generator

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

## **Generato**

r

## **Section Summary**

• The strength of the magnetic field created by current in a long straight wire is given by

## **Equation:**

$$B = \frac{\mu_0 I}{2\pi r} (\text{long straight wire}),$$

where I is the current, r is the shortest distance to the wire, and the constant  $\mu_0 = 4\pi \times 10^{-7} \ \mathrm{T \cdot m/A}$  is the permeability of free space.

- The direction of the magnetic field created by a long straight wire is given by right hand rule 2 (RHR-2): *Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops* created by it.
- The magnetic field created by current following any path is the sum (or integral) of the fields due to segments along the path (magnitude and direction as for a straight wire), resulting in a general relationship between current and field known as Ampere's law.
- The magnetic field strength at the center of a circular loop is given by **Equation:**

$$B = \frac{\mu_0 I}{2R} \text{(at center of loop)},$$

where R is the radius of the loop. This equation becomes  $B = \mu_0 \mathrm{nI}/(2R)$  for a flat coil of N loops. RHR-2 gives the direction of the field about the loop. A long coil is called a solenoid.

• The magnetic field strength inside a solenoid is **Equation:** 

$$B = \mu_0 \text{nI}$$
 (inside a solenoid),

where n is the number of loops per unit length of the solenoid. The field inside is very uniform in magnitude and direction.

## **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

Make a drawing and use RHR-2 to find the direction of the magnetic field of a current loop in a motor (such as in [link]). Then show that the direction of the torque on the loop is the same as produced by like poles repelling and unlike poles attracting.

## Glossary

right hand rule 2 (RHR-2)

a rule to determine the direction of the magnetic field induced by a current-carrying wire: Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops

magnetic field strength (magnitude) produced by a long straight currentcarrying wire

defined as  $B=\frac{\mu_0I}{2\pi r}$ , where I is the current, r is the shortest distance to the wire, and  $\mu_0$  is the permeability of free space

permeability of free space

the measure of the ability of a material, in this case free space, to support a magnetic field; the constant  $\mu_0=4\pi\times 10^{-7}~{
m T\cdot m/A}$ 

magnetic field strength at the center of a circular loop defined as  $B=rac{\mu_0 I}{2R}$  where R is the radius of the loop

solenoid

a thin wire wound into a coil that produces a magnetic field when an electric current is passed through it

magnetic field strength inside a solenoid

defined as  $B=\mu_0 nI$  where n is the number of loops per unit length of the solenoid (n=N/l, with N being the number of loops and l the length)

## Biot-Savart law

a physical law that describes the magnetic field generated by an electric current in terms of a specific equation

## Ampere's law

the physical law that states that the magnetic field around an electric current is proportional to the current; each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment

## Maxwell's equations

a set of four equations that describe electromagnetic phenomena

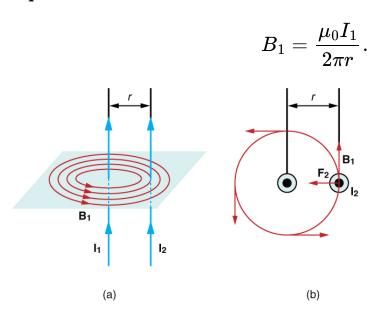
## Magnetic Force between Two Parallel Conductors

- Describe the effects of the magnetic force between two conductors.
- Calculate the force between two parallel conductors.

You might expect that there are significant forces between current-carrying wires, since ordinary currents produce significant magnetic fields and these fields exert significant forces on ordinary currents. But you might not expect that the force between wires is used to *define* the ampere. It might also surprise you to learn that this force has something to do with why large circuit breakers burn up when they attempt to interrupt large currents.

The force between two long straight and parallel conductors separated by a distance r can be found by applying what we have developed in preceding sections. [link] shows the wires, their currents, the fields they create, and the subsequent forces they exert on one another. Let us consider the field produced by wire 1 and the force it exerts on wire 2 (call the force  $F_2$ ). The field due to  $I_1$  at a distance r is given to be

## **Equation:**



(a) The magnetic field produced by a long straight conductor is perpendicular to a parallel conductor, as indicated by RHR-2. (b) A view

from above of the two wires shown in (a), with one magnetic field line shown for each wire. RHR-1 shows that the force between the parallel conductors is attractive when the currents are in the same direction. A similar analysis shows that the force is repulsive between currents in opposite directions.

This field is uniform along wire 2 and perpendicular to it, and so the force  $F_2$  it exerts on wire 2 is given by  $F = \text{IIB sin } \theta$  with  $\sin \theta = 1$ :

## **Equation:**

$$F_2 = I_2 l B_1$$
.

By Newton's third law, the forces on the wires are equal in magnitude, and so we just write F for the magnitude of  $F_2$ . (Note that  $F_1 = -F_2$ .) Since the wires are very long, it is convenient to think in terms of F/l, the force per unit length. Substituting the expression for  $B_1$  into the last equation and rearranging terms gives

## **Equation:**

$$rac{F}{l}=rac{\mu_0 I_1 I_2}{2\pi r}.$$

F/l is the force per unit length between two parallel currents  $I_1$  and  $I_2$  separated by a distance r. The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

This force is responsible for the *pinch effect* in electric arcs and plasmas. The force exists whether the currents are in wires or not. In an electric arc, where currents are moving parallel to one another, there is an attraction that squeezes currents into a smaller tube. In large circuit breakers, like those

used in neighborhood power distribution systems, the pinch effect can concentrate an arc between plates of a switch trying to break a large current, burn holes, and even ignite the equipment. Another example of the pinch effect is found in the solar plasma, where jets of ionized material, such as solar flares, are shaped by magnetic forces.

The *operational definition of the ampere* is based on the force between current-carrying wires. Note that for parallel wires separated by 1 meter with each carrying 1 ampere, the force per meter is

## **Equation:**

$$rac{F}{l} = rac{ig(4\pi imes 10^{-7}~{
m T}\cdot{
m m/A}ig)(1~{
m A})^2}{(2\pi)(1~{
m m})} = 2 imes 10^{-7}~{
m N/m}.$$

Since  $\mu_0$  is exactly  $4\pi \times 10^{-7}~T \cdot m/A$  by definition, and because  $1~T=1~N/(A\cdot m)$ , the force per meter is exactly  $2\times 10^{-7}~N/m$ . This is the basis of the operational definition of the ampere.

#### Note:

## The Ampere

The official definition of the ampere is:

One ampere of current through each of two parallel conductors of infinite length, separated by one meter in empty space free of other magnetic fields, causes a force of exactly  $2 \times 10^{-7} \ \mathrm{N/m}$  on each conductor.

Infinite-length straight wires are impractical and so, in practice, a current balance is constructed with coils of wire separated by a few centimeters. Force is measured to determine current. This also provides us with a method for measuring the coulomb. We measure the charge that flows for a current of one ampere in one second. That is,  $1 \, C = 1 \, A \cdot s$ . For both the ampere and the coulomb, the method of measuring force between conductors is the most accurate in practice.

## **Section Summary**

• The force between two parallel currents  $I_1$  and  $I_2$ , separated by a distance r, has a magnitude per unit length given by **Equation:** 

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}.$$

• The force is attractive if the currents are in the same direction, repulsive if they are in opposite directions.

## **Conceptual Questions**

## **Exercise:**

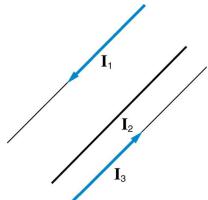
## **Problem:**

Is the force attractive or repulsive between the hot and neutral lines hung from power poles? Why?

#### **Exercise:**

#### **Problem:**

If you have three parallel wires in the same plane, as in [link], with currents in the outer two running in opposite directions, is it possible for the middle wire to be repelled by both? Attracted by both? Explain.



Three parallel

coplanar wires with currents in the outer two in opposite directions.

## **Exercise:**

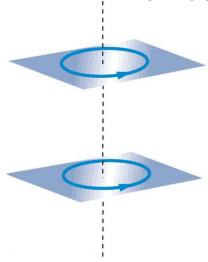
#### **Problem:**

Suppose two long straight wires run perpendicular to one another without touching. Does one exert a net force on the other? If so, what is its direction? Does one exert a net torque on the other? If so, what is its direction? Justify your responses by using the right hand rules.

## **Exercise:**

## **Problem:**

Use the right hand rules to show that the force between the two loops in [link] is attractive if the currents are in the same direction and repulsive if they are in opposite directions. Is this consistent with like poles of the loops repelling and unlike poles of the loops attracting? Draw sketches to justify your answers.



Two loops of wire carrying currents

can exert forces and torques on one another.

#### **Exercise:**

## **Problem:**

If one of the loops in [link] is tilted slightly relative to the other and their currents are in the same direction, what are the directions of the torques they exert on each other? Does this imply that the poles of the bar magnet-like fields they create will line up with each other if the loops are allowed to rotate?

## **Exercise:**

#### **Problem:**

Electric field lines can be shielded by the Faraday cage effect. Can we have magnetic shielding? Can we have gravitational shielding?

## **Problems & Exercises**

## **Exercise:**

## **Problem:**

- (a) The hot and neutral wires supplying DC power to a light-rail commuter train carry 800 A and are separated by 75.0 cm. What is the magnitude and direction of the force between 50.0 m of these wires?
- (b) Discuss the practical consequences of this force, if any.

## **Solution:**

- (a) 8.53 N, repulsive
- (b) This force is repulsive and therefore there is never a risk that the two wires will touch and short circuit.

#### **Exercise:**

## **Problem:**

The force per meter between the two wires of a jumper cable being used to start a stalled car is 0.225 N/m. (a) What is the current in the wires, given they are separated by 2.00 cm? (b) Is the force attractive or repulsive?

#### **Exercise:**

## **Problem:**

A 2.50-m segment of wire supplying current to the motor of a submerged submarine carries 1000 A and feels a 4.00-N repulsive force from a parallel wire 5.00 cm away. What is the direction and magnitude of the current in the other wire?

## **Solution:**

400 A in the opposite direction

#### **Exercise:**

#### **Problem:**

The wire carrying 400 A to the motor of a commuter train feels an attractive force of  $4.00 \times 10^{-3} \ \mathrm{N/m}$  due to a parallel wire carrying 5.00 A to a headlight. (a) How far apart are the wires? (b) Are the currents in the same direction?

#### **Exercise:**

#### **Problem:**

An AC appliance cord has its hot and neutral wires separated by 3.00 mm and carries a 5.00-A current. (a) What is the average force per meter between the wires in the cord? (b) What is the maximum force per meter between the wires? (c) Are the forces attractive or repulsive? (d) Do appliance cords need any special design features to compensate for these forces?

**Solution:** 

(a) 
$$1.67 \times 10^{-3} \; \text{N/m}$$

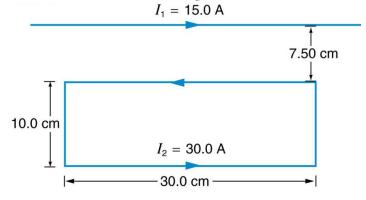
(b) 
$$3.33 \times 10^{-3} \text{ N/m}$$

- (c) Repulsive
- (d) No, these are very small forces

## **Exercise:**

## **Problem:**

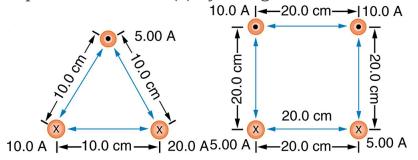
[link] shows a long straight wire near a rectangular current loop. What is the direction and magnitude of the total force on the loop?



## **Exercise:**

## **Problem:**

Find the direction and magnitude of the force that each wire experiences in [link](a) by, using vector addition.



## **Solution:**

(a) Top wire:  $2.65 \times 10^{-4}~\mathrm{N/m}$  s,  $10.9^{\mathrm{o}}$  to left of up

(b) Lower left wire:  $3.61 \times 10^{-4} \; N/m$ ,  $13.9^{o}$  down from right

(c) Lower right wire:  $3.46 \times 10^{-4} \ N/m$ ,  $30.0^{\circ}$  down from left

## **Exercise:**

## **Problem:**

Find the direction and magnitude of the force that each wire experiences in [link](b), using vector addition.

# More Applications of Magnetism

• Describe some applications of magnetism.

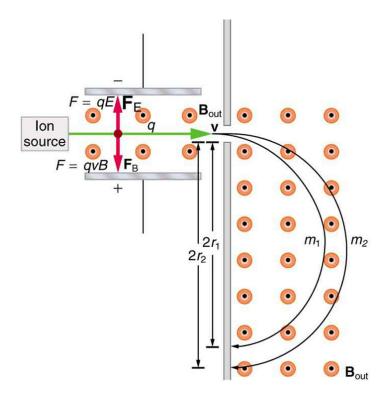
# **Mass Spectrometry**

The curved paths followed by charged particles in magnetic fields can be put to use. A charged particle moving perpendicular to a magnetic field travels in a circular path having a radius r.

### **Equation:**

$$r=rac{\mathrm{mv}}{\mathrm{qB}}$$

It was noted that this relationship could be used to measure the mass of charged particles such as ions. A mass spectrometer is a device that measures such masses. Most mass spectrometers use magnetic fields for this purpose, although some of them have extremely sophisticated designs. Since there are five variables in the relationship, there are many possibilities. However, if v, q, and B can be fixed, then the radius of the path r is simply proportional to the mass m of the charged particle. Let us examine one such mass spectrometer that has a relatively simple design. (See [link].) The process begins with an ion source, a device like an electron gun. The ion source gives ions their charge, accelerates them to some velocity v, and directs a beam of them into the next stage of the spectrometer. This next region is a *velocity selector* that only allows particles with a particular value of v to get through.



This mass spectrometer uses a velocity selector to fix v so that the radius of the path is proportional to mass.

The velocity selector has both an electric field and a magnetic field, perpendicular to one another, producing forces in opposite directions on the ions. Only those ions for which the forces balance travel in a straight line into the next region. If the forces balance, then the electric force F=qE equals the magnetic force F=qvB, so that qE=qvB. Noting that q cancels, we see that

# **Equation:**

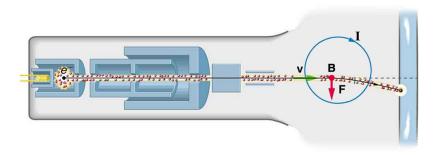
$$v=rac{E}{B}$$

is the velocity particles must have to make it through the velocity selector, and further, that v can be selected by varying E and B. In the final region, there is only a uniform magnetic field, and so the charged particles move in circular arcs with radii proportional to particle mass. The paths also depend on charge q, but since q is in multiples of electron charges, it is easy to determine and to discriminate between ions in different charge states.

Mass spectrometry today is used extensively in chemistry and biology laboratories to identify chemical and biological substances according to their mass-to-charge ratios. In medicine, mass spectrometers are used to measure the concentration of isotopes used as tracers. Usually, biological molecules such as proteins are very large, so they are broken down into smaller fragments before analyzing. Recently, large virus particles have been analyzed as a whole on mass spectrometers. Sometimes a gas chromatograph or high-performance liquid chromatograph provides an initial separation of the large molecules, which are then input into the mass spectrometer.

# Cathode Ray Tubes—CRTs—and the Like

What do non-flat-screen TVs, old computer monitors, x-ray machines, and the 2-mile-long Stanford Linear Accelerator have in common? All of them accelerate electrons, making them different versions of the electron gun. Many of these devices use magnetic fields to steer the accelerated electrons. [link] shows the construction of the type of cathode ray tube (CRT) found in some TVs, oscilloscopes, and old computer monitors. Two pairs of coils are used to steer the electrons, one vertically and the other horizontally, to their desired destination.



The cathode ray tube (CRT) is so named because rays of electrons originate at the cathode in the electron gun. Magnetic coils are used to steer the beam in many CRTs. In this case, the beam is moved down. Another pair of horizontal coils would steer the beam horizontally.

# **Magnetic Resonance Imaging**

**Magnetic resonance imaging (MRI)** is one of the most useful and rapidly growing medical imaging tools. It non-invasively produces two-dimensional and three-dimensional images of the body that provide important medical information with none of the hazards of x-rays. MRI is based on an effect called **nuclear magnetic resonance (NMR)** in which an externally applied magnetic field interacts with the nuclei of certain atoms, particularly those of hydrogen (protons). These nuclei possess their own small magnetic fields, similar to those of electrons and the current loops discussed earlier in this chapter.

When placed in an external magnetic field, such nuclei experience a torque that pushes or aligns the nuclei into one of two new energy states—depending on the orientation of its spin (analogous to the N pole and S pole in a bar magnet). Transitions from the lower to higher energy state can be achieved by using an external radio frequency signal to "flip" the orientation of the small magnets. (This is actually a quantum mechanical process. The direction of the nuclear magnetic field is quantized as is energy in the radio waves. We will return to these topics in later chapters.) The specific frequency of the radio waves that are absorbed and reemitted depends sensitively on the type of nucleus, the chemical environment, and the external magnetic field strength. Therefore, this is a *resonance* phenomenon in which *nuclei* in a *magnetic* field act like resonators (analogous to those discussed in the treatment of sound in <u>Oscillatory</u>

<u>Motion and Waves</u>) that absorb and reemit only certain frequencies. Hence, the phenomenon is named *nuclear magnetic resonance (NMR)*.

NMR has been used for more than 50 years as an analytical tool. It was formulated in 1946 by F. Bloch and E. Purcell, with the 1952 Nobel Prize in Physics going to them for their work. Over the past two decades, NMR has been developed to produce detailed images in a process now called magnetic resonance imaging (MRI), a name coined to avoid the use of the word "nuclear" and the concomitant implication that nuclear radiation is involved. (It is not.) The 2003 Nobel Prize in Medicine went to P. Lauterbur and P. Mansfield for their work with MRI applications.

The largest part of the MRI unit is a superconducting magnet that creates a magnetic field, typically between 1 and 2 T in strength, over a relatively large volume. MRI images can be both highly detailed and informative about structures and organ functions. It is helpful that normal and nonnormal tissues respond differently for slight changes in the magnetic field. In most medical images, the protons that are hydrogen nuclei are imaged. (About 2/3 of the atoms in the body are hydrogen.) Their location and density give a variety of medically useful information, such as organ function, the condition of tissue (as in the brain), and the shape of structures, such as vertebral disks and knee-joint surfaces. MRI can also be used to follow the movement of certain ions across membranes, yielding information on active transport, osmosis, dialysis, and other phenomena. With excellent spatial resolution, MRI can provide information about tumors, strokes, shoulder injuries, infections, etc.

An image requires position information as well as the density of a nuclear type (usually protons). By varying the magnetic field slightly over the volume to be imaged, the resonant frequency of the protons is made to vary with position. Broadcast radio frequencies are swept over an appropriate range and nuclei absorb and reemit them only if the nuclei are in a magnetic field with the correct strength. The imaging receiver gathers information through the body almost point by point, building up a tissue map. The reception of reemitted radio waves as a function of frequency thus gives position information. These "slices" or cross sections through the body are only several mm thick. The intensity of the reemitted radio waves is

proportional to the concentration of the nuclear type being flipped, as well as information on the chemical environment in that area of the body. Various techniques are available for enhancing contrast in images and for obtaining more information. Scans called T1, T2, or proton density scans rely on different relaxation mechanisms of nuclei. Relaxation refers to the time it takes for the protons to return to equilibrium after the external field is turned off. This time depends upon tissue type and status (such as inflammation).

While MRI images are superior to x rays for certain types of tissue and have none of the hazards of x rays, they do not completely supplant x-ray images. MRI is less effective than x rays for detecting breaks in bone, for example, and in imaging breast tissue, so the two diagnostic tools complement each other. MRI images are also expensive compared to simple x-ray images and tend to be used most often where they supply information not readily obtained from x rays. Another disadvantage of MRI is that the patient is totally enclosed with detectors close to the body for about 30 minutes or more, leading to claustrophobia. It is also difficult for the obese patient to be in the magnet tunnel. New "open-MRI" machines are now available in which the magnet does not completely surround the patient.

Over the last decade, the development of much faster scans, called "functional MRI" (fMRI), has allowed us to map the functioning of various regions in the brain responsible for thought and motor control. This technique measures the change in blood flow for activities (thought, experiences, action) in the brain. The nerve cells increase their consumption of oxygen when active. Blood hemoglobin releases oxygen to active nerve cells and has somewhat different magnetic properties when oxygenated than when deoxygenated. With MRI, we can measure this and detect a blood oxygen-dependent signal. Most of the brain scans today use fMRI.

# **Other Medical Uses of Magnetic Fields**

Currents in nerve cells and the heart create magnetic fields like any other currents. These can be measured but with some difficulty since their strengths are about  $10^{-6}$  to  $10^{-8}$  *less* than the Earth's magnetic field. Recording of the heart's magnetic field as it beats is called a

**magnetocardiogram (MCG)**, while measurements of the brain's magnetic field is called a **magnetoencephalogram (MEG)**. Both give information that differs from that obtained by measuring the electric fields of these organs (ECGs and EEGs), but they are not yet of sufficient importance to make these difficult measurements common.

In both of these techniques, the sensors do not touch the body. MCG can be used in fetal studies, and is probably more sensitive than echocardiography. MCG also looks at the heart's electrical activity whose voltage output is too small to be recorded by surface electrodes as in EKG. It has the potential of being a rapid scan for early diagnosis of cardiac ischemia (obstruction of blood flow to the heart) or problems with the fetus.

MEG can be used to identify abnormal electrical discharges in the brain that produce weak magnetic signals. Therefore, it looks at brain activity, not just brain structure. It has been used for studies of Alzheimer's disease and epilepsy. Advances in instrumentation to measure very small magnetic fields have allowed these two techniques to be used more in recent years. What is used is a sensor called a SQUID, for superconducting quantum interference device. This operates at liquid helium temperatures and can measure magnetic fields thousands of times smaller than the Earth's.

Finally, there is a burgeoning market for magnetic cures in which magnets are applied in a variety of ways to the body, from magnetic bracelets to magnetic mattresses. The best that can be said for such practices is that they are apparently harmless, unless the magnets get close to the patient's computer or magnetic storage disks. Claims are made for a broad spectrum of benefits from cleansing the blood to giving the patient more energy, but clinical studies have not verified these claims, nor is there an identifiable mechanism by which such benefits might occur.

#### **Note:**

# PhET Explorations: Magnet and Compass

Ever wonder how a compass worked to point you to the Arctic? Explore the interactions between a compass and bar magnet, and then add the Earth and find the surprising answer! Vary the magnet's strength, and see how

things change both inside and outside. Use the field meter to measure how the magnetic field changes.

https://archive.cnx.org/specials/5ca3e2cc-ae74-11e5-b6d3-f3c228f04b5c/magnet-and-compass/#sim-bar-magnet

# **Section Summary**

 Crossed (perpendicular) electric and magnetic fields act as a velocity filter, giving equal and opposite forces on any charge with velocity perpendicular to the fields and of magnitude
 Equation:

$$v = \frac{E}{B}.$$

# **Conceptual Questions**

#### Exercise:

#### **Problem:**

Measurements of the weak and fluctuating magnetic fields associated with brain activity are called magnetoencephalograms (MEGs). Do the brain's magnetic fields imply coordinated or uncoordinated nerve impulses? Explain.

#### **Exercise:**

#### **Problem:**

Discuss the possibility that a Hall voltage would be generated on the moving heart of a patient during MRI imaging. Also discuss the same effect on the wires of a pacemaker. (The fact that patients with pacemakers are not given MRIs is significant.)

A patient in an MRI unit turns his head quickly to one side and experiences momentary dizziness and a strange taste in his mouth. Discuss the possible causes.

#### **Exercise:**

#### **Problem:**

You are told that in a certain region there is either a uniform electric or magnetic field. What measurement or observation could you make to determine the type? (Ignore the Earth's magnetic field.)

#### **Exercise:**

#### **Problem:**

An example of magnetohydrodynamics (MHD) comes from the flow of a river (salty water). This fluid interacts with the Earth's magnetic field to produce a potential difference between the two river banks. How would you go about calculating the potential difference?

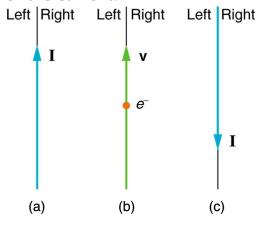
#### **Exercise:**

#### **Problem:**

Draw gravitational field lines between 2 masses, electric field lines between a positive and a negative charge, electric field lines between 2 positive charges and magnetic field lines around a magnet. Qualitatively describe the differences between the fields and the entities responsible for the field lines.

#### **Problems & Exercises**

Indicate whether the magnetic field created in each of the three situations shown in [link] is into or out of the page on the left and right of the current.



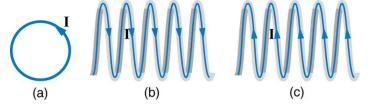
# **Solution:**

- (a) right-into page, left-out of page
- (b) right-out of page, left-into page
- (c) right-out of page, left-into page

### **Exercise:**

### **Problem:**

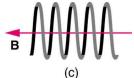
What are the directions of the fields in the center of the loop and coils shown in [link]?



What are the directions of the currents in the loop and coils shown in

[link]?





# **Solution:**

- (a) clockwise
- (b) clockwise as seen from the left
- (c) clockwise as seen from the right

#### **Exercise:**

#### **Problem:**

To see why an MRI utilizes iron to increase the magnetic field created by a coil, calculate the current needed in a 400-loop-per-meter circular coil 0.660 m in radius to create a 1.20-T field (typical of an MRI instrument) at its center with no iron present. The magnetic field of a proton is approximately like that of a circular current loop  $0.650 \times 10^{-15}$  m in radius carrying  $1.05 \times 10^4$  A. What is the field at the center of such a loop?

# **Solution:**

$$1.01 \times 10^{13} \mathrm{T}$$

## **Exercise:**

# Problem:

Inside a motor, 30.0 A passes through a 250-turn circular loop that is 10.0 cm in radius. What is the magnetic field strength created at its center?

#### **Exercise:**

#### **Problem:**

Nonnuclear submarines use batteries for power when submerged. (a) Find the magnetic field 50.0 cm from a straight wire carrying 1200 A from the batteries to the drive mechanism of a submarine. (b) What is the field if the wires to and from the drive mechanism are side by side? (c) Discuss the effects this could have for a compass on the submarine that is not shielded.

#### **Solution:**

- (a)  $4.80 \times 10^{-4} \text{ T}$
- (b) Zero
- (c) If the wires are not paired, the field is about 10 times stronger than Earth's magnetic field and so could severely disrupt the use of a compass.

#### **Exercise:**

#### **Problem:**

How strong is the magnetic field inside a solenoid with 10,000 turns per meter that carries 20.0 A?

#### **Exercise:**

#### **Problem:**

What current is needed in the solenoid described in [link] to produce a magnetic field  $10^4$  times the Earth's magnetic field of  $5.00 \times 10^{-5}$  T?

#### **Solution:**

39.8 A

How far from the starter cable of a car, carrying 150 A, must you be to experience a field less than the Earth's  $(5.00 \times 10^{-5} \text{ T})$ ? Assume a long straight wire carries the current. (In practice, the body of your car shields the dashboard compass.)

#### **Exercise:**

#### **Problem:**

Measurements affect the system being measured, such as the current loop in [link]. (a) Estimate the field the loop creates by calculating the field at the center of a circular loop 20.0 cm in diameter carrying 5.00 A. (b) What is the smallest field strength this loop can be used to measure, if its field must alter the measured field by less than 0.0100%?

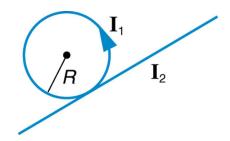
#### **Solution:**

- (a)  $3.14 \times 10^{-5} \text{ T}$
- (b) 0.314 T

#### **Exercise:**

#### **Problem:**

[link] shows a long straight wire just touching a loop carrying a current  $I_1$ . Both lie in the same plane. (a) What direction must the current  $I_2$  in the straight wire have to create a field at the center of the loop in the direction opposite to that created by the loop? (b) What is the ratio of  $I_1/I_2$  that gives zero field strength at the center of the loop? (c) What is the direction of the field directly above the loop under this circumstance?



#### **Exercise:**

#### **Problem:**

Find the magnitude and direction of the magnetic field at the point equidistant from the wires in [link](a), using the rules of vector addition to sum the contributions from each wire.

#### **Solution:**

 $7.55 \times 10^{-5} \text{ T}, 23.4^{\circ}$ 

#### **Exercise:**

#### **Problem:**

Find the magnitude and direction of the magnetic field at the point equidistant from the wires in [link](b), using the rules of vector addition to sum the contributions from each wire.

#### **Exercise:**

#### **Problem:**

What current is needed in the top wire in [link](a) to produce a field of zero at the point equidistant from the wires, if the currents in the bottom two wires are both 10.0 A into the page?

#### **Solution:**

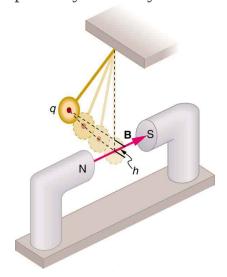
10.0 A

Calculate the size of the magnetic field 20 m below a high voltage power line. The line carries 450 MW at a voltage of 300,000 V.

#### **Exercise:**

# **Problem:** Integrated Concepts

(a) A pendulum is set up so that its bob (a thin copper disk) swings between the poles of a permanent magnet as shown in [link]. What is the magnitude and direction of the magnetic force on the bob at the lowest point in its path, if it has a positive  $0.250~\mu C$  charge and is released from a height of 30.0 cm above its lowest point? The magnetic field strength is 1.50 T. (b) What is the acceleration of the bob at the bottom of its swing if its mass is 30.0 grams and it is hung from a flexible string? Be certain to include a free-body diagram as part of your analysis.



### **Solution:**

(a) 
$$9.09 \times 10^{-7} \ N$$
 upward

(b) 
$$3.03 \times 10^{-5} \; \mathrm{m/s}^2$$

# **Problem:** Integrated Concepts

(a) What voltage will accelerate electrons to a speed of  $6.00 \times 10^{-7}$  m/s? (b) Find the radius of curvature of the path of a *proton* accelerated through this potential in a 0.500-T field and compare this with the radius of curvature of an electron accelerated through the same potential.

#### **Exercise:**

# **Problem:** Integrated Concepts

Find the radius of curvature of the path of a 25.0-MeV proton moving perpendicularly to the 1.20-T field of a cyclotron.

#### **Solution:**

60.2 cm

#### **Exercise:**

# **Problem:** Integrated Concepts

To construct a nonmechanical water meter, a 0.500-T magnetic field is placed across the supply water pipe to a home and the Hall voltage is recorded. (a) Find the flow rate in liters per second through a 3.00-cm-diameter pipe if the Hall voltage is 60.0 mV. (b) What would the Hall voltage be for the same flow rate through a 10.0-cm-diameter pipe with the same field applied?

#### **Exercise:**

# **Problem:** Integrated Concepts

(a) Using the values given for an MHD drive in [link], and assuming the force is uniformly applied to the fluid, calculate the pressure created in  $N/m^2$ . (b) Is this a significant fraction of an atmosphere?

### **Solution:**

- (a)  $1.02 \times 10^3 \text{ N/m}^2$
- (b) Not a significant fraction of an atmosphere

#### **Exercise:**

# **Problem:** Integrated Concepts

(a) Calculate the maximum torque on a 50-turn, 1.50 cm radius circular current loop carrying  $50~\mu A$  in a 0.500-T field. (b) If this coil is to be used in a galvanometer that reads  $50~\mu A$  full scale, what force constant spring must be used, if it is attached 1.00 cm from the axis of rotation and is stretched by the  $60^{\circ}$  arc moved?

#### **Exercise:**

# **Problem:** Integrated Concepts

A current balance used to define the ampere is designed so that the current through it is constant, as is the distance between wires. Even so, if the wires change length with temperature, the force between them will change. What percent change in force per degree will occur if the wires are copper?

#### **Solution:**

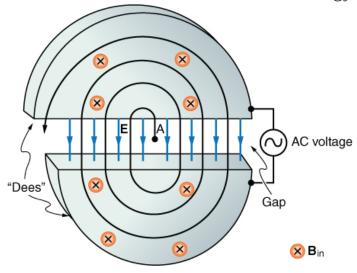
$$17.0 \times 10^{-4} \% / ^{\circ} \text{C}$$

#### **Exercise:**

# **Problem:**Integrated Concepts

(a) Show that the period of the circular orbit of a charged particle moving perpendicularly to a uniform magnetic field is  $T=2\pi m/({\rm qB})$ . (b) What is the frequency f? (c) What is the angular

velocity  $\omega$ ? Note that these results are independent of the velocity and radius of the orbit and, hence, of the energy of the particle. ([link].)



Cyclotrons accelerate charged particles orbiting in a magnetic field by placing an AC voltage on the metal Dees, between which the particles move, so that energy is added twice each orbit. The frequency is constant, since it is independent of the particle energy—the radius of the orbit simply increases with energy until the particles approach the edge and are extracted for various experiments and applications.

#### **Exercise:**

# **Problem:** Integrated Concepts

A cyclotron accelerates charged particles as shown in [link]. Using the results of the previous problem, calculate the frequency of the accelerating voltage needed for a proton in a 1.20-T field.

#### **Solution:**

18.3 MHz

#### **Exercise:**

# **Problem:** Integrated Concepts

(a) A 0.140-kg baseball, pitched at 40.0 m/s horizontally and perpendicular to the Earth's horizontal  $5.00 \times 10^{-5}$  T field, has a 100-nC charge on it. What distance is it deflected from its path by the magnetic force, after traveling 30.0 m horizontally? (b) Would you suggest this as a secret technique for a pitcher to throw curve balls?

#### **Exercise:**

# **Problem:** Integrated Concepts

(a) What is the direction of the force on a wire carrying a current due east in a location where the Earth's field is due north? Both are parallel to the ground. (b) Calculate the force per meter if the wire carries 20.0 A and the field strength is  $3.00 \times 10^{-5}$  T. (c) What diameter copper wire would have its weight supported by this force? (d) Calculate the resistance per meter and the voltage per meter needed.

### **Solution:**

- (a) Straight up
- (b)  $6.00 \times 10^{-4} \text{ N/m}$
- (c)  $94.1 \, \mu m$
- (d)2.47  $\Omega/m$ , 49.4 V/m

#### **Exercise:**

**Problem:** Integrated Concepts

One long straight wire is to be held directly above another by repulsion between their currents. The lower wire carries 100 A and the wire 7.50 cm above it is 10-gauge (2.588 mm diameter) copper wire. (a) What current must flow in the upper wire, neglecting the Earth's field? (b) What is the smallest current if the Earth's  $3.00 \times 10^{-5}$  T field is parallel to the ground and is not neglected? (c) Is the supported wire in a stable or unstable equilibrium if displaced vertically? If displaced horizontally?

### **Exercise:**

#### **Problem:** Unreasonable Results

(a) Find the charge on a baseball, thrown at 35.0 m/s perpendicular to the Earth's  $5.00 \times 10^{-5}$  T field, that experiences a 1.00-N magnetic force. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

#### **Solution:**

- (a) 571 C
- (b) Impossible to have such a large separated charge on such a small object.
- (c) The 1.00-N force is much too great to be realistic in the Earth's field.

#### **Exercise:**

#### **Problem:** Unreasonable Results

A charged particle having mass  $6.64 \times 10^{-27}$  kg (that of a helium atom) moving at  $8.70 \times 10^5$  m/s perpendicular to a 1.50-T magnetic field travels in a circular path of radius 16.0 mm. (a) What is the charge of the particle? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

#### **Problem:** Unreasonable Results

An inventor wants to generate 120-V power by moving a 1.00-m-long wire perpendicular to the Earth's  $5.00 \times 10^{-5}$  T field. (a) Find the speed with which the wire must move. (b) What is unreasonable about this result? (c) Which assumption is responsible?

#### **Solution:**

- (a)  $2.40 \times 10^6 \text{ m/s}$
- (b) The speed is too high to be practical  $\leq 1\%$  speed of light
- (c) The assumption that you could reasonably generate such a voltage with a single wire in the Earth's field is unreasonable

#### **Exercise:**

#### **Problem:** Unreasonable Results

Frustrated by the small Hall voltage obtained in blood flow measurements, a medical physicist decides to increase the applied magnetic field strength to get a 0.500-V output for blood moving at 30.0 cm/s in a 1.50-cm-diameter vessel. (a) What magnetic field strength is needed? (b) What is unreasonable about this result? (c) Which premise is responsible?

#### **Exercise:**

#### **Problem:** Unreasonable Results

A surveyor 100 m from a long straight 200-kV DC power line suspects that its magnetic field may equal that of the Earth and affect compass readings. (a) Calculate the current in the wire needed to create a  $5.00 \times 10^{-5}$  T field at this distance. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

#### **Solution:**

- (a) 25.0 kA
- (b) This current is unreasonably high. It implies a total power delivery in the line of 50.0x10<sup>9</sup> W, which is much too high for standard transmission lines.
- (c)100 meters is a long distance to obtain the required field strength. Also coaxial cables are used for transmission lines so that there is virtually no field for DC power lines, because of cancellation from opposing currents. The surveyor's concerns are not a problem for his magnetic field measurements.

#### **Exercise:**

#### **Problem:**Construct Your Own Problem

Consider a mass separator that applies a magnetic field perpendicular to the velocity of ions and separates the ions based on the radius of curvature of their paths in the field. Construct a problem in which you calculate the magnetic field strength needed to separate two ions that differ in mass, but not charge, and have the same initial velocity. Among the things to consider are the types of ions, the velocities they can be given before entering the magnetic field, and a reasonable value for the radius of curvature of the paths they follow. In addition, calculate the separation distance between the ions at the point where they are detected.

#### **Exercise:**

#### **Problem:** Construct Your Own Problem

Consider using the torque on a current-carrying coil in a magnetic field to detect relatively small magnetic fields (less than the field of the Earth, for example). Construct a problem in which you calculate the maximum torque on a current-carrying loop in a magnetic field. Among the things to be considered are the size of the coil, the number

of loops it has, the current you pass through the coil, and the size of the field you wish to detect. Discuss whether the torque produced is large enough to be effectively measured. Your instructor may also wish for you to consider the effects, if any, of the field produced by the coil on the surroundings that could affect detection of the small field.

# Glossary

magnetic resonance imaging (MRI)

a medical imaging technique that uses magnetic fields create detailed images of internal tissues and organs

nuclear magnetic resonance (NMR)

a phenomenon in which an externally applied magnetic field interacts with the nuclei of certain atoms

magnetocardiogram (MCG)

a recording of the heart's magnetic field as it beats

magnetoencephalogram (MEG)

a measurement of the brain's magnetic field

# Introduction to Waves class="introduction"

```
There
 are at
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  of
waves
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water
waves
  are
evident
. There
are also
sound
waves,
 light
waves,
 and
waves
on the
guitar
strings.
(credit:
 John
Norton
   )
```



What do an ocean buoy, a child in a swing, the cone inside a speaker, a guitar, atoms in a crystal, the motion of chest cavities, and the beating of hearts all have in common? They all **oscillate**—that is, they move back and forth between two points. Many systems oscillate, and they have certain characteristics in common. All oscillations involve force and energy. You push a child in a swing to get the motion started. The energy of atoms vibrating in a crystal can be increased with heat. You put energy into a guitar string when you pluck it.

Some oscillations create **waves**. A guitar creates sound waves. You can make water waves in a swimming pool by slapping the water with your hand. You can no doubt think of other types of waves. Some, such as water waves, are visible. Some, such as sound waves, are not. But *every wave is a disturbance that moves from its source and carries energy*. Other examples of waves include earthquakes and visible light. Even subatomic particles, such as electrons, can behave like waves.

By studying oscillatory motion and waves, we shall find that a small number of underlying principles describe all of them and that wave phenomena are more common than you have ever imagined. We begin by studying the type of force that underlies the simplest oscillations and waves. We will then expand our exploration of oscillatory motion and waves to include concepts such as simple harmonic motion, uniform circular motion, and damped harmonic motion. Finally, we will explore what happens when two or more waves share the same space, in the phenomena known as superposition and interference.

# **Glossary**

oscillate

moving back and forth regularly between two points

wave

a disturbance that moves from its source and carries energy

# Period and Frequency in Oscillations

- Observe the vibrations of a guitar string.
- Determine the frequency of oscillations.



The strings on this guitar vibrate at regular time intervals. (credit: JAR)

When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each successive vibration of the string takes the same time as the previous one. We define **periodic motion** to be a motion that repeats itself at regular time intervals, such as exhibited by the guitar string or by an object on a spring moving up and down. The time to complete one oscillation remains constant and is called the **period** T. Its units are usually seconds, but may be any convenient unit of time. The word period refers to the time for some event whether repetitive or not; but we shall be primarily interested in periodic motion, which is by definition repetitive. A concept closely related to period is the frequency of an event. For example, if you get a paycheck twice a month, the frequency of payment is two per month and the period between checks is half a month. **Frequency** f is defined to be the number of events per unit time. For periodic motion, frequency is the number of oscillations per unit time. The relationship between frequency and period is

# **Equation:**

$$f = \frac{1}{T}$$
.

The SI unit for frequency is the *cycle per second*, which is defined to be a *hertz* (Hz):

### **Equation:**

$$1 \text{ Hz} = 1 \frac{\text{cycle}}{\text{sec}} \text{ or } 1 \text{ Hz} = \frac{1}{\text{s}}$$

A cycle is one complete oscillation. Note that a vibration can be a single or multiple event, whereas oscillations are usually repetitive for a significant number of cycles.

# **Example:**

# Determine the Frequency of Two Oscillations: Medical Ultrasound and the Period of Middle C

We can use the formulas presented in this module to determine both the frequency based on known oscillations and the oscillation based on a known frequency. Let's try one example of each. (a) A medical imaging device produces ultrasound by oscillating with a period of 0.400 µs. What is the frequency of this oscillation? (b) The frequency of middle C on a typical musical instrument is 264 Hz. What is the time for one complete oscillation?

# Strategy

Both questions (a) and (b) can be answered using the relationship between period and frequency. In question (a), the period T is given and we are asked to find frequency f. In question (b), the frequency f is given and we are asked to find the period T.

#### Solution a

1. Substitute 0.400  $\mu$ s for T in  $f = \frac{1}{T}$ : **Equation:** 

$$f = \frac{1}{T} = \frac{1}{0.400 \times 10^{-6} \text{ s}}.$$

Solve to find **Equation:** 

$$f = 2.50 \times 10^6 \; {\rm Hz}.$$

#### Discussion a

The frequency of sound found in (a) is much higher than the highest frequency that humans can hear and, therefore, is called ultrasound. Appropriate oscillations at this frequency generate ultrasound used for noninvasive medical diagnoses, such as observations of a fetus in the womb.

#### Solution b

1. Identify the known values:

The time for one complete oscillation is the period T:

**Equation:** 

$$f = \frac{1}{T}$$
.

2. Solve for T:

**Equation:** 

$$T = \frac{1}{f}.$$

3. Substitute the given value for the frequency into the resulting expression:

**Equation:** 

$$T = \frac{1}{f} = \frac{1}{264 \text{ Hz}} = \frac{1}{264 \text{ cycles/s}} = 3.79 \times 10^{-3} \text{ s} = 3.79 \text{ ms}.$$

#### Discussion

The period found in (b) is the time per cycle, but this value is often quoted as simply the time in convenient units (ms or milliseconds in this case).

### **Exercise:**

# **Check your Understanding**

#### **Problem:**

Identify an event in your life (such as receiving a paycheck) that occurs regularly. Identify both the period and frequency of this event.

#### **Solution:**

I visit my parents for dinner every other Sunday. The frequency of my visits is 26 per calendar year. The period is two weeks.

# **Section Summary**

- Periodic motion is a repetitious oscillation.
- The time for one oscillation is the period T.
- The number of oscillations per unit time is the frequency f.
- These quantities are related by Equation:

$$f = \frac{1}{T}$$
.

# **Problems & Exercises**

#### **Exercise:**

**Problem:** What is the period of 60.0 Hz electrical power?

Solution:	
16.7 ms	
Exercise:	
Problem:	
If your heart rate is 150 beats per minute during strenuous exercise, what is the time per beat in units of seconds?	
Solution:	
$0.400~\mathrm{s/beats}$	
Exercise:	
Problem:	
Find the frequency of a tuning fork that takes $2.50\times 10^{-3}\ \mathrm{s}$ to complete one oscillation.	
Solution:	
400 Hz	
Exercise:	
Problem:	
A stroboscope is set to flash every $8.00 \times 10^{-5} \ \mathrm{s}$ . What is the frequency of the flashes?	
Solution:	
12,500 Hz	
Exercise:	

A tire has a tread pattern with a crevice every 2.00 cm. Each crevice makes a single vibration as the tire moves. What is the frequency of these vibrations if the car moves at 30.0 m/s?

#### **Solution:**

1.50 kHz

#### **Exercise:**

#### **Problem:** Engineering Application

Each piston of an engine makes a sharp sound every other revolution of the engine. (a) How fast is a race car going if its eight-cylinder engine emits a sound of frequency 750 Hz, given that the engine makes 2000 revolutions per kilometer? (b) At how many revolutions per minute is the engine rotating?

#### **Solution:**

- (a) 93.8 m/s
- (b)  $11.3 \times 10^3 \ \mathrm{rev/min}$

# **Glossary**

period

time it takes to complete one oscillation

periodic motion

motion that repeats itself at regular time intervals

frequency

number of events per unit of time

#### Waves

- State the characteristics of a wave.
- Calculate the velocity of wave propagation.



Waves in the ocean behave similarly to all other types of waves. (credit: Steve Jurveston, Flickr)

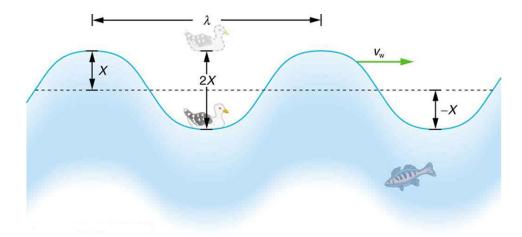
What do we mean when we say something is a wave? The most intuitive and easiest wave to imagine is the familiar water wave. More precisely, a wave is a disturbance that propagates, or moves from the place it was created. For water waves, the disturbance is in the surface of the water, perhaps created by a rock thrown into a pond or by a swimmer splashing the surface repeatedly. For sound waves, the disturbance is a change in air pressure, perhaps created by the oscillating cone inside a speaker. For earthquakes, there are several types of disturbances, including disturbance of Earth's surface and pressure disturbances under the surface. Even radio waves are most easily understood using an analogy with water waves. Visualizing water waves is useful because there is more to it than just a mental image. Water waves exhibit characteristics common to all waves, such as amplitude, period, frequency and energy. All wave characteristics can be described by a small set of underlying principles.

A wave is a disturbance that propagates, or moves from the place it was created. The simplest waves repeat themselves for several cycles and are associated with simple harmonic motion. Let us start by considering the simplified water wave in [link]. The wave is an up and down disturbance of the water surface. It causes a sea gull to move up and down in simple harmonic motion as the wave crests and troughs (peaks and valleys) pass under the bird. The time for one complete up and down motion is the wave's period T. The wave's frequency is f=1/T, as usual. The wave itself moves to the right in the figure. This movement of the wave is actually the disturbance moving to the right, not the water itself (or the bird would move to the right). We define **wave velocity**  $v_{\rm w}$  to be the speed at which the disturbance moves. Wave velocity is sometimes also called the *propagation velocity or propagation speed*, because the disturbance propagates from one location to another.

#### Note:

# Misconception Alert

Many people think that water waves push water from one direction to another. In fact, the particles of water tend to stay in one location, save for moving up and down due to the energy in the wave. The energy moves forward through the water, but the water stays in one place. If you feel yourself pushed in an ocean, what you feel is the energy of the wave, not a rush of water.



An idealized ocean wave passes under a sea gull that bobs up and down in simple harmonic motion. The wave has a wavelength  $\lambda$ , which is the distance between adjacent identical parts of the wave. The up and down disturbance of the surface propagates parallel to the surface at a speed  $v_{\rm w}$ .

The water wave in the figure also has a length associated with it, called its **wavelength**  $\lambda$ , the distance between adjacent identical parts of a wave. ( $\lambda$  is the distance parallel to the direction of propagation.) The speed of propagation  $v_{\rm w}$  is the distance the wave travels in a given time, which is one wavelength in the time of one period. In equation form, that is

# **Equation:**

$$v_{
m w}=rac{\lambda}{T}$$

or

# **Equation:**

$$v_{
m w}=f\lambda.$$

This fundamental relationship holds for all types of waves. For water waves,  $v_{\rm w}$  is the speed of a surface wave; for sound,  $v_{\rm w}$  is the speed of sound; and for visible light,  $v_{\rm w}$  is the speed of light, for example.

#### Note:

# Take-Home Experiment: Waves in a Bowl

Fill a large bowl or basin with water and wait for the water to settle so there are no ripples. Gently drop a cork into the middle of the bowl. Estimate the wavelength and period of oscillation of the water wave that propagates away from the cork. Remove the cork from the bowl and wait

for the water to settle again. Gently drop the cork at a height that is different from the first drop. Does the wavelength depend upon how high above the water the cork is dropped?

# **Example:**

# Calculate the Velocity of Wave Propagation: Gull in the Ocean

Calculate the wave velocity of the ocean wave in [link] if the distance between wave crests is 10.0 m and the time for a sea gull to bob up and down is 5.00 s.

# Strategy

We are asked to find  $v_{\rm w}$ . The given information tells us that  $\lambda=10.0~{\rm m}$  and  $T=5.00~{\rm s}$ . Therefore, we can use  $v_{\rm w}=\frac{\lambda}{T}$  to find the wave velocity.

### Solution

1. Enter the known values into  $v_{\mathrm{w}}=\frac{\lambda}{T}$ : **Equation:** 

$$v_{
m w} = rac{10.0 \ 
m m}{5.00 \ 
m s}.$$

2. Solve for  $v_{\rm w}$  to find  $v_{\rm w}$ = 2.00 m/s.

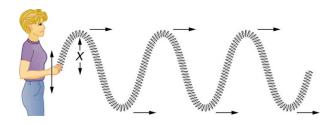
#### Discussion

This slow speed seems reasonable for an ocean wave. Note that the wave moves to the right in the figure at this speed, not the varying speed at which the sea gull moves up and down.

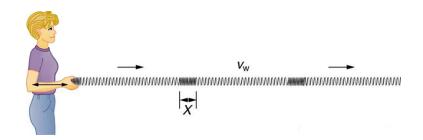
# **Transverse and Longitudinal Waves**

A simple wave consists of a periodic disturbance that propagates from one place to another. The wave in [link] propagates in the horizontal direction while the surface is disturbed in the vertical direction. Such a wave is called a **transverse wave** or shear wave; in such a wave, the disturbance is perpendicular to the direction of propagation. In contrast, in a **longitudinal** 

**wave** or compressional wave, the disturbance is parallel to the direction of propagation. [link] shows an example of a longitudinal wave. The size of the disturbance is its amplitude X and is completely independent of the speed of propagation  $v_{\rm w}$ .



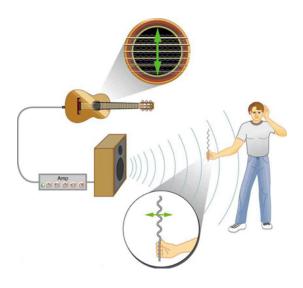
In this example of a transverse wave, the wave propagates horizontally, and the disturbance in the cord is in the vertical direction.



In this example of a longitudinal wave, the wave propagates horizontally, and the disturbance in the cord is also in the horizontal direction.

Waves may be transverse, longitudinal, or *a combination of the two*. (Water waves are actually a combination of transverse and longitudinal. The simplified water wave illustrated in [link] shows no longitudinal motion of the bird.) The waves on the strings of musical instruments are transverse—so are electromagnetic waves, such as visible light.

Sound waves in air and water are longitudinal. Their disturbances are periodic variations in pressure that are transmitted in fluids. Fluids do not have appreciable shear strength, and thus the sound waves in them must be longitudinal or compressional. Sound in solids can be both longitudinal and transverse.



The wave on a guitar string is transverse. The sound wave rattles a sheet of paper in a direction that shows the sound wave is longitudinal.

Earthquake waves under Earth's surface also have both longitudinal and transverse components (called compressional or P-waves and shear or S-waves, respectively). These components have important individual characteristics—they propagate at different speeds, for example.

Earthquakes also have surface waves that are similar to surface waves on water.

#### **Exercise:**

# **Check Your Understanding**

### **Problem:**

Why is it important to differentiate between longitudinal and transverse waves?

#### **Solution:**

In the different types of waves, energy can propagate in a different direction relative to the motion of the wave. This is important to understand how different types of waves affect the materials around them.

#### Note:

## PhET Explorations: Wave on a String

Watch a string vibrate in slow motion. Wiggle the end of the string and make waves, or adjust the frequency and amplitude of an oscillator. Adjust the damping and tension. The end can be fixed, loose, or open. <a href="https://phet.colorado.edu/sims/html/wave-on-a-string/latest/wave-on-a-string\_en.html">https://phet.colorado.edu/sims/html/wave-on-a-string/latest/wave-on-a-string\_en.html</a>

# **Section Summary**

- A wave is a disturbance that moves from the point of creation with a wave velocity  $v_{\mathrm{w}}$ .
- A wave has a wavelength  $\lambda$ , which is the distance between adjacent identical parts of the wave.
- Wave velocity and wavelength are related to the wave's frequency and period by  $v_{\rm w}=\frac{\lambda}{T}$  or  $v_{\rm w}=f\lambda$ .

• A transverse wave has a disturbance perpendicular to its direction of propagation, whereas a longitudinal wave has a disturbance parallel to its direction of propagation.

# **Conceptual Questions**

#### **Exercise:**

### **Problem:**

Give one example of a transverse wave and another of a longitudinal wave, being careful to note the relative directions of the disturbance and wave propagation in each.

#### **Exercise:**

### **Problem:**

What is the difference between propagation speed and the frequency of a wave? Does one or both affect wavelength? If so, how?

## **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

Storms in the South Pacific can create waves that travel all the way to the California coast, which are 12,000 km away. How long does it take them if they travel at 15.0 m/s?

# **Solution:**

**Equation:** 

$$t = 9.26 \; \mathrm{d}$$

### **Exercise:**

### **Problem:**

Waves on a swimming pool propagate at 0.750 m/s. You splash the water at one end of the pool and observe the wave go to the opposite end, reflect, and return in 30.0 s. How far away is the other end of the pool?

### **Exercise:**

## **Problem:**

Wind gusts create ripples on the ocean that have a wavelength of 5.00 cm and propagate at 2.00 m/s. What is their frequency?

# **Solution:**

**Equation:** 

$$f=40.0~\mathrm{Hz}$$

### **Exercise:**

#### Problem:

How many times a minute does a boat bob up and down on ocean waves that have a wavelength of 40.0 m and a propagation speed of 5.00 m/s?

#### **Exercise:**

#### **Problem:**

Scouts at a camp shake the rope bridge they have just crossed and observe the wave crests to be 8.00 m apart. If they shake it the bridge twice per second, what is the propagation speed of the waves?

### **Solution:**

## **Equation:**

$$v_{
m w}=16.0~{
m m/s}$$

#### **Exercise:**

#### **Problem:**

What is the wavelength of the waves you create in a swimming pool if you splash your hand at a rate of 2.00 Hz and the waves propagate at 0.800 m/s?

### **Exercise:**

### **Problem:**

What is the wavelength of an earthquake that shakes you with a frequency of 10.0 Hz and gets to another city 84.0 km away in 12.0 s?

**Solution:** 

**Equation:** 

 $\lambda = 700 \text{ m}$ 

#### **Exercise:**

#### **Problem:**

Radio waves transmitted through space at  $3.00\times10^8~m/s$  by the Voyager spacecraft have a wavelength of 0.120 m. What is their frequency?

#### **Exercise:**

#### Problem:

Your ear is capable of differentiating sounds that arrive at the ear just 1.00 ms apart. What is the minimum distance between two speakers that produce sounds that arrive at noticeably different times on a day when the speed of sound is 340 m/s?

**Solution:** 

**Equation:** 

#### **Exercise:**

#### **Problem:**

(a) Seismographs measure the arrival times of earthquakes with a precision of 0.100 s. To get the distance to the epicenter of the quake, they compare the arrival times of S- and P-waves, which travel at different speeds. [link]) If S- and P-waves travel at 4.00 and 7.20 km/s, respectively, in the region considered, how precisely can the distance to the source of the earthquake be determined? (b) Seismic waves from underground detonations of nuclear bombs can be used to locate the test site and detect violations of test bans. Discuss whether your answer to (a) implies a serious limit to such detection. (Note also that the uncertainty is greater if there is an uncertainty in the propagation speeds of the S- and P-waves.)



A seismograph as described in above problem.(credit: Oleg Alexandrov)

# **Glossary**

longitudinal wave

a wave in which the disturbance is parallel to the direction of propagation

#### transverse wave

a wave in which the disturbance is perpendicular to the direction of propagation

## wave velocity

the speed at which the disturbance moves. Also called the propagation velocity or propagation speed

## wavelength

the distance between adjacent identical parts of a wave

# Superposition and Interference

- Explain standing waves.
- Describe the mathematical representation of overtones and beat frequency.



These waves result from the superposition of several waves from different sources, producing a complex pattern. (credit: waterborough, Wikimedia Commons)

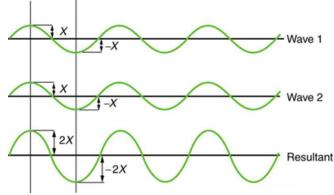
Most waves do not look very simple. They look more like the waves in [link] than like the simple water wave considered in Waves. (Simple waves may be created by a simple harmonic oscillation, and thus have a sinusoidal shape). Complex waves are more interesting, even beautiful, but they look formidable. Most waves appear complex because they result from several simple waves adding together. Luckily, the rules for adding waves are quite simple.

When two or more waves arrive at the same point, they superimpose themselves on one another. More specifically, the disturbances of waves are superimposed when they come together—a phenomenon called **superposition**. Each disturbance corresponds to a force, and forces add. If the disturbances are along the same line, then the resulting wave is a simple

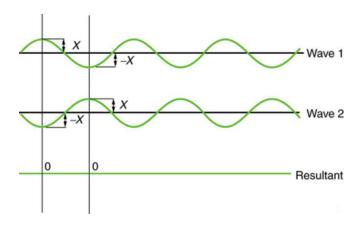
addition of the disturbances of the individual waves—that is, their amplitudes add. [link] and [link] illustrate superposition in two special cases, both of which produce simple results.

[link] shows two identical waves that arrive at the same point exactly in phase. The crests of the two waves are precisely aligned, as are the troughs. This superposition produces pure **constructive interference**. Because the disturbances add, pure constructive interference produces a wave that has twice the amplitude of the individual waves, but has the same wavelength.

[link] shows two identical waves that arrive exactly out of phase—that is, precisely aligned crest to trough—producing pure **destructive interference**. Because the disturbances are in the opposite direction for this superposition, the resulting amplitude is zero for pure destructive interference—the waves completely cancel.



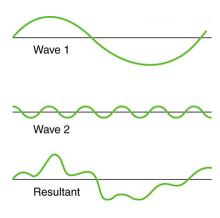
Pure constructive interference of two identical waves produces one with twice the amplitude, but the same wavelength.



Pure destructive interference of two identical waves produces zero amplitude, or complete cancellation.

While pure constructive and pure destructive interference do occur, they require precisely aligned identical waves. The superposition of most waves produces a combination of constructive and destructive interference and can vary from place to place and time to time. Sound from a stereo, for example, can be loud in one spot and quiet in another. Varying loudness means the sound waves add partially constructively and partially destructively at different locations. A stereo has at least two speakers creating sound waves, and waves can reflect from walls. All these waves superimpose. An example of sounds that vary over time from constructive to destructive is found in the combined whine of airplane jets heard by a stationary passenger. The combined sound can fluctuate up and down in volume as the sound from the two engines varies in time from constructive to destructive. These examples are of waves that are similar.

An example of the superposition of two dissimilar waves is shown in [link]. Here again, the disturbances add and subtract, producing a more complicated looking wave.

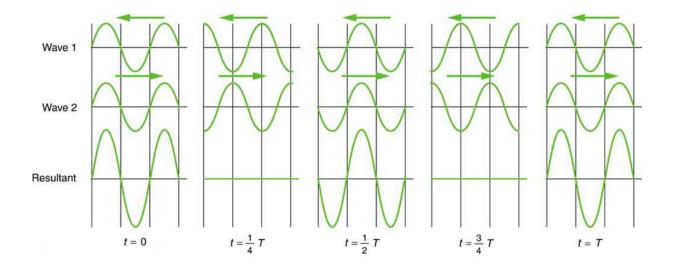


Superposition of non-identical waves exhibits both constructive and destructive interference.

# **Standing Waves**

Sometimes waves do not seem to move; rather, they just vibrate in place. Unmoving waves can be seen on the surface of a glass of milk in a refrigerator, for example. Vibrations from the refrigerator motor create waves on the milk that oscillate up and down but do not seem to move across the surface. These waves are formed by the superposition of two or more moving waves, such as illustrated in [link] for two identical waves moving in opposite directions. The waves move through each other with their disturbances adding as they go by. If the two waves have the same amplitude and wavelength, then they alternate between constructive and destructive interference. The resultant looks like a wave standing in place and, thus, is called a **standing wave**. Waves on the glass of milk are one example of standing waves. There are other standing waves, such as on guitar strings and in organ pipes. With the glass of milk, the two waves that produce standing waves may come from reflections from the side of the glass.

A closer look at earthquakes provides evidence for conditions appropriate for resonance, standing waves, and constructive and destructive interference. A building may be vibrated for several seconds with a driving frequency matching that of the natural frequency of vibration of the building—producing a resonance resulting in one building collapsing while neighboring buildings do not. Often buildings of a certain height are devastated while other taller buildings remain intact. The building height matches the condition for setting up a standing wave for that particular height. As the earthquake waves travel along the surface of Earth and reflect off denser rocks, constructive interference occurs at certain points. Often areas closer to the epicenter are not damaged while areas farther away are damaged.

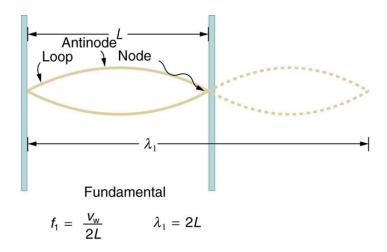


Standing wave created by the superposition of two identical waves moving in opposite directions. The oscillations are at fixed locations in space and result from alternately constructive and destructive interference.

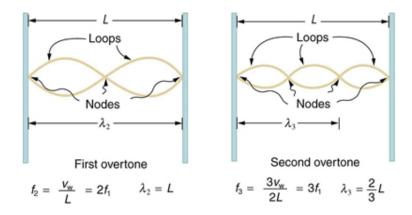
Standing waves are also found on the strings of musical instruments and are due to reflections of waves from the ends of the string. [link] and [link] show three standing waves that can be created on a string that is fixed at both ends. **Nodes** are the points where the string does not move; more

generally, nodes are where the wave disturbance is zero in a standing wave. The fixed ends of strings must be nodes, too, because the string cannot move there. The word **antinode** is used to denote the location of maximum amplitude in standing waves. Standing waves on strings have a frequency that is related to the propagation speed  $v_{\rm w}$  of the disturbance on the string. The wavelength  $\lambda$  is determined by the distance between the points where the string is fixed in place.

The lowest frequency, called the **fundamental frequency**, is thus for the longest wavelength, which is seen to be  $\lambda_1=2L$ . Therefore, the fundamental frequency is  $f_1=v_{\rm w}/\lambda_1=v_{\rm w}/2L$ . In this case, the **overtones** or harmonics are multiples of the fundamental frequency. As seen in [link], the first harmonic can easily be calculated since  $\lambda_2=L$ . Thus,  $f_2=v_{\rm w}/\lambda_2=v_{\rm w}/2L=2f_1$ . Similarly,  $f_3=3f_1$ , and so on. All of these frequencies can be changed by adjusting the tension in the string. The greater the tension, the greater  $v_{\rm w}$  is and the higher the frequencies. This observation is familiar to anyone who has ever observed a string instrument being tuned. We will see in later chapters that standing waves are crucial to many resonance phenomena, such as in sounding boxes on string instruments.



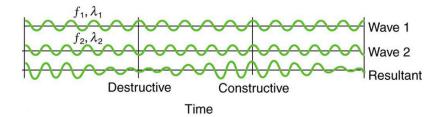
The figure shows a string oscillating at its fundamental frequency.



First and second harmonic frequencies are shown.

### **Beats**

Striking two adjacent keys on a piano produces a warbling combination usually considered to be unpleasant. The superposition of two waves of similar but not identical frequencies is the culprit. Another example is often noticeable in jet aircraft, particularly the two-engine variety, while taxiing. The combined sound of the engines goes up and down in loudness. This varying loudness happens because the sound waves have similar but not identical frequencies. The discordant warbling of the piano and the fluctuating loudness of the jet engine noise are both due to alternately constructive and destructive interference as the two waves go in and out of phase. [link] illustrates this graphically.



Beats are produced by the superposition of two waves of slightly different frequencies but identical amplitudes. The waves alternate in time between constructive interference and destructive interference, giving the resulting wave a time-varying amplitude.

The wave resulting from the superposition of two similar-frequency waves has a frequency that is the average of the two. This wave fluctuates in amplitude, or *beats*, with a frequency called the **beat frequency**. We can determine the beat frequency by adding two waves together mathematically. Note that a wave can be represented at one point in space as

# **Equation:**

$$x = X \cos\!\left(rac{2\pi\,t}{T}
ight) = X \cos(2\pi\,{
m ft}),$$

where f=1/T is the frequency of the wave. Adding two waves that have different frequencies but identical amplitudes produces a resultant **Equation:** 

$$x = x_1 + x_2.$$

More specifically,

# **Equation:**

$$x = X \cos(2\pi f_1 t) + X \cos(2\pi f_2 t).$$

Using a trigonometric identity, it can be shown that

# **Equation:**

$$x = 2X\cos(\pi f_{\rm B}t)\cos(2\pi f_{\rm ave}t),$$

where

## **Equation:**

$$f_{
m B}=\mid f_1-f_2\mid$$

is the beat frequency, and  $f_{\rm ave}$  is the average of  $f_1$  and  $f_2$ . These results mean that the resultant wave has twice the amplitude and the average frequency of the two superimposed waves, but it also fluctuates in overall amplitude at the beat frequency  $f_{\rm B}$ . The first cosine term in the expression effectively causes the amplitude to go up and down. The second cosine term is the wave with frequency  $f_{\rm ave}$ . This result is valid for all types of waves. However, if it is a sound wave, providing the two frequencies are similar, then what we hear is an average frequency that gets louder and softer (or warbles) at the beat frequency.

#### Note:

# Making Career Connections

Piano tuners use beats routinely in their work. When comparing a note with a tuning fork, they listen for beats and adjust the string until the beats go away (to zero frequency). For example, if the tuning fork has a 256 Hz frequency and two beats per second are heard, then the other frequency is either 254 or 258 Hz. Most keys hit multiple strings, and these strings are actually adjusted until they have nearly the same frequency and give a slow beat for richness. Twelve-string guitars and mandolins are also tuned using beats.

While beats may sometimes be annoying in audible sounds, we will find that beats have many applications. Observing beats is a very useful way to compare similar frequencies. There are applications of beats as apparently disparate as in ultrasonic imaging and radar speed traps.

#### **Exercise:**

## **Check Your Understanding**

#### **Problem:**

Imagine you are holding one end of a jump rope, and your friend holds the other. If your friend holds her end still, you can move your end up and down, creating a transverse wave. If your friend then begins to move her end up and down, generating a wave in the opposite direction, what resultant wave forms would you expect to see in the jump rope?

#### **Solution:**

The rope would alternate between having waves with amplitudes two times the original amplitude and reaching equilibrium with no amplitude at all. The wavelengths will result in both constructive and destructive interference

#### **Exercise:**

## **Check Your Understanding**

**Problem:** Define nodes and antinodes.

### **Solution:**

Nodes are areas of wave interference where there is no motion. Antinodes are areas of wave interference where the motion is at its maximum point.

#### **Exercise:**

**Check Your Understanding** 

### **Problem:**

You hook up a stereo system. When you test the system, you notice that in one corner of the room, the sounds seem dull. In another area, the sounds seem excessively loud. Describe how the sound moving about the room could result in these effects.

### **Solution:**

With multiple speakers putting out sounds into the room, and these sounds bouncing off walls, there is bound to be some wave interference. In the dull areas, the interference is probably mostly destructive. In the louder areas, the interference is probably mostly constructive.

### Note:

PhET Explorations: Wave Interference

Make waves with a dripping faucet, audio speaker, or laser! Add a second source or a pair of slits to create an interference pattern.

<u>Wave</u> <u>Interferenc</u> <u>e</u>

# **Section Summary**

- Superposition is the combination of two waves at the same location.
- Constructive interference occurs when two identical waves are superimposed in phase.

- Destructive interference occurs when two identical waves are superimposed exactly out of phase.
- A standing wave is one in which two waves superimpose to produce a wave that varies in amplitude but does not propagate.
- Nodes are points of no motion in standing waves.
- An antinode is the location of maximum amplitude of a standing wave.
- Waves on a string are resonant standing waves with a fundamental frequency and can occur at higher multiples of the fundamental, called overtones or harmonics.
- Beats occur when waves of similar frequencies  $f_1$  and  $f_2$  are superimposed. The resulting amplitude oscillates with a beat frequency given by

**Equation:** 

$$f_{\rm B} = \mid f_1 - f_2 \mid$$
.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

Speakers in stereo systems have two color-coded terminals to indicate how to hook up the wires. If the wires are reversed, the speaker moves in a direction opposite that of a properly connected speaker. Explain why it is important to have both speakers connected the same way.

### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

A car has two horns, one emitting a frequency of 199 Hz and the other emitting a frequency of 203 Hz. What beat frequency do they produce?

#### **Solution:**

 $f=4~\mathrm{Hz}$ 

#### **Exercise:**

#### **Problem:**

The middle-C hammer of a piano hits two strings, producing beats of 1.50 Hz. One of the strings is tuned to 260.00 Hz. What frequencies could the other string have?

### **Exercise:**

#### **Problem:**

Two tuning forks having frequencies of 460 and 464 Hz are struck simultaneously. What average frequency will you hear, and what will the beat frequency be?

### **Solution:**

462 Hz,

4 Hz

#### **Exercise:**

#### **Problem:**

Twin jet engines on an airplane are producing an average sound frequency of 4100 Hz with a beat frequency of 0.500 Hz. What are their individual frequencies?

#### **Exercise:**

### **Problem:**

A wave traveling on a Slinky® that is stretched to 4 m takes 2.4 s to travel the length of the Slinky and back again. (a) What is the speed of the wave? (b) Using the same Slinky stretched to the same length, a standing wave is created which consists of three antinodes and four nodes. At what frequency must the Slinky be oscillating?

### **Solution:**

- (a) 3.33 m/s
- (b) 1.25 Hz

#### **Exercise:**

#### **Problem:**

Three adjacent keys on a piano (F, F-sharp, and G) are struck simultaneously, producing frequencies of 349, 370, and 392 Hz. What beat frequencies are produced by this discordant combination?

# Glossary

#### antinode

the location of maximum amplitude in standing waves

# beat frequency

the frequency of the amplitude fluctuations of a wave

### constructive interference

when two waves arrive at the same point exactly in phase; that is, the crests of the two waves are precisely aligned, as are the troughs

### destructive interference

when two identical waves arrive at the same point exactly out of phase; that is, precisely aligned crest to trough

# fundamental frequency

the lowest frequency of a periodic waveform

#### nodes

the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave

#### overtones

# multiples of the fundamental frequency of a sound

# superposition

the phenomenon that occurs when two or more waves arrive at the same point

Introduction to Geometric Optics class="introduction"

# **Geometric Optics**

Light from this page or screen is formed into an image by the lens of your eye, much as the lens of the camera that made this photograph. Mirrors, like lenses, can also form images that in turn are captured by your eye.

**Image** seen as a result of reflectio n of light on a plane smooth surface. (credit: **NASA** Goddard Photo and Video, via Flickr)



Our lives are filled with light. Through vision, the most valued of our senses, light can evoke spiritual emotions, such as when we view a magnificent sunset or glimpse a rainbow breaking through the clouds. Light can also simply amuse us in a theater, or warn us to stop at an intersection. It has innumerable uses beyond vision. Light can carry telephone signals through glass fibers or cook a meal in a solar oven. Life itself could not exist without light's energy. From photosynthesis in plants to the sun warming a cold-blooded animal, its supply of energy is vital.



Double Rainbow over the bay

of Pocitos in Montevideo, Uruguay. (credit: Madrax, Wikimedia Commons)

We already know that visible light is the type of electromagnetic waves to which our eyes respond. That knowledge still leaves many questions regarding the nature of light and vision. What is color, and how do our eyes detect it? Why do diamonds sparkle? How does light travel? How do lenses and mirrors form images? These are but a few of the questions that are answered by the study of optics. Optics is the branch of physics that deals with the behavior of visible light and other electromagnetic waves. In particular, optics is concerned with the generation and propagation of light and its interaction with matter. What we have already learned about the generation of light in our study of heat transfer by radiation will be expanded upon in later topics, especially those on atomic physics. Now, we will concentrate on the propagation of light and its interaction with matter.

It is convenient to divide optics into two major parts based on the size of objects that light encounters. When light interacts with an object that is several times as large as the light's wavelength, its observable behavior is like that of a ray; it does not prominently display its wave characteristics. We call this part of optics "geometric optics." This chapter will concentrate on such situations. When light interacts with smaller objects, it has very prominent wave characteristics, such as constructive and destructive interference. Wave Optics will concentrate on such situations.

# The Ray Aspect of Light

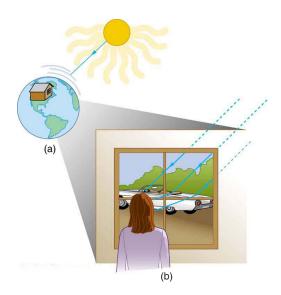
• List the ways by which light travels from a source to another location.

There are three ways in which light can travel from a source to another location. (See [link].) It can come directly from the source through empty space, such as from the Sun to Earth. Or light can travel through various media, such as air and glass, to the person. Light can also arrive after being reflected, such as by a mirror. In all of these cases, light is modeled as traveling in straight lines called rays. Light may change direction when it encounters objects (such as a mirror) or in passing from one material to another (such as in passing from air to glass), but it then continues in a straight line or as a ray. The word **ray** comes from mathematics and here means a straight line that originates at some point. It is acceptable to visualize light rays as laser rays (or even science fiction depictions of ray guns).

### Note:

## Ray

The word "ray" comes from mathematics and here means a straight line that originates at some point.



Three methods for light to travel from a source to another location. (a) Light reaches the upper atmosphere of Earth traveling through empty space directly from the source. (b) Light can reach a person in one of two ways. It can travel through media like air and glass. It can also reflect from an object like a mirror. In the situations shown here, light interacts with objects large enough that it travels in straight lines, like a ray.

Experiments, as well as our own experiences, show that when light interacts with objects several times as large as its wavelength, it travels in straight lines and acts like a ray. Its wave characteristics are not pronounced in such situations. Since the wavelength of light is less than a micron (a thousandth of a millimeter), it acts like a ray in the many common situations in which it encounters objects larger than a micron. For example, when light encounters anything we can observe with unaided eyes, such as a mirror, it acts like a ray, with only subtle wave characteristics. We will concentrate on the ray characteristics in this chapter.

Since light moves in straight lines, changing directions when it interacts with materials, it is described by geometry and simple trigonometry. This part of optics, where the ray aspect of light dominates, is therefore called **geometric optics**. There are two laws that govern how light changes direction when it interacts with matter. These are the law of reflection, for

situations in which light bounces off matter, and the law of refraction, for situations in which light passes through matter.

#### Note:

## **Geometric Optics**

The part of optics dealing with the ray aspect of light is called geometric optics.

# **Section Summary**

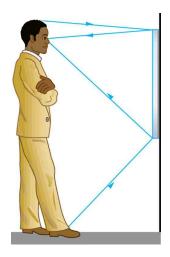
- A straight line that originates at some point is called a ray.
- The part of optics dealing with the ray aspect of light is called geometric optics.
- Light can travel in three ways from a source to another location: (1) directly from the source through empty space; (2) through various media; (3) after being reflected from a mirror.

### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

Suppose a man stands in front of a mirror as shown in [link]. His eyes are 1.65 m above the floor, and the top of his head is 0.13 m higher. Find the height above the floor of the top and bottom of the smallest mirror in which he can see both the top of his head and his feet. How is this distance related to the man's height?



A full-length mirror is one in which you can see all of yourself. It need not be as big as you, and its size is independent of your distance from it.

### **Solution:**

Top 1.715 m from floor, bottom 0.825 m from floor. Height of mirror is 0.890 m, or precisely one-half the height of the person.

# Glossary

ray

straight line that originates at some point

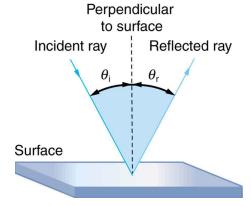
geometric optics part of optics dealing with the ray aspect of light

### The Law of Reflection

• Explain reflection of light from polished and rough surfaces.

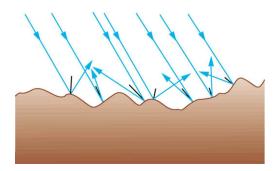
Whenever we look into a mirror, or squint at sunlight glinting from a lake, we are seeing a reflection. When you look at this page, too, you are seeing light reflected from it. Large telescopes use reflection to form an image of stars and other astronomical objects.

The law of reflection is illustrated in [link], which also shows how the angles are measured relative to the perpendicular to the surface at the point where the light ray strikes. We expect to see reflections from smooth surfaces, but [link] illustrates how a rough surface reflects light. Since the light strikes different parts of the surface at different angles, it is reflected in many different directions, or diffused. Diffused light is what allows us to see a sheet of paper from any angle, as illustrated in [link]. Many objects, such as people, clothing, leaves, and walls, have rough surfaces and can be seen from all sides. A mirror, on the other hand, has a smooth surface (compared with the wavelength of light) and reflects light at specific angles, as illustrated in [link]. When the moon reflects from a lake, as shown in [link], a combination of these effects takes place.

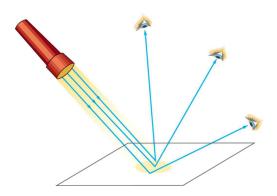


The law of reflection states that the angle of reflection equals the angle of incidence— $\theta_{\rm r}=\theta_{\rm i}$ . The angles are measured relative to the perpendicular to

the surface at the point where the ray strikes the surface.

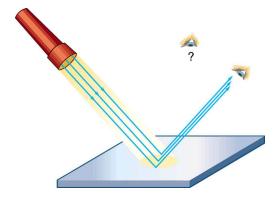


Light is diffused when it reflects from a rough surface. Here many parallel rays are incident, but they are reflected at many different angles since the surface is rough.



When a sheet of paper is illuminated with many parallel incident rays, it can be seen at many different angles, because

its surface is rough and diffuses the light.



A mirror illuminated by many parallel rays reflects them in only one direction, since its surface is very smooth. Only the observer at a particular angle will see the reflected light.



Moonlight is spread out when it is reflected by the lake, since the surface is shiny but uneven. (credit:

# Diego Torres Silvestre, Flickr)

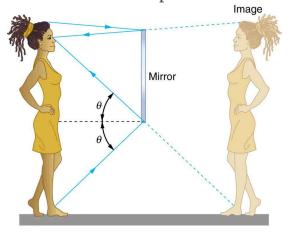
The law of reflection is very simple: The angle of reflection equals the angle of incidence.

### Note:

### The Law of Reflection

The angle of reflection equals the angle of incidence.

When we see ourselves in a mirror, it appears that our image is actually behind the mirror. This is illustrated in [link]. We see the light coming from a direction determined by the law of reflection. The angles are such that our image is exactly the same distance behind the mirror as we stand away from the mirror. If the mirror is on the wall of a room, the images in it are all behind the mirror, which can make the room seem bigger. Although these mirror images make objects appear to be where they cannot be (like behind a solid wall), the images are not figments of our imagination. Mirror images can be photographed and videotaped by instruments and look just as they do with our eyes (optical instruments themselves). The precise manner in which images are formed by mirrors and lenses will be treated in later sections of this chapter.



Our image in a mirror is behind the mirror. The two rays shown are those that strike the mirror at just the correct angles to be reflected into the eyes of the person. The image appears to be in the direction the rays are coming from when they enter the eyes.

#### Note:

## Take-Home Experiment: Law of Reflection

Take a piece of paper and shine a flashlight at an angle at the paper, as shown in [link]. Now shine the flashlight at a mirror at an angle. Do your observations confirm the predictions in [link] and [link]? Shine the flashlight on various surfaces and determine whether the reflected light is diffuse or not. You can choose a shiny metallic lid of a pot or your skin. Using the mirror and flashlight, can you confirm the law of reflection? You will need to draw lines on a piece of paper showing the incident and reflected rays. (This part works even better if you use a laser pencil.)

# **Section Summary**

- The angle of reflection equals the angle of incidence.
- A mirror has a smooth surface and reflects light at specific angles.
- Light is diffused when it reflects from a rough surface.
- Mirror images can be photographed and videotaped by instruments.

# **Conceptual Questions**

## **Exercise:**

#### **Problem:**

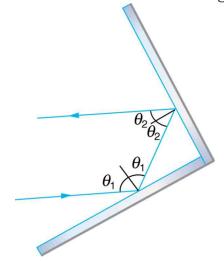
Using the law of reflection, explain how powder takes the shine off of a person's nose. What is the name of the optical effect?

## **Problems & Exercises**

#### **Exercise:**

## **Problem:**

Show that when light reflects from two mirrors that meet each other at a right angle, the outgoing ray is parallel to the incoming ray, as illustrated in the following figure.



A corner reflector sends the reflected ray back in a direction parallel to the incident ray, independent of incoming direction.

#### **Exercise:**

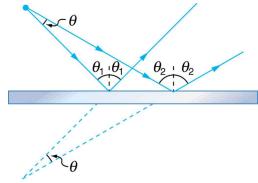
## **Problem:**

Light shows staged with lasers use moving mirrors to swing beams and create colorful effects. Show that a light ray reflected from a mirror changes direction by  $2\theta$  when the mirror is rotated by an angle  $\theta$ .

#### **Exercise:**

### **Problem:**

A flat mirror is neither converging nor diverging. To prove this, consider two rays originating from the same point and diverging at an angle  $\theta$ . Show that after striking a plane mirror, the angle between their directions remains  $\theta$ .



A flat mirror neither converges nor diverges light rays. Two rays continue to diverge at the same angle after reflection.

# **Glossary**

mirror

smooth surface that reflects light at specific angles, forming an image of the person or object in front of it

law of reflection angle of reflection equals the angle of incidence

#### The Law of Refraction

• Determine the index of refraction, given the speed of light in a medium.

It is easy to notice some odd things when looking into a fish tank. For example, you may see the same fish appearing to be in two different places. (See [link].) This is because light coming from the fish to us changes direction when it leaves the tank, and in this case, it can travel two different paths to get to our eyes. The changing of a light ray's direction (loosely called bending) when it passes through variations in matter is called **refraction**. Refraction is responsible for a tremendous range of optical phenomena, from the action of lenses to voice transmission through optical fibers.

#### Note:

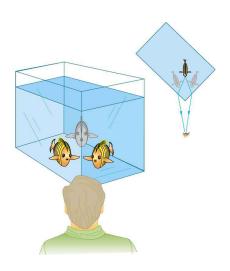
#### Refraction

The changing of a light ray's direction (loosely called bending) when it passes through variations in matter is called refraction.

#### Note:

# Speed of Light

The speed of light c not only affects refraction, it is one of the central concepts of Einstein's theory of relativity. As the accuracy of the measurements of the speed of light were improved, c was found not to depend on the velocity of the source or the observer. However, the speed of light does vary in a precise manner with the material it traverses. These facts have far-reaching implications, as we will see in <u>Special Relativity</u>. It makes connections between space and time and alters our expectations that all observers measure the same time for the same event, for example. The speed of light is so important that its value in a vacuum is one of the most fundamental constants in nature as well as being one of the four fundamental SI units.



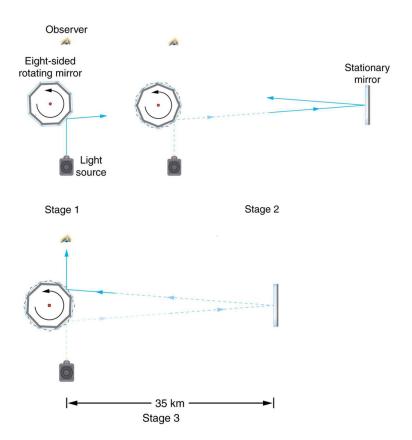
Looking at the fish tank as shown, we can see the same fish in two different locations, because light changes directions when it passes from water to air. In this case, the light can reach the observer by two different paths, and so the fish seems to be in two different places. This bending of light is called refraction and is responsible for many optical phenomena.

Why does light change direction when passing from one material (medium) to another? It is because light changes speed when going from one material

to another. So before we study the law of refraction, it is useful to discuss the speed of light and how it varies in different media.

# The Speed of Light

Early attempts to measure the speed of light, such as those made by Galileo, determined that light moved extremely fast, perhaps instantaneously. The first real evidence that light traveled at a finite speed came from the Danish astronomer Ole Roemer in the late 17th century. Roemer had noted that the average orbital period of one of Jupiter's moons, as measured from Earth, varied depending on whether Earth was moving toward or away from Jupiter. He correctly concluded that the apparent change in period was due to the change in distance between Earth and Jupiter and the time it took light to travel this distance. From his 1676 data, a value of the speed of light was calculated to be  $2.26 \times 10^8$  m/s (only 25% different from today's accepted value). In more recent times, physicists have measured the speed of light in numerous ways and with increasing accuracy. One particularly direct method, used in 1887 by the American physicist Albert Michelson (1852–1931), is illustrated in [link]. Light reflected from a rotating set of mirrors was reflected from a stationary mirror 35 km away and returned to the rotating mirrors. The time for the light to travel can be determined by how fast the mirrors must rotate for the light to be returned to the observer's eye.



A schematic of early apparatus used by Michelson and others to determine the speed of light. As the mirrors rotate, the reflected ray is only briefly directed at the stationary mirror. The returning ray will be reflected into the observer's eye only if the next mirror has rotated into the correct position just as the ray returns. By measuring the correct rotation rate, the time for the round trip can be measured and the speed of light calculated. Michelson's calculated value of the speed of light was only 0.04% different from the value used today.

The speed of light is now known to great precision. In fact, the speed of light in a vacuum c is so important that it is accepted as one of the basic physical quantities and has the fixed value

# **Equation:**

$$c = 2.99792458 \times 10^8 \, \mathrm{m/s} pprox 3.00 \times 10^8 \, \mathrm{m/s},$$

where the approximate value of  $3.00 \times 10^8$  m/s is used whenever three-digit accuracy is sufficient. The speed of light through matter is less than it is in a vacuum, because light interacts with atoms in a material. The speed of light depends strongly on the type of material, since its interaction with different atoms, crystal lattices, and other substructures varies. We define the **index of refraction** n of a material to be

## **Equation:**

$$n = \frac{c}{v},$$

where v is the observed speed of light in the material. Since the speed of light is always less than c in matter and equals c only in a vacuum, the index of refraction is always greater than or equal to one.

Note:

Value of the Speed of Light

**Equation:** 

$$c = 2.99792458 imes 10^8 \, \mathrm{m/s} pprox 3.00 imes 10^8 \, \mathrm{m/s}$$

Note:

Index of Refraction

$$n=rac{c}{v}$$

That is,  $n \geq 1$ . [link] gives the indices of refraction for some representative substances. The values are listed for a particular wavelength of light, because they vary slightly with wavelength. (This can have important effects, such as colors produced by a prism.) Note that for gases, n is close to 1.0. This seems reasonable, since atoms in gases are widely separated and light travels at c in the vacuum between atoms. It is common to take n=1 for gases unless great precision is needed. Although the speed of light v in a medium varies considerably from its value c in a vacuum, it is still a large speed.

Medium	n
Gases at 0°C, 1 atm	
Air	1.000293
Carbon dioxide	1.00045
Hydrogen	1.000139
Oxygen	1.000271
Liquids at $20^{ m oC}$	
Benzene	1.501
Carbon disulfide	1.628

Medium	n
Carbon tetrachloride	1.461
Ethanol	1.361
Glycerine	1.473
Water, fresh	1.333
Solids at 20°C	
Diamond	2.419
Fluorite	1.434
Glass, crown	1.52
Glass, flint	1.66
Ice at $20^{\circ}\mathrm{C}$	1.309
Polystyrene	1.49
Plexiglas	1.51
Quartz, crystalline	1.544
Quartz, fused	1.458
Sodium chloride	1.544
Zircon	1.923

Index of Refraction in Various Media

## **Example:**

# **Speed of Light in Matter**

Calculate the speed of light in zircon, a material used in jewelry to imitate diamond.

## Strategy

The speed of light in a material, v, can be calculated from the index of refraction n of the material using the equation n = c/v.

#### Solution

The equation for index of refraction states that n=c/v. Rearranging this to determine v gives

## **Equation:**

$$v = \frac{c}{n}$$
.

The index of refraction for zircon is given as 1.923 in [link], and c is given in the equation for speed of light. Entering these values in the last expression gives

## **Equation:**

$$egin{array}{lll} v & = & rac{3.00 imes 10^8 \, ext{m/s}}{1.923} \ & = & 1.56 imes 10^8 \, ext{m/s}. \end{array}$$

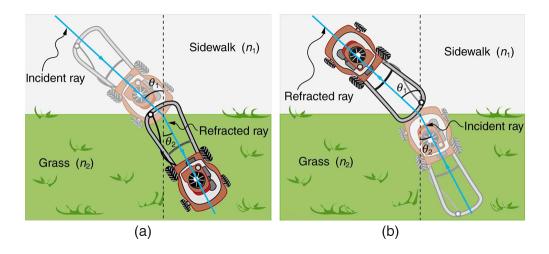
#### Discussion

This speed is slightly larger than half the speed of light in a vacuum and is still high compared with speeds we normally experience. The only substance listed in [link] that has a greater index of refraction than zircon is diamond. We shall see later that the large index of refraction for zircon makes it sparkle more than glass, but less than diamond.

# **Law of Refraction**

[link] shows how a ray of light changes direction when it passes from one medium to another. As before, the angles are measured relative to a perpendicular to the surface at the point where the light ray crosses it.

(Some of the incident light will be reflected from the surface, but for now we will concentrate on the light that is transmitted.) The change in direction of the light ray depends on how the speed of light changes. The change in the speed of light is related to the indices of refraction of the media involved. In the situations shown in [link], medium 2 has a greater index of refraction than medium 1. This means that the speed of light is less in medium 2 than in medium 1. Note that as shown in [link](a), the direction of the ray moves closer to the perpendicular when it slows down. Conversely, as shown in [link](b), the direction of the ray moves away from the perpendicular when it speeds up. The path is exactly reversible. In both cases, you can imagine what happens by thinking about pushing a lawn mower from a footpath onto grass, and vice versa. Going from the footpath to grass, the front wheels are slowed and pulled to the side as shown. This is the same change in direction as for light when it goes from a fast medium to a slow one. When going from the grass to the footpath, the front wheels can move faster and the mower changes direction as shown. This, too, is the same change in direction as for light going from slow to fast.



The change in direction of a light ray depends on how the speed of light changes when it crosses from one medium to another. The speed of light is greater in medium 1 than in medium 2 in the situations shown here.

(a) A ray of light moves closer to the perpendicular when it slows down. This is analogous to what happens when a lawn mower goes from a footpath to grass. (b) A ray of

light moves away from the perpendicular when it speeds up. This is analogous to what happens when a lawn mower goes from grass to footpath. The paths are exactly reversible.

The amount that a light ray changes its direction depends both on the incident angle and the amount that the speed changes. For a ray at a given incident angle, a large change in speed causes a large change in direction, and thus a large change in angle. The exact mathematical relationship is the **law of refraction**, or "Snell's Law," which is stated in equation form as **Equation**:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
.

Here  $n_1$  and  $n_2$  are the indices of refraction for medium 1 and 2, and  $\theta_1$  and  $\theta_2$  are the angles between the rays and the perpendicular in medium 1 and 2, as shown in [link]. The incoming ray is called the incident ray and the outgoing ray the refracted ray, and the associated angles the incident angle and the refracted angle. The law of refraction is also called Snell's law after the Dutch mathematician Willebrord Snell (1591–1626), who discovered it in 1621. Snell's experiments showed that the law of refraction was obeyed and that a characteristic index of refraction n could be assigned to a given medium. Snell was not aware that the speed of light varied in different media, but through experiments he was able to determine indices of refraction from the way light rays changed direction.

Note:

The Law of Refraction

$$n_1\sin\theta_1=n_2\sin\theta_2$$

#### Note:

## Take-Home Experiment: A Broken Pencil

A classic observation of refraction occurs when a pencil is placed in a glass half filled with water. Do this and observe the shape of the pencil when you look at the pencil sideways, that is, through air, glass, water. Explain your observations. Draw ray diagrams for the situation.

## **Example:**

#### Determine the Index of Refraction from Refraction Data

Find the index of refraction for medium 2 in [link](a), assuming medium 1 is air and given the incident angle is 30.0° and the angle of refraction is 22.0°.

## Strategy

The index of refraction for air is taken to be 1 in most cases (and up to four significant figures, it is 1.000). Thus  $n_1=1.00$  here. From the given information,  $\theta_1=30.0^\circ$  and  $\theta_2=22.0^\circ$ . With this information, the only unknown in Snell's law is  $n_2$ , so that it can be used to find this unknown.

#### **Solution**

Snell's law is

## **Equation:**

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
.

Rearranging to isolate  $n_2$  gives

# **Equation:**

$$n_2 = n_1 rac{\sin heta_1}{\sin heta_2}.$$

Entering known values,

$$n_2 = 1.00 \frac{\sin 30.0^{\circ}}{\sin 22.0^{\circ}} = \frac{0.500}{0.375}$$
  
= 1.33.

#### Discussion

This is the index of refraction for water, and Snell could have determined it by measuring the angles and performing this calculation. He would then have found 1.33 to be the appropriate index of refraction for water in all other situations, such as when a ray passes from water to glass. Today we can verify that the index of refraction is related to the speed of light in a medium by measuring that speed directly.

# **Example:**

# A Larger Change in Direction

Suppose that in a situation like that in [link], light goes from air to diamond and that the incident angle is  $30.0^{\circ}$ . Calculate the angle of refraction  $\theta_2$  in the diamond.

## Strategy

Again the index of refraction for air is taken to be  $n_1 = 1.00$ , and we are given  $\theta_1 = 30.0^{\circ}$ . We can look up the index of refraction for diamond in [link], finding  $n_2 = 2.419$ . The only unknown in Snell's law is  $\theta_2$ , which we wish to determine.

#### Solution

Solving Snell's law for  $\sin \theta_2$  yields

# **Equation:**

$$\sin heta_2 = rac{n_1}{n_2} \sin heta_1.$$

Entering known values,

# **Equation:**

$$\sin heta_2 = rac{1.00}{2.419} \sin 30.0^{\circ} = \left(0.413\right)(0.500) = 0.207.$$

The angle is thus

$$heta_2 = \sin^{-1}\!0.207 = 11.9^{
m o}.$$

#### Discussion

For the same  $30^{\circ}$  angle of incidence, the angle of refraction in diamond is significantly smaller than in water ( $11.9^{\circ}$  rather than  $22^{\circ}$ —see the preceding example). This means there is a larger change in direction in diamond. The cause of a large change in direction is a large change in the index of refraction (or speed). In general, the larger the change in speed, the greater the effect on the direction of the ray.

# **Section Summary**

- The changing of a light ray's direction when it passes through variations in matter is called refraction.
- The speed of light in vacuum  $c=2.99792458 imes 10^8 \, \mathrm{m/s} pprox 3.00 imes 10^8 \, \mathrm{m/s}.$
- Index of refraction  $n=\frac{c}{v}$ , where v is the speed of light in the material, c is the speed of light in vacuum, and n is the index of refraction.
- Snell's law, the law of refraction, is stated in equation form as  $n_1 \sin \theta_1 = n_2 \sin \theta_2$ .

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

Diffusion by reflection from a rough surface is described in this chapter. Light can also be diffused by refraction. Describe how this occurs in a specific situation, such as light interacting with crushed ice.

#### **Exercise:**

#### **Problem:**

Why is the index of refraction always greater than or equal to 1?

#### **Exercise:**

#### **Problem:**

Does the fact that the light flash from lightning reaches you before its sound prove that the speed of light is extremely large or simply that it is greater than the speed of sound? Discuss how you could use this effect to get an estimate of the speed of light.

#### **Exercise:**

#### **Problem:**

Will light change direction toward or away from the perpendicular when it goes from air to water? Water to glass? Glass to air?

#### **Exercise:**

#### **Problem:**

Explain why an object in water always appears to be at a depth shallower than it actually is? Why do people sometimes sustain neck and spinal injuries when diving into unfamiliar ponds or waters?

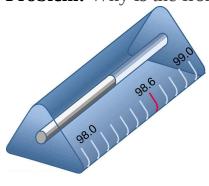
#### **Exercise:**

#### **Problem:**

Explain why a person's legs appear very short when wading in a pool. Justify your explanation with a ray diagram showing the path of rays from the feet to the eye of an observer who is out of the water.

#### **Exercise:**

**Problem:** Why is the front surface of a thermometer curved as shown?



The curved surface of the thermometer serves a purpose.

#### **Exercise:**

#### **Problem:**

Suppose light were incident from air onto a material that had a negative index of refraction, say -1.3; where does the refracted light ray go?

## **Problems & Exercises**

#### **Exercise:**

**Problem:** What is the speed of light in water? In glycerine?

#### **Solution:**

 $2.25 imes 10^8 \, \mathrm{m/s}$  in water

 $2.04 \times 10^8 \ m/s$  in glycerine

#### **Exercise:**

**Problem:** What is the speed of light in air? In crown glass?

#### **Exercise:**

#### **Problem:**

Calculate the index of refraction for a medium in which the speed of light is  $2.012 \times 10^8 \, \mathrm{m/s}$ , and identify the most likely substance based on [link].

#### **Solution:**

1.490, polystyrene

#### **Exercise:**

#### **Problem:**

In what substance in [link] is the speed of light  $2.290 \times 10^8 \ m/s$ ?

#### **Exercise:**

#### **Problem:**

There was a major collision of an asteroid with the Moon in medieval times. It was described by monks at Canterbury Cathedral in England as a red glow on and around the Moon. How long after the asteroid hit the Moon, which is  $3.84\times10^5$  km away, would the light first arrive on Earth?

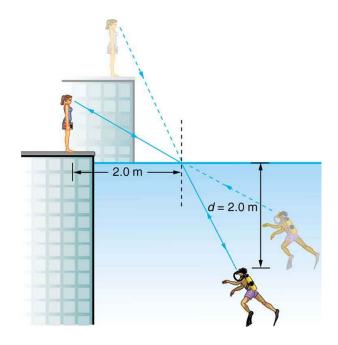
#### **Solution:**

 $1.28 \mathrm{s}$ 

#### **Exercise:**

#### **Problem:**

A scuba diver training in a pool looks at his instructor as shown in  $[\underline{link}]$ . What angle does the ray from the instructor's face make with the perpendicular to the water at the point where the ray enters? The angle between the ray in the water and the perpendicular to the water is  $25.0^{\circ}$ .



A scuba diver in a pool and his trainer look at each other.

## **Exercise:**

## **Problem:**

Components of some computers communicate with each other through optical fibers having an index of refraction n=1.55. What time in nanoseconds is required for a signal to travel 0.200 m through such a fiber?

# **Solution:**

 $1.03 \, \mathrm{ns}$ 

## **Exercise:**

#### **Problem:**

(a) Given that the angle between the ray in the water and the perpendicular to the water is 25.0°, and using information in [link], find the height of the instructor's head above the water, noting that you will first have to calculate the angle of incidence. (b) Find the apparent depth of the diver's head below water as seen by the instructor.

#### **Exercise:**

#### **Problem:**

Suppose you have an unknown clear substance immersed in water, and you wish to identify it by finding its index of refraction. You arrange to have a beam of light enter it at an angle of  $45.0^{\circ}$ , and you observe the angle of refraction to be  $40.3^{\circ}$ . What is the index of refraction of the substance and its likely identity?

#### **Solution:**

n = 1.46, fused quartz

#### **Exercise:**

#### **Problem:**

On the Moon's surface, lunar astronauts placed a corner reflector, off which a laser beam is periodically reflected. The distance to the Moon is calculated from the round-trip time. What percent correction is needed to account for the delay in time due to the slowing of light in Earth's atmosphere? Assume the distance to the Moon is precisely  $3.84 \times 10^8$  m, and Earth's atmosphere (which varies in density with altitude) is equivalent to a layer 30.0 km thick with a constant index of refraction n=1.000293.

#### **Exercise:**

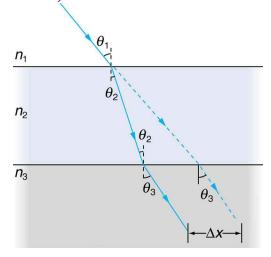
#### **Problem:**

Suppose [link] represents a ray of light going from air through crown glass into water, such as going into a fish tank. Calculate the amount the ray is displaced by the glass ( $\Delta x$ ), given that the incident angle is  $40.0^{\circ}$  and the glass is 1.00 cm thick.

#### **Exercise:**

#### **Problem:**

[link] shows a ray of light passing from one medium into a second and then a third. Show that  $\theta_3$  is the same as it would be if the second medium were not present (provided total internal reflection does not occur).



A ray of light passes from one medium to a third by traveling through a second. The final direction is the same as if the second medium were not present, but the ray is displaced by  $\Delta x$  (shown exaggerated).

#### **Exercise:**

#### **Problem: Unreasonable Results**

Suppose light travels from water to another substance, with an angle of incidence of 10.0° and an angle of refraction of 14.9°. (a) What is the index of refraction of the other substance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

#### **Solution:**

- (a) 0.898
- (b) Can't have n < 1.00 since this would imply a speed greater than c.
- (c) Refracted angle is too big relative to the angle of incidence.

#### **Exercise:**

#### **Problem:** Construct Your Own Problem

Consider sunlight entering the Earth's atmosphere at sunrise and sunset—that is, at a 90° incident angle. Taking the boundary between nearly empty space and the atmosphere to be sudden, calculate the angle of refraction for sunlight. This lengthens the time the Sun appears to be above the horizon, both at sunrise and sunset. Now construct a problem in which you determine the angle of refraction for different models of the atmosphere, such as various layers of varying density. Your instructor may wish to guide you on the level of complexity to consider and on how the index of refraction varies with air density.

#### **Exercise:**

#### **Problem:** Unreasonable Results

Light traveling from water to a gemstone strikes the surface at an angle of  $80.0^{\circ}$  and has an angle of refraction of  $15.2^{\circ}$ . (a) What is the speed

of light in the gemstone? (b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

## **Solution:**

- (a)  $\frac{c}{5.00}$
- (b) Speed of light too slow, since index is much greater than that of diamond.
- (c) Angle of refraction is unreasonable relative to the angle of incidence.

# **Glossary**

## refraction

changing of a light ray's direction when it passes through variations in matter

## index of refraction

for a material, the ratio of the speed of light in vacuum to that in the material

#### **Total Internal Reflection**

- Explain the phenomenon of total internal reflection.
- Describe the workings and uses of fiber optics.
- Analyze the reason for the sparkle of diamonds.

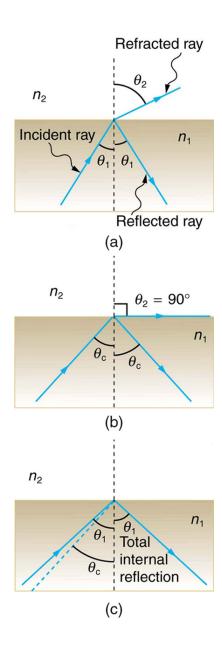
A good-quality mirror may reflect more than 90% of the light that falls on it, absorbing the rest. But it would be useful to have a mirror that reflects all of the light that falls on it. Interestingly, we can produce *total reflection* using an aspect of *refraction*.

Consider what happens when a ray of light strikes the surface between two materials, such as is shown in [link](a). Part of the light crosses the boundary and is refracted; the rest is reflected. If, as shown in the figure, the index of refraction for the second medium is less than for the first, the ray bends away from the perpendicular. (Since  $n_1 > n_2$ , the angle of refraction is greater than the angle of incidence—that is,  $\theta_2 > \theta_1$ .) Now imagine what happens as the incident angle is increased. This causes  $\theta_2$  to increase also. The largest the angle of refraction  $\theta_2$ can be is 90°, as shown in [link](b). The **critical angle**  $\theta_2$  for a combination of materials is defined to be the incident angle  $\theta_1$  that produces an angle of refraction of 90°. That is,  $\theta_c$  is the incident angle for which  $\theta_2 = 90$ °. If the incident angle  $\theta_1$  is greater than the critical angle, as shown in [link](c), then all of the light is reflected back into medium 1, a condition called **total internal reflection**.

#### Note:

## Critical Angle

The incident angle  $\theta_1$  that produces an angle of refraction of  $90^\circ$  is called the critical angle,  $\theta_c$ .



(a) A ray of light crosses a boundary where the speed of light increases and the index of refraction decreases. That is,  $n_2 < n_1$ . The ray bends away from the perpendicular. (b) The critical

angle  $\theta_c$  is the one for which the angle of refraction is . (c) Total internal reflection occurs when the incident angle is greater than the critical angle.

Snell's law states the relationship between angles and indices of refraction. It is given by

## **Equation:**

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
.

When the incident angle equals the critical angle ( $\theta_1 = \theta_c$ ), the angle of refraction is  $90^{\circ}$  ( $\theta_2 = 90^{\circ}$ ). Noting that  $\sin 90^{\circ} = 1$ , Snell's law in this case becomes

## **Equation:**

$$n_1 \sin \theta_1 = n_2$$
.

The critical angle  $\theta_c$  for a given combination of materials is thus **Equation:** 

$$heta_c = \sin^{-1}(n_2/n_1) ext{ for } n_1 > n_2.$$

Total internal reflection occurs for any incident angle greater than the critical angle  $\theta_c$ , and it can only occur when the second medium has an index of refraction less than the first. Note the above equation is written for a light ray that travels in medium 1 and reflects from medium 2, as shown in the figure.

## **Example:**

# **How Big is the Critical Angle Here?**

What is the critical angle for light traveling in a polystyrene (a type of plastic) pipe surrounded by air?

## **Strategy**

The index of refraction for polystyrene is found to be 1.49 in [link], and the index of refraction of air can be taken to be 1.00, as before. Thus, the condition that the second medium (air) has an index of refraction less than the first (plastic) is satisfied, and the equation  $\theta_c = \sin^{-1}(n_2/n_1)$  can be used to find the critical angle  $\theta_c$ . Here, then,  $n_2 = 1.00$  and  $n_1 = 1.49$ .

#### Solution

The critical angle is given by

## **Equation:**

$$heta_c = \sin^{-1}(n_2/n_1).$$

Substituting the identified values gives

# **Equation:**

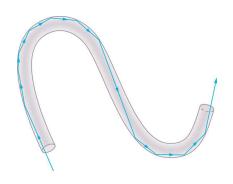
$$heta_c = \sin^{-1}(1.00/1.49) = \sin^{-1}(0.671) \ 42.2^{\circ}.$$

#### Discussion

This means that any ray of light inside the plastic that strikes the surface at an angle greater than  $42.2^{\circ}$  will be totally reflected. This will make the inside surface of the clear plastic a perfect mirror for such rays without any need for the silvering used on common mirrors. Different combinations of materials have different critical angles, but any combination with  $n_1 > n_2$  can produce total internal reflection. The same calculation as made here shows that the critical angle for a ray going from water to air is  $48.6^{\circ}$ , while that from diamond to air is  $24.4^{\circ}$ , and that from flint glass to crown glass is  $66.3^{\circ}$ . There is no total reflection for rays going in the other direction—for example, from air to water—since the condition that the second medium must have a smaller index of refraction is not satisfied. A number of interesting applications of total internal reflection follow.

# **Fiber Optics: Endoscopes to Telephones**

Fiber optics is one application of total internal reflection that is in wide use. In communications, it is used to transmit telephone, internet, and cable TV signals. **Fiber optics** employs the transmission of light down fibers of plastic or glass. Because the fibers are thin, light entering one is likely to strike the inside surface at an angle greater than the critical angle and, thus, be totally reflected (See [link].) The index of refraction outside the fiber must be smaller than inside, a condition that is easily satisfied by coating the outside of the fiber with a material having an appropriate refractive index. In fact, most fibers have a varying refractive index to allow more light to be guided along the fiber through total internal refraction. Rays are reflected around corners as shown, making the fibers into tiny light pipes.

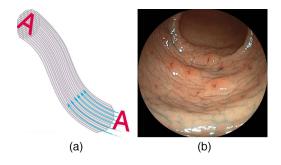


Light entering a
thin fiber may
strike the inside
surface at large or
grazing angles and
is completely
reflected if these
angles exceed the
critical angle. Such
rays continue down
the fiber, even
following it around
corners, since the
angles of reflection

and incidence remain large.

Bundles of fibers can be used to transmit an image without a lens, as illustrated in [link]. The output of a device called an **endoscope** is shown in [link](b). Endoscopes are used to explore the body through various orifices or minor incisions. Light is transmitted down one fiber bundle to illuminate internal parts, and the reflected light is transmitted back out through another to be observed. Surgery can be performed, such as arthroscopic surgery on the knee joint, employing cutting tools attached to and observed with the endoscope. Samples can also be obtained, such as by lassoing an intestinal polyp for external examination.

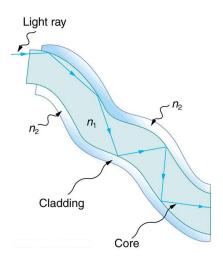
Fiber optics has revolutionized surgical techniques and observations within the body. There are a host of medical diagnostic and therapeutic uses. The flexibility of the fiber optic bundle allows it to navigate around difficult and small regions in the body, such as the intestines, the heart, blood vessels, and joints. Transmission of an intense laser beam to burn away obstructing plaques in major arteries as well as delivering light to activate chemotherapy drugs are becoming commonplace. Optical fibers have in fact enabled microsurgery and remote surgery where the incisions are small and the surgeon's fingers do not need to touch the diseased tissue.



(a) An image is transmitted by a bundle of fibers that have fixed

neighbors. (b) An endoscope is used to probe the body, both transmitting light to the interior and returning an image such as the one shown. (credit: Med\_Chaos, Wikimedia Commons)

Fibers in bundles are surrounded by a cladding material that has a lower index of refraction than the core. (See [link].) The cladding prevents light from being transmitted between fibers in a bundle. Without cladding, light could pass between fibers in contact, since their indices of refraction are identical. Since no light gets into the cladding (there is total internal reflection back into the core), none can be transmitted between clad fibers that are in contact with one another. The cladding prevents light from escaping out of the fiber; instead most of the light is propagated along the length of the fiber, minimizing the loss of signal and ensuring that a quality image is formed at the other end. The cladding and an additional protective layer make optical fibers flexible and durable.



Fibers in bundles are clad by a material that has a lower index of refraction than the core to ensure total internal reflection, even when fibers are in contact with one another. This shows a single fiber with its cladding.

#### Note:

## Cladding

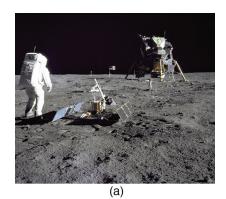
The cladding prevents light from being transmitted between fibers in a bundle.

Special tiny lenses that can be attached to the ends of bundles of fibers are being designed and fabricated. Light emerging from a fiber bundle can be focused and a tiny spot can be imaged. In some cases the spot can be scanned, allowing quality imaging of a region inside the body. Special minute optical filters inserted at the end of the fiber bundle have the capacity to image tens of microns below the surface without cutting the surface—non-intrusive diagnostics. This is particularly useful for determining the extent of cancers in the stomach and bowel.

Most telephone conversations and Internet communications are now carried by laser signals along optical fibers. Extensive optical fiber cables have been placed on the ocean floor and underground to enable optical communications. Optical fiber communication systems offer several advantages over electrical (copper) based systems, particularly for long distances. The fibers can be made so transparent that light can travel many kilometers before it becomes dim enough to require amplification—much superior to copper conductors. This property of optical fibers is called *low loss*. Lasers emit light with characteristics that allow far more conversations in one fiber than are possible with electric signals on a single conductor. This property of optical fibers is called *high bandwidth*. Optical signals in one fiber do not produce undesirable effects in other adjacent fibers. This property of optical fibers is called *reduced crosstalk*. We shall explore the unique characteristics of laser radiation in a later chapter.

# **Corner Reflectors and Diamonds**

A light ray that strikes an object consisting of two mutually perpendicular reflecting surfaces is reflected back exactly parallel to the direction from which it came. This is true whenever the reflecting surfaces are perpendicular, and it is independent of the angle of incidence. Such an object, shown in [link], is called a **corner reflector**, since the light bounces from its inside corner. Many inexpensive reflector buttons on bicycles, cars, and warning signs have corner reflectors designed to return light in the direction from which it originated. It was more expensive for astronauts to place one on the moon. Laser signals can be bounced from that corner reflector to measure the gradually increasing distance to the moon with great precision.





(a) Astronauts placed a corner reflector on the moon to measure its gradually increasing orbital distance. (credit: NASA) (b) The bright spots on these bicycle safety reflectors are reflections of the flash of the camera that took this picture on a dark night. (credit: Julo, Wikimedia Commons)

Corner reflectors are perfectly efficient when the conditions for total internal reflection are satisfied. With common materials, it is easy to obtain a critical angle that is less than  $45^{\circ}$ . One use of these perfect mirrors is in binoculars, as shown in [link]. Another use is in periscopes found in submarines.

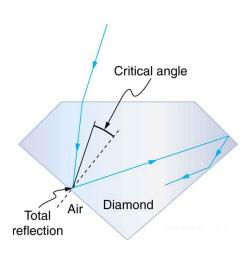


These binoculars employ corner reflectors with total internal reflection to get light to the observer's eyes.

# The Sparkle of Diamonds

Total internal reflection, coupled with a large index of refraction, explains why diamonds sparkle more than other materials. The critical angle for a diamond-to-air surface is only 24.4°, and so when light enters a diamond, it has trouble getting back out. (See [link].) Although light freely enters the diamond, it can exit only if it makes an angle less than 24.4°. Facets on diamonds are specifically intended to make this unlikely, so that the light can exit only in certain places. Good diamonds are very clear, so that the light makes many internal reflections and is concentrated at the few places it can exit—hence the sparkle. (Zircon is a natural gemstone that has an exceptionally large index of refraction, but not as large as diamond, so it is

not as highly prized. Cubic zirconia is manufactured and has an even higher index of refraction ( $\approx 2.17$ ), but still less than that of diamond.) The colors you see emerging from a sparkling diamond are not due to the diamond's color, which is usually nearly colorless. Those colors result from dispersion, the topic of Dispersion: The Rainbow and Prisms. Colored diamonds get their color from structural defects of the crystal lattice and the inclusion of minute quantities of graphite and other materials. The Argyle Mine in Western Australia produces around 90% of the world's pink, red, champagne, and cognac diamonds, while around 50% of the world's clear diamonds come from central and southern Africa.



Light cannot easily escape a diamond, because its critical angle with air is so small. Most reflections are total, and the facets are placed so that light can exit only in particular ways—thus concentrating the light and making the diamond sparkle.

# Note:

# PhET Explorations: Bending Light

Explore bending of light between two media with different indices of refraction. See how changing from air to water to glass changes the bending angle. Play with prisms of different shapes and make rainbows.

https://phet.colorado.edu/sims/html/bending-light/latest/bending-light\_en.html

# **Section Summary**

- The incident angle that produces an angle of refraction of 90° is called critical angle.
- Total internal reflection is a phenomenon that occurs at the boundary between two mediums, such that if the incident angle in the first medium is greater than the critical angle, then all the light is reflected back into that medium.
- Fiber optics involves the transmission of light down fibers of plastic or glass, applying the principle of total internal reflection.
- Endoscopes are used to explore the body through various orifices or minor incisions, based on the transmission of light through optical fibers.
- Cladding prevents light from being transmitted between fibers in a bundle.
- Diamonds sparkle due to total internal reflection coupled with a large index of refraction.

# **Conceptual Questions**

#### **Exercise:**

### **Problem:**

A ring with a colorless gemstone is dropped into water. The gemstone becomes invisible when submerged. Can it be a diamond? Explain.

# **Exercise:**

# **Problem:**

A high-quality diamond may be quite clear and colorless, transmitting all visible wavelengths with little absorption. Explain how it can sparkle with flashes of brilliant color when illuminated by white light.

# **Exercise:**

# **Problem:**

Is it possible that total internal reflection plays a role in rainbows? Explain in terms of indices of refraction and angles, perhaps referring to [link]. Some of us have seen the formation of a double rainbow. Is it physically possible to observe a triple rainbow?



Double rainbows are not a very common observance. (credit: InvictusOU812, Flickr)

# **Exercise:**

# **Problem:**

The most common type of mirage is an illusion that light from faraway objects is reflected by a pool of water that is not really there. Mirages are generally observed in deserts, when there is a hot layer of air near the ground. Given that the refractive index of air is lower for air at higher temperatures, explain how mirages can be formed.

# **Problems & Exercises**

### **Exercise:**

### **Problem:**

Verify that the critical angle for light going from water to air is 48.6°, as discussed at the end of [link], regarding the critical angle for light traveling in a polystyrene (a type of plastic) pipe surrounded by air.

### **Exercise:**

### **Problem:**

(a) At the end of [link], it was stated that the critical angle for light going from diamond to air is  $24.4^{\circ}$ . Verify this. (b) What is the critical angle for light going from zircon to air?

#### **Exercise:**

### **Problem:**

An optical fiber uses flint glass clad with crown glass. What is the critical angle?

### **Solution:**

 $66.3^{\circ}$ 

#### **Exercise:**

# **Problem:**

At what minimum angle will you get total internal reflection of light traveling in water and reflected from ice?

# **Exercise:**

#### **Problem:**

Suppose you are using total internal reflection to make an efficient corner reflector. If there is air outside and the incident angle is 45.0°, what must be the minimum index of refraction of the material from which the reflector is made?

# **Solution:**

> 1.414

## **Exercise:**

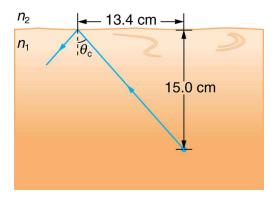
# **Problem:**

You can determine the index of refraction of a substance by determining its critical angle. (a) What is the index of refraction of a substance that has a critical angle of 68.4° when submerged in water? What is the substance, based on [link]? (b) What would the critical angle be for this substance in air?

### **Exercise:**

#### **Problem:**

A ray of light, emitted beneath the surface of an unknown liquid with air above it, undergoes total internal reflection as shown in [link]. What is the index of refraction for the liquid and its likely identification?



A light ray inside a liquid strikes the surface at the critical angle and undergoes total internal reflection.

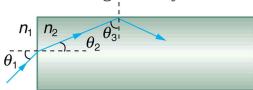
# **Solution:**

1.50, benzene

# **Exercise:**

# **Problem:**

A light ray entering an optical fiber surrounded by air is first refracted and then reflected as shown in [link]. Show that if the fiber is made from crown glass, any incident ray will be totally internally reflected.



A light ray enters the end of a fiber, the surface of which is perpendicular to its sides. Examine the conditions under which it

# may be totally internally reflected.

# Glossary

# critical angle

incident angle that produces an angle of refraction of  $90^{\circ}$ 

# fiber optics

transmission of light down fibers of plastic or glass, applying the principle of total internal reflection

# corner reflector

an object consisting of two mutually perpendicular reflecting surfaces, so that the light that enters is reflected back exactly parallel to the direction from which it came

### zircon

natural gemstone with a large index of refraction

# Introduction to Wave Optics class="introduction"

```
The colors
reflected
 by this
 compact
disc vary
with angle
 and are
not caused
    by
pigments.
 Colors
 such as
these are
  direct
evidence
  of the
  wave
character
 of light.
 (credit:
 Infopro,
Wikimedi
    a
Commons
     )
```



Examine a compact disc under white light, noting the colors observed and locations of the colors. Determine if the spectra are formed by diffraction from circular lines centered at the middle of the disc and, if so, what is their spacing. If not, determine the type of spacing. Also with the CD, explore the spectra of a few light sources, such as a candle flame, incandescent bulb, halogen light, and fluorescent light. Knowing the spacing of the rows of pits in the compact disc, estimate the maximum spacing that will allow the given number of megabytes of information to be stored.

If you have ever looked at the reds, blues, and greens in a sunlit soap bubble and wondered how straw-colored soapy water could produce them, you have hit upon one of the many phenomena that can only be explained by the wave character of light (see [link]). The same is true for the colors seen in an oil slick or in the light reflected from a compact disc. These and other interesting phenomena, such as the dispersion of white light into a rainbow of colors when passed through a narrow slit, cannot be explained fully by geometric optics. In these cases, light interacts with small objects and exhibits its wave characteristics. The branch of optics that considers the

behavior of light when it exhibits wave characteristics (particularly when it interacts with small objects) is called wave optics (sometimes called physical optics). It is the topic of this chapter.



These soap bubbles exhibit brilliant colors when exposed to sunlight. How are the colors produced if they are not pigments in the soap? (credit: Scott Robinson, Flickr)

# The Wave Aspect of Light: Interference

- Discuss the wave character of light.
- Identify the changes when light enters a medium.

We know that visible light is the type of electromagnetic wave to which our eyes respond. Like all other electromagnetic waves, it obeys the equation **Equation:** 

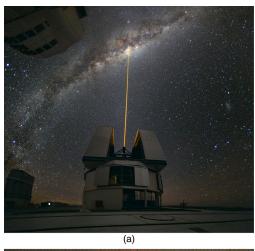
$$c = f\lambda$$
,

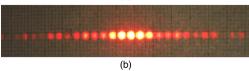
where  $c=3\times10^8~{\rm m/s}$  is the speed of light in vacuum, f is the frequency of the electromagnetic waves, and  $\lambda$  is its wavelength. The range of visible wavelengths is approximately 380 to 760 nm. As is true for all waves, light travels in straight lines and acts like a ray when it interacts with objects several times as large as its wavelength. However, when it interacts with smaller objects, it displays its wave characteristics prominently. Interference is the hallmark of a wave, and in [link] both the ray and wave characteristics of light can be seen. The laser beam emitted by the observatory epitomizes a ray, traveling in a straight line. However, passing a pure-wavelength beam through vertical slits with a size close to the wavelength of the beam reveals the wave character of light, as the beam spreads out horizontally into a pattern of bright and dark regions caused by systematic constructive and destructive interference. Rather than spreading out, a ray would continue traveling straight ahead after passing through slits.

#### Note:

# Making Connections: Waves

The most certain indication of a wave is interference. This wave characteristic is most prominent when the wave interacts with an object that is not large compared with the wavelength. Interference is observed for water waves, sound waves, light waves, and (as we will see in <a href="Special Relativity">Special Relativity</a>) for matter waves, such as electrons scattered from a crystal.





(a) The laser beam emitted by an observatory acts like a ray, traveling in a straight line. This laser beam is from the Paranal Observatory of the European Southern Observatory. (credit: Yuri Beletsky, European Southern Observatory) (b) A laser beam passing through a grid of vertical slits produces an interference pattern characteristic of a wave. (credit: Shim'on and Slava Rybka, Wikimedia Commons)

Light has wave characteristics in various media as well as in a vacuum. When light goes from a vacuum to some medium, like water, its speed and

wavelength change, but its frequency f remains the same. (We can think of light as a forced oscillation that must have the frequency of the original source.) The speed of light in a medium is v=c/n, where n is its index of refraction. If we divide both sides of equation  $c=f\lambda$  by n, we get  $c/n=v=f\lambda/n$ . This implies that  $v=f\lambda_{\rm n}$ , where  $\lambda_{\rm n}$  is the **wavelength** in a medium and that

# **Equation:**

$$\lambda_{
m n}=rac{\lambda}{n},$$

where  $\lambda$  is the wavelength in vacuum and n is the medium's index of refraction. Therefore, the wavelength of light is smaller in any medium than it is in vacuum. In water, for example, which has n=1.333, the range of visible wavelengths is (380 nm)/1.333 to (760 nm)/1.333, or  $\lambda_n=285$  to 570 nm. Although wavelengths change while traveling from one medium to another, colors do not, since colors are associated with frequency.

# **Section Summary**

- Wave optics is the branch of optics that must be used when light interacts with small objects or whenever the wave characteristics of light are considered.
- Wave characteristics are those associated with interference and diffraction.
- Visible light is the type of electromagnetic wave to which our eyes respond and has a wavelength in the range of 380 to 760 nm.
- Like all EM waves, the following relationship is valid in vacuum:  $c=f\lambda$ , where  $c=3\times 10^8~\text{m/s}$  is the speed of light, f is the frequency of the electromagnetic wave, and  $\lambda$  is its wavelength in vacuum.
- The wavelength  $\lambda_{\rm n}$  of light in a medium with index of refraction n is  $\lambda_{\rm n}=\lambda/n$ . Its frequency is the same as in vacuum.

# **Conceptual Questions**

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# **Problem:**

What type of experimental evidence indicates that light is a wave?

## **Exercise:**

# **Problem:**

Give an example of a wave characteristic of light that is easily observed outside the laboratory.

# **Problems & Exercises**

# **Exercise:**

# **Problem:**

Show that when light passes from air to water, its wavelength decreases to 0.750 times its original value.

## **Solution:**

$$1/1.333 = 0.750$$

# **Exercise:**

### **Problem:**

Find the range of visible wavelengths of light in crown glass.

# **Exercise:**

# **Problem:**

What is the index of refraction of a material for which the wavelength of light is 0.671 times its value in a vacuum? Identify the likely substance.

# **Solution:**

1.49, Polystyrene

# **Exercise:**

# **Problem:**

Analysis of an interference effect in a clear solid shows that the wavelength of light in the solid is 329 nm. Knowing this light comes from a He-Ne laser and has a wavelength of 633 nm in air, is the substance zircon or diamond?

# **Exercise:**

# **Problem:**

What is the ratio of thicknesses of crown glass and water that would contain the same number of wavelengths of light?

# **Solution:**

0.877 glass to water

# **Glossary**

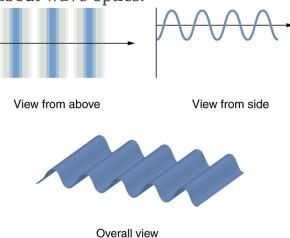
wavelength in a medium

 $\lambda_{\rm n}=\lambda/n$ , where  $\lambda$  is the wavelength in vacuum, and n is the index of refraction of the medium

# Huygens's Principle: Diffraction

- Discuss the propagation of transverse waves.
- Discuss Huygens's principle.
- Explain the bending of light.

[link] shows how a transverse wave looks as viewed from above and from the side. A light wave can be imagined to propagate like this, although we do not actually see it wiggling through space. From above, we view the wavefronts (or wave crests) as we would by looking down on the ocean waves. The side view would be a graph of the electric or magnetic field. The view from above is perhaps the most useful in developing concepts about wave optics.



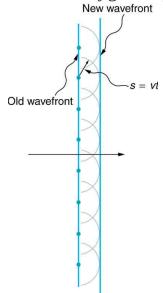
A transverse wave, such as an electromagnetic wave like light, as viewed from above and from the side. The direction of propagation is perpendicular to the wavefronts (or wave crests) and is represented by an arrow like a ray.

The Dutch scientist Christiaan Huygens (1629–1695) developed a useful technique for determining in detail how and where waves propagate.

Starting from some known position, **Huygens's principle** states that:

Every point on a wavefront is a source of wavelets that spread out in the forward direction at the same speed as the wave itself. The new wavefront is a line tangent to all of the wavelets.

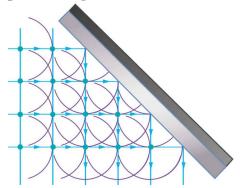
[link] shows how Huygens's principle is applied. A wavefront is the long edge that moves, for example, the crest or the trough. Each point on the wavefront emits a semicircular wave that moves at the propagation speed v. These are drawn at a time t later, so that they have moved a distance  $s=\mathrm{vt}$ . The new wavefront is a line tangent to the wavelets and is where we would expect the wave to be a time t later. Huygens's principle works for all types of waves, including water waves, sound waves, and light waves. We will find it useful not only in describing how light waves propagate, but also in explaining the laws of reflection and refraction. In addition, we will see that Huygens's principle tells us how and where light rays interfere.



Huygens's principle applied to a straight wavefront. Each point on the wavefront

emits a semicircular wavelet that moves a distance s = vt. The new wavefront is a line tangent to the wavelets.

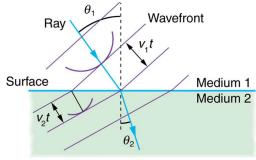
[link] shows how a mirror reflects an incoming wave at an angle equal to the incident angle, verifying the law of reflection. As the wavefront strikes the mirror, wavelets are first emitted from the left part of the mirror and then the right. The wavelets closer to the left have had time to travel farther, producing a wavefront traveling in the direction shown.



Huygens's principle applied to a straight wavefront striking a mirror. The wavelets shown were emitted as each point on the wavefront struck the mirror. The tangent to these wavelets shows

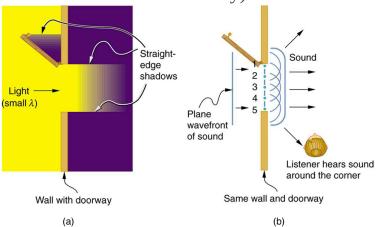
that the new wavefront has been reflected at an angle equal to the incident angle. The direction of propagation is perpendicular to the wavefront, as shown by the downward-pointing arrows.

The law of refraction can be explained by applying Huygens's principle to a wavefront passing from one medium to another (see [link]). Each wavelet in the figure was emitted when the wavefront crossed the interface between the media. Since the speed of light is smaller in the second medium, the waves do not travel as far in a given time, and the new wavefront changes direction as shown. This explains why a ray changes direction to become closer to the perpendicular when light slows down. Snell's law can be derived from the geometry in [link], but this is left as an exercise for ambitious readers.



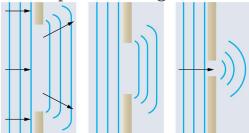
Huygens's principle applied to a straight wavefront traveling from one medium to another where its speed is less. The ray bends toward the perpendicular, since the wavelets have a lower speed in the second medium.

What happens when a wave passes through an opening, such as light shining through an open door into a dark room? For light, we expect to see a sharp shadow of the doorway on the floor of the room, and we expect no light to bend around corners into other parts of the room. When sound passes through a door, we expect to hear it everywhere in the room and, thus, expect that sound spreads out when passing through such an opening (see [link]). What is the difference between the behavior of sound waves and light waves in this case? The answer is that light has very short wavelengths and acts like a ray. Sound has wavelengths on the order of the size of the door and bends around corners (for frequency of 1000 Hz,  $\lambda = c/f = (330 \text{ m/s})/(1000 \text{ s}^{-1}) = 0.33 \text{ m}$ , about three times smaller than the width of the doorway).



(a) Light passing through a doorway makes a sharp outline on the floor. Since light's wavelength is very small compared with the size of the door, it acts like a ray. (b) Sound waves bend into all parts of the room, a wave effect, because their wavelength is similar to the size of the door.

If we pass light through smaller openings, often called slits, we can use Huygens's principle to see that light bends as sound does (see [link]). The bending of a wave around the edges of an opening or an obstacle is called **diffraction**. Diffraction is a wave characteristic and occurs for all types of waves. If diffraction is observed for some phenomenon, it is evidence that the phenomenon is a wave. Thus the horizontal diffraction of the laser beam after it passes through slits in [link] is evidence that light is a wave.



Huygens's principle applied to a straight wavefront striking an opening. The edges of the wavefront bend after passing through the opening, a process called diffraction. The amount of bending is more extreme for a small opening, consistent with the fact that wave characteristics are most noticeable for interactions with objects about the same size as the wavelength.

# **Section Summary**

- An accurate technique for determining how and where waves propagate is given by Huygens's principle: Every point on a wavefront is a source of wavelets that spread out in the forward direction at the same speed as the wave itself. The new wavefront is a line tangent to all of the wavelets.
- Diffraction is the bending of a wave around the edges of an opening or other obstacle.

# **Conceptual Questions**

# **Exercise:**

### **Problem:**

How do wave effects depend on the size of the object with which the wave interacts? For example, why does sound bend around the corner of a building while light does not?

### **Exercise:**

# **Problem:**

Under what conditions can light be modeled like a ray? Like a wave?

### **Exercise:**

#### **Problem:**

Go outside in the sunlight and observe your shadow. It has fuzzy edges even if you do not. Is this a diffraction effect? Explain.

### **Exercise:**

### **Problem:**

Why does the wavelength of light decrease when it passes from vacuum into a medium? State which attributes change and which stay the same and, thus, require the wavelength to decrease.

#### **Exercise:**

**Problem:** Does Huygens's principle apply to all types of waves?

# Glossary

# diffraction

the bending of a wave around the edges of an opening or an obstacle

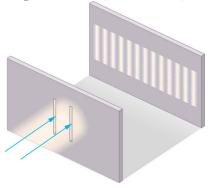
# Huygens's principle

every point on a wavefront is a source of wavelets that spread out in the forward direction at the same speed as the wave itself. The new wavefront is a line tangent to all of the wavelets

# Young's Double Slit Experiment

- Explain the phenomena of interference.
- Define constructive interference for a double slit and destructive interference for a double slit.

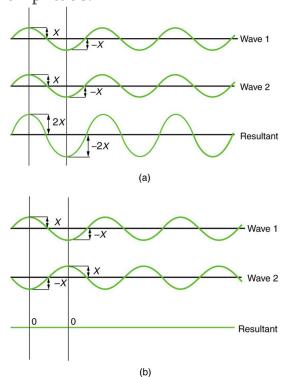
Although Christiaan Huygens thought that light was a wave, Isaac Newton did not. Newton felt that there were other explanations for color, and for the interference and diffraction effects that were observable at the time. Owing to Newton's tremendous stature, his view generally prevailed. The fact that Huygens's principle worked was not considered evidence that was direct enough to prove that light is a wave. The acceptance of the wave character of light came many years later when, in 1801, the English physicist and physician Thomas Young (1773–1829) did his now-classic double slit experiment (see [link]).



Young's double slit experiment. Here pure-wavelength light sent through a pair of vertical slits is diffracted into a pattern on the screen of numerous vertical lines spread out horizontally. Without diffraction and interference, the light would

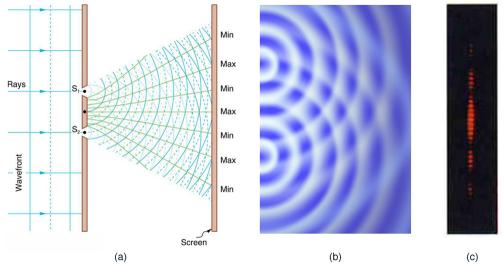
simply make two lines on the screen.

Why do we not ordinarily observe wave behavior for light, such as observed in Young's double slit experiment? First, light must interact with something small, such as the closely spaced slits used by Young, to show pronounced wave effects. Furthermore, Young first passed light from a single source (the Sun) through a single slit to make the light somewhat coherent. By **coherent**, we mean waves are in phase or have a definite phase relationship. **Incoherent** means the waves have random phase relationships. Why did Young then pass the light through a double slit? The answer to this question is that two slits provide two coherent light sources that then interfere constructively or destructively. Young used sunlight, where each wavelength forms its own pattern, making the effect more difficult to see. We illustrate the double slit experiment with monochromatic (single  $\lambda$ ) light to clarify the effect. [link] shows the pure constructive and destructive interference of two waves having the same wavelength and amplitude.



The amplitudes of waves add. (a) Pure constructive interference is obtained when identical waves are in phase. (b) Pure destructive interference occurs when identical waves are exactly out of phase, or shifted by half a wavelength.

When light passes through narrow slits, it is diffracted into semicircular waves, as shown in [link](a). Pure constructive interference occurs where the waves are crest to crest or trough to trough. Pure destructive interference occurs where they are crest to trough. The light must fall on a screen and be scattered into our eyes for us to see the pattern. An analogous pattern for water waves is shown in [link](b). Note that regions of constructive and destructive interference move out from the slits at well-defined angles to the original beam. These angles depend on wavelength and the distance between the slits, as we shall see below.



Double slits produce two coherent sources of waves that interfere. (a) Light spreads out (diffracts) from each slit, because the slits are narrow. These waves overlap and

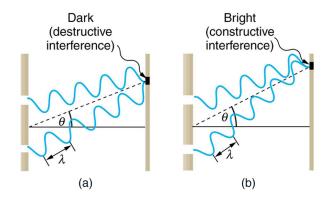
interfere constructively (bright lines) and destructively (dark regions). We can only see this if the light falls onto a screen and is scattered into our eyes. (b) Double slit interference pattern for water waves are nearly identical to that for light. Wave action is greatest in regions of constructive interference and least in regions of destructive interference. (c) When light that has passed through double slits falls on a screen, we see a pattern such as this. (credit: PASCO)

To understand the double slit interference pattern, we consider how two waves travel from the slits to the screen, as illustrated in [link]. Each slit is a different distance from a given point on the screen. Thus different numbers of wavelengths fit into each path. Waves start out from the slits in phase (crest to crest), but they may end up out of phase (crest to trough) at the screen if the paths differ in length by half a wavelength, interfering destructively as shown in [link](a). If the paths differ by a whole wavelength, then the waves arrive in phase (crest to crest) at the screen, interfering constructively as shown in [link](b). More generally, if the paths taken by the two waves differ by any half-integral number of wavelengths [ $(1/2)\lambda$ ,  $(3/2)\lambda$ ,  $(5/2)\lambda$ , etc.], then destructive interference occurs. Similarly, if the paths taken by the two waves differ by any integral number of wavelengths ( $\lambda$ ,  $2\lambda$ ,  $3\lambda$ , etc.), then constructive interference occurs.

### Note:

# Take-Home Experiment: Using Fingers as Slits

Look at a light, such as a street lamp or incandescent bulb, through the narrow gap between two fingers held close together. What type of pattern do you see? How does it change when you allow the fingers to move a little farther apart? Is it more distinct for a monochromatic source, such as the yellow light from a sodium vapor lamp, than for an incandescent bulb?



Waves follow different paths from the slits to a common point on a screen. (a)

Destructive interference occurs here, because one path is a half wavelength longer than the other. The waves start in phase but arrive out of phase. (b)

Constructive interference occurs here because one path is a whole wavelength longer than the other. The waves start out and arrive in phase.

[link] shows how to determine the path length difference for waves traveling from two slits to a common point on a screen. If the screen is a large distance away compared with the distance between the slits, then the angle  $\theta$  between the path and a line from the slits to the screen (see the figure) is nearly the same for each path. The difference between the paths is shown in the figure; simple trigonometry shows it to be  $d \sin \theta$ , where d is the distance between the slits. To obtain **constructive interference for a double slit**, the path length difference must be an integral multiple of the wavelength, or

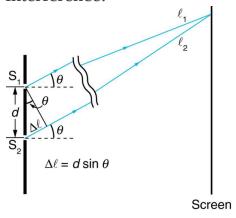
# **Equation:**

$$d \sin \theta = m\lambda$$
, for  $m = 0, 1, -1, 2, -2, \dots$  (constructive).

Similarly, to obtain **destructive interference for a double slit**, the path length difference must be a half-integral multiple of the wavelength, or **Equation:** 

$$d \sin heta = \left(m + rac{1}{2}
ight) \lambda, ext{ for } m = 0, 1, \; -1, 2, \; -2, \; \dots \; ext{ (destructive)},$$

where  $\lambda$  is the wavelength of the light, d is the distance between slits, and  $\theta$  is the angle from the original direction of the beam as discussed above. We call m the **order** of the interference. For example, m=4 is fourth-order interference.



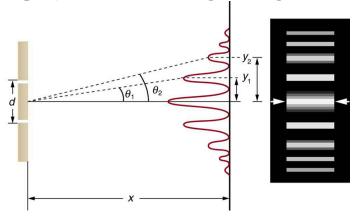
The paths from each slit to a common point on the screen differ by an amount  $d \sin \theta$ , assuming the distance to the screen is much greater than the distance between slits (not to scale here).

The equations for double slit interference imply that a series of bright and dark lines are formed. For vertical slits, the light spreads out horizontally on

either side of the incident beam into a pattern called interference fringes, illustrated in [link]. The intensity of the bright fringes falls off on either side, being brightest at the center. The closer the slits are, the more is the spreading of the bright fringes. We can see this by examining the equation **Equation:** 

$$d \sin \theta = m\lambda, \, {
m for} \ m = 0, \, 1, \, -1, \, 2, \, -2, \, \ldots$$

For fixed  $\lambda$  and m, the smaller d is, the larger  $\theta$  must be, since  $\sin \theta = m \lambda / d$ . This is consistent with our contention that wave effects are most noticeable when the object the wave encounters (here, slits a distance d apart) is small. Small d gives large  $\theta$ , hence a large effect.



The interference pattern for a double slit has an intensity that falls off with angle. The photograph shows multiple bright and dark lines, or fringes, formed by light passing through a double slit.

# Example:

Finding a Wavelength from an Interference Pattern

Suppose you pass light from a He-Ne laser through two slits separated by 0.0100 mm and find that the third bright line on a screen is formed at an angle of  $10.95^{\circ}$  relative to the incident beam. What is the wavelength of the light?

# Strategy

The third bright line is due to third-order constructive interference, which means that m=3. We are given d=0.0100 mm and  $\theta=10.95^{\circ}$ . The wavelength can thus be found using the equation  $d\sin\theta=m\lambda$  for constructive interference.

# Solution

The equation is  $d \sin \theta = m\lambda$ . Solving for the wavelength  $\lambda$  gives **Equation:** 

$$\lambda = \frac{d \sin \theta}{m}.$$

Substituting known values yields

# **Equation:**

$$\lambda = \frac{(0.0100 \text{ mm})(\sin 10.95^{\circ})}{3} \ = 6.33 \times 10^{-4} \text{ mm} = 633 \text{ nm}.$$

#### Discussion

To three digits, this is the wavelength of light emitted by the common He-Ne laser. Not by coincidence, this red color is similar to that emitted by neon lights. More important, however, is the fact that interference patterns can be used to measure wavelength. Young did this for visible wavelengths. This analytical technique is still widely used to measure electromagnetic spectra. For a given order, the angle for constructive interference increases with  $\lambda$ , so that spectra (measurements of intensity versus wavelength) can be obtained.

# **Example:**

# **Calculating Highest Order Possible**

Interference patterns do not have an infinite number of lines, since there is a limit to how big m can be. What is the highest-order constructive interference possible with the system described in the preceding example?

# **Strategy and Concept**

The equation  $d \sin \theta = m\lambda$  (for m = 0, 1, -1, 2, -2, ...) describes constructive interference. For fixed values of d and  $\lambda$ , the larger m is, the larger  $\sin \theta$  is. However, the maximum value that  $\sin \theta$  can have is 1, for an angle of 90°. (Larger angles imply that light goes backward and does not reach the screen at all.) Let us find which m corresponds to this maximum diffraction angle.

### Solution

Solving the equation  $d \sin \theta = m\lambda$  for m gives

# **Equation:**

$$m=rac{d\sin heta}{\lambda}.$$

Taking  $\sin \theta = 1$  and substituting the values of d and  $\lambda$  from the preceding example gives

# **Equation:**

$$m = \frac{(0.0100 \text{ mm})(1)}{633 \text{ nm}} \approx 15.8.$$

Therefore, the largest integer m can be is 15, or

# **Equation:**

$$m = 15$$
.

### Discussion

The number of fringes depends on the wavelength and slit separation. The number of fringes will be very large for large slit separations. However, if the slit separation becomes much greater than the wavelength, the intensity of the interference pattern changes so that the screen has two bright lines cast by the slits, as expected when light behaves like a ray. We also note that the fringes get fainter further away from the center. Consequently, not all 15 fringes may be observable.

# **Section Summary**

- Young's double slit experiment gave definitive proof of the wave character of light.
- An interference pattern is obtained by the superposition of light from two slits.
- There is constructive interference when  $d \sin \theta = m\lambda$  (for  $m = 0, 1, -1, 2, -2, \ldots$ ), where d is the distance between the slits,  $\theta$  is the angle relative to the incident direction, and m is the order of the interference.
- There is destructive interference when  $d \sin \theta = \left(m + \frac{1}{2}\right) \lambda$  (for  $m = 0, 1, -1, 2, -2, \ldots$ ).

# **Conceptual Questions**

#### **Exercise:**

### **Problem:**

Young's double slit experiment breaks a single light beam into two sources. Would the same pattern be obtained for two independent sources of light, such as the headlights of a distant car? Explain.

#### **Exercise:**

## **Problem:**

Suppose you use the same double slit to perform Young's double slit experiment in air and then repeat the experiment in water. Do the angles to the same parts of the interference pattern get larger or smaller? Does the color of the light change? Explain.

### **Exercise:**

### **Problem:**

Is it possible to create a situation in which there is only destructive interference? Explain.

#### **Exercise:**

# **Problem:**

[link] shows the central part of the interference pattern for a pure wavelength of red light projected onto a double slit. The pattern is actually a combination of single slit and double slit interference. Note that the bright spots are evenly spaced. Is this a double slit or single slit characteristic? Note that some of the bright spots are dim on either side of the center. Is this a single slit or double slit characteristic? Which is smaller, the slit width or the separation between slits? Explain your responses.



This double slit interference pattern also shows signs of single slit interference. (credit: PASCO)

# **Problems & Exercises**

# **Exercise:**

# **Problem:**

At what angle is the first-order maximum for 450-nm wavelength blue light falling on double slits separated by 0.0500 mm?

# **Solution:**

 $0.516^{\circ}$ 

# **Exercise:**

# **Problem:**

Calculate the angle for the third-order maximum of 580-nm wavelength yellow light falling on double slits separated by 0.100 mm.

## **Exercise:**

## **Problem:**

What is the separation between two slits for which 610-nm orange light has its first maximum at an angle of 30.0°?

# **Solution:**

$$1.22 \times 10^{-6} \,\mathrm{m}$$

## **Exercise:**

## **Problem:**

Find the distance between two slits that produces the first minimum for 410-nm violet light at an angle of  $45.0^{\circ}$ .

#### **Exercise:**

#### **Problem:**

Calculate the wavelength of light that has its third minimum at an angle of  $30.0^{\circ}$  when falling on double slits separated by  $3.00~\mu m$ . Explicitly, show how you follow the steps in Problem-Solving Strategies for Wave Optics.

# **Solution:**

600 nm

# **Exercise:**

### **Problem:**

What is the wavelength of light falling on double slits separated by  $2.00 \ \mu m$  if the third-order maximum is at an angle of  $60.0^{\circ}$ ?

# **Exercise:**

# **Problem:**

At what angle is the fourth-order maximum for the situation in [link]?

# **Solution:**

 $2.06^{\circ}$ 

# **Exercise:**

### **Problem:**

What is the highest-order maximum for 400-nm light falling on double slits separated by  $25.0~\mu m$ ?

### **Exercise:**

## **Problem:**

Find the largest wavelength of light falling on double slits separated by  $1.20~\mu m$  for which there is a first-order maximum. Is this in the visible part of the spectrum?

# **Solution:**

1200 nm (not visible)

### **Exercise:**

### **Problem:**

What is the smallest separation between two slits that will produce a second-order maximum for 720-nm red light?

### **Exercise:**

# **Problem:**

(a) What is the smallest separation between two slits that will produce a second-order maximum for any visible light? (b) For all visible light?

# **Solution:**

- (a) 760 nm
- (b) 1520 nm

# **Exercise:**

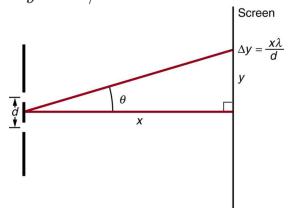
## **Problem:**

(a) If the first-order maximum for pure-wavelength light falling on a double slit is at an angle of 10.0°, at what angle is the second-order maximum? (b) What is the angle of the first minimum? (c) What is the highest-order maximum possible here?

## **Exercise:**

# **Problem:**

[link] shows a double slit located a distance x from a screen, with the distance from the center of the screen given by y. When the distance d between the slits is relatively large, there will be numerous bright spots, called fringes. Show that, for small angles (where  $\sin\theta\approx\theta$ , with  $\theta$  in radians), the distance between fringes is given by  $\Delta y = x\lambda/d$ .



The distance between adjacent fringes is  $\Delta y = x \lambda/d$ , assuming the

slit separation d is large compared with  $\lambda$ .

## **Solution:**

For small angles  $\sin \theta - \tan \theta \approx \theta$  (in radians).

For two adjacent fringes we have,

## **Equation:**

$$d \sin \theta_{\rm m} = m\lambda$$

and

## **Equation:**

$$d \sin \theta_{\mathrm{m}+1} = (m+1)\lambda$$

Subtracting these equations gives

# **Equation:**

$$egin{aligned} d(\sin heta_{ ext{m}+1}-\sin heta_{ ext{m}}) &= [(m+1)-m]\lambda \ d( heta_{ ext{m}+1}- heta_{ ext{m}}) &= \lambda \ an heta_{ ext{m}} &= rac{y_{ ext{m}}}{x} pprox heta_{ ext{m}} \Rightarrow dig(rac{y_{ ext{m}+1}}{x}-rac{y_{ ext{m}}}{x}ig) &= \lambda \ drac{\Delta y}{x} &= \lambda \Rightarrow \Delta y = rac{x\lambda}{d} \end{aligned}$$

#### **Exercise:**

#### **Problem:**

Using the result of the problem above, calculate the distance between fringes for 633-nm light falling on double slits separated by 0.0800 mm, located 3.00 m from a screen as in [link].

Using the result of the problem two problems prior, find the wavelength of light that produces fringes 7.50 mm apart on a screen 2.00 m from double slits separated by 0.120 mm (see [link]).

#### **Solution:**

450 nm

# **Glossary**

#### coherent

waves are in phase or have a definite phase relationship

constructive interference for a double slit the path length difference must be an integral multiple of the wavelength

destructive interference for a double slit the path length difference must be a half-integral multiple of the wavelength

#### incoherent

waves have random phase relationships

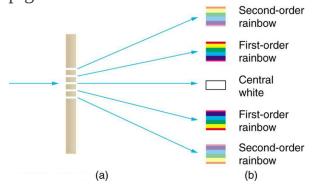
#### order

the integer m used in the equations for constructive and destructive interference for a double slit

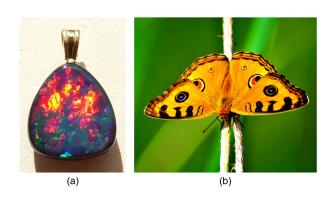
# Multiple Slit Diffraction

- Discuss the pattern obtained from diffraction grating.
- Explain diffraction grating effects.

An interesting thing happens if you pass light through a large number of evenly spaced parallel slits, called a **diffraction grating**. An interference pattern is created that is very similar to the one formed by a double slit (see [link]). A diffraction grating can be manufactured by scratching glass with a sharp tool in a number of precisely positioned parallel lines, with the untouched regions acting like slits. These can be photographically mass produced rather cheaply. Diffraction gratings work both for transmission of light, as in [link], and for reflection of light, as on butterfly wings and the Australian opal in [link] or the CD pictured in the opening photograph of this chapter, [link]. In addition to their use as novelty items, diffraction gratings are commonly used for spectroscopic dispersion and analysis of light. What makes them particularly useful is the fact that they form a sharper pattern than double slits do. That is, their bright regions are narrower and brighter, while their dark regions are darker. [link] shows idealized graphs demonstrating the sharper pattern. Natural diffraction gratings occur in the feathers of certain birds. Tiny, finger-like structures in regular patterns act as reflection gratings, producing constructive interference that gives the feathers colors not solely due to their pigmentation. This is called iridescence.

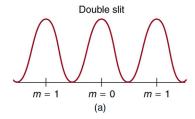


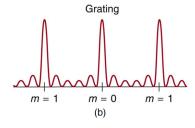
A diffraction grating is a large number of evenly spaced parallel slits. (a) Light passing through is diffracted in a pattern similar to a double slit, with bright regions at various angles.
(b) The pattern obtained for
white light incident on a
grating. The central maximum
is white, and the higher-order
maxima disperse white light
into a rainbow of colors.



(a) This Australian opal and (b) the butterfly wings have rows of reflectors that act like reflection gratings, reflecting different colors at different angles.

(credits: (a) Opals-On-Black.com, via Flickr (b) whologwhy, Flickr)



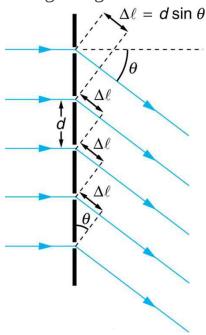


Idealized graphs of the intensity of light passing through a double slit (a) and a diffraction grating (b) for monochromatic light. Maxima can be produced at the same angles, but those for the diffraction grating are narrower and hence sharper. The maxima become narrower and the regions between darker as the number of slits is increased.

The analysis of a diffraction grating is very similar to that for a double slit (see [link]). As we know from our discussion of double slits in Young's Double Slit Experiment, light is diffracted by each slit and spreads out after passing through. Rays traveling in the same direction (at an angle  $\theta$  relative to the incident direction) are shown in the figure. Each of these rays travels a different distance to a common point on a screen far away. The rays start in phase, and they can be in or out of phase when they reach a screen, depending on the difference in the path lengths traveled. As seen in the figure, each ray travels a distance  $d \sin \theta$  different from that of its neighbor, where d is the distance between slits. If this distance equals an integral number of wavelengths, the rays all arrive in phase, and constructive interference (a maximum) is obtained. Thus, the condition necessary to obtain constructive interference for a diffraction grating is Equation:

$$d \sin \theta = m\lambda$$
, for  $m = 0, 1, -1, 2, -2, \dots$  (constructive),

where d is the distance between slits in the grating,  $\lambda$  is the wavelength of light, and m is the order of the maximum. Note that this is exactly the same equation as for double slits separated by d. However, the slits are usually closer in diffraction gratings than in double slits, producing fewer maxima at larger angles.



Diffraction grating showing light rays from each slit traveling in the same direction. Each ray travels a different distance to reach a common point on a screen (not shown). Each ray travels a distance  $d \sin \theta$  different from that of its neighbor.

Where are diffraction gratings used? Diffraction gratings are key components of monochromators used, for example, in optical imaging of particular wavelengths from biological or medical samples. A diffraction grating can be chosen to specifically analyze a wavelength emitted by molecules in diseased cells in a biopsy sample or to help excite strategic molecules in the sample with a selected frequency of light. Another vital use is in optical fiber technologies where fibers are designed to provide optimum performance at specific wavelengths. A range of diffraction gratings are available for selecting specific wavelengths for such use.

#### Note:

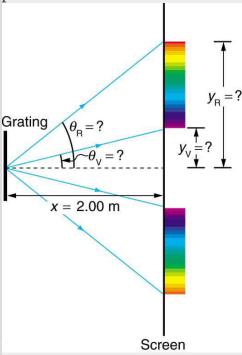
# Take-Home Experiment: Rainbows on a CD

The spacing d of the grooves in a CD or DVD can be well determined by using a laser and the equation  $d \sin \theta = m\lambda$ , for  $m = 0, 1, -1, 2, -2, \ldots$ . However, we can still make a good estimate of this spacing by using white light and the rainbow of colors that comes from the interference. Reflect sunlight from a CD onto a wall and use your best judgment of the location of a strongly diffracted color to find the separation d.

## **Example:**

# **Calculating Typical Diffraction Grating Effects**

Diffraction gratings with 10,000 lines per centimeter are readily available. Suppose you have one, and you send a beam of white light through it to a screen 2.00 m away. (a) Find the angles for the first-order diffraction of the shortest and longest wavelengths of visible light (380 and 760 nm). (b) What is the distance between the ends of the rainbow of visible light produced on the screen for first-order interference? (See [link].)



The diffraction grating considered in this example produces a rainbow of colors on a screen a distance x=2.00 m from the grating. The distances along the screen are measured perpendicular to the x-direction. In other words, the rainbow pattern extends out of the page.

## Strategy

The angles can be found using the equation

## **Equation:**

$$d \sin \theta = m\lambda \text{ (for } m = 0, 1, -1, 2, -2, \ldots)$$

once a value for the slit spacing d has been determined. Since there are 10,000 lines per centimeter, each line is separated by 1/10,000 of a centimeter. Once the angles are found, the distances along the screen can be found using simple trigonometry.

## Solution for (a)

The distance between slits is  $d=(1~{\rm cm})/10,000=1.00\times 10^{-4}~{\rm cm}$  or  $1.00\times 10^{-6}~{\rm m}$ . Let us call the two angles  $\theta_{\rm V}$  for violet (380 nm) and  $\theta_{\rm R}$  for red (760 nm). Solving the equation  $d\sin\theta_{\rm V}=m\lambda$  for  $\sin\theta_{\rm V}$ ,

## **Equation:**

$$\sin heta_{
m V} = rac{m \lambda_{
m V}}{d},$$

where m=1 for first order and  $\lambda_{\rm V}=380~{
m nm}=3.80\times 10^{-7}~{
m m}.$  Substituting these values gives

# **Equation:**

$$\sin heta_{
m V} = rac{3.80 imes 10^{-7} \, 
m m}{1.00 imes 10^{-6} \, 
m m} = 0.380.$$

Thus the angle  $\theta_{
m V}$  is

# **Equation:**

$$heta_{
m V} = \sin^{-1} 0.380 = 22.33^{
m o}.$$

Similarly,

# **Equation:**

$$\sin heta_{
m R} = rac{7.60 imes10^{-7} {
m m}}{1.00 imes10^{-6} {
m m}}.$$

Thus the angle  $heta_{
m R}$  is

## **Equation:**

$$\theta_{\rm R} = \sin^{-1} 0.760 = 49.46^{\rm o}.$$

Notice that in both equations, we reported the results of these intermediate calculations to four significant figures to use with the calculation in part (b).

## Solution for (b)

The distances on the screen are labeled  $y_{\rm V}$  and  $y_{\rm R}$  in [link]. Noting that  $\tan \theta = y/x$ , we can solve for  $y_{\rm V}$  and  $y_{\rm R}$ . That is,

## **Equation:**

$$y_{
m V} = x an heta_{
m V} = (2.00 ext{ m})( an 22.33^{
m o}) = 0.815 ext{ m}$$

and

## **Equation:**

$$y_{\rm R} = x \tan \theta_{\rm R} = (2.00 \text{ m})(\tan 49.46^{\circ}) = 2.338 \text{ m}.$$

The distance between them is therefore

## **Equation:**

$$y_{
m R}-y_{
m V}=1.52~{
m m}.$$

#### Discussion

The large distance between the red and violet ends of the rainbow produced from the white light indicates the potential this diffraction grating has as a spectroscopic tool. The more it can spread out the wavelengths (greater dispersion), the more detail can be seen in a spectrum. This depends on the quality of the diffraction grating—it must be very precisely made in addition to having closely spaced lines.

# **Section Summary**

- A diffraction grating is a large collection of evenly spaced parallel slits that produces an interference pattern similar to but sharper than that of a double slit.
- There is constructive interference for a diffraction grating when  $d \sin \theta = m\lambda$  (for  $m = 0, 1, -1, 2, -2, \ldots$ ), where d is the distance between slits in the grating,  $\lambda$  is the wavelength of light, and m is the order of the maximum.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

What is the advantage of a diffraction grating over a double slit in dispersing light into a spectrum?

#### **Exercise:**

#### **Problem:**

What are the advantages of a diffraction grating over a prism in dispersing light for spectral analysis?

#### **Exercise:**

#### **Problem:**

Can the lines in a diffraction grating be too close together to be useful as a spectroscopic tool for visible light? If so, what type of EM radiation would the grating be suitable for? Explain.

#### **Exercise:**

#### **Problem:**

If a beam of white light passes through a diffraction grating with vertical lines, the light is dispersed into rainbow colors on the right and left. If a glass prism disperses white light to the right into a rainbow, how does the sequence of colors compare with that produced on the right by a diffraction grating?

#### **Exercise:**

#### **Problem:**

Suppose pure-wavelength light falls on a diffraction grating. What happens to the interference pattern if the same light falls on a grating that has more lines per centimeter? What happens to the interference pattern if a longer-wavelength light falls on the same grating? Explain how these two effects are consistent in terms of the relationship of wavelength to the distance between slits.

#### **Exercise:**

#### **Problem:**

Suppose a feather appears green but has no green pigment. Explain in terms of diffraction.

#### **Exercise:**

#### **Problem:**

It is possible that there is no minimum in the interference pattern of a single slit. Explain why. Is the same true of double slits and diffraction gratings?

## **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

A diffraction grating has 2000 lines per centimeter. At what angle will the first-order maximum be for 520-nm-wavelength green light?

#### **Solution:**

 $5.97^{\circ}$ 

Find the angle for the third-order maximum for 580-nm-wavelength yellow light falling on a diffraction grating having 1500 lines per centimeter.

#### **Exercise:**

#### **Problem:**

How many lines per centimeter are there on a diffraction grating that gives a first-order maximum for 470-nm blue light at an angle of  $25.0^{\circ}$ 

#### **Solution:**

 $8.99 \times 10^{3}$ 

#### **Exercise:**

#### **Problem:**

What is the distance between lines on a diffraction grating that produces a second-order maximum for 760-nm red light at an angle of 60.0°?

#### **Exercise:**

#### **Problem:**

Calculate the wavelength of light that has its second-order maximum at 45.0° when falling on a diffraction grating that has 5000 lines per centimeter.

#### **Solution:**

707 nm

An electric current through hydrogen gas produces several distinct wavelengths of visible light. What are the wavelengths of the hydrogen spectrum, if they form first-order maxima at angles of 24.2°, 25.7°, 29.1°, and 41.0° when projected on a diffraction grating having 10,000 lines per centimeter? Explicitly show how you follow the steps in <a href="Problem-Solving Strategies for Wave Optics">Problem-Solving Strategies for Wave Optics</a>

#### **Exercise:**

#### **Problem:**

(a) What do the four angles in the above problem become if a 5000-line-per-centimeter diffraction grating is used? (b) Using this grating, what would the angles be for the second-order maxima? (c) Discuss the relationship between integral reductions in lines per centimeter and the new angles of various order maxima.

#### **Solution:**

- (a) 11.8°, 12.5°, 14.1°, 19.2°
- (b) 24.2°, 25.7°, 29.1°, 41.0°
- (c) Decreasing the number of lines per centimeter by a factor of x means that the angle for the x-order maximum is the same as the original angle for the first- order maximum.

#### **Exercise:**

#### **Problem:**

What is the maximum number of lines per centimeter a diffraction grating can have and produce a complete first-order spectrum for visible light?

The yellow light from a sodium vapor lamp *seems* to be of pure wavelength, but it produces two first-order maxima at 36.093° and 36.129° when projected on a 10,000 line per centimeter diffraction grating. What are the two wavelengths to an accuracy of 0.1 nm?

#### **Solution:**

589.1 nm and 589.6 nm

#### **Exercise:**

#### **Problem:**

What is the spacing between structures in a feather that acts as a reflection grating, given that they produce a first-order maximum for 525-nm light at a  $30.0^{\circ}$  angle?

#### **Exercise:**

## **Problem:**

Structures on a bird feather act like a reflection grating having 8000 lines per centimeter. What is the angle of the first-order maximum for 600-nm light?

#### **Solution:**

 $28.7^{\circ}$ 

#### **Exercise:**

#### **Problem:**

An opal such as that shown in  $[\underline{link}]$  acts like a reflection grating with rows separated by about 8  $\mu$ m. If the opal is illuminated normally, (a) at what angle will red light be seen and (b) at what angle will blue light be seen?

At what angle does a diffraction grating produces a second-order maximum for light having a first-order maximum at  $20.0^{\circ}$ ?

#### **Solution:**

 $43.2^{\circ}$ 

#### **Exercise:**

#### **Problem:**

Show that a diffraction grating cannot produce a second-order maximum for a given wavelength of light unless the first-order maximum is at an angle less than  $30.0^{\circ}$ .

#### **Exercise:**

## **Problem:**

If a diffraction grating produces a first-order maximum for the shortest wavelength of visible light at 30.0°, at what angle will the first-order maximum be for the longest wavelength of visible light?

#### **Solution:**

 $90.0^{\circ}$ 

#### **Exercise:**

#### **Problem:**

(a) Find the maximum number of lines per centimeter a diffraction grating can have and produce a maximum for the smallest wavelength of visible light. (b) Would such a grating be useful for ultraviolet spectra? (c) For infrared spectra?

(a) Show that a 30,000-line-per-centimeter grating will not produce a maximum for visible light. (b) What is the longest wavelength for which it does produce a first-order maximum? (c) What is the greatest number of lines per centimeter a diffraction grating can have and produce a complete second-order spectrum for visible light?

#### **Solution:**

- (a) The longest wavelength is 333.3 nm, which is not visible.
- (b) 333 nm (UV)
- (c)  $6.58 \times 10^3$  cm

#### **Exercise:**

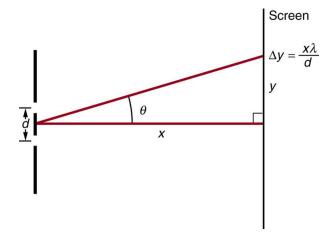
#### **Problem:**

A He—Ne laser beam is reflected from the surface of a CD onto a wall. The brightest spot is the reflected beam at an angle equal to the angle of incidence. However, fringes are also observed. If the wall is 1.50 m from the CD, and the first fringe is 0.600 m from the central maximum, what is the spacing of grooves on the CD?

#### **Exercise:**

#### **Problem:**

The analysis shown in the figure below also applies to diffraction gratings with lines separated by a distance d. What is the distance between fringes produced by a diffraction grating having 125 lines per centimeter for 600-nm light, if the screen is 1.50 m away?



The distance between adjacent fringes is  $\Delta y = x \lambda/d$ , assuming the slit separation d is large compared with  $\lambda$ .

## **Solution:**

$$1.13 \times 10^{-2} \,\mathrm{m}$$

#### **Exercise:**

#### **Problem:** Unreasonable Results

Red light of wavelength of 700 nm falls on a double slit separated by 400 nm. (a) At what angle is the first-order maximum in the diffraction pattern? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

## **Exercise:**

## **Problem:** Unreasonable Results

(a) What visible wavelength has its fourth-order maximum at an angle of  $25.0^{\circ}$  when projected on a 25,000-line-per-centimeter diffraction

grating? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

## **Solution:**

- (a) 42.3 nm
- (b) Not a visible wavelength

The number of slits in this diffraction grating is too large. Etching in integrated circuits can be done to a resolution of 50 nm, so slit separations of 400 nm are at the limit of what we can do today. This line spacing is too small to produce diffraction of light.

#### **Exercise:**

#### **Problem:** Construct Your Own Problem

Consider a spectrometer based on a diffraction grating. Construct a problem in which you calculate the distance between two wavelengths of electromagnetic radiation in your spectrometer. Among the things to be considered are the wavelengths you wish to be able to distinguish, the number of lines per meter on the diffraction grating, and the distance from the grating to the screen or detector. Discuss the practicality of the device in terms of being able to discern between wavelengths of interest.

# Glossary

constructive interference for a diffraction grating occurs when the condition  $d \sin \theta = m\lambda$  (for  $m=0,1,-1,2,-2,\ldots$ ) is satisfied, where d is the distance between slits in the grating,  $\lambda$  is the wavelength of light, and m is the order of the maximum

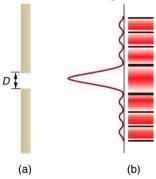
## diffraction grating

a large number of evenly spaced parallel slits

## Single Slit Diffraction

• Discuss the single slit diffraction pattern.

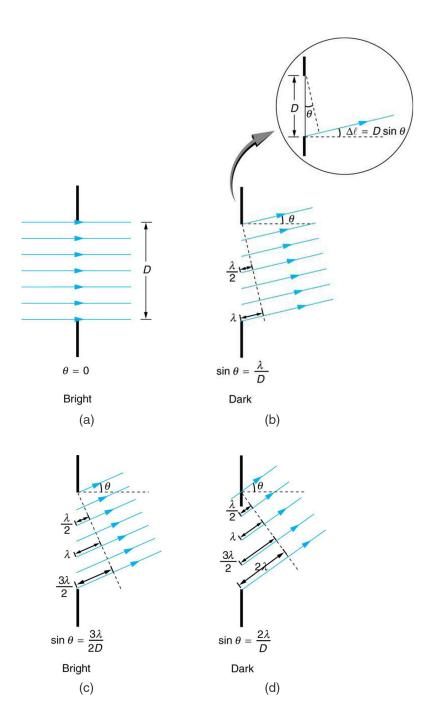
Light passing through a single slit forms a diffraction pattern somewhat different from those formed by double slits or diffraction gratings. [link] shows a single slit diffraction pattern. Note that the central maximum is larger than those on either side, and that the intensity decreases rapidly on either side. In contrast, a diffraction grating produces evenly spaced lines that dim slowly on either side of center.



(a) Single slit diffraction pattern. Monochromati c light passing through a single slit has a central maximum and many smaller and dimmer maxima on either side. The central maximum is six times higher than shown. (b) The drawing shows

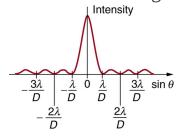
the bright
central
maximum and
dimmer and
thinner maxima
on either side.

The analysis of single slit diffraction is illustrated in [link]. Here we consider light coming from different parts of the same slit. According to Huygens's principle, every part of the wavefront in the slit emits wavelets. These are like rays that start out in phase and head in all directions. (Each ray is perpendicular to the wavefront of a wavelet.) Assuming the screen is very far away compared with the size of the slit, rays heading toward a common destination are nearly parallel. When they travel straight ahead, as in [link](a), they remain in phase, and a central maximum is obtained. However, when rays travel at an angle  $\theta$  relative to the original direction of the beam, each travels a different distance to a common location, and they can arrive in or out of phase. In [link](b), the ray from the bottom travels a distance of one wavelength  $\lambda$  farther than the ray from the top. Thus a ray from the center travels a distance  $\lambda/2$  farther than the one on the left, arrives out of phase, and interferes destructively. A ray from slightly above the center and one from slightly above the bottom will also cancel one another. In fact, each ray from the slit will have another to interfere destructively, and a minimum in intensity will occur at this angle. There will be another minimum at the same angle to the right of the incident direction of the light.



Light passing through a single slit is diffracted in all directions and may interfere constructively or destructively, depending on the angle. The difference in path length for rays from either side of the slit is seen to be  $D \sin \theta$ .

At the larger angle shown in [link](c), the path lengths differ by  $3\lambda/2$  for rays from the top and bottom of the slit. One ray travels a distance  $\lambda$  different from the ray from the bottom and arrives in phase, interfering constructively. Two rays, each from slightly above those two, will also add constructively. Most rays from the slit will have another to interfere with constructively, and a maximum in intensity will occur at this angle. However, all rays do not interfere constructively for this situation, and so the maximum is not as intense as the central maximum. Finally, in [link](d), the angle shown is large enough to produce a second minimum. As seen in the figure, the difference in path length for rays from either side of the slit is  $D \sin \theta$ , and we see that a destructive minimum is obtained when this distance is an integral multiple of the wavelength.



A graph of single slit diffraction intensity showing the central maximum to be wider and much more intense than those to the sides. In fact the central maximum is six times higher than shown here.

Thus, to obtain **destructive interference for a single slit**, **Equation:** 

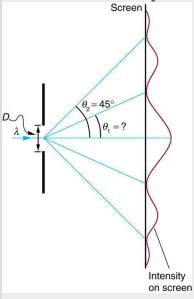
$$D \sin \theta = m\lambda$$
, for  $m = 1, -1, 2, -2, 3, \dots$  (destructive),

where D is the slit width,  $\lambda$  is the light's wavelength,  $\theta$  is the angle relative to the original direction of the light, and m is the order of the minimum. [link] shows a graph of intensity for single slit interference, and it is apparent that the maxima on either side of the central maximum are much less intense and not as wide. This is consistent with the illustration in [link] (b).

## **Example:**

# **Calculating Single Slit Diffraction**

Visible light of wavelength 550 nm falls on a single slit and produces its second diffraction minimum at an angle of 45.0° relative to the incident direction of the light. (a) What is the width of the slit? (b) At what angle is the first minimum produced?



A graph of the

single slit diffraction pattern is analyzed in this example.

## Strategy

From the given information, and assuming the screen is far away from the slit, we can use the equation  $D \sin \theta = m\lambda$  first to find D, and again to find the angle for the first minimum  $\theta_1$ .

## Solution for (a)

We are given that  $\lambda = 550$  nm, m = 2, and  $\theta_2 = 45.0^\circ$ . Solving the equation  $D \sin \theta = m\lambda$  for D and substituting known values gives

# **Equation:**

$$D = \frac{m\lambda}{\sin \theta_2} = \frac{2(550 \text{ nm})}{\sin 45.0^{\circ}}$$
$$= \frac{1100 \times 10^{-9}}{0.707}$$
$$= 1.56 \times 10^{-6}.$$

# Solution for (b)

Solving the equation  $D \sin \theta = m\lambda$  for  $\sin \theta_1$  and substituting the known values gives

# **Equation:**

$$\sin heta_1 = rac{m \lambda}{D} = rac{1 ig( 550 imes 10^{-9} \; ext{m} ig)}{1.56 imes 10^{-6} \; ext{m}}.$$

Thus the angle  $\theta_1$  is

# **Equation:**

$$\theta_1 = \sin^{-1} 0.354 = 20.7^{\circ}.$$

#### **Discussion**

We see that the slit is narrow (it is only a few times greater than the wavelength of light). This is consistent with the fact that light must interact with an object comparable in size to its wavelength in order to exhibit

significant wave effects such as this single slit diffraction pattern. We also see that the central maximum extends  $20.7^{\circ}$  on either side of the original beam, for a width of about  $41^{\circ}$ . The angle between the first and second minima is only about  $24^{\circ}$  ( $45.0^{\circ}-20.7^{\circ}$ ). Thus the second maximum is only about half as wide as the central maximum.

# **Section Summary**

- A single slit produces an interference pattern characterized by a broad central maximum with narrower and dimmer maxima to the sides.
- There is destructive interference for a single slit when  $D \sin \theta = m\lambda$ , (for  $m = 1, -1, 2, -2, 3, \ldots$ ), where D is the slit width,  $\lambda$  is the light's wavelength,  $\theta$  is the angle relative to the original direction of the light, and m is the order of the minimum. Note that there is no m = 0 minimum.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

As the width of the slit producing a single-slit diffraction pattern is reduced, how will the diffraction pattern produced change?

## **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

(a) At what angle is the first minimum for 550-nm light falling on a single slit of width  $1.00 \mu m$ ? (b) Will there be a second minimum?

#### **Solution:**

- (a)  $33.4^{\circ}$
- (b) No

#### **Exercise:**

## **Problem:**

(a) Calculate the angle at which a 2.00- $\mu m$ -wide slit produces its first minimum for 410-nm violet light. (b) Where is the first minimum for 700-nm red light?

#### **Exercise:**

## **Problem:**

(a) How wide is a single slit that produces its first minimum for 633-nm light at an angle of 28.0°? (b) At what angle will the second minimum be?

#### **Solution:**

- (a)  $1.35 \times 10^{-6}$  m
- (b)  $69.9^{\circ}$

#### **Exercise:**

#### **Problem:**

(a) What is the width of a single slit that produces its first minimum at  $60.0^{\circ}$  for 600-nm light? (b) Find the wavelength of light that has its first minimum at  $62.0^{\circ}$ .

## **Exercise:**

#### **Problem:**

Find the wavelength of light that has its third minimum at an angle of  $48.6^{\circ}$  when it falls on a single slit of width  $3.00 \ \mu m$ .

#### **Solution:**

750 nm

#### **Exercise:**

#### Problem:

Calculate the wavelength of light that produces its first minimum at an angle of  $36.9^{\circ}$  when falling on a single slit of width  $1.00 \, \mu m$ .

#### **Exercise:**

#### Problem:

(a) Sodium vapor light averaging 589 nm in wavelength falls on a single slit of width  $7.50~\mu m$ . At what angle does it produces its second minimum? (b) What is the highest-order minimum produced?

## **Solution:**

- (a)  $9.04^{\circ}$
- (b) 12

## **Exercise:**

#### **Problem:**

(a) Find the angle of the third diffraction minimum for 633-nm light falling on a slit of width 20.0  $\mu m$ . (b) What slit width would place this minimum at 85.0°? Explicitly show how you follow the steps in Problem-Solving Strategies for Wave Optics

#### **Exercise:**

#### **Problem:**

(a) Find the angle between the first minima for the two sodium vapor lines, which have wavelengths of 589.1 and 589.6 nm, when they fall upon a single slit of width  $2.00~\mu m$ . (b) What is the distance between these minima if the diffraction pattern falls on a screen 1.00 m from the slit? (c) Discuss the ease or difficulty of measuring such a distance.

#### **Solution:**

- (a)  $0.0150^{\circ}$
- (b) 0.262 mm
- (c) This distance is not easily measured by human eye, but under a microscope or magnifying glass it is quite easily measurable.

#### **Exercise:**

#### **Problem:**

(a) What is the minimum width of a single slit (in multiples of  $\lambda$ ) that will produce a first minimum for a wavelength  $\lambda$ ? (b) What is its minimum width if it produces 50 minima? (c) 1000 minima?

#### **Exercise:**

#### **Problem:**

(a) If a single slit produces a first minimum at 14.5°, at what angle is the second-order minimum? (b) What is the angle of the third-order minimum? (c) Is there a fourth-order minimum? (d) Use your answers to illustrate how the angular width of the central maximum is about twice the angular width of the next maximum (which is the angle between the first and second minima).

#### **Solution:**

- (a)  $30.1^{\circ}$
- (b)  $48.7^{\circ}$
- (c) No

(d) 
$$2\theta_1=(2)(14.5^\circ)=29^\circ,\, \theta_2-\theta_1=30.05^\circ-14.5^\circ{=}15.56^\circ.$$
 Thus,  $29^\circ\approx(2)(15.56^\circ)=31.1^\circ.$ 

A double slit produces a diffraction pattern that is a combination of single and double slit interference. Find the ratio of the width of the slits to the separation between them, if the first minimum of the single slit pattern falls on the fifth maximum of the double slit pattern. (This will greatly reduce the intensity of the fifth maximum.)

#### **Exercise:**

# **Problem:** Integrated Concepts

A water break at the entrance to a harbor consists of a rock barrier with a 50.0-m-wide opening. Ocean waves of 20.0-m wavelength approach the opening straight on. At what angle to the incident direction are the boats inside the harbor most protected against wave action?

#### **Solution:**

 $23.6^{\circ}$  and  $53.1^{\circ}$ 

#### **Exercise:**

# **Problem:** Integrated Concepts

An aircraft maintenance technician walks past a tall hangar door that acts like a single slit for sound entering the hangar. Outside the door, on a line perpendicular to the opening in the door, a jet engine makes a 600-Hz sound. At what angle with the door will the technician observe the first minimum in sound intensity if the vertical opening is 0.800 m wide and the speed of sound is 340 m/s?

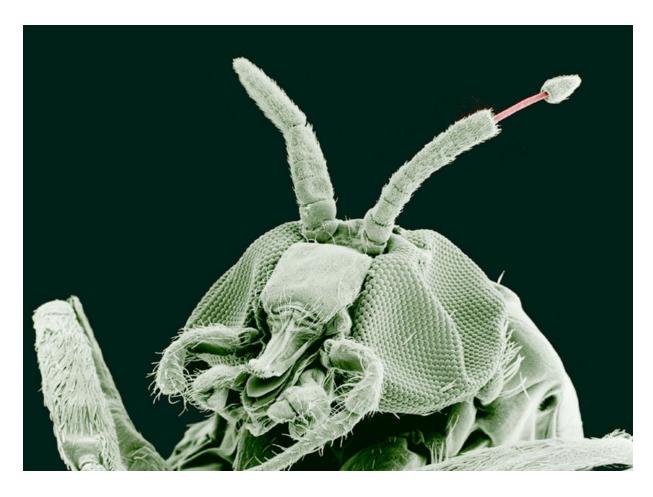
# **Glossary**

destructive interference for a single slit occurs when  $D \sin \theta = m\lambda$ , (for  $m=1,-1,2,-2,3,\ldots$ ), where D is the slit width,  $\lambda$  is the light's wavelength,  $\theta$  is the angle relative to

the original direction of the light, and  $\boldsymbol{m}$  is the order of the minimum

# Introduction to Quantum Physics class="introduction"

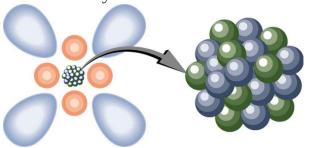
```
A black fly
imaged by
an electron
microscope
   is as
monstrous
  as any
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Quantum mechanics is the branch of physics needed to deal with submicroscopic objects. Because these objects are smaller than we can observe directly with our senses and generally must be observed with the aid of instruments, parts of quantum mechanics seem as foreign and bizarre as parts of relativity. But, like relativity, quantum mechanics has been shown to be valid—truth is often stranger than fiction.

Certain aspects of quantum mechanics are familiar to us. We accept as fact that matter is composed of atoms, the smallest unit of an element, and that these atoms combine to form molecules, the smallest unit of a compound. (See [link].) While we cannot see the individual water molecules in a stream, for example, we are aware that this is because molecules are so small and so numerous in that stream. When introducing atoms, we commonly say that electrons orbit atoms in discrete shells around a tiny nucleus, itself composed of smaller particles called protons and neutrons. We are also aware that electric charge comes in tiny units carried almost entirely by electrons and protons. As with water molecules in a stream, we

do not notice individual charges in the current through a lightbulb, because the charges are so small and so numerous in the macroscopic situations we sense directly.



Atoms and their substructure are familiar examples of objects that require quantum mechanics to be fully explained. Certain of their characteristics, such as the discrete electron shells, are classical physics explanations.

In quantum mechanics we conceptualize discrete "electron clouds" around the nucleus.

## Note:

Making Connections: Realms of Physics

Classical physics is a good approximation of modern physics under conditions first discussed in the <u>The Nature of Science and Physics</u>. Quantum mechanics is valid in general, and it must be used rather than classical physics to describe small objects, such as atoms.

Atoms, molecules, and fundamental electron and proton charges are all examples of physical entities that are **quantized**—that is, they appear only in certain discrete values and do not have every conceivable value.

Quantized is the opposite of continuous. We cannot have a fraction of an atom, or part of an electron's charge, or 14-1/3 cents, for example. Rather, everything is built of integral multiples of these substructures. Quantum physics is the branch of physics that deals with small objects and the quantization of various entities, including energy and angular momentum. Just as with classical physics, quantum physics has several subfields, such as mechanics and the study of electromagnetic forces. The **correspondence principle** states that in the classical limit (large, slow-moving objects), **quantum mechanics** becomes the same as classical physics. In this chapter, we begin the development of quantum mechanics and its description of the strange submicroscopic world. In later chapters, we will examine many areas, such as atomic and nuclear physics, in which quantum mechanics is crucial.

# **Glossary**

## quantized

the fact that certain physical entities exist only with particular discrete values and not every conceivable value

# correspondence principle

in the classical limit (large, slow-moving objects), quantum mechanics becomes the same as classical physics

# quantum mechanics

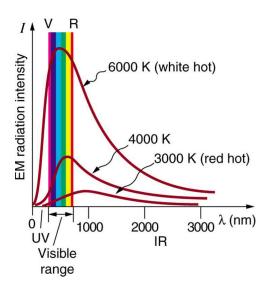
the branch of physics that deals with small objects and with the quantization of various entities, especially energy

## Quantization of Energy

- Explain Max Planck's contribution to the development of quantum mechanics.
- Explain why atomic spectra indicate quantization.

## **Planck's Contribution**

Energy is quantized in some systems, meaning that the system can have only certain energies and not a continuum of energies, unlike the classical case. This would be like having only certain speeds at which a car can travel because its kinetic energy can have only certain values. We also find that some forms of energy transfer take place with discrete lumps of energy. While most of us are familiar with the quantization of matter into lumps called atoms, molecules, and the like, we are less aware that energy, too, can be quantized. Some of the earliest clues about the necessity of quantum mechanics over classical physics came from the quantization of energy.



Graphs of blackbody radiation (from an ideal radiator) at three different radiator temperatures. The intensity or rate of

radiation emission increases dramatically with temperature, and the peak of the spectrum shifts toward the visible and ultraviolet parts of the spectrum. The shape of the spectrum cannot be described with classical physics.

Where is the quantization of energy observed? Let us begin by considering the emission and absorption of electromagnetic (EM) radiation. The EM spectrum radiated by a hot solid is linked directly to the solid's temperature. (See [link].) An ideal radiator is one that has an emissivity of 1 at all wavelengths and, thus, is jet black. Ideal radiators are therefore called **blackbodies**, and their EM radiation is called **blackbody radiation**. It was discussed that the total intensity of the radiation varies as  $T^4$ , the fourth power of the absolute temperature of the body, and that the peak of the spectrum shifts to shorter wavelengths at higher temperatures. All of this seems quite continuous, but it was the curve of the spectrum of intensity versus wavelength that gave a clue that the energies of the atoms in the solid are quantized. In fact, providing a theoretical explanation for the experimentally measured shape of the spectrum was a mystery at the turn of the century. When this "ultraviolet catastrophe" was eventually solved, the answers led to new technologies such as computers and the sophisticated imaging techniques described in earlier chapters. Once again, physics as an enabling science changed the way we live.

The German physicist Max Planck (1858–1947) used the idea that atoms and molecules in a body act like oscillators to absorb and emit radiation. The energies of the oscillating atoms and molecules had to be quantized to correctly describe the shape of the blackbody spectrum. Planck deduced that the energy of an oscillator having a frequency f is given by

# **Equation:**

$$E = \left(n + rac{1}{2}
ight) ext{hf.}$$

Here n is any nonnegative integer (0, 1, 2, 3, ...). The symbol h stands for **Planck's constant**, given by

# **Equation:**

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s.}$$

The equation  $E=\left(n+\frac{1}{2}\right)$ hf means that an oscillator having a frequency f (emitting and absorbing EM radiation of frequency f) can have its energy increase or decrease only in *discrete* steps of size

# **Equation:**

$$\Delta E = \mathrm{hf}$$
.

It might be helpful to mention some macroscopic analogies of this quantization of energy phenomena. This is like a pendulum that has a characteristic oscillation frequency but can swing with only certain amplitudes. Quantization of energy also resembles a standing wave on a string that allows only particular harmonics described by integers. It is also similar to going up and down a hill using discrete stair steps rather than being able to move up and down a continuous slope. Your potential energy takes on discrete values as you move from step to step.

Using the quantization of oscillators, Planck was able to correctly describe the experimentally known shape of the blackbody spectrum. This was the first indication that energy is sometimes quantized on a small scale and earned him the Nobel Prize in Physics in 1918. Although Planck's theory comes from observations of a macroscopic object, its analysis is based on atoms and molecules. It was such a revolutionary departure from classical physics that Planck himself was reluctant to accept his own idea that energy states are not continuous. The general acceptance of Planck's energy quantization was greatly enhanced by Einstein's explanation of the photoelectric effect (discussed in the next section), which took energy

quantization a step further. Planck was fully involved in the development of both early quantum mechanics and relativity. He quickly embraced Einstein's special relativity, published in 1905, and in 1906 Planck was the first to suggest the correct formula for relativistic momentum,  $p = \gamma mu$ .



The German physicist Max
Planck had a major influence on
the early development of
quantum mechanics, being the
first to recognize that energy is
sometimes quantized. Planck
also made important
contributions to special
relativity and classical physics.
(credit: Library of Congress,
Prints and Photographs Division
via Wikimedia Commons)

Note that Planck's constant h is a very small number. So for an infrared frequency of  $10^{14}$  Hz being emitted by a blackbody, for example, the difference between energy levels is only

 $\Delta E = \text{hf} = (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(10^{14} \text{ Hz}) = 6.63 \times 10^{-20} \text{ J}$ , or about 0.4 eV. This 0.4 eV of energy is significant compared with typical atomic

energies, which are on the order of an electron volt, or thermal energies, which are typically fractions of an electron volt. But on a macroscopic or classical scale, energies are typically on the order of joules. Even if macroscopic energies are quantized, the quantum steps are too small to be noticed. This is an example of the correspondence principle. For a large object, quantum mechanics produces results indistinguishable from those of classical physics.

# **Atomic Spectra**

Now let us turn our attention to the *emission and absorption of EM* radiation by gases. The Sun is the most common example of a body containing gases emitting an EM spectrum that includes visible light. We also see examples in neon signs and candle flames. Studies of emissions of hot gases began more than two centuries ago, and it was soon recognized that these emission spectra contained huge amounts of information. The type of gas and its temperature, for example, could be determined. We now know that these EM emissions come from electrons transitioning between energy levels in individual atoms and molecules; thus, they are called **atomic spectra**. Atomic spectra remain an important analytical tool today. [link] shows an example of an emission spectrum obtained by passing an electric discharge through a material. One of the most important characteristics of these spectra is that they are discrete. By this we mean that only certain wavelengths, and hence frequencies, are emitted. This is called a line spectrum. If frequency and energy are associated as  $\Delta E = hf$ , the energies of the electrons in the emitting atoms and molecules are quantized. This is discussed in more detail later in this chapter.



Emission spectrum of oxygen. When an electrical discharge is passed through a substance, its atoms and molecules absorb energy, which is reemitted as EM radiation. The discrete nature of these emissions implies that the energy states of the atoms

and molecules are quantized. Such atomic spectra were used as analytical tools for many decades before it was understood why they are quantized. (credit: Teravolt, Wikimedia Commons)

It was a major puzzle that atomic spectra are quantized. Some of the best minds of 19th-century science failed to explain why this might be. Not until the second decade of the 20th century did an answer based on quantum mechanics begin to emerge. Again a macroscopic or classical body of gas was involved in the studies, but the effect, as we shall see, is due to individual atoms and molecules.

# Note:

# PhET Explorations: Models of the Hydrogen Atom

How did scientists figure out the structure of atoms without looking at them? Try out different models by shooting light at the atom. Check how the prediction of the model matches the experimental results.

https://archive.cnx.org/specials/d77cc1d0-33e4-11e6-b016-6726afecd2be/hydrogen-atom/#sim-hydrogen-atom

# **Section Summary**

- The first indication that energy is sometimes quantized came from blackbody radiation, which is the emission of EM radiation by an object with an emissivity of 1.
- Planck recognized that the energy levels of the emitting atoms and molecules were quantized, with only the allowed values of  $E = \left(n + \frac{1}{2}\right) hf$ , where n is any non-negative integer (0, 1, 2, 3, ...).
- h is Planck's constant, whose value is  $h = 6.626 \times 10^{-34} \, \mathrm{J \cdot s}$ .
- Thus, the oscillatory absorption and emission energies of atoms and molecules in a blackbody could increase or decrease only in steps of

- size  $\Delta E = \mathrm{hf}$  where f is the frequency of the oscillatory nature of the absorption and emission of EM radiation.
- Another indication of energy levels being quantized in atoms and molecules comes from the lines in atomic spectra, which are the EM emissions of individual atoms and molecules.

# **Conceptual Questions**

## **Exercise:**

### **Problem:**

Give an example of a physical entity that is quantized. State specifically what the entity is and what the limits are on its values.

### **Exercise:**

# **Problem:**

Give an example of a physical entity that is not quantized, in that it is continuous and may have a continuous range of values.

### **Exercise:**

### **Problem:**

What aspect of the blackbody spectrum forced Planck to propose quantization of energy levels in its atoms and molecules?

### **Exercise:**

### Problem:

If Planck's constant were large, say  $10^{34}$  times greater than it is, we would observe macroscopic entities to be quantized. Describe the motions of a child's swing under such circumstances.

# **Exercise:**

**Problem:** Why don't we notice quantization in everyday events?

# **Problems & Exercises**

# **Exercise:**

# **Problem:**

A LiBr molecule oscillates with a frequency of  $1.7 \times 10^{13}$  Hz. (a) What is the difference in energy in eV between allowed oscillator states? (b) What is the approximate value of n for a state having an energy of 1.0 eV?

# **Solution:**

- (a) 0.070 eV
- (b) 14

### **Exercise:**

## **Problem:**

The difference in energy between allowed oscillator states in HBr molecules is 0.330 eV. What is the oscillation frequency of this molecule?

### **Exercise:**

# **Problem:**

A physicist is watching a 15-kg orangutan at a zoo swing lazily in a tire at the end of a rope. He (the physicist) notices that each oscillation takes 3.00 s and hypothesizes that the energy is quantized. (a) What is the difference in energy in joules between allowed oscillator states? (b) What is the value of n for a state where the energy is 5.00 J? (c) Can the quantization be observed?

# **Solution:**

- (a)  $2.21 \times 10^{34} \ \mathrm{J}$
- (b)  $2.26 \times 10^{34}$

# Glossary

blackbody

an ideal radiator, which can radiate equally well at all wavelengths

blackbody radiation

the electromagnetic radiation from a blackbody

Planck's constant

$$h=6.626 imes10^{-34}~\mathrm{J\cdot s}$$

atomic spectra

the electromagnetic emission from atoms and molecules

# The Photoelectric Effect

- Describe a typical photoelectric-effect experiment.
- Determine the maximum kinetic energy of photoelectrons ejected by photons of one energy or wavelength, when given the maximum kinetic energy of photoelectrons for a different photon energy or wavelength.

When light strikes materials, it can eject electrons from them. This is called the **photoelectric effect**, meaning that light (*photo*) produces electricity. One common use of the photoelectric effect is in light meters, such as those that adjust the automatic iris on various types of cameras. In a similar way, another use is in solar cells, as you probably have in your calculator or have seen on a roof top or a roadside sign. These make use of the photoelectric effect to convert light into electricity for running different devices.



The photoelectric effect can be observed by allowing light to fall on the metal plate in this evacuated tube. Electrons ejected by the light are collected on the collector wire and

measured as a current. A retarding voltage between the collector wire and plate can then be adjusted so as to determine the energy of the ejected electrons. For example, if it is sufficiently negative, no electrons will reach the wire. (credit: P.P. Urone)

This effect has been known for more than a century and can be studied using a device such as that shown in [link]. This figure shows an evacuated tube with a metal plate and a collector wire that are connected by a variable voltage source, with the collector more negative than the plate. When light (or other EM radiation) strikes the plate in the evacuated tube, it may eject electrons. If the electrons have energy in electron volts (eV) greater than the potential difference between the plate and the wire in volts, some electrons will be collected on the wire. Since the electron energy in eV is qV, where q is the electron charge and V is the potential difference, the electron energy can be measured by adjusting the retarding voltage between the wire and the plate. The voltage that stops the electrons from reaching the wire equals the energy in eV. For example, if -3.00 V barely stops the electrons,

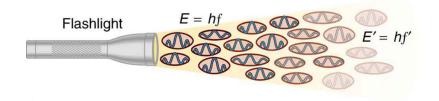
their energy is 3.00 eV. The number of electrons ejected can be determined by measuring the current between the wire and plate. The more light, the more electrons; a little circuitry allows this device to be used as a light meter.

What is really important about the photoelectric effect is what Albert Einstein deduced from it. Einstein realized that there were several characteristics of the photoelectric effect that could be explained only if *EM radiation is itself quantized*: the apparently continuous stream of energy in an EM wave is actually composed of energy quanta called photons. In his explanation of the photoelectric effect, Einstein defined a quantized unit or quantum of EM energy, which we now call a **photon**, with an energy proportional to the frequency of EM radiation. In equation form, the **photon energy** is

# **Equation:**

$$E = hf$$
,

where E is the energy of a photon of frequency f and h is Planck's constant. This revolutionary idea looks similar to Planck's quantization of energy states in blackbody oscillators, but it is quite different. It is the quantization of EM radiation itself. EM waves are composed of photons and are not continuous smooth waves as described in previous chapters on optics. Their energy is absorbed and emitted in lumps, not continuously. This is exactly consistent with Planck's quantization of energy levels in blackbody oscillators, since these oscillators increase and decrease their energy in steps of hf by absorbing and emitting photons having  $E=\mathrm{hf}$ . We do not observe this with our eyes, because there are so many photons in common light sources that individual photons go unnoticed. (See [link].) The next section of the text (Photon Energies and the Electromagnetic Spectrum) is devoted to a discussion of photons and some of their characteristics and implications. For now, we will use the photon concept to explain the photoelectric effect, much as Einstein did.



An EM wave of frequency f is composed of photons, or individual quanta of EM radiation. The energy of each photon is  $E=\mathrm{hf}$ , where h is Planck's constant and f is the frequency of the EM radiation. Higher intensity means more photons per unit area. The flashlight emits large numbers of photons of many different frequencies, hence others have energy  $E\prime=\mathrm{hf}\prime$ , and so on.

The photoelectric effect has the properties discussed below. All these properties are consistent with the idea that individual photons of EM radiation are absorbed by individual electrons in a material, with the electron gaining the photon's energy. Some of these properties are inconsistent with the idea that EM radiation is a simple wave. For simplicity, let us consider what happens with monochromatic EM radiation in which all photons have the same energy hf.

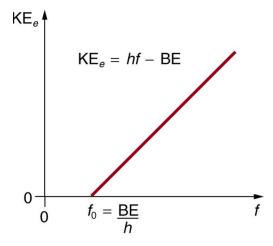
- 1. If we vary the frequency of the EM radiation falling on a material, we find the following: For a given material, there is a threshold frequency  $f_0$  for the EM radiation below which no electrons are ejected, regardless of intensity. Individual photons interact with individual electrons. Thus if the photon energy is too small to break an electron away, no electrons will be ejected. If EM radiation was a simple wave, sufficient energy could be obtained by increasing the intensity.
- 2. Once EM radiation falls on a material, electrons are ejected without delay. As soon as an individual photon of a sufficiently high frequency is absorbed by an individual electron, the electron is ejected. If the EM

- radiation were a simple wave, several minutes would be required for sufficient energy to be deposited to the metal surface to eject an electron.
- 3. The number of electrons ejected per unit time is proportional to the intensity of the EM radiation and to no other characteristic. High-intensity EM radiation consists of large numbers of photons per unit area, with all photons having the same characteristic energy hf.
- 4. If we vary the intensity of the EM radiation and measure the energy of ejected electrons, we find the following: *The maximum kinetic energy of ejected electrons is independent of the intensity of the EM radiation.* Since there are so many electrons in a material, it is extremely unlikely that two photons will interact with the same electron at the same time, thereby increasing the energy given it. Instead (as noted in 3 above), increased intensity results in more electrons of the same energy being ejected. If EM radiation were a simple wave, a higher intensity could give more energy, and higher-energy electrons would be ejected.
- 5. The kinetic energy of an ejected electron equals the photon energy minus the binding energy of the electron in the specific material. An individual photon can give all of its energy to an electron. The photon's energy is partly used to break the electron away from the material. The remainder goes into the ejected electron's kinetic energy. In equation form, this is given by

# **Equation:**

$$KE_{e} = hf - BE$$

where  $KE_e$  is the maximum kinetic energy of the ejected electron, hf is the photon's energy, and BE is the **binding energy** of the electron to the particular material. (BE is sometimes called the *work function* of the material.) This equation, due to Einstein in 1905, explains the properties of the photoelectric effect quantitatively. An individual photon of EM radiation (it does not come any other way) interacts with an individual electron, supplying enough energy, BE, to break it away, with the remainder going to kinetic energy. The binding energy is  $BE = hf_0$ , where  $f_0$  is the threshold frequency for the particular material. [link] shows a graph of maximum  $KE_e$  versus the frequency of incident EM radiation falling on a particular material.



Photoelectric effect. A graph of the kinetic energy of an ejected electron,  $KE_e$ , versus the frequency of EM radiation impinging on a certain material. There is a threshold frequency below which no electrons are ejected, because the individual photon interacting with an individual electron has insufficient energy to break it away. Above the threshold energy,  $KE_e$ increases linearly with f, consistent with  $KE_e = hf - BE$ . The slope of this line is h the data can be used to determine Planck's constant experimentally. Einstein gave the first successful explanation of such data by proposing

the idea of photons—quanta of EM radiation.

Einstein's idea that EM radiation is quantized was crucial to the beginnings of quantum mechanics. It is a far more general concept than its explanation of the photoelectric effect might imply. All EM radiation can also be modeled in the form of photons, and the characteristics of EM radiation are entirely consistent with this fact. (As we will see in the next section, many aspects of EM radiation, such as the hazards of ultraviolet (UV) radiation, can be explained *only* by photon properties.) More famous for modern relativity, Einstein planted an important seed for quantum mechanics in 1905, the same year he published his first paper on special relativity. His explanation of the photoelectric effect was the basis for the Nobel Prize awarded to him in 1921. Although his other contributions to theoretical physics were also noted in that award, special and general relativity were not fully recognized in spite of having been partially verified by experiment by 1921. Although hero-worshipped, this great man never received Nobel recognition for his most famous work—relativity.

# **Example:**

# Calculating Photon Energy and the Photoelectric Effect: A Violet Light

(a) What is the energy in joules and electron volts of a photon of 420-nm violet light? (b) What is the maximum kinetic energy of electrons ejected from calcium by 420-nm violet light, given that the binding energy (or work function) of electrons for calcium metal is 2.71 eV?

# Strategy

To solve part (a), note that the energy of a photon is given by  $E=\mathrm{hf}$ . For part (b), once the energy of the photon is calculated, it is a straightforward application of  $\mathrm{KE}_e=\mathrm{hf}\mathrm{-BE}$  to find the ejected electron's maximum kinetic energy, since BE is given.

# Solution for (a)

Photon energy is given by

# **Equation:**

$$E = hf$$

Since we are given the wavelength rather than the frequency, we solve the familiar relationship  $c=f\lambda$  for the frequency, yielding

# **Equation:**

$$f = \frac{c}{\lambda}$$
.

Combining these two equations gives the useful relationship

# **Equation:**

$$E = rac{\mathrm{hc}}{\lambda}$$
.

Now substituting known values yields

# **Equation:**

$$E = rac{\left(6.63 imes 10^{-34} ext{ J} \cdot ext{s}
ight) \left(3.00 imes 10^8 ext{ m/s}
ight)}{420 imes 10^{-9} ext{ m}} = 4.74 imes 10^{-19} ext{ J}.$$

Converting to eV, the energy of the photon is

# **Equation:**

$$E = \left(4.74 imes 10^{-19} \; \mathrm{J}
ight) rac{1 \; \mathrm{eV}}{1.6 imes 10^{-19} \; \mathrm{J}} = 2.96 \; \mathrm{eV}.$$

# Solution for (b)

Finding the kinetic energy of the ejected electron is now a simple application of the equation  $KE_e = hf-BE$ . Substituting the photon energy and binding energy yields

# **Equation:**

$$KE_e = hf-BE = 2.96 \text{ eV} - 2.71 \text{ eV} = 0.246 \text{ eV}.$$

### Discussion

The energy of this 420-nm photon of violet light is a tiny fraction of a joule, and so it is no wonder that a single photon would be difficult for us to sense directly—humans are more attuned to energies on the order of joules. But looking at the energy in electron volts, we can see that this photon has enough energy to affect atoms and molecules. A DNA molecule can be broken with about 1 eV of energy, for example, and typical atomic and molecular energies are on the order of eV, so that the UV photon in this example could have biological effects. The ejected electron (called a photoelectron) has a rather low energy, and it would not travel far, except in a vacuum. The electron would be stopped by a retarding potential of but 0.26 eV. In fact, if the photon wavelength were longer and its energy less than 2.71 eV, then the formula would give a negative kinetic energy, an impossibility. This simply means that the 420-nm photons with their 2.96eV energy are not much above the frequency threshold. You can show for yourself that the threshold wavelength is 459 nm (blue light). This means that if calcium metal is used in a light meter, the meter will be insensitive to wavelengths longer than those of blue light. Such a light meter would be completely insensitive to red light, for example.

# Note:

PhET Explorations: Photoelectric Effect

See how light knocks electrons off a metal target, and recreate the experiment that spawned the field of quantum mechanics.

https://archive.cnx.org/specials/cf1152da-eae8-11e5-b874-f779884a9994/photoelectric-effect/#sim-photoelectric-effect

# **Section Summary**

- The photoelectric effect is the process in which EM radiation ejects electrons from a material.
- Einstein proposed photons to be quanta of EM radiation having energy  $E = \mathrm{hf}$ , where f is the frequency of the radiation.

- All EM radiation is composed of photons. As Einstein explained, all characteristics of the photoelectric effect are due to the interaction of individual photons with individual electrons.
- The maximum kinetic energy  $KE_e$  of ejected electrons (photoelectrons) is given by  $KE_e = hf BE$ , where hf is the photon energy and BE is the binding energy (or work function) of the electron to the particular material.

# **Conceptual Questions**

# **Exercise:**

### **Problem:**

Is visible light the only type of EM radiation that can cause the photoelectric effect?

# **Exercise:**

## **Problem:**

Which aspects of the photoelectric effect cannot be explained without photons? Which can be explained without photons? Are the latter inconsistent with the existence of photons?

### **Exercise:**

### **Problem:**

Is the photoelectric effect a direct consequence of the wave character of EM radiation or of the particle character of EM radiation? Explain briefly.

### **Exercise:**

### **Problem:**

Insulators (nonmetals) have a higher BE than metals, and it is more difficult for photons to eject electrons from insulators. Discuss how this relates to the free charges in metals that make them good conductors.

### **Exercise:**

### **Problem:**

If you pick up and shake a piece of metal that has electrons in it free to move as a current, no electrons fall out. Yet if you heat the metal, electrons can be boiled off. Explain both of these facts as they relate to the amount and distribution of energy involved with shaking the object as compared with heating it.

# **Problems & Exercises**

# **Exercise:**

# **Problem:**

What is the longest-wavelength EM radiation that can eject a photoelectron from silver, given that the binding energy is 4.73 eV? Is this in the visible range?

### **Solution:**

263 nm

### **Exercise:**

### **Problem:**

Find the longest-wavelength photon that can eject an electron from potassium, given that the binding energy is 2.24 eV. Is this visible EM radiation?

### Exercise:

### **Problem:**

What is the binding energy in eV of electrons in magnesium, if the longest-wavelength photon that can eject electrons is 337 nm?

### **Solution:**

3.69 eV

# **Exercise:**

# **Problem:**

Calculate the binding energy in eV of electrons in aluminum, if the longest-wavelength photon that can eject them is 304 nm.

### **Exercise:**

# **Problem:**

What is the maximum kinetic energy in eV of electrons ejected from sodium metal by 450-nm EM radiation, given that the binding energy is 2.28 eV?

# **Solution:**

0.483 eV

# **Exercise:**

### **Problem:**

UV radiation having a wavelength of 120 nm falls on gold metal, to which electrons are bound by 4.82 eV. What is the maximum kinetic energy of the ejected photoelectrons?

# **Exercise:**

# **Problem:**

Violet light of wavelength 400 nm ejects electrons with a maximum kinetic energy of 0.860 eV from sodium metal. What is the binding energy of electrons to sodium metal?

### **Solution:**

 $2.25 \, \mathrm{eV}$ 

### **Exercise:**

# **Problem:**

UV radiation having a 300-nm wavelength falls on uranium metal, ejecting 0.500-eV electrons. What is the binding energy of electrons to uranium metal?

# **Exercise:**

# **Problem:**

What is the wavelength of EM radiation that ejects 2.00-eV electrons from calcium metal, given that the binding energy is 2.71 eV? What type of EM radiation is this?

# **Solution:**

- (a) 264 nm
- (b) Ultraviolet

## **Exercise:**

### **Problem:**

Find the wavelength of photons that eject 0.100-eV electrons from potassium, given that the binding energy is 2.24 eV. Are these photons visible?

### **Exercise:**

### **Problem:**

What is the maximum velocity of electrons ejected from a material by 80-nm photons, if they are bound to the material by 4.73 eV?

### **Solution:**

$$1.95 \times 10^6 \mathrm{\ m/s}$$

# **Exercise:**

# **Problem:**

Photoelectrons from a material with a binding energy of 2.71 eV are ejected by 420-nm photons. Once ejected, how long does it take these electrons to travel 2.50 cm to a detection device?

# **Exercise:**

# **Problem:**

A laser with a power output of 2.00 mW at a wavelength of 400 nm is projected onto calcium metal. (a) How many electrons per second are ejected? (b) What power is carried away by the electrons, given that the binding energy is 2.71 eV?

# **Solution:**

- (a)  $4.02 \times 10^{15} / s$
- (b) 0.256 mW

### **Exercise:**

### **Problem:**

(a) Calculate the number of photoelectrons per second ejected from a  $1.00\text{-mm}^2$  area of sodium metal by 500-nm EM radiation having an intensity of  $1.30 \text{ kW/m}^2$  (the intensity of sunlight above the Earth's atmosphere). (b) Given that the binding energy is 2.28 eV, what power is carried away by the electrons? (c) The electrons carry away less power than brought in by the photons. Where does the other power go? How can it be recovered?

### Exercise:

### **Problem:** Unreasonable Results

Red light having a wavelength of 700 nm is projected onto magnesium metal to which electrons are bound by 3.68 eV. (a) Use  $KE_e = hf-BE$  to calculate the kinetic energy of the ejected electrons.

(b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

# **Solution:**

- (a) -1.90 eV
- (b) Negative kinetic energy
- (c) That the electrons would be knocked free.

# **Exercise:**

# **Problem:** Unreasonable Results

(a) What is the binding energy of electrons to a material from which 4.00-eV electrons are ejected by 400-nm EM radiation? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

# **Glossary**

# photoelectric effect

the phenomenon whereby some materials eject electrons when light is shined on them

# photon

a quantum, or particle, of electromagnetic radiation

# photon energy

the amount of energy a photon has; E = hf

# binding energy

also called the *work function*; the amount of energy necessary to eject an electron from a material

### Photon Energies and the Electromagnetic Spectrum

- Explain the relationship between the energy of a photon in joules or electron volts and its wavelength or frequency.
- Calculate the number of photons per second emitted by a monochromatic source of specific wavelength and power.

### **Ionizing Radiation**

A photon is a quantum of EM radiation. Its energy is given by  $E=\mathrm{hf}$  and is related to the frequency f and wavelength  $\lambda$  of the radiation by

### **Equation:**

$$E= ext{hf}=rac{ ext{hc}}{\lambda} ext{(energy of a photon)},$$

where E is the energy of a single photon and c is the speed of light. When working with small systems, energy in eV is often useful. Note that Planck's constant in these units is

### **Equation:**

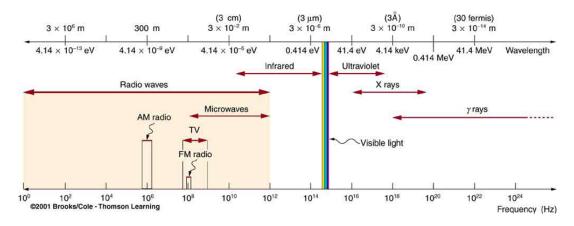
$$h = 4.14 \times 10^{-15} \; \mathrm{eV \cdot s}.$$

Since many wavelengths are stated in nanometers (nm), it is also useful to know that **Equation:** 

$$hc = 1240 eV \cdot nm$$
.

These will make many calculations a little easier.

All EM radiation is composed of photons. [link] shows various divisions of the EM spectrum plotted against wavelength, frequency, and photon energy. Previously in this book, photon characteristics were alluded to in the discussion of some of the characteristics of UV, x rays, and  $\gamma$  rays, the first of which start with frequencies just above violet in the visible spectrum. It was noted that these types of EM radiation have characteristics much different than visible light. We can now see that such properties arise because photon energy is larger at high frequencies.



The EM spectrum, showing major categories as a function of photon energy in eV, as well as wavelength and frequency. Certain characteristics of EM radiation are directly attributable to photon energy alone.

Rotational energies of molecules	$10^{-5}\mathrm{eV}$
Vibrational energies of molecules	0.1 eV
Energy between outer electron shells in atoms	1 eV
Binding energy of a weakly bound molecule	1 eV
Energy of red light	2 eV
Binding energy of a tightly bound molecule	10 eV
Energy to ionize atom or molecule	10 to 1000 eV

### Representative Energies for Submicroscopic Effects (Order of Magnitude Only)

Photons act as individual quanta and interact with individual electrons, atoms, molecules, and so on. The energy a photon carries is, thus, crucial to the effects it has. [link] lists representative submicroscopic energies in eV. When we compare photon energies from the EM spectrum in [link] with energies in the table, we can see how effects vary with the type of EM radiation.

**Gamma rays**, a form of nuclear and cosmic EM radiation, can have the highest frequencies and, hence, the highest photon energies in the EM spectrum. For example, a  $\gamma$ -ray photon with  $f=10^{21}$  Hz has an energy  $E=\mathrm{hf}=6.63\times10^{-13}~\mathrm{J}=4.14~\mathrm{MeV}$ . This is sufficient energy to ionize thousands of atoms and molecules, since only 10 to 1000 eV are needed per ionization. In fact,  $\gamma$  rays are one type of **ionizing radiation**, as are x rays and UV, because they produce ionization in materials that absorb

them. Because so much ionization can be produced, a single  $\gamma$ -ray photon can cause significant damage to biological tissue, killing cells or damaging their ability to properly reproduce. When cell reproduction is disrupted, the result can be cancer, one of the known effects of exposure to ionizing radiation. Since cancer cells are rapidly reproducing, they are exceptionally sensitive to the disruption produced by ionizing radiation. This means that ionizing radiation has positive uses in cancer treatment as well as risks in producing cancer.



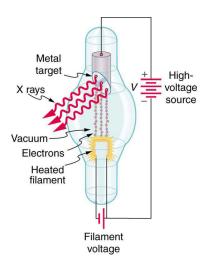
One of the first x-ray images, taken by Röentgen himself. The hand belongs to Bertha Röentgen, his wife. (credit: Wilhelm Conrad Röntgen, via Wikimedia Commons)

High photon energy also enables  $\gamma$  rays to penetrate materials, since a collision with a single atom or molecule is unlikely to absorb all the  $\gamma$  ray's energy. This can make  $\gamma$  rays useful as a probe, and they are sometimes used in medical imaging.  $\bf x$  rays, as you can see in [link], overlap with the low-frequency end of the  $\gamma$  ray range. Since  $\bf x$  rays have energies of keV and up, individual  $\bf x$ -ray photons also can produce large amounts of ionization. At lower photon energies,  $\bf x$  rays are not as penetrating as  $\bf \gamma$  rays and are slightly less hazardous.  $\bf X$  rays are ideal for medical imaging, their most common use, and a fact that was recognized immediately upon their discovery in 1895 by the German physicist  $\bf W$ . C. Roentgen (1845–1923). (See [link].) Within one year of their discovery,  $\bf x$  rays (for a time called Roentgen rays) were used for medical diagnostics. Roentgen received the 1901 Nobel Prize for the discovery of  $\bf x$  rays.

### Note:

### Connections: Conservation of Energy

Once again, we find that conservation of energy allows us to consider the initial and final forms that energy takes, without having to make detailed calculations of the intermediate steps. [link] is solved by considering only the initial and final forms of energy.



X rays are produced when energetic electrons strike the copper anode of this cathode ray tube (CRT). Electrons (shown here as separate particles) interact individually with the material they strike, sometimes producing photons of EM radiation.

While  $\gamma$  rays originate in nuclear decay, x rays are produced by the process shown in [link]. Electrons ejected by thermal agitation from a hot filament in a vacuum tube are accelerated through a high voltage, gaining kinetic energy from the electrical potential energy. When they strike the anode, the electrons convert their kinetic energy to a variety of forms, including thermal energy. But since an accelerated charge radiates EM waves, and since the electrons act individually, photons are also produced. Some of these x-ray photons obtain the kinetic energy of the electron. The accelerated electrons originate at the cathode, so such a tube is called a cathode ray tube (CRT), and various versions of them are found in older TV and computer screens as well as in x-ray machines.

### **Example:**

### X-ray Photon Energy and X-ray Tube Voltage

Find the maximum energy in eV of an x-ray photon produced by electrons accelerated through a potential difference of 50.0 kV in a CRT like the one in [link]. **Strategy** 

Electrons can give all of their kinetic energy to a single photon when they strike the anode of a CRT. (This is something like the photoelectric effect in reverse.) The kinetic energy of the electron comes from electrical potential energy. Thus we can simply equate the maximum photon energy to the electrical potential energy—that is, hf = qV. (We do not have to calculate each step from beginning to end if we know that all of the starting energy qV is converted to the final form hf.)

#### Solution

The maximum photon energy is hf = qV, where q is the charge of the electron and V is the accelerating voltage. Thus,

### **Equation:**

$$\mathrm{hf} = (1.60 \times 10^{-19} \; \mathrm{C})(50.0 \times 10^{3} \; \mathrm{V}).$$

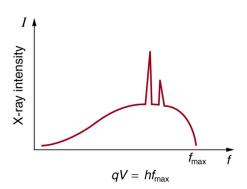
From the definition of the electron volt, we know  $1~eV=1.60\times 10^{-19}~J$ , where  $1~J=1~C\cdot V$ . Gathering factors and converting energy to eV yields

### **Equation:**

$$\mathrm{hf} = (50.0 imes 10^3)(1.60 imes 10^{-19} \ \mathrm{C \cdot V}) igg( rac{1 \ \mathrm{eV}}{1.60 imes 10^{-19} \ \mathrm{C \cdot V}} igg) = (50.0 imes 10^3)(1 \ \mathrm{eV}) = 50.0 \ \mathrm{keV}.$$

#### Discussion

This example produces a result that can be applied to many similar situations. If you accelerate a single elementary charge, like that of an electron, through a potential given in volts, then its energy in eV has the same numerical value. Thus a 50.0-kV potential generates 50.0 keV electrons, which in turn can produce photons with a maximum energy of 50 keV. Similarly, a 100-kV potential in an x-ray tube can generate up to 100-keV x-ray photons. Many x-ray tubes have adjustable voltages so that various energy x rays with differing energies, and therefore differing abilities to penetrate, can be generated.



X-ray spectrum obtained when energetic electrons strike a material. The smooth part of the spectrum is bremsstrahlung, while the peaks are characteristic of the anode material. Both are atomic processes that produce energetic

photons known as x-ray photons.

[link] shows the spectrum of x rays obtained from an x-ray tube. There are two distinct features to the spectrum. First, the smooth distribution results from electrons being decelerated in the anode material. A curve like this is obtained by detecting many photons, and it is apparent that the maximum energy is unlikely. This decelerating process produces radiation that is called **bremsstrahlung** (German for *braking radiation*). The second feature is the existence of sharp peaks in the spectrum; these are called **characteristic x rays**, since they are characteristic of the anode material. Characteristic x rays come from atomic excitations unique to a given type of anode material. They are akin to lines in atomic spectra, implying the energy levels of atoms are quantized. Phenomena such as discrete atomic spectra and characteristic x rays are explored further in <u>Atomic Physics</u>.

**Ultraviolet radiation** (approximately 4 eV to 300 eV) overlaps with the low end of the energy range of x rays, but UV is typically lower in energy. UV comes from the de-excitation of atoms that may be part of a hot solid or gas. These atoms can be given energy that they later release as UV by numerous processes, including electric discharge, nuclear explosion, thermal agitation, and exposure to x rays. A UV photon has sufficient energy to ionize atoms and molecules, which makes its effects different from those of visible light. UV thus has some of the same biological effects as  $\gamma$  rays and x rays. For example, it can cause skin cancer and is used as a sterilizer. The major difference is that several UV photons are required to disrupt cell reproduction or kill a bacterium, whereas single  $\gamma$ -ray and X-ray photons can do the same damage. But since UV does have the energy to alter molecules, it can do what visible light cannot. One of the beneficial aspects of UV is that it triggers the production of vitamin D in the skin, whereas visible light has insufficient energy per photon to alter the molecules that trigger this production. Infantile jaundice is treated by exposing the baby to UV (with eye protection), called phototherapy, the beneficial effects of which are thought to be related to its ability to help prevent the buildup of potentially toxic bilirubin in the blood.

### **Example:**

### Photon Energy and Effects for UV

Short-wavelength UV is sometimes called vacuum UV, because it is strongly absorbed by air and must be studied in a vacuum. Calculate the photon energy in eV for 100-nm vacuum UV, and estimate the number of molecules it could ionize or break apart.

### Strategy

Using the equation E = hf and appropriate constants, we can find the photon energy and compare it with energy information in [link].

### Solution

The energy of a photon is given by

### **Equation:**

$$E = \mathrm{hf} = \frac{\mathrm{hc}}{\lambda}.$$

Using  $hc = 1240 \text{ eV} \cdot \text{nm}$ , we find that

### **Equation:**

$$E = \frac{\text{hc}}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{100 \text{ nm}} = 12.4 \text{ eV}.$$

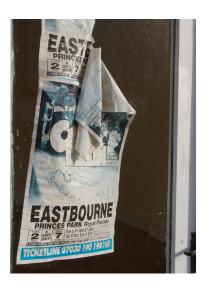
#### Discussion

According to [link], this photon energy might be able to ionize an atom or molecule, and it is about what is needed to break up a tightly bound molecule, since they are bound by approximately 10 eV. This photon energy could destroy about a dozen weakly bound molecules. Because of its high photon energy, UV disrupts atoms and molecules it interacts with. One good consequence is that all but the longest-wavelength UV is strongly absorbed and is easily blocked by sunglasses. In fact, most of the Sun's UV is absorbed by a thin layer of ozone in the upper atmosphere, protecting sensitive organisms on Earth. Damage to our ozone layer by the addition of such chemicals as CFC's has reduced this protection for us.

### Visible Light

The range of photon energies for **visible light** from red to violet is 1.63 to 3.26 eV, respectively (left for this chapter's Problems and Exercises to verify). These energies are on the order of those between outer electron shells in atoms and molecules. This means that these photons can be absorbed by atoms and molecules. A *single* photon can actually stimulate the retina, for example, by altering a receptor molecule that then triggers a nerve impulse. Photons can be absorbed or emitted only by atoms and molecules that have precisely the correct quantized energy step to do so. For example, if a red photon of frequency f encounters a molecule that has an energy step,  $\Delta E$ , equal to f, then the photon can be absorbed. Violet flowers absorb red and reflect violet; this implies there is no energy step between levels in the receptor molecule equal to the violet photon's energy, but there is an energy step for the red.

There are some noticeable differences in the characteristics of light between the two ends of the visible spectrum that are due to photon energies. Red light has insufficient photon energy to expose most black-and-white film, and it is thus used to illuminate darkrooms where such film is developed. Since violet light has a higher photon energy, dyes that absorb violet tend to fade more quickly than those that do not. (See [link].) Take a look at some faded color posters in a storefront some time, and you will notice that the blues and violets are the last to fade. This is because other dyes, such as red and green dyes, absorb blue and violet photons, the higher energies of which break up their weakly bound molecules. (Complex molecules such as those in dyes and DNA tend to be weakly bound.) Blue and violet dyes reflect those colors and, therefore, do not absorb these more energetic photons, thus suffering less molecular damage.



Why do the reds, yellows, and greens fade before the blues and violets when exposed to the Sun, as with this poster? The answer is related to photon energy. (credit: Deb Collins, Flickr)

Transparent materials, such as some glasses, do not absorb any visible light, because there is no energy step in the atoms or molecules that could absorb the light. Since individual photons interact with individual atoms, it is nearly impossible to have two photons absorbed simultaneously to reach a large energy step. Because of its lower photon energy, visible light can sometimes pass through many kilometers of a substance, while higher frequencies like UV, x ray, and  $\gamma$  rays are absorbed, because they have sufficient photon energy to ionize the material.

### **Example:**

### How Many Photons per Second Does a Typical Light Bulb Produce?

Assuming that 10.0% of a 100-W light bulb's energy output is in the visible range (typical for incandescent bulbs) with an average wavelength of 580 nm, calculate the number of visible photons emitted per second.

#### Strategy

Power is energy per unit time, and so if we can find the energy per photon, we can determine the number of photons per second. This will best be done in joules, since power is given in watts, which are joules per second.

### Solution

The power in visible light production is 10.0% of 100 W, or 10.0 J/s. The energy of the average visible photon is found by substituting the given average wavelength into the formula

### **Equation:**

$$E = rac{ ext{hc}}{\lambda}.$$

This produces

### **Equation:**

$$E = rac{(6.63 imes 10^{-34} ext{ J} \cdot ext{s})(3.00 imes 10^8 ext{ m/s})}{580 imes 10^{-9} ext{ m}} = 3.43 imes 10^{-19} ext{ J}.$$

The number of visible photons per second is thus

### **Equation:**

$${
m photon/s} = rac{10.0 \ {
m J/s}}{3.43 imes 10^{-19} \ {
m J/photon}} = 2.92 imes 10^{19} \ {
m photon/s}.$$

#### Discussion

This incredible number of photons per second is verification that individual photons are insignificant in ordinary human experience. It is also a verification of the correspondence principle—on the macroscopic scale, quantization becomes essentially continuous or classical. Finally, there are so many photons emitted by a 100-W lightbulb that it can be seen by the unaided eye many kilometers away.

# **Lower-Energy Photons**

**Infrared radiation (IR)** has even lower photon energies than visible light and cannot significantly alter atoms and molecules. IR can be absorbed and emitted by atoms and molecules, particularly between closely spaced states. IR is extremely strongly absorbed by water, for example, because water molecules have many states separated by energies on the order of  $10^{-5}$  eV to  $10^{-2}$  eV, well within the IR and microwave energy ranges. This is why in the IR range, skin is almost jet black, with an emissivity near 1—there are many states in water molecules in the skin that can absorb a large range of IR photon energies. Not all molecules have this property. Air, for example, is nearly transparent to many IR frequencies.

**Microwaves** are the highest frequencies that can be produced by electronic circuits, although they are also produced naturally. Thus microwaves are similar to IR but do not extend to as high frequencies. There are states in water and other molecules that have the same frequency and energy as microwaves, typically about  $10^{-5}$  eV. This is one reason why food absorbs microwaves more strongly than many other materials, making microwave ovens an efficient way of putting energy directly into food.

Photon energies for both IR and microwaves are so low that huge numbers of photons are involved in any significant energy transfer by IR or microwaves (such as warming yourself with a heat lamp or cooking pizza in the microwave). Visible light, IR, microwaves, and all lower frequencies cannot produce ionization with single photons and do not ordinarily have the hazards of higher frequencies. When visible, IR, or microwave radiation *is* hazardous, such as the inducement of cataracts by microwaves, the hazard is due to huge numbers of photons acting together (not to an accumulation of photons, such as sterilization by weak UV). The negative effects of visible, IR, or microwave radiation can be thermal effects, which could be produced by any heat source. But one difference is that at very high intensity, strong electric and magnetic fields can be produced by photons acting together. Such electromagnetic fields (EMF) can actually ionize materials.

#### Note:

### Misconception Alert: High-Voltage Power Lines

Although some people think that living near high-voltage power lines is hazardous to one's health, ongoing studies of the transient field effects produced by these lines show their strengths to be insufficient to cause damage. Demographic studies also fail to show significant correlation of ill effects with high-voltage power lines. The American Physical Society issued a report over 10 years ago on power-line fields, which concluded that the scientific literature and reviews of panels show no consistent, significant link between cancer and power-line fields. They also felt that the "diversion of resources to eliminate a threat which has no persuasive scientific basis is disturbing."

It is virtually impossible to detect individual photons having frequencies below microwave frequencies, because of their low photon energy. But the photons are there. A continuous EM wave can be modeled as photons. At low frequencies, EM waves are generally treated as time- and position-varying electric and magnetic fields with no discernible quantization. This is another example of the correspondence principle in situations involving huge numbers of photons.

### Note:

### PhET Explorations: Color Vision

Make a whole rainbow by mixing red, green, and blue light. Change the wavelength of a monochromatic beam or filter white light. View the light as a solid beam, or see the individual photons.

https://phet.colorado.edu/sims/html/color-vision/latest/color-vision\_en.html

### **Section Summary**

- Photon energy is responsible for many characteristics of EM radiation, being particularly noticeable at high frequencies.
- Photons have both wave and particle characteristics.

### **Conceptual Questions**

### **Exercise:**

**Problem:** Why are UV, x rays, and  $\gamma$  rays called ionizing radiation?

**Exercise:** 

### **Problem:**

How can treating food with ionizing radiation help keep it from spoiling? UV is not very penetrating. What else could be used?

### **Exercise:**

#### **Problem:**

Some television tubes are CRTs. They use an approximately 30-kV accelerating potential to send electrons to the screen, where the electrons stimulate phosphors to emit the light that forms the pictures we watch. Would you expect x rays also to be created?

#### **Exercise:**

### **Problem:**

Tanning salons use "safe" UV with a longer wavelength than some of the UV in sunlight. This "safe" UV has enough photon energy to trigger the tanning mechanism. Is it likely to be able to cause cell damage and induce cancer with prolonged exposure?

### **Exercise:**

### **Problem:**

Your pupils dilate when visible light intensity is reduced. Does wearing sunglasses that lack UV blockers increase or decrease the UV hazard to your eyes? Explain.

#### **Exercise:**

#### **Problem:**

One could feel heat transfer in the form of infrared radiation from a large nuclear bomb detonated in the atmosphere 75 km from you. However, none of the profusely emitted x rays or  $\gamma$  rays reaches you. Explain.

#### **Exercise:**

**Problem:** Can a single microwave photon cause cell damage? Explain.

### **Exercise:**

### **Problem:**

In an x-ray tube, the maximum photon energy is given by hf = qV. Would it be technically more correct to say hf = qV + BE, where BE is the binding energy of electrons in the target anode? Why isn't the energy stated the latter way?

### **Problems & Exercises**

#### **Exercise:**

### **Problem:**

What is the energy in joules and eV of a photon in a radio wave from an AM station that has a 1530-kHz broadcast frequency?

### **Solution:**

$$6.34 \times 10^{-9} \; \mathrm{eV}$$
,  $1.01 \times 10^{-27} \; \mathrm{J}$ 

### **Exercise:**

### **Problem:**

(a) Find the energy in joules and eV of photons in radio waves from an FM station that has a 90.0-MHz broadcast frequency. (b) What does this imply about the number of photons per second that the radio station must broadcast?

#### **Exercise:**

**Problem:** Calculate the frequency in hertz of a 1.00-MeV  $\gamma$ -ray photon.

### **Solution:**

$$2.42 imes 10^{20}~\mathrm{Hz}$$

#### **Exercise:**

#### **Problem:**

(a) What is the wavelength of a 1.00-eV photon? (b) Find its frequency in hertz. (c) Identify the type of EM radiation.

### **Exercise:**

#### **Problem:**

Do the unit conversions necessary to show that  $hc = 1240 \text{ eV} \cdot \text{nm}$ , as stated in the text.

#### **Solution:**

### **Equation:**

$$\begin{array}{ll} hc &=& \left(6.62607\times 10^{-34}\; J\cdot s\right) \left(2.99792\times 10^8\; m/s\right) \left(\frac{10^9\; nm}{1\; m}\right) \left(\frac{1.00000\; eV}{1.60218\times 10^{-19}\; J}\right) \\ &=& 1239.84\; eV\cdot nm \\ &\approx& 1240\; eV\cdot nm \end{array}$$

#### **Exercise:**

#### **Problem:**

Confirm the statement in the text that the range of photon energies for visible light is 1.63 to 3.26 eV, given that the range of visible wavelengths is 380 to 760 nm.

### **Exercise:**

### **Problem:**

(a) Calculate the energy in eV of an IR photon of frequency  $2.00 \times 10^{13}~{\rm Hz}$ . (b) How many of these photons would need to be absorbed simultaneously by a tightly bound molecule to break it apart? (c) What is the energy in eV of a  $\gamma$  ray of frequency  $3.00 \times 10^{20}~{\rm Hz}$ ? (d) How many tightly bound molecules could a single such  $\gamma$  ray break apart?

### **Solution:**

(a) 0.0829 eV

- (b) 121
- (c) 1.24 MeV
- (d)  $1.24 \times 10^5$

### Exercise:

**Problem:** Prove that, to three-digit accuracy,  $h = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$ , as stated in the text.

#### **Exercise:**

#### **Problem:**

(a) What is the maximum energy in eV of photons produced in a CRT using a 25.0-kV accelerating potential, such as a color TV? (b) What is their frequency?

### **Solution:**

- (a)  $25.0 \times 10^3 \text{ eV}$
- (b)  $6.04 \times 10^{18} \text{ Hz}$

### **Exercise:**

#### **Problem:**

What is the accelerating voltage of an x-ray tube that produces x rays with a shortest wavelength of 0.0103 nm?

#### **Exercise:**

### **Problem:**

(a) What is the ratio of power outputs by two microwave ovens having frequencies of 950 and 2560 MHz, if they emit the same number of photons per second? (b) What is the ratio of photons per second if they have the same power output?

#### **Solution:**

- (a) 2.69
- (b) 0.371

### **Exercise:**

### **Problem:**

How many photons per second are emitted by the antenna of a microwave oven, if its power output is 1.00 kW at a frequency of 2560 MHz?

### **Exercise:**

#### **Problem:**

Some satellites use nuclear power. (a) If such a satellite emits a 1.00-W flux of  $\gamma$  rays having an average energy of 0.500 MeV, how many are emitted per second? (b) These  $\gamma$  rays affect other satellites. How far away must another satellite be to only receive one  $\gamma$  ray per second per square meter?

#### Solution:

- (a)  $1.25 \times 10^{13}$  photons/s
- (b) 997 km

#### **Exercise:**

#### **Problem:**

(a) If the power output of a 650-kHz radio station is 50.0 kW, how many photons per second are produced? (b) If the radio waves are broadcast uniformly in all directions, find the number of photons per second per square meter at a distance of 100 km. Assume no reflection from the ground or absorption by the air.

#### **Exercise:**

#### **Problem:**

How many x-ray photons per second are created by an x-ray tube that produces a flux of x rays having a power of 1.00 W? Assume the average energy per photon is 75.0 keV.

#### **Solution:**

 $8.33 \times 10^{13} \text{ photons/s}$ 

#### **Exercise:**

#### **Problem:**

(a) How far away must you be from a 650-kHz radio station with power 50.0 kW for there to be only one photon per second per square meter? Assume no reflections or absorption, as if you were in deep outer space. (b) Discuss the implications for detecting intelligent life in other solar systems by detecting their radio broadcasts.

#### **Exercise:**

#### **Problem:**

Assuming that 10.0% of a 100-W light bulb's energy output is in the visible range (typical for incandescent bulbs) with an average wavelength of 580 nm, and that the photons spread out uniformly and are not absorbed by the atmosphere, how far away would you be if 500 photons per second enter the 3.00-mm diameter pupil of your eye? (This number easily stimulates the retina.)

#### **Solution:**

181 km

#### **Exercise:**

#### **Problem:**Construct Your Own Problem

Consider a laser pen. Construct a problem in which you calculate the number of photons per second emitted by the pen. Among the things to be considered are the laser pen's wavelength and power output. Your instructor may also wish for you to determine the minimum diffraction spreading in the beam and the number of photons per square centimeter the pen can project at some large distance. In this latter case, you will also need to consider the output size of the laser beam, the distance to the object being illuminated, and any absorption or scattering along the way.

### **Glossary**

gamma ray

also  $\gamma$ -ray; highest-energy photon in the EM spectrum

ionizing radiation

radiation that ionizes materials that absorb it

x ray

EM photon between  $\gamma$ -ray and UV in energy

bremsstrahlung

German for *braking radiation*; produced when electrons are decelerated

characteristic x rays

x rays whose energy depends on the material they were produced in

ultraviolet radiation

UV; ionizing photons slightly more energetic than violet light

visible light

the range of photon energies the human eye can detect

infrared radiation

photons with energies slightly less than red light

microwaves

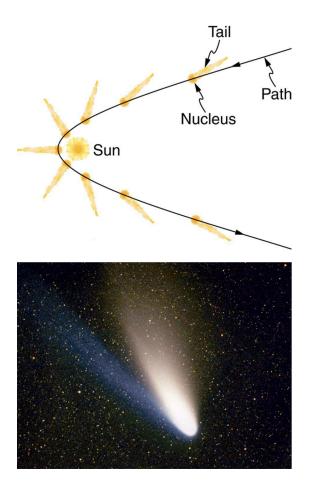
photons with wavelengths on the order of a micron (µm)

#### Photon Momentum

- Relate the linear momentum of a photon to its energy or wavelength, and apply linear momentum conservation to simple processes involving the emission, absorption, or reflection of photons.
- Account qualitatively for the increase of photon wavelength that is observed, and explain the significance of the Compton wavelength.

# **Measuring Photon Momentum**

The quantum of EM radiation we call a **photon** has properties analogous to those of particles we can see, such as grains of sand. A photon interacts as a unit in collisions or when absorbed, rather than as an extensive wave. Massive quanta, like electrons, also act like macroscopic particles—something we expect, because they are the smallest units of matter. Particles carry momentum as well as energy. Despite photons having no mass, there has long been evidence that EM radiation carries momentum. (Maxwell and others who studied EM waves predicted that they would carry momentum.) It is now a well-established fact that photons *do* have momentum. In fact, photon momentum is suggested by the photoelectric effect, where photons knock electrons out of a substance. [link] shows macroscopic evidence of photon momentum.



The tails of the Hale-Bopp comet point away from the Sun, evidence that light has momentum. Dust emanating from the body of the comet forms this tail. Particles of dust are pushed away from the Sun by light reflecting from them. The blue ionized gas tail is also produced by photons interacting with atoms in the comet material. (credit: Geoff Chester, U.S. Navy, via Wikimedia Commons)

[link] shows a comet with two prominent tails. What most people do not know about the tails is that they always point *away* from the Sun rather than trailing behind the comet (like the tail of Bo Peep's sheep). Comet tails are composed of gases and dust evaporated from the body of the comet and ionized gas. The dust particles recoil away from the Sun when photons scatter from them. Evidently, photons carry momentum in the direction of their motion (away from the Sun), and some of this momentum is transferred to dust particles in collisions. Gas atoms and molecules in the blue tail are most affected by other particles of radiation, such as protons and electrons emanating from the Sun, rather than by the momentum of photons.

#### Note:

### Connections: Conservation of Momentum

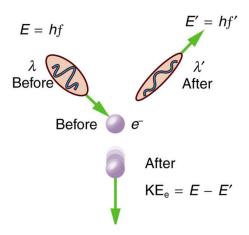
Not only is momentum conserved in all realms of physics, but all types of particles are found to have momentum. We expect particles with mass to have momentum, but now we see that massless particles including photons also carry momentum.

Momentum is conserved in quantum mechanics just as it is in relativity and classical physics. Some of the earliest direct experimental evidence of this came from scattering of x-ray photons by electrons in substances, named Compton scattering after the American physicist Arthur H. Compton (1892–1962). Around 1923, Compton observed that x rays scattered from materials had a decreased energy and correctly analyzed this as being due to the scattering of photons from electrons. This phenomenon could be handled as a collision between two particles—a photon and an electron at rest in the material. Energy and momentum are conserved in the collision. (See [link]) He won a Nobel Prize in 1929 for the discovery of this scattering, now called the **Compton effect**, because it helped prove that **photon momentum** is given by

# **Equation:**

$$p=rac{h}{\lambda},$$

where h is Planck's constant and  $\lambda$  is the photon wavelength. (Note that relativistic momentum given as  $p=\gamma mu$  is valid only for particles having mass.)



The Compton effect is the name given to the scattering of a photon by an electron. Energy and momentum are conserved, resulting in a reduction of both for the scattered photon. Studying this effect, Compton verified that photons have momentum.

We can see that photon momentum is small, since  $p=h/\lambda$  and h is very small. It is for this reason that we do not ordinarily observe photon

momentum. Our mirrors do not recoil when light reflects from them (except perhaps in cartoons). Compton saw the effects of photon momentum because he was observing x rays, which have a small wavelength and a relatively large momentum, interacting with the lightest of particles, the electron.

# **Example:**

# **Electron and Photon Momentum Compared**

(a) Calculate the momentum of a visible photon that has a wavelength of 500 nm. (b) Find the velocity of an electron having the same momentum.

(c) What is the energy of the electron, and how does it compare with the energy of the photon?

# Strategy

Finding the photon momentum is a straightforward application of its definition:  $p = \frac{h}{\lambda}$ . If we find the photon momentum is small, then we can assume that an electron with the same momentum will be nonrelativistic, making it easy to find its velocity and kinetic energy from the classical formulas.

# Solution for (a)

Photon momentum is given by the equation:

# **Equation:**

$$p = \frac{h}{\lambda}.$$

Entering the given photon wavelength yields

# **Equation:**

$$p = rac{6.63 imes 10^{-34} ext{ J} \cdot ext{s}}{500 imes 10^{-9} ext{ m}} = 1.33 imes 10^{-27} ext{ kg} \cdot ext{m/s}.$$

# Solution for (b)

Since this momentum is indeed small, we will use the classical expression p = mv to find the velocity of an electron with this momentum. Solving for v and using the known value for the mass of an electron gives

# **Equation:**

$$v = rac{p}{m} = rac{1.33 imes 10^{-27} ext{ kg} \cdot ext{m/s}}{9.11 imes 10^{-31} ext{ kg}} = 1460 ext{ m/s} pprox 1460 ext{ m/s}.$$

# Solution for (c)

The electron has kinetic energy, which is classically given by

# **Equation:**

$$ext{KE}_e = rac{1}{2} m v^2.$$

Thus,

# **Equation:**

$${
m KE}_e = rac{1}{2} (9.11 imes 10^{-3} {
m \, kg}) (1455 {
m \, m/s})^2 = 9.64 imes 10^{-25} {
m \, J}.$$

Converting this to eV by multiplying by  $(1~{\rm eV})/(1.602\times 10^{-19}~{\rm J})$  yields **Equation:** 

$$\mathrm{KE}_e = 6.02 imes 10^{-6} \ \mathrm{eV}.$$

The photon energy E is

# **Equation:**

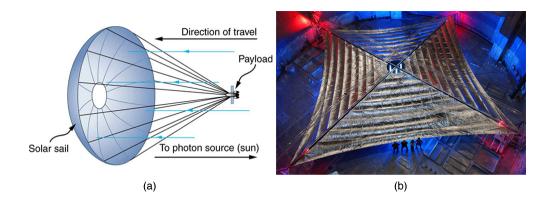
$$E = rac{{
m hc}}{\lambda} = rac{1240~{
m eV} \cdot {
m nm}}{500~{
m nm}} = 2.48~{
m eV},$$

which is about five orders of magnitude greater.

## **Discussion**

Photon momentum is indeed small. Even if we have huge numbers of them, the total momentum they carry is small. An electron with the same momentum has a 1460 m/s velocity, which is clearly nonrelativistic. A more massive particle with the same momentum would have an even smaller velocity. This is borne out by the fact that it takes far less energy to give an electron the same momentum as a photon. But on a quantum-mechanical scale, especially for high-energy photons interacting with small

masses, photon momentum is significant. Even on a large scale, photon momentum can have an effect if there are enough of them and if there is nothing to prevent the slow recoil of matter. Comet tails are one example, but there are also proposals to build space sails that use huge low-mass mirrors (made of aluminized Mylar) to reflect sunlight. In the vacuum of space, the mirrors would gradually recoil and could actually take spacecraft from place to place in the solar system. (See [link].)



(a) Space sails have been proposed that use the momentum of sunlight reflecting from gigantic low-mass sails to propel spacecraft about the solar system. A Russian test model of this (the Cosmos 1) was launched in 2005, but did not make it into orbit due to a rocket failure. (b) A U.S. version of this, labeled LightSail-1, is scheduled for trial launches in the first part of this decade. It will have a 40-m<sup>2</sup> sail. (credit: Kim Newton/NASA)

# **Relativistic Photon Momentum**

There is a relationship between photon momentum p and photon energy E that is consistent with the relation given previously for the relativistic total energy of a particle as  $E^2 = (pc)^2 + (mc)^2$ . We know m is zero for a photon, but p is not, so that  $E^2 = (pc)^2 + (mc)^2$  becomes

# **Equation:**

$$E = pc$$
,

or

# **Equation:**

$$p = \frac{E}{c}$$
 (photons).

To check the validity of this relation, note that  $E=\mathrm{hc}/\lambda$  for a photon. Substituting this into p=E/c yields

# **Equation:**

$$p=(\mathrm{hc}/\lambda)/c=rac{h}{\lambda},$$

as determined experimentally and discussed above. Thus, p=E/c is equivalent to Compton's result  $p=h/\lambda$ . For a further verification of the relationship between photon energy and momentum, see [link].

#### Note:

### **Photon Detectors**

Almost all detection systems talked about thus far—eyes, photographic plates, photomultiplier tubes in microscopes, and CCD cameras—rely on particle-like properties of photons interacting with a sensitive area. A change is caused and either the change is cascaded or zillions of points are recorded to form an image we detect. These detectors are used in biomedical imaging systems, and there is ongoing research into improving the efficiency of receiving photons, particularly by cooling detection systems and reducing thermal effects.

# **Example:**

# **Photon Energy and Momentum**

Show that p = E/c for the photon considered in the [link].

# **Strategy**

We will take the energy E found in [link], divide it by the speed of light, and see if the same momentum is obtained as before.

#### Solution

Given that the energy of the photon is 2.48 eV and converting this to joules, we get

# **Equation:**

$$p = rac{E}{c} = rac{(2.48 ext{ eV})(1.60 imes 10^{-19} ext{ J/eV})}{3.00 imes 10^8 ext{ m/s}} = 1.33 imes 10^{-27} ext{ kg} \cdot ext{m/s}.$$

#### Discussion

This value for momentum is the same as found before (note that unrounded values are used in all calculations to avoid even small rounding errors), an expected verification of the relationship p=E/c. This also means the relationship between energy, momentum, and mass given by  $E^2=(\mathrm{pc})^2+(\mathrm{mc})^2$  applies to both matter and photons. Once again, note that p is not zero, even when m is.

## Note:

# **Problem-Solving Suggestion**

Note that the forms of the constants  $h=4.14\times 10^{-15}~{\rm eV\cdot s}$  and  $hc=1240~{\rm eV\cdot nm}$  may be particularly useful for this section's Problems and Exercises.

# **Section Summary**

• Photons have momentum, given by  $p=\frac{h}{\lambda}$ , where  $\lambda$  is the photon wavelength.

• Photon energy and momentum are related by  $p=\frac{E}{c}$ , where  $E=\mathrm{hf}=\mathrm{hc}/\lambda$  for a photon.

# **Conceptual Questions**

### **Exercise:**

#### **Problem:**

Which formula may be used for the momentum of all particles, with or without mass?

# **Exercise:**

### **Problem:**

Is there any measurable difference between the momentum of a photon and the momentum of matter?

### **Exercise:**

### **Problem:**

Why don't we feel the momentum of sunlight when we are on the beach?

# **Problems & Exercises**

### **Exercise:**

## **Problem:**

- (a) Find the momentum of a 4.00-cm-wavelength microwave photon.
- (b) Discuss why you expect the answer to (a) to be very small.

# **Solution:**

(a) 
$$1.66\times 10^{-32}~kg\cdot m/s$$

(b) The wavelength of microwave photons is large, so the momentum they carry is very small.

### **Exercise:**

#### **Problem:**

(a) What is the momentum of a 0.0100-nm-wavelength photon that could detect details of an atom? (b) What is its energy in MeV?

## **Exercise:**

### **Problem:**

(a) What is the wavelength of a photon that has a momentum of  $5.00 \times 10^{-29} \text{ kg} \cdot \text{m/s}$ ? (b) Find its energy in eV.

### **Solution:**

- (a)  $13.3 \, \mu m$
- (b)  $9.38 \times 10^{-2} \text{ eV}$

#### **Exercise:**

#### **Problem:**

(a) A  $\gamma$ -ray photon has a momentum of  $8.00 \times 10^{-21} \text{ kg} \cdot \text{m/s}$ . What is its wavelength? (b) Calculate its energy in MeV.

#### **Exercise:**

#### **Problem:**

(a) Calculate the momentum of a photon having a wavelength of  $2.50~\mu m$ . (b) Find the velocity of an electron having the same momentum. (c) What is the kinetic energy of the electron, and how does it compare with that of the photon?

#### **Solution:**

(a) 
$$2.65 imes 10^{-28} ext{ kg} \cdot ext{m/s}$$

- (b) 291 m/s
- (c) electron  $3.86 \times 10^{-26}$  J, photon  $7.96 \times 10^{-20}$  J, ratio  $2.06 \times 10^6$

# **Exercise:**

# **Problem:**

Repeat the previous problem for a 10.0-nm-wavelength photon.

#### **Exercise:**

## **Problem:**

(a) Calculate the wavelength of a photon that has the same momentum as a proton moving at 1.00% of the speed of light. (b) What is the energy of the photon in MeV? (c) What is the kinetic energy of the proton in MeV?

### **Solution:**

- (a)  $1.32 \times 10^{-13} \text{ m}$
- (b) 9.39 MeV
- (c)  $4.70 \times 10^{-2} \text{ MeV}$

#### **Exercise:**

#### **Problem:**

(a) Find the momentum of a 100-keV x-ray photon. (b) Find the equivalent velocity of a neutron with the same momentum. (c) What is the neutron's kinetic energy in keV?

### **Exercise:**

### **Problem:**

Take the ratio of relativistic rest energy,  $E=\gamma mc^2$ , to relativistic momentum,  $p=\gamma mu$ , and show that in the limit that mass approaches zero, you find E/p=c.

#### **Solution:**

$$E=\gamma mc^2$$
 and  $P=\gamma mu$ , so

# **Equation:**

$$rac{E}{P} = rac{\gamma mc^2}{\gamma mu} = rac{c^2}{u}.$$

As the mass of particle approaches zero, its velocity u will approach c, so that the ratio of energy to momentum in this limit is

# **Equation:**

$$\lim_{m o 0}rac{E}{P}=rac{c^2}{c}=c$$

which is consistent with the equation for photon energy.

### **Exercise:**

## **Problem:** Construct Your Own Problem

Consider a space sail such as mentioned in [link]. Construct a problem in which you calculate the light pressure on the sail in  $N/m^2$  produced by reflecting sunlight. Also calculate the force that could be produced and how much effect that would have on a spacecraft. Among the things to be considered are the intensity of sunlight, its average wavelength, the number of photons per square meter this implies, the area of the space sail, and the mass of the system being accelerated.

#### **Exercise:**

## **Problem:** Unreasonable Results

A car feels a small force due to the light it sends out from its headlights, equal to the momentum of the light divided by the time in which it is emitted. (a) Calculate the power of each headlight, if they exert a total force of  $2.00\times10^{-2}~\rm N$  backward on the car. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

# **Solution:**

- (a)  $3.00 \times 10^6 \; {
  m W}$
- (b) Headlights are way too bright.
- (c) Force is too large.

# Glossary

# photon momentum

the amount of momentum a photon has, calculated by  $p=rac{h}{\lambda}=rac{E}{c}$ 

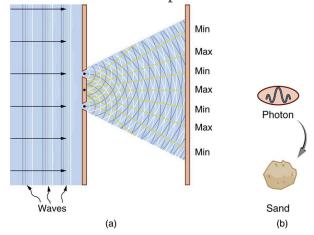
# Compton effect

the phenomenon whereby x rays scattered from materials have decreased energy

# The Particle-Wave Duality

• Explain what the term particle-wave duality means, and why it is applied to EM radiation.

We have long known that EM radiation is a wave, capable of interference and diffraction. We now see that light can be modeled as photons, which are massless particles. This may seem contradictory, since we ordinarily deal with large objects that never act like both wave and particle. An ocean wave, for example, looks nothing like a rock. To understand small-scale phenomena, we make analogies with the large-scale phenomena we observe directly. When we say something behaves like a wave, we mean it shows interference effects analogous to those seen in overlapping water waves. (See [link].) Two examples of waves are sound and EM radiation. When we say something behaves like a particle, we mean that it interacts as a discrete unit with no interference effects. Examples of particles include electrons, atoms, and photons of EM radiation. How do we talk about a phenomenon that acts like both a particle and a wave?



(a) The interference pattern for light through a double slit is a wave property understood by analogy to water waves. (b) The properties of photons having quantized energy and momentum and acting as a concentrated unit are

understood by analogy to macroscopic particles.

There is no doubt that EM radiation interferes and has the properties of wavelength and frequency. There is also no doubt that it behaves as particles—photons with discrete energy. We call this twofold nature the **particle-wave duality**, meaning that EM radiation has both particle and wave properties. This so-called duality is simply a term for properties of the photon analogous to phenomena we can observe directly, on a macroscopic scale. If this term seems strange, it is because we do not ordinarily observe details on the quantum level directly, and our observations yield either particle *or* wavelike properties, but never both simultaneously.

Since we have a particle-wave duality for photons, and since we have seen connections between photons and matter in that both have momentum, it is reasonable to ask whether there is a particle-wave duality for matter as well. If the EM radiation we once thought to be a pure wave has particle properties, is it possible that matter has wave properties? The answer is yes. The consequences are tremendous, as we will begin to see in the next section.

#### Note:

# PhET Explorations: Quantum Wave Interference

When do photons, electrons, and atoms behave like particles and when do they behave like waves? Watch waves spread out and interfere as they pass through a double slit, then get detected on a screen as tiny dots. Use quantum detectors to explore how measurements change the waves and the patterns they produce on the screen.

Quantum Wave Interferenc

# **Section Summary**

- EM radiation can behave like either a particle or a wave.
- This is termed particle-wave duality.

# Glossary

particle-wave duality

the property of behaving like either a particle or a wave; the term for the phenomenon that all particles have wave characteristics

### The Wave Nature of Matter

• Describe the Davisson-Germer experiment, and explain how it provides evidence for the wave nature of electrons.

# **De Broglie Wavelength**

In 1923 a French physics graduate student named Prince Louis-Victor de Broglie (1892–1987) made a radical proposal based on the hope that nature is symmetric. If EM radiation has both particle and wave properties, then nature would be symmetric if matter also had both particle and wave properties. If what we once thought of as an unequivocal wave (EM radiation) is also a particle, then what we think of as an unequivocal particle (matter) may also be a wave. De Broglie's suggestion, made as part of his doctoral thesis, was so radical that it was greeted with some skepticism. A copy of his thesis was sent to Einstein, who said it was not only probably correct, but that it might be of fundamental importance. With the support of Einstein and a few other prominent physicists, de Broglie was awarded his doctorate.

De Broglie took both relativity and quantum mechanics into account to develop the proposal that *all particles have a wavelength*, given by **Equation:** 

$$\lambda = \frac{h}{p}$$
 (matter and photons),

where h is Planck's constant and p is momentum. This is defined to be the **de Broglie wavelength**. (Note that we already have this for photons, from the equation  $p = h/\lambda$ .) The hallmark of a wave is interference. If matter is a wave, then it must exhibit constructive and destructive interference. Why isn't this ordinarily observed? The answer is that in order to see significant interference effects, a wave must interact with an object about the same size as its wavelength. Since h is very small,  $\lambda$  is also small, especially for macroscopic objects. A 3-kg bowling ball moving at 10 m/s, for example, has

# **Equation:**

$$\lambda = h/p = (6.63 imes 10^{-34} \ {
m J\cdot s})/[(3 \ {
m kg})(10 \ {
m m/s} \ )] = 2 imes 10^{-35} \ {
m m}.$$

This means that to see its wave characteristics, the bowling ball would have to interact with something about  $10^{-35}\,\mathrm{m}$  in size—far smaller than anything known. When waves interact with objects much larger than their wavelength, they show negligible interference effects and move in straight lines (such as light rays in geometric optics). To get easily observed interference effects from particles of matter, the longest wavelength and hence smallest mass possible would be useful. Therefore, this effect was first observed with electrons.

American physicists Clinton J. Davisson and Lester H. Germer in 1925 and, independently, British physicist G. P. Thomson (son of J. J. Thomson, discoverer of the electron) in 1926 scattered electrons from crystals and found diffraction patterns. These patterns are exactly consistent with interference of electrons having the de Broglie wavelength and are somewhat analogous to light interacting with a diffraction grating. (See [link].)

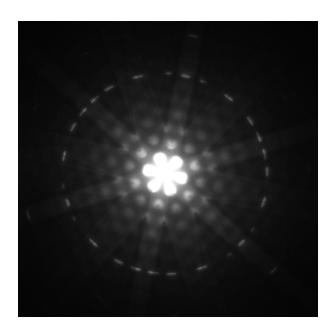
#### Note:

Connections: Waves

All microscopic particles, whether massless, like photons, or having mass, like electrons, have wave properties. The relationship between momentum and wavelength is fundamental for all particles.

De Broglie's proposal of a wave nature for all particles initiated a remarkably productive era in which the foundations for quantum mechanics were laid. In 1926, the Austrian physicist Erwin Schrödinger (1887–1961) published four papers in which the wave nature of particles was treated explicitly with wave equations. At the same time, many others began important work. Among them was German physicist Werner Heisenberg

(1901–1976) who, among many other contributions to quantum mechanics, formulated a mathematical treatment of the wave nature of matter that used matrices rather than wave equations. We will deal with some specifics in later sections, but it is worth noting that de Broglie's work was a watershed for the development of quantum mechanics. De Broglie was awarded the Nobel Prize in 1929 for his vision, as were Davisson and G. P. Thomson in 1937 for their experimental verification of de Broglie's hypothesis.



This diffraction pattern was obtained for electrons diffracted by crystalline silicon. Bright regions are those of constructive interference, while dark regions are those of destructive interference. (credit: Ndthe, Wikimedia Commons)

# **Example:**

# **Electron Wavelength versus Velocity and Energy**

For an electron having a de Broglie wavelength of 0.167 nm (appropriate for interacting with crystal lattice structures that are about this size): (a) Calculate the electron's velocity, assuming it is nonrelativistic. (b) Calculate the electron's kinetic energy in eV.

# Strategy

For part (a), since the de Broglie wavelength is given, the electron's velocity can be obtained from  $\lambda = h/p$  by using the nonrelativistic formula for momentum, p = mv. For part (b), once v is obtained (and it has been verified that v is nonrelativistic), the classical kinetic energy is simply  $(1/2)mv^2$ .

# Solution for (a)

Substituting the nonrelativistic formula for momentum (p = mv) into the de Broglie wavelength gives

# **Equation:**

$$\lambda = rac{h}{p} = rac{h}{ ext{mv}}.$$

Solving for v gives

# **Equation:**

$$v = \frac{h}{m\lambda}.$$

Substituting known values yields

# **Equation:**

$$v = rac{6.63 imes 10^{-34} ext{ J} \cdot ext{s}}{(9.11 imes 10^{-31} ext{ kg})(0.167 imes 10^{-9} ext{ m})} = 4.36 imes 10^6 ext{ m/s}.$$

# Solution for (b)

While fast compared with a car, this electron's speed is not highly relativistic, and so we can comfortably use the classical formula to find the electron's kinetic energy and convert it to eV as requested.

# **Equation:**

$$\begin{array}{lll} \mathrm{KE} &=& \frac{1}{2} m v^2 \\ &=& \frac{1}{2} (9.11 \times 10^{-31} \ \mathrm{kg}) (4.36 \times 10^6 \ \mathrm{m/s})^2 \\ &=& (86.4 \times 10^{-18} \ \mathrm{J}) \Big( \frac{1 \ \mathrm{eV}}{1.602 \times 10^{-19} \ \mathrm{J}} \Big) \\ &=& 54.0 \ \mathrm{eV} \end{array}$$

#### Discussion

This low energy means that these 0.167-nm electrons could be obtained by accelerating them through a 54.0-V electrostatic potential, an easy task. The results also confirm the assumption that the electrons are nonrelativistic, since their velocity is just over 1% of the speed of light and the kinetic energy is about 0.01% of the rest energy of an electron (0.511 MeV). If the electrons had turned out to be relativistic, we would have had to use more involved calculations employing relativistic formulas.

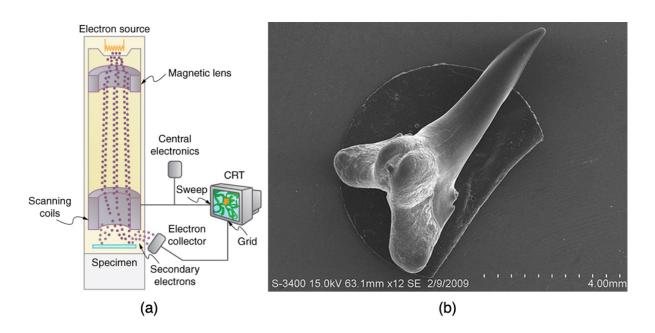
# **Electron Microscopes**

One consequence or use of the wave nature of matter is found in the electron microscope. As we have discussed, there is a limit to the detail observed with any probe having a wavelength. Resolution, or observable detail, is limited to about one wavelength. Since a potential of only 54 V can produce electrons with sub-nanometer wavelengths, it is easy to get electrons with much smaller wavelengths than those of visible light (hundreds of nanometers). Electron microscopes can, thus, be constructed to detect much smaller details than optical microscopes. (See [link].)

There are basically two types of electron microscopes. The transmission electron microscope (TEM) accelerates electrons that are emitted from a hot filament (the cathode). The beam is broadened and then passes through the sample. A magnetic lens focuses the beam image onto a fluorescent screen, a photographic plate, or (most probably) a CCD (light sensitive camera), from which it is transferred to a computer. The TEM is similar to the optical microscope, but it requires a thin sample examined in a vacuum. However it can resolve details as small as  $0.1 \text{ nm} (10^{-10} \text{ m})$ , providing magnifications

of 100 million times the size of the original object. The TEM has allowed us to see individual atoms and structure of cell nuclei.

The scanning electron microscope (SEM) provides images by using secondary electrons produced by the primary beam interacting with the surface of the sample (see [link]). The SEM also uses magnetic lenses to focus the beam onto the sample. However, it moves the beam around electrically to "scan" the sample in the *x* and *y* directions. A CCD detector is used to process the data for each electron position, producing images like the one at the beginning of this chapter. The SEM has the advantage of not requiring a thin sample and of providing a 3-D view. However, its resolution is about ten times less than a TEM.



Schematic of a scanning electron microscope (SEM) (a) used to observe small details, such as those seen in this image of a tooth of a *Himipristis*, a type of shark (b). (credit: Dallas Krentzel, Flickr)

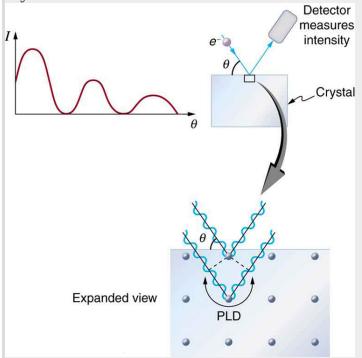
Electrons were the first particles with mass to be directly confirmed to have the wavelength proposed by de Broglie. Subsequently, protons, helium nuclei, neutrons, and many others have been observed to exhibit interference when they interact with objects having sizes similar to their de Broglie wavelength. The de Broglie wavelength for massless particles was well established in the 1920s for photons, and it has since been observed that all massless particles have a de Broglie wavelength  $\lambda = h/p$ . The wave nature of all particles is a universal characteristic of nature. We shall see in following sections that implications of the de Broglie wavelength include the quantization of energy in atoms and molecules, and an alteration of our basic view of nature on the microscopic scale. The next section, for example, shows that there are limits to the precision with which we may make predictions, regardless of how hard we try. There are even limits to the precision with which we may measure an object's location or energy.

#### Note:

# Making Connections: A Submicroscopic Diffraction Grating

The wave nature of matter allows it to exhibit all the characteristics of other, more familiar, waves. Diffraction gratings, for example, produce diffraction patterns for light that depend on grating spacing and the wavelength of the light. This effect, as with most wave phenomena, is most pronounced when the wave interacts with objects having a size similar to its wavelength. For gratings, this is the spacing between multiple slits.) When electrons interact with a system having a spacing similar to the electron wavelength, they show the same types of interference patterns as light does for diffraction gratings, as shown at top left in [link]. Atoms are spaced at regular intervals in a crystal as parallel planes, as shown in the bottom part of [link]. The spacings between these planes act like the openings in a diffraction grating. At certain incident angles, the paths of electrons scattering from successive planes differ by one wavelength and, thus, interfere constructively. At other angles, the path length differences are not an integral wavelength, and there is partial to total destructive interference. This type of scattering from a large crystal with well-defined lattice planes can produce dramatic interference patterns. It is called *Bragg reflection*, for the father-and-son team who first explored and analyzed it in some detail. The expanded view also shows the pathlength differences and indicates how these depend on incident angle  $\theta$  in a

manner similar to the diffraction patterns for x rays reflecting from a crystal.



The diffraction pattern at top left is produced by scattering electrons from a crystal and is graphed as a function of incident angle relative to the regular array of atoms in a crystal, as shown at bottom. Electrons scattering from the second layer of atoms travel farther than those scattered from the top layer. If the path length difference (PLD) is an integral wavelength, there is constructive interference.

Let us take the spacing between parallel planes of atoms in the crystal to be d. As mentioned, if the path length difference (PLD) for the electrons is a whole number of wavelengths, there will be constructive interference—that is,  $PLD = n\lambda(n = 1, 2, 3, ...)$ . Because  $AB = BC = d\sin\theta$ , we have constructive interference when  $n\lambda = 2d\sin\theta$ . This relationship is

called the *Bragg equation* and applies not only to electrons but also to x rays.

The wavelength of matter is a submicroscopic characteristic that explains a macroscopic phenomenon such as Bragg reflection. Similarly, the wavelength of light is a submicroscopic characteristic that explains the macroscopic phenomenon of diffraction patterns.

# **Section Summary**

- Particles of matter also have a wavelength, called the de Broglie wavelength, given by  $\lambda = \frac{h}{p}$ , where p is momentum.
- Matter is found to have the same *interference characteristics* as any other wave.

# **Conceptual Questions**

## **Exercise:**

### **Problem:**

How does the interference of water waves differ from the interference of electrons? How are they analogous?

### **Exercise:**

**Problem:** Describe one type of evidence for the wave nature of matter.

## **Exercise:**

#### **Problem:**

Describe one type of evidence for the particle nature of EM radiation.

# **Problems & Exercises**

### **Exercise:**

## **Problem:**

At what velocity will an electron have a wavelength of 1.00 m?

# **Solution:**

$$7.28 \times 10^{-4} \mathrm{\ m}$$

### **Exercise:**

## **Problem:**

What is the wavelength of an electron moving at 3.00% of the speed of light?

### **Exercise:**

### **Problem:**

At what velocity does a proton have a 6.00-fm wavelength (about the size of a nucleus)? Assume the proton is nonrelativistic. (1 femtometer =  $10^{-15}$  m.)

# **Solution:**

$$6.62\times10^7~\mathrm{m/s}$$

## **Exercise:**

### **Problem:**

What is the velocity of a 0.400-kg billiard ball if its wavelength is 7.50 cm (large enough for it to interfere with other billiard balls)?

# **Exercise:**

#### **Problem:**

Find the wavelength of a proton moving at 1.00% of the speed of light.

### **Solution:**

$$1.32\times10^{-13}~\mathrm{m}$$

# **Exercise:**

### **Problem:**

Experiments are performed with ultracold neutrons having velocities as small as 1.00 m/s. (a) What is the wavelength of such a neutron? (b) What is its kinetic energy in eV?

## **Exercise:**

### **Problem:**

(a) Find the velocity of a neutron that has a 6.00-fm wavelength (about the size of a nucleus). Assume the neutron is nonrelativistic. (b) What is the neutron's kinetic energy in MeV?

# **Solution:**

- (a)  $6.62 \times 10^7 \text{ m/s}$
- (b) 22.9 MeV

### **Exercise:**

## **Problem:**

What is the wavelength of an electron accelerated through a 30.0-kV potential, as in a TV tube?

### **Exercise:**

### **Problem:**

What is the kinetic energy of an electron in a TEM having a 0.0100-nm wavelength?

# **Solution:**

**Equation:**15.1 keV

#### **Exercise:**

### **Problem:**

(a) Calculate the velocity of an electron that has a wavelength of  $1.00~\mu m$ . (b) Through what voltage must the electron be accelerated to have this velocity?

### **Exercise:**

#### **Problem:**

The velocity of a proton emerging from a Van de Graaff accelerator is 25.0% of the speed of light. (a) What is the proton's wavelength? (b) What is its kinetic energy, assuming it is nonrelativistic? (c) What was the equivalent voltage through which it was accelerated?

# **Solution:**

- (a) 5.29 fm
- (b)  $4.70 \times 10^{-12} \text{ J}$
- (c) 29.4 MV

### **Exercise:**

### **Problem:**

The kinetic energy of an electron accelerated in an x-ray tube is 100 keV. Assuming it is nonrelativistic, what is its wavelength?

#### **Exercise:**

#### **Problem:** Unreasonable Results

(a) Assuming it is nonrelativistic, calculate the velocity of an electron with a 0.100-fm wavelength (small enough to detect details of a nucleus). (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

### **Solution:**

(a) 
$$7.28 \times 10^{12} \mathrm{\ m/s}$$

- (b) This is thousands of times the speed of light (an impossibility).
- (c) The assumption that the electron is non-relativistic is unreasonable at this wavelength.

# Glossary

de Broglie wavelength

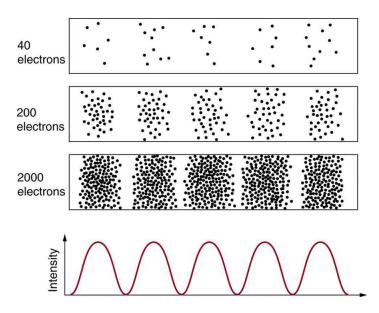
the wavelength possessed by a particle of matter, calculated by  $\lambda = h/p$ 

# Probability: The Heisenberg Uncertainty Principle

- Use both versions of Heisenberg's uncertainty principle in calculations.
- Explain the implications of Heisenberg's uncertainty principle for measurements.

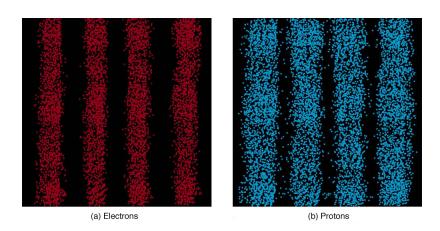
# **Probability Distribution**

Matter and photons are waves, implying they are spread out over some distance. What is the position of a particle, such as an electron? Is it at the center of the wave? The answer lies in how you measure the position of an electron. Experiments show that you will find the electron at some definite location, unlike a wave. But if you set up exactly the same situation and measure it again, you will find the electron in a different location, often far outside any experimental uncertainty in your measurement. Repeated measurements will display a statistical distribution of locations that appears wavelike. (See [link].)



The building up of the diffraction pattern of electrons scattered from a crystal surface. Each electron arrives

at a definite location, which cannot be precisely predicted. The overall distribution shown at the bottom can be predicted as the diffraction of waves having the de Broglie wavelength of the electrons.



Double-slit interference for electrons (a) and protons (b) is identical for equal wavelengths and equal slit separations. Both patterns are probability distributions in the sense that they are built up by individual particles traversing the apparatus, the paths of which are not individually predictable.

After de Broglie proposed the wave nature of matter, many physicists, including Schrödinger and Heisenberg, explored the consequences. The idea quickly emerged that, *because of its wave character, a particle's trajectory and destination cannot be precisely predicted for each particle individually*. However, each particle goes to a definite place (as illustrated in [link]). After compiling enough data, you get a distribution related to the

particle's wavelength and diffraction pattern. There is a certain *probability* of finding the particle at a given location, and the overall pattern is called a **probability distribution**. Those who developed quantum mechanics devised equations that predicted the probability distribution in various circumstances.

It is somewhat disquieting to think that you cannot predict exactly where an individual particle will go, or even follow it to its destination. Let us explore what happens if we try to follow a particle. Consider the double-slit patterns obtained for electrons and photons in [link]. First, we note that these patterns are identical, following  $d \sin \theta = m\lambda$ , the equation for double-slit constructive interference developed in Photon Energies and the Electromagnetic Spectrum, where d is the slit separation and  $\lambda$  is the electron or photon wavelength.

Both patterns build up statistically as individual particles fall on the detector. This can be observed for photons or electrons—for now, let us concentrate on electrons. You might imagine that the electrons are interfering with one another as any waves do. To test this, you can lower the intensity until there is never more than one electron between the slits and the screen. The same interference pattern builds up! This implies that a particle's probability distribution spans both slits, and the particles actually interfere with themselves. Does this also mean that the electron goes through both slits? An electron is a basic unit of matter that is not divisible. But it is a fair question, and so we should look to see if the electron traverses one slit or the other, or both. One possibility is to have coils around the slits that detect charges moving through them. What is observed is that an electron always goes through one slit or the other; it does not split to go through both. But there is a catch. If you determine that the electron went through one of the slits, you no longer get a double slit pattern instead, you get single slit interference. There is no escape by using another method of determining which slit the electron went through. Knowing the particle went through one slit forces a single-slit pattern. If you do not observe which slit the electron goes through, you obtain a double-slit pattern.

# **Heisenberg Uncertainty**

How does knowing which slit the electron passed through change the pattern? The answer is fundamentally important—*measurement affects the system being observed*. Information can be lost, and in some cases it is impossible to measure two physical quantities simultaneously to exact precision. For example, you can measure the position of a moving electron by scattering light or other electrons from it. Those probes have momentum themselves, and by scattering from the electron, they change its momentum *in a manner that loses information*. There is a limit to absolute knowledge, even in principle.



Werner Heisenberg was one of the best of those physicists who developed early quantum mechanics. Not only did his work enable a description of nature on the very small scale, it also changed our

view of the availability of knowledge. Although he is universally recognized for his brilliance and the importance of his work (he received the Nobel Prize in 1932, for example), Heisenberg remained in Germany during World War II and headed the German effort to build a nuclear bomb, permanently alienating himself from most of the scientific community. (credit: Author Unknown, via Wikimedia Commons)

It was Werner Heisenberg who first stated this limit to knowledge in 1929 as a result of his work on quantum mechanics and the wave characteristics of all particles. (See [link]). Specifically, consider simultaneously measuring the position and momentum of an electron (it could be any particle). There is an **uncertainty in position**  $\Delta x$  that is approximately equal to the wavelength of the particle. That is,

### **Equation:**

$$\Delta x \approx \lambda$$
.

As discussed above, a wave is not located at one point in space. If the electron's position is measured repeatedly, a spread in locations will be observed, implying an uncertainty in position  $\Delta x$ . To detect the position of the particle, we must interact with it, such as having it collide with a detector. In the collision, the particle will lose momentum. This change in momentum could be anywhere from close to zero to the total momentum of the particle,  $p = h/\lambda$ . It is not possible to tell how much momentum will be transferred to a detector, and so there is an **uncertainty in momentum**  $\Delta p$ , too. In fact, the uncertainty in momentum may be as large as the momentum itself, which in equation form means that

### **Equation:**

$$\Delta p pprox rac{h}{\lambda}.$$

The uncertainty in position can be reduced by using a shorter-wavelength electron, since  $\Delta x \approx \lambda$ . But shortening the wavelength increases the uncertainty in momentum, since  $\Delta p \approx h/\lambda$ . Conversely, the uncertainty in momentum can be reduced by using a longer-wavelength electron, but this increases the uncertainty in position. Mathematically, you can express this trade-off by multiplying the uncertainties. The wavelength cancels, leaving **Equation:** 

$$\Delta x \Delta p \approx h.$$

So if one uncertainty is reduced, the other must increase so that their product is  $\approx h$ .

With the use of advanced mathematics, Heisenberg showed that the best that can be done in a *simultaneous measurement of position and momentum* is

### **Equation:**

$$\Delta x \Delta p \geq rac{h}{4\pi}.$$

This is known as the **Heisenberg uncertainty principle**. It is impossible to measure position x and momentum p simultaneously with uncertainties  $\Delta x$ and  $\Delta p$  that multiply to be less than  $h/4\pi$ . Neither uncertainty can be zero. Neither uncertainty can become small without the other becoming large. A small wavelength allows accurate position measurement, but it increases the momentum of the probe to the point that it further disturbs the momentum of a system being measured. For example, if an electron is scattered from an atom and has a wavelength small enough to detect the position of electrons in the atom, its momentum can knock the electrons from their orbits in a manner that loses information about their original motion. It is therefore impossible to follow an electron in its orbit around an atom. If you measure the electron's position, you will find it in a definite location, but the atom will be disrupted. Repeated measurements on identical atoms will produce interesting probability distributions for electrons around the atom, but they will not produce motion information. The probability distributions are referred to as electron clouds or orbitals. The shapes of these orbitals are often shown in general chemistry texts and are discussed in **The Wave** Nature of Matter Causes Quantization.

# **Example:**

# Heisenberg Uncertainty Principle in Position and Momentum for an Atom

(a) If the position of an electron in an atom is measured to an accuracy of 0.0100 nm, what is the electron's uncertainty in velocity? (b) If the electron has this velocity, what is its kinetic energy in eV?

# Strategy

The uncertainty in position is the accuracy of the measurement, or  $\Delta x=0.0100$  nm. Thus the smallest uncertainty in momentum  $\Delta p$  can be calculated using  $\Delta x \Delta p \geq h/4\pi$ . Once the uncertainty in momentum  $\Delta p$  is found, the uncertainty in velocity can be found from  $\Delta p=m\Delta v$ .

### Solution for (a)

Using the equals sign in the uncertainty principle to express the minimum uncertainty, we have

### **Equation:**

$$\Delta x \Delta p = rac{h}{4\pi}.$$

Solving for  $\Delta p$  and substituting known values gives

### **Equation:**

$$\Delta p = rac{h}{4\pi \Delta x} = rac{6.63 imes 10^{-34} \, \mathrm{J \cdot s}}{4\pi (1.00 imes 10^{-11} \, \mathrm{m})} = 5.28 imes 10^{-24} \, \mathrm{kg \cdot m/s}.$$

Thus,

### **Equation:**

$$\Delta p = 5.28 imes 10^{-24} \ ext{kg} \cdot ext{m/s} = m \Delta v.$$

Solving for  $\Delta v$  and substituting the mass of an electron gives

### **Equation:**

$$\Delta v = rac{\Delta p}{m} = rac{5.28 imes 10^{-24} ext{ kg} \cdot ext{m/s}}{9.11 imes 10^{-31} ext{ kg}} = 5.79 imes 10^6 ext{ m/s}.$$

### Solution for (b)

Although large, this velocity is not highly relativistic, and so the electron's kinetic energy is

# **Equation:**

$$egin{array}{lll} {
m KE}_e &=& rac{1}{2} m v^2 \ &=& rac{1}{2} (9.11 imes 10^{-31} {
m \, kg}) (5.79 imes 10^6 {
m \, m/s})^2 \ &=& (1.53 imes 10^{-17} {
m \, J}) \Big( rac{1 {
m \, eV}}{1.60 imes 10^{-19} {
m \, J}} \Big) = 95.5 {
m \, eV}. \end{array}$$

### **Discussion**

Since atoms are roughly 0.1 nm in size, knowing the position of an electron to 0.0100 nm localizes it reasonably well inside the atom. This

would be like being able to see details one-tenth the size of the atom. But the consequent uncertainty in velocity is large. You certainly could not follow it very well if its velocity is so uncertain. To get a further idea of how large the uncertainty in velocity is, we assumed the velocity of the electron was equal to its uncertainty and found this gave a kinetic energy of 95.5 eV. This is significantly greater than the typical energy difference between levels in atoms (see [link]), so that it is impossible to get a meaningful energy for the electron if we know its position even moderately well.

Why don't we notice Heisenberg's uncertainty principle in everyday life? The answer is that Planck's constant is very small. Thus the lower limit in the uncertainty of measuring the position and momentum of large objects is negligible. We can detect sunlight reflected from Jupiter and follow the planet in its orbit around the Sun. The reflected sunlight alters the momentum of Jupiter and creates an uncertainty in its momentum, but this is totally negligible compared with Jupiter's huge momentum. The correspondence principle tells us that the predictions of quantum mechanics become indistinguishable from classical physics for large objects, which is the case here.

# **Heisenberg Uncertainty for Energy and Time**

There is another form of **Heisenberg's uncertainty principle** for *simultaneous measurements of energy and time*. In equation form, **Equation:** 

$$\Delta E \Delta t \geq rac{h}{4\pi},$$

where  $\Delta E$  is the **uncertainty in energy** and  $\Delta t$  is the **uncertainty in time**. This means that within a time interval  $\Delta t$ , it is not possible to measure energy precisely—there will be an uncertainty  $\Delta E$  in the measurement. In order to measure energy more precisely (to make  $\Delta E$  smaller), we must

increase  $\Delta t$ . This time interval may be the amount of time we take to make the measurement, or it could be the amount of time a particular state exists, as in the next [link].

### **Example:**

### Heisenberg Uncertainty Principle for Energy and Time for an Atom

An atom in an excited state temporarily stores energy. If the lifetime of this excited state is measured to be  $1.0 \times 10^{-10}$  s, what is the minimum uncertainty in the energy of the state in eV?

### Strategy

The minimum uncertainty in energy  $\Delta E$  is found by using the equals sign in  $\Delta E \Delta t \geq h/4\pi$  and corresponds to a reasonable choice for the uncertainty in time. The largest the uncertainty in time can be is the full lifetime of the excited state, or  $\Delta t = 1.0 \times 10^{-10} \, \mathrm{s}$ .

#### Solution

Solving the uncertainty principle for  $\Delta E$  and substituting known values gives

# **Equation:**

$$\Delta E = rac{h}{4\pi\Delta {
m t}} = rac{6.63 imes 10^{-34}~{
m J\cdot s}}{4\pi (1.0 imes 10^{-10}~{
m s})} = 5.3 imes 10^{-25}~{
m J}.$$

Now converting to eV yields

# **Equation:**

$$\Delta E = (5.3 imes 10^{-25} \, \mathrm{J}) igg( rac{1 \, \mathrm{eV}}{1.6 imes 10^{-19} \, \mathrm{J}} igg) = 3.3 imes 10^{-6} \, \mathrm{eV}.$$

#### Discussion

The lifetime of  $10^{-10}$  s is typical of excited states in atoms—on human time scales, they quickly emit their stored energy. An uncertainty in energy of only a few millionths of an eV results. This uncertainty is small compared with typical excitation energies in atoms, which are on the order of 1 eV. So here the uncertainty principle limits the accuracy with which

we can measure the lifetime and energy of such states, but not very significantly.

The uncertainty principle for energy and time can be of great significance if the lifetime of a system is very short. Then  $\Delta t$  is very small, and  $\Delta E$  is consequently very large. Some nuclei and exotic particles have extremely short lifetimes (as small as  $10^{-25}$  s), causing uncertainties in energy as great as many GeV ( $10^9$  eV). Stored energy appears as increased rest mass, and so this means that there is significant uncertainty in the rest mass of shortlived particles. When measured repeatedly, a spread of masses or decay energies are obtained. The spread is  $\Delta E$ . You might ask whether this uncertainty in energy could be avoided by not measuring the lifetime. The answer is no. Nature knows the lifetime, and so its brevity affects the energy of the particle. This is so well established experimentally that the uncertainty in decay energy is used to calculate the lifetime of short-lived states. Some nuclei and particles are so short-lived that it is difficult to measure their lifetime. But if their decay energy can be measured, its spread is  $\Delta E$ , and this is used in the uncertainty principle ( $\Delta E \Delta t \geq h/4\pi$ ) to calculate the lifetime  $\Delta t$ .

There is another consequence of the uncertainty principle for energy and time. If energy is uncertain by  $\Delta E$ , then conservation of energy can be violated by  $\Delta E$  for a time  $\Delta t$ . Neither the physicist nor nature can tell that conservation of energy has been violated, if the violation is temporary and smaller than the uncertainty in energy. While this sounds innocuous enough, we shall see in later chapters that it allows the temporary creation of matter from nothing and has implications for how nature transmits forces over very small distances.

Finally, note that in the discussion of particles and waves, we have stated that individual measurements produce precise or particle-like results. A definite position is determined each time we observe an electron, for example. But repeated measurements produce a spread in values consistent with wave characteristics. The great theoretical physicist Richard Feynman (1918–1988) commented, "What there are, are particles." When you

observe enough of them, they distribute themselves as you would expect for a wave phenomenon. However, what there are as they travel we cannot tell because, when we do try to measure, we affect the traveling.

# **Section Summary**

- Matter is found to have the same interference characteristics as any other wave.
- There is now a probability distribution for the location of a particle rather than a definite position.
- Another consequence of the wave character of all particles is the Heisenberg uncertainty principle, which limits the precision with which certain physical quantities can be known simultaneously. For position and momentum, the uncertainty principle is  $\Delta x \Delta p \geq \frac{h}{4\pi}$ , where  $\Delta x$  is the uncertainty in position and  $\Delta p$  is the uncertainty in momentum.
- For energy and time, the uncertainty principle is  $\Delta E \Delta t \geq \frac{h}{4\pi}$  where  $\Delta E$  is the uncertainty in energy and  $\Delta t$  is the uncertainty in time.
- These small limits are fundamentally important on the quantummechanical scale.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

What is the Heisenberg uncertainty principle? Does it place limits on what can be known?

#### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

(a) If the position of an electron in a membrane is measured to an accuracy of  $1.00~\mu m$ , what is the electron's minimum uncertainty in velocity? (b) If the electron has this velocity, what is its kinetic energy in eV? (c) What are the implications of this energy, comparing it to typical molecular binding energies?

#### **Solution:**

- (a) 57.9 m/s
- (b)  $9.55 \times 10^{-9} \text{ eV}$
- (c) From [link], we see that typical molecular binding energies range from about 1eV to 10 eV, therefore the result in part (b) is approximately 9 orders of magnitude smaller than typical molecular binding energies.

#### **Exercise:**

#### **Problem:**

(a) If the position of a chlorine ion in a membrane is measured to an accuracy of  $1.00~\mu m$ , what is its minimum uncertainty in velocity, given its mass is  $5.86\times 10^{-26}~kg?$  (b) If the ion has this velocity, what is its kinetic energy in eV, and how does this compare with typical molecular binding energies?

#### **Exercise:**

#### **Problem:**

Suppose the velocity of an electron in an atom is known to an accuracy of  $2.0 \times 10^3 \, \mathrm{m/s}$  (reasonably accurate compared with orbital velocities). What is the electron's minimum uncertainty in position, and how does this compare with the approximate 0.1-nm size of the atom?

#### **Solution:**

29 nm,

290 times greater

#### **Exercise:**

#### **Problem:**

The velocity of a proton in an accelerator is known to an accuracy of 0.250% of the speed of light. (This could be small compared with its velocity.) What is the smallest possible uncertainty in its position?

#### **Exercise:**

#### **Problem:**

A relatively long-lived excited state of an atom has a lifetime of 3.00 ms. What is the minimum uncertainty in its energy?

### **Solution:**

$$1.10 \times 10^{-13} \, \mathrm{eV}$$

#### **Exercise:**

#### **Problem:**

(a) The lifetime of a highly unstable nucleus is  $10^{-20}$  s. What is the smallest uncertainty in its decay energy? (b) Compare this with the rest energy of an electron.

#### **Exercise:**

#### **Problem:**

The decay energy of a short-lived particle has an uncertainty of 1.0 MeV due to its short lifetime. What is the smallest lifetime it can have?

#### **Solution:**

$$3.3 \times 10^{-22} \, \mathrm{s}$$

### **Problem:**

The decay energy of a short-lived nuclear excited state has an uncertainty of 2.0 eV due to its short lifetime. What is the smallest lifetime it can have?

#### **Exercise:**

#### **Problem:**

What is the approximate uncertainty in the mass of a muon, as determined from its decay lifetime?

### **Solution:**

$$2.66 \times 10^{-46} \text{ kg}$$

#### **Exercise:**

#### **Problem:**

Derive the approximate form of Heisenberg's uncertainty principle for energy and time,  $\Delta E \Delta t \approx h$ , using the following arguments: Since the position of a particle is uncertain by  $\Delta x \approx \lambda$ , where  $\lambda$  is the wavelength of the photon used to examine it, there is an uncertainty in the time the photon takes to traverse  $\Delta x$ . Furthermore, the photon has an energy related to its wavelength, and it can transfer some or all of this energy to the object being examined. Thus the uncertainty in the energy of the object is also related to  $\lambda$ . Find  $\Delta t$  and  $\Delta E$ ; then multiply them to give the approximate uncertainty principle.

# Glossary

Heisenberg's uncertainty principle

a fundamental limit to the precision with which pairs of quantities (momentum and position, and energy and time) can be measured

### uncertainty in energy

lack of precision or lack of knowledge of precise results in measurements of energy

### uncertainty in time

lack of precision or lack of knowledge of precise results in measurements of time

### uncertainty in momentum

lack of precision or lack of knowledge of precise results in measurements of momentum

### uncertainty in position

lack of precision or lack of knowledge of precise results in measurements of position

### probability distribution

the overall spatial distribution of probabilities to find a particle at a given location

### The Particle-Wave Duality Reviewed

• Explain the concept of particle-wave duality, and its scope.

**Particle-wave duality**—the fact that all particles have wave properties—is one of the cornerstones of quantum mechanics. We first came across it in the treatment of photons, those particles of EM radiation that exhibit both particle and wave properties, but not at the same time. Later it was noted that particles of matter have wave properties as well. The dual properties of particles and waves are found for all particles, whether massless like photons, or having a mass like electrons. (See [link].)

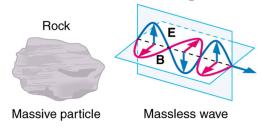




On a quantummechanical scale (i.e., very small), particles with and without mass have wave properties. For example, both electrons and photons have wavelengths but also behave as particles.

There are many submicroscopic particles in nature. Most have mass and are expected to act as particles, or the smallest units of matter. All these masses have wave properties, with wavelengths given by the de Broglie relationship  $\lambda=h/p$ . So, too, do combinations of these particles, such as nuclei, atoms, and molecules. As a combination of masses becomes large, particularly if it is large enough to be called macroscopic, its wave nature becomes difficult to observe. This is consistent with our common experience with matter.

Some particles in nature are massless. We have only treated the photon so far, but all massless entities travel at the speed of light, have a wavelength, and exhibit particle and wave behaviors. They have momentum given by a rearrangement of the de Broglie relationship,  $p = h/\lambda$ . In large combinations of these massless particles (such large combinations are common only for photons or EM waves), there is mostly wave behavior upon detection, and the particle nature becomes difficult to observe. This is also consistent with experience. (See [link].)



On a classical scale (macroscopic), particles with mass behave as particles and not as waves. Particles without mass act as waves and not as particles.

The particle-wave duality is a universal attribute. It is another connection between matter and energy. Not only has modern physics been able to

describe nature for high speeds and small sizes, it has also discovered new connections and symmetries. There is greater unity and symmetry in nature than was known in the classical era—but they were dreamt of. A beautiful poem written by the English poet William Blake some two centuries ago contains the following four lines:

To see the World in a Grain of Sand

And a Heaven in a Wild Flower

Hold Infinity in the palm of your hand

And Eternity in an hour

# **Integrated Concepts**

The problem set for this section involves concepts from this chapter and several others. Physics is most interesting when applied to general situations involving more than a narrow set of physical principles. For example, photons have momentum, hence the relevance of <a href="Linear"><u>Linear</u></a> <a href="Momentum and Collisions">Momentum and Collisions</a>. The following topics are involved in some or all of the problems in this section:

- <u>Dynamics: Newton's Laws of Motion</u>
- Work, Energy, and Energy Resources
- <u>Linear Momentum and Collisions</u>
- Heat and Heat Transfer Methods
- Electric Potential and Electric Field
- Electric Current, Resistance, and Ohm's Law
- Wave Optics
- Special Relativity

#### Note:

Problem-Solving Strategy

1. Identify which physical principles are involved.

2. Solve the problem using strategies outlined in the text.

[link] illustrates how these strategies are applied to an integrated-concept problem.

# **Example:**

### Recoil of a Dust Particle after Absorbing a Photon

The following topics are involved in this integrated concepts worked example:

Photons (quantum mechanics)

Linear Momentum

# **Topics**

A 550-nm photon (visible light) is absorbed by a 1.00-µg particle of dust in outer space. (a) Find the momentum of such a photon. (b) What is the recoil velocity of the particle of dust, assuming it is initially at rest? **Strategy Step 1** 

To solve an *integrated-concept problem*, such as those following this example, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example asks for the *momentum of a photon*, a topic of the present chapter. Part (b) considers *recoil following a collision*, a topic of <u>Linear Momentum and</u> Collisions.

**Strategy Step 2** 

The following solutions to each part of the example illustrate how specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so on.

### Solution for (a)

The momentum of a photon is related to its wavelength by the equation:

### **Equation:**

$$p=rac{h}{\lambda}.$$

Entering the known value for Planck's constant h and given the wavelength  $\lambda$ , we obtain

### **Equation:**

$$egin{array}{lcl} p & = & rac{6.63 imes 10^{-34} \; \mathrm{J \cdot s}}{550 imes 10^{-9} \; \mathrm{m}} \ & = & 1.21 imes 10^{-27} \; \mathrm{kg \cdot m/s}. \end{array}$$

### Discussion for (a)

This momentum is small, as expected from discussions in the text and the fact that photons of visible light carry small amounts of energy and momentum compared with those carried by macroscopic objects.

# Solution for (b)

Conservation of momentum in the absorption of this photon by a grain of dust can be analyzed using the equation:

# **Equation:**

$$p_1 + p_2 = p\prime_1 + p\prime_2 (F_{
m net} = 0).$$

The net external force is zero, since the dust is in outer space. Let 1 represent the photon and 2 the dust particle. Before the collision, the dust is at rest (relative to some observer); after the collision, there is no photon (it is absorbed). So conservation of momentum can be written

# **Equation:**

$$p_1 = p \prime_2 = \text{mv},$$

where  $p_1$  is the photon momentum before the collision and  $p_2$  is the dust momentum after the collision. The mass and recoil velocity of the dust are

m and v, respectively. Solving this for v, the requested quantity, yields **Equation:** 

$$v = \frac{p}{m},$$

where p is the photon momentum found in part (a). Entering known values (noting that a microgram is  $10^{-9}$  kg) gives

# **Equation:**

$$egin{array}{lll} v & = & rac{1.21 imes 10^{-27} \ {
m kg \cdot m/s}}{1.00 imes 10^{-9} \ {
m kg}} \ & = & 1.21 imes 10^{-18} \ {
m m/s}. \end{array}$$

#### Discussion

The recoil velocity of the particle of dust is extremely small. As we have noted, however, there are immense numbers of photons in sunlight and other macroscopic sources. In time, collisions and absorption of many photons could cause a significant recoil of the dust, as observed in comet tails.

# **Section Summary**

- The particle-wave duality refers to the fact that all particles—those with mass and those without mass—have wave characteristics.
- This is a further connection between mass and energy.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

In what ways are matter and energy related that were not known before the development of relativity and quantum mechanics?

### **Problems & Exercises**

#### **Exercise:**

**Problem: Integrated Concepts** 

The 54.0-eV electron in [link] has a 0.167-nm wavelength. If such electrons are passed through a double slit and have their first maximum at an angle of  $25.0^{\circ}$ , what is the slit separation d?

#### **Solution:**

0.395 nm

#### **Exercise:**

**Problem:** Integrated Concepts

An electron microscope produces electrons with a 2.00-pm wavelength. If these are passed through a 1.00-nm single slit, at what angle will the first diffraction minimum be found?

#### **Exercise:**

**Problem:** Integrated Concepts

A certain heat lamp emits 200 W of mostly IR radiation averaging 1500 nm in wavelength. (a) What is the average photon energy in joules? (b) How many of these photons are required to increase the temperature of a person's shoulder by 2.0°C, assuming the affected mass is 4.0 kg with a specific heat of 0.83 kcal/kg·°C. Also assume no other significant heat transfer. (c) How long does this take?

### **Solution:**

(a) 
$$1.3 \times 10^{-19} \text{ J}$$

(b) 
$$2.1 \times 10^{23}$$

(c) 
$$1.4 \times 10^2 \text{ s}$$

### **Problem:** Integrated Concepts

On its high power setting, a microwave oven produces 900 W of 2560 MHz microwaves. (a) How many photons per second is this? (b) How many photons are required to increase the temperature of a 0.500-kg mass of pasta by  $45.0^{\circ}$ C, assuming a specific heat of 0.900 kcal/kg·°C? Neglect all other heat transfer. (c) How long must the microwave operator wait for their pasta to be ready?

#### **Exercise:**

### **Problem:** Integrated Concepts

- (a) Calculate the amount of microwave energy in joules needed to raise the temperature of 1.00 kg of soup from 20.0°C to 100°C. (b) What is the total momentum of all the microwave photons it takes to do this?
- (c) Calculate the velocity of a 1.00-kg mass with the same momentum.
- (d) What is the kinetic energy of this mass?

### **Solution:**

(a) 
$$3.35 \times 10^5 \ {
m J}$$

(b) 
$$1.12 \times 10^{-3} \; \mathrm{kg \cdot m/s}$$

(c) 
$$1.12 \times 10^{-3} \text{ m/s}$$

(d) 
$$6.23 \times 10^{-7} \text{ J}$$

### **Exercise:**

**Problem:** Integrated Concepts

- (a) What is  $\gamma$  for an electron emerging from the Stanford Linear Accelerator with a total energy of 50.0 GeV? (b) Find its momentum.
- (c) What is the electron's wavelength?

# **Problem:** Integrated Concepts

(a) What is  $\gamma$  for a proton having an energy of 1.00 TeV, produced by the Fermilab accelerator? (b) Find its momentum. (c) What is the proton's wavelength?

### **Solution:**

- (a)  $1.06 \times 10^3$
- (b)  $5.33 \times 10^{-16} \text{ kg} \cdot \text{m/s}$
- (c)  $1.24 \times 10^{-18}$  m

#### **Exercise:**

# **Problem:** Integrated Concepts

An electron microscope passes 1.00-pm-wavelength electrons through a circular aperture  $2.00 \ \mu m$  in diameter. What is the angle between two just-resolvable point sources for this microscope?

#### **Exercise:**

### **Problem:** Integrated Concepts

(a) Calculate the velocity of electrons that form the same pattern as 450-nm light when passed through a double slit. (b) Calculate the kinetic energy of each and compare them. (c) Would either be easier to generate than the other? Explain.

#### **Solution:**

(a) 
$$1.62 \times 10^3 \text{ m/s}$$

- (b)  $4.42\times10^{-19}$  J for photon,  $1.19\times10^{-24}$  J for electron, photon energy is  $3.71\times10^5$  times greater
- (c) The light is easier to make because 450-nm light is blue light and therefore easy to make. Creating electrons with  $7.43~\mu eV$  of energy would not be difficult, but would require a vacuum.

### **Problem:** Integrated Concepts

(a) What is the separation between double slits that produces a second-order minimum at  $45.0^{\circ}$  for 650-nm light? (b) What slit separation is needed to produce the same pattern for 1.00-keV protons.

#### **Solution:**

(a) 
$$2.30 \times 10^{-6} \text{ m}$$

(b) 
$$3.20 \times 10^{-12}$$
 m

#### **Exercise:**

### **Problem:** Integrated Concepts

A laser with a power output of 2.00 mW at a wavelength of 400 nm is projected onto calcium metal. (a) How many electrons per second are ejected? (b) What power is carried away by the electrons, given that the binding energy is 2.71 eV? (c) Calculate the current of ejected electrons. (d) If the photoelectric material is electrically insulated and acts like a 2.00-pF capacitor, how long will current flow before the capacitor voltage stops it?

#### **Exercise:**

**Problem:** Integrated Concepts

One problem with x rays is that they are not sensed. Calculate the temperature increase of a researcher exposed in a few seconds to a nearly fatal accidental dose of x rays under the following conditions. The energy of the x-ray photons is 200 keV, and  $4.00 \times 10^{13}$  of them are absorbed per kilogram of tissue, the specific heat of which is  $0.830~\rm kcal/kg \cdot ^{\circ}C$ . (Note that medical diagnostic x-ray machines cannot produce an intensity this great.)

#### **Solution:**

 $3.69 \times 10^{-4} \, {}^{\circ}\mathrm{C}$ 

#### **Exercise:**

### **Problem:** Integrated Concepts

A 1.00-fm photon has a wavelength short enough to detect some information about nuclei. (a) What is the photon momentum? (b) What is its energy in joules and MeV? (c) What is the (relativistic) velocity of an electron with the same momentum? (d) Calculate the electron's kinetic energy.

#### **Exercise:**

# **Problem:** Integrated Concepts

The momentum of light is exactly reversed when reflected straight back from a mirror, assuming negligible recoil of the mirror. Thus the change in momentum is twice the photon momentum. Suppose light of intensity  $1.00~{\rm kW/m^2}$  reflects from a mirror of area  $2.00~{\rm m^2}$ . (a) Calculate the energy reflected in  $1.00~{\rm s.}$  (b) What is the momentum imparted to the mirror? (c) Using the most general form of Newton's second law, what is the force on the mirror? (d) Does the assumption of no mirror recoil seem reasonable?

#### **Solution:**

(a) 2.00 kJ

(b) 
$$1.33 \times 10^{-5} \text{ kg} \cdot \text{m/s}$$

(c) 
$$1.33 \times 10^{-5} \text{ N}$$

(d) yes

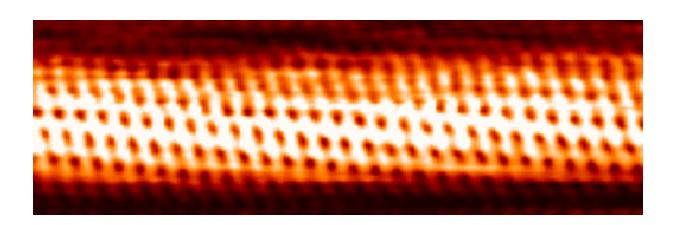
#### **Exercise:**

### **Problem:** Integrated Concepts

Sunlight above the Earth's atmosphere has an intensity of  $1.30~\mathrm{kW/m^2}$ . If this is reflected straight back from a mirror that has only a small recoil, the light's momentum is exactly reversed, giving the mirror twice the incident momentum. (a) Calculate the force per square meter of mirror. (b) Very low mass mirrors can be constructed in the near weightlessness of space, and attached to a spaceship to sail it. Once done, the average mass per square meter of the spaceship is  $0.100~\mathrm{kg}$ . Find the acceleration of the spaceship if all other forces are balanced. (c) How fast is it moving 24 hours later?

# Introduction to Atomic Physics class="introduction"

Individual carbon atoms are visible in this image of a carbon nanotube made by a scanning tunneling electron microscope . (credit: Taner Yildirim, National Institute of Standards and Technology , via Wikimedia Commons)



From childhood on, we learn that atoms are a substructure of all things around us, from the air we breathe to the autumn leaves that blanket a forest trail. Invisible to the eye, the existence and properties of atoms are used to explain many phenomena—a theme found throughout this text. In this chapter, we discuss the discovery of atoms and their own substructures; we then apply quantum mechanics to the description of atoms, and their properties and interactions. Along the way, we will find, much like the scientists who made the original discoveries, that new concepts emerge with applications far beyond the boundaries of atomic physics.

### Discovery of the Atom

• Describe the basic structure of the atom, the substructure of all matter.

How do we know that atoms are really there if we cannot see them with our eyes? A brief account of the progression from the proposal of atoms by the Greeks to the first direct evidence of their existence follows.

People have long speculated about the structure of matter and the existence of atoms. The earliest significant ideas to survive are due to the ancient Greeks in the fifth century BCE, especially those of the philosophers Leucippus and Democritus. (There is some evidence that philosophers in both India and China made similar speculations, at about the same time.) They considered the question of whether a substance can be divided without limit into ever smaller pieces. There are only a few possible answers to this question. One is that infinitesimally small subdivision is possible. Another is what Democritus in particular believed—that there is a smallest unit that cannot be further subdivided. Democritus called this the **atom**. We now know that atoms themselves can be subdivided, but their identity is destroyed in the process, so the Greeks were correct in a respect. The Greeks also felt that atoms were in constant motion, another correct notion.

The Greeks and others speculated about the properties of atoms, proposing that only a few types existed and that all matter was formed as various combinations of these types. The famous proposal that the basic elements were earth, air, fire, and water was brilliant, but incorrect. The Greeks had identified the most common examples of the four states of matter (solid, gas, plasma, and liquid), rather than the basic elements. More than 2000 years passed before observations could be made with equipment capable of revealing the true nature of atoms.

Over the centuries, discoveries were made regarding the properties of substances and their chemical reactions. Certain systematic features were recognized, but similarities between common and rare elements resulted in efforts to transmute them (lead into gold, in particular) for financial gain. Secrecy was endemic. Alchemists discovered and rediscovered many facts but did not make them broadly available. As the Middle Ages ended, alchemy gradually faded, and the science of chemistry arose. It was no

longer possible, nor considered desirable, to keep discoveries secret. Collective knowledge grew, and by the beginning of the 19th century, an important fact was well established—the masses of reactants in specific chemical reactions always have a particular mass ratio. This is very strong indirect evidence that there are basic units (atoms and molecules) that have these same mass ratios. The English chemist John Dalton (1766–1844) did much of this work, with significant contributions by the Italian physicist Amedeo Avogadro (1776–1856). It was Avogadro who developed the idea of a fixed number of atoms and molecules in a mole, and this special number is called Avogadro's number in his honor. The Austrian physicist Johann Josef Loschmidt was the first to measure the value of the constant in 1865 using the kinetic theory of gases.

#### Note:

### Patterns and Systematics

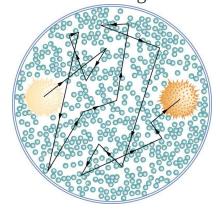
The recognition and appreciation of patterns has enabled us to make many discoveries. The periodic table of elements was proposed as an organized summary of the known elements long before all elements had been discovered, and it led to many other discoveries. We shall see in later chapters that patterns in the properties of subatomic particles led to the proposal of quarks as their underlying structure, an idea that is still bearing fruit.

Knowledge of the properties of elements and compounds grew, culminating in the mid-19th-century development of the periodic table of the elements by Dmitri Mendeleev (1834–1907), the great Russian chemist. Mendeleev proposed an ingenious array that highlighted the periodic nature of the properties of elements. Believing in the systematics of the periodic table, he also predicted the existence of then-unknown elements to complete it. Once these elements were discovered and determined to have properties predicted by Mendeleev, his periodic table became universally accepted.

Also during the 19th century, the kinetic theory of gases was developed. Kinetic theory is based on the existence of atoms and molecules in random

thermal motion and provides a microscopic explanation of the gas laws, heat transfer, and thermodynamics (see <u>Introduction to Temperature</u>, <u>Kinetic Theory</u>, <u>and the Gas Laws</u> and <u>Introduction to Laws of Thermodynamics</u>). Kinetic theory works so well that it is another strong indication of the existence of atoms. But it is still indirect evidence—individual atoms and molecules had not been observed. There were heated debates about the validity of kinetic theory until direct evidence of atoms was obtained.

The first truly direct evidence of atoms is credited to Robert Brown, a Scottish botanist. In 1827, he noticed that tiny pollen grains suspended in still water moved about in complex paths. This can be observed with a microscope for any small particles in a fluid. The motion is caused by the random thermal motions of fluid molecules colliding with particles in the fluid, and it is now called **Brownian motion**. (See [link].) Statistical fluctuations in the numbers of molecules striking the sides of a visible particle cause it to move first this way, then that. Although the molecules cannot be directly observed, their effects on the particle can be. By examining Brownian motion, the size of molecules can be calculated. The smaller and more numerous they are, the smaller the fluctuations in the numbers striking different sides.



The position of a pollen grain in water, measured every few seconds under a microscope,

exhibits Brownian motion. Brownian motion is due to fluctuations in the number of atoms and molecules colliding with a small mass, causing it to move about in complex paths. This is nearly direct evidence for the existence of atoms. providing a satisfactory alternative explanation cannot be found.

It was Albert Einstein who, starting in his epochal year of 1905, published several papers that explained precisely how Brownian motion could be used to measure the size of atoms and molecules. (In 1905 Einstein created special relativity, proposed photons as quanta of EM radiation, and produced a theory of Brownian motion that allowed the size of atoms to be determined. All of this was done in his spare time, since he worked days as a patent examiner. Any one of these very basic works could have been the crowning achievement of an entire career—yet Einstein did even more in later years.) Their sizes were only approximately known to be  $10^{-10}$  m, based on a comparison of latent heat of vaporization and surface tension made in about 1805 by Thomas Young of double-slit fame and the famous astronomer and mathematician Simon Laplace.

Using Einstein's ideas, the French physicist Jean-Baptiste Perrin (1870–1942) carefully observed Brownian motion; not only did he confirm Einstein's theory, he also produced accurate sizes for atoms and molecules.

Since molecular weights and densities of materials were well established, knowing atomic and molecular sizes allowed a precise value for Avogadro's number to be obtained. (If we know how big an atom is, we know how many fit into a certain volume.) Perrin also used these ideas to explain atomic and molecular agitation effects in sedimentation, and he received the 1926 Nobel Prize for his achievements. Most scientists were already convinced of the existence of atoms, but the accurate observation and analysis of Brownian motion was conclusive—it was the first truly direct evidence.

A huge array of direct and indirect evidence for the existence of atoms now exists. For example, it has become possible to accelerate ions (much as electrons are accelerated in cathode-ray tubes) and to detect them individually as well as measure their masses (see <a href="More Applications of Magnetism">More Applications of Magnetism</a> for a discussion of mass spectrometers). Other devices that observe individual atoms, such as the scanning tunneling electron microscope, will be discussed elsewhere. (See <a href="Link">[link</a>].) All of our understanding of the properties of matter is based on and consistent with the atom. The atom's substructures, such as electron shells and the nucleus, are both interesting and important. The nucleus in turn has a substructure, as do the particles of which it is composed. These topics, and the question of whether there is a smallest basic structure to matter, will be explored in later parts of the text.

Individual atoms can be detected with devices such as the scanning tunneling electron

microscope that
produced this
image of individual
gold atoms on a
graphite substrate.
(credit: Erwin
Rossen, Eindhoven
University of
Technology, via
Wikimedia
Commons)

# **Section Summary**

- Atoms are the smallest unit of elements; atoms combine to form molecules, the smallest unit of compounds.
- The first direct observation of atoms was in Brownian motion.
- Analysis of Brownian motion gave accurate sizes for atoms ( $10^{-10}$  m on average) and a precise value for Avogadro's number.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

Name three different types of evidence for the existence of atoms.

#### **Exercise:**

#### **Problem:**

Explain why patterns observed in the periodic table of the elements are evidence for the existence of atoms, and why Brownian motion is a more direct type of evidence for their existence.

#### **Exercise:**

**Problem:** If atoms exist, why can't we see them with visible light?

### **Problems & Exercises**

#### **Exercise:**

#### **Problem:**

Using the given charge-to-mass ratios for electrons and protons, and knowing the magnitudes of their charges are equal, what is the ratio of the proton's mass to the electron's? (Note that since the charge-to-mass ratios are given to only three-digit accuracy, your answer may differ from the accepted ratio in the fourth digit.)

#### **Solution:**

 $1.84 \times 10^{3}$ 

#### **Exercise:**

#### **Problem:**

(a) Calculate the mass of a proton using the charge-to-mass ratio given for it in this chapter and its known charge. (b) How does your result compare with the proton mass given in this chapter?

#### **Exercise:**

#### **Problem:**

If someone wanted to build a scale model of the atom with a nucleus 1.00 m in diameter, how far away would the nearest electron need to be?

#### **Solution:**

50 km

# Glossary

#### atom

basic unit of matter, which consists of a central, positively charged nucleus surrounded by negatively charged electrons

### Brownian motion

the continuous random movement of particles of matter suspended in a liquid or gas

### Discovery of the Parts of the Atom: Electrons and Nuclei

- Describe how electrons were discovered.
- Explain the Millikan oil drop experiment.
- Describe Rutherford's gold foil experiment.
- Describe Rutherford's planetary model of the atom.

Just as atoms are a substructure of matter, electrons and nuclei are substructures of the atom. The experiments that were used to discover electrons and nuclei reveal some of the basic properties of atoms and can be readily understood using ideas such as electrostatic and magnetic force, already covered in previous chapters.

#### Note:

### Charges and Electromagnetic Forces

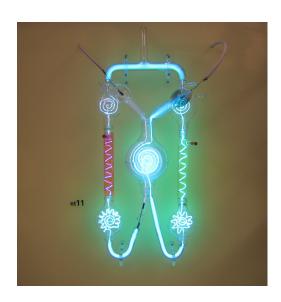
In previous discussions, we have noted that positive charge is associated with nuclei and negative charge with electrons. We have also covered many aspects of the electric and magnetic forces that affect charges. We will now explore the discovery of the electron and nucleus as substructures of the atom and examine their contributions to the properties of atoms.

### The Electron

Gas discharge tubes, such as that shown in [link], consist of an evacuated glass tube containing two metal electrodes and a rarefied gas. When a high voltage is applied to the electrodes, the gas glows. These tubes were the precursors to today's neon lights. They were first studied seriously by Heinrich Geissler, a German inventor and glassblower, starting in the 1860s. The English scientist William Crookes, among others, continued to study what for some time were called Crookes tubes, wherein electrons are freed from atoms and molecules in the rarefied gas inside the tube and are accelerated from the cathode (negative) to the anode (positive) by the high potential. These "cathode rays" collide with the gas atoms and molecules and excite them, resulting in the emission of electromagnetic (EM)

radiation that makes the electrons' path visible as a ray that spreads and fades as it moves away from the cathode.

Gas discharge tubes today are most commonly called **cathode-ray tubes**, because the rays originate at the cathode. Crookes showed that the electrons carry momentum (they can make a small paddle wheel rotate). He also found that their normally straight path is bent by a magnet in the direction expected for a negative charge moving away from the cathode. These were the first direct indications of electrons and their charge.



A gas discharge tube glows when a high voltage is applied to it. Electrons emitted from the cathode are accelerated toward the anode; they excite atoms and molecules in the gas, which glow in response. Once called Geissler tubes and later Crookes tubes, they are now known as cathode-ray

tubes (CRTs) and are found in older TVs, computer screens, and x-ray machines. When a magnetic field is applied, the beam bends in the direction expected for negative charge. (credit: Paul Downey, Flickr)

The English physicist J. J. Thomson (1856–1940) improved and expanded the scope of experiments with gas discharge tubes. (See [link] and [link].) He verified the negative charge of the cathode rays with both magnetic and electric fields. Additionally, he collected the rays in a metal cup and found an excess of negative charge. Thomson was also able to measure the ratio of the charge of the electron to its mass,  $q_e/m_e$ —an important step to finding the actual values of both  $q_e$  and  $m_e$ . [link] shows a cathode-ray tube, which produces a narrow beam of electrons that passes through charging plates connected to a high-voltage power supply. An electric field  ${\bf E}$  is produced between the charging plates, and the cathode-ray tube is placed between the poles of a magnet so that the electric field  ${f E}$  is perpendicular to the magnetic field **B** of the magnet. These fields, being perpendicular to each other, produce opposing forces on the electrons. As discussed for mass spectrometers in More Applications of Magnetism, if the net force due to the fields vanishes, then the velocity of the charged particle is v = E/B. In this manner, Thomson determined the velocity of the electrons and then moved the beam up and down by adjusting the electric field.



J. J. Thomson (credit: www.firstworldwar.com , via Wikimedia Commons)

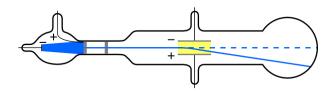
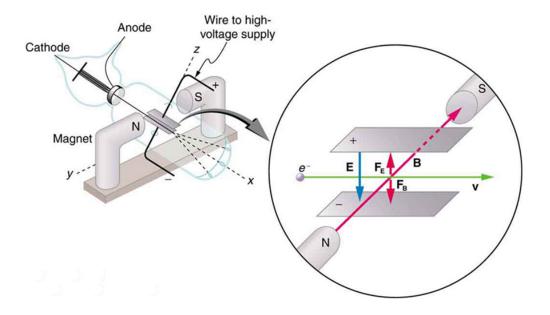


Diagram of Thomson's CRT. (credit: Kurzon, Wikimedia Commons)



This schematic shows the electron beam in a CRT passing through crossed electric and magnetic fields and causing phosphor to glow when striking the end of the tube.

To see how the amount of deflection is used to calculate  $q_e/m_e$ , note that the deflection is proportional to the electric force on the electron:

# **Equation:**

$$F = q_e E$$
.

But the vertical deflection is also related to the electron's mass, since the electron's acceleration is

# **Equation:**

$$a = rac{F}{m_e}.$$

The value of F is not known, since  $q_e$  was not yet known. Substituting the expression for electric force into the expression for acceleration yields

# **Equation:**

$$a=rac{F}{m_e}=rac{q_e E}{m_e}.$$

Gathering terms, we have

# **Equation:**

$$rac{q_e}{m_e} = rac{a}{E}.$$

The deflection is analyzed to get a, and E is determined from the applied voltage and distance between the plates; thus,  $\frac{q_e}{m_e}$  can be determined. With the velocity known, another measurement of  $\frac{q_e}{m_e}$  can be obtained by bending the beam of electrons with the magnetic field. Since  $F_{\rm mag}=q_e{\rm vB}=m_ea$ , we have  $q_e/m_e=a/{\rm vB}$ . Consistent results are obtained using magnetic deflection.

What is so important about  $q_e/m_e$ , the ratio of the electron's charge to its mass? The value obtained is

# **Equation:**

$$rac{q_e}{m_e} = -1.76 imes 10^{11} \ \mathrm{C/kg} \ \mathrm{(electron)}.$$

This is a huge number, as Thomson realized, and it implies that the electron has a very small mass. It was known from electroplating that about  $10^8~{\rm C/kg}$  is needed to plate a material, a factor of about 1000 less than the charge per kilogram of electrons. Thomson went on to do the same experiment for positively charged hydrogen ions (now known to be bare protons) and found a charge per kilogram about 1000 times smaller than that for the electron, implying that the proton is about 1000 times more massive than the electron. Today, we know more precisely that

# **Equation:**

$$rac{q_p}{m_p} = 9.58 imes 10^7 ext{ C/kg (proton)},$$

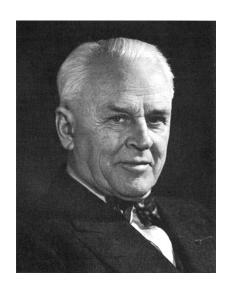
where  $q_p$  is the charge of the proton and  $m_p$  is its mass. This ratio (to four significant figures) is 1836 times less charge per kilogram than for the electron. Since the charges of electrons and protons are equal in magnitude, this implies  $m_p=1836m_e$ .

Thomson performed a variety of experiments using differing gases in discharge tubes and employing other methods, such as the photoelectric effect, for freeing electrons from atoms. He always found the same properties for the electron, proving it to be an independent particle. For his work, the important pieces of which he began to publish in 1897, Thomson was awarded the 1906 Nobel Prize in Physics. In retrospect, it is difficult to appreciate how astonishing it was to find that the atom has a substructure. Thomson himself said, "It was only when I was convinced that the experiment left no escape from it that I published my belief in the existence of bodies smaller than atoms."

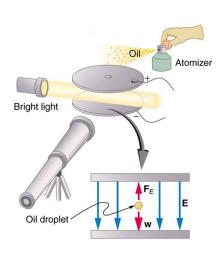
Thomson attempted to measure the charge of individual electrons, but his method could determine its charge only to the order of magnitude expected.

Since Faraday's experiments with electroplating in the 1830s, it had been known that about 100,000 C per mole was needed to plate singly ionized ions. Dividing this by the number of ions per mole (that is, by Avogadro's number), which was approximately known, the charge per ion was calculated to be about  $1.6 \times 10^{-19}$  C, close to the actual value.

An American physicist, Robert Millikan (1868–1953) (see [link]), decided to improve upon Thomson's experiment for measuring  $q_e$  and was eventually forced to try another approach, which is now a classic experiment performed by students. The Millikan oil drop experiment is shown in [link].



Robert Millikan (credit: Unknown Author, via Wikimedia Commons)



The Millikan oil drop experiment produced the first accurate direct measurement of the

charge on electrons, one of the most fundamental constants in nature. Fine drops of oil become charged when sprayed. Their movement is observed between metal plates with a potential applied to oppose the gravitational force. The balance of gravitational and electric forces allows the calculation of the charge on a drop. The charge is found to be quantized in units of  $-1.6 \times 10^{-19} \text{ C}.$ thus determining directly the charge of the excess and missing electrons on the oil drops.

In the Millikan oil drop experiment, fine drops of oil are sprayed from an atomizer. Some of these are charged by the process and can then be suspended between metal plates by a voltage between the plates. In this situation, the weight of the drop is balanced by the electric force:

# **Equation:**

$$m_{
m drop} g = q_e E$$

The electric field is produced by the applied voltage, hence, E=V/d, and V is adjusted to just balance the drop's weight. The drops can be seen as points of reflected light using a microscope, but they are too small to directly measure their size and mass. The mass of the drop is determined by observing how fast it falls when the voltage is turned off. Since air resistance is very significant for these submicroscopic drops, the more massive drops fall faster than the less massive, and sophisticated sedimentation calculations can reveal their mass. Oil is used rather than water, because it does not readily evaporate, and so mass is nearly constant. Once the mass of the drop is known, the charge of the electron is given by rearranging the previous equation:

# **Equation:**

$$q = rac{m_{
m drop} g}{E} = rac{m_{
m drop} {
m gd}}{V},$$

where d is the separation of the plates and V is the voltage that holds the drop motionless. (The same drop can be observed for several hours to see that it really is motionless.) By 1913 Millikan had measured the charge of the electron  $q_e$  to an accuracy of 1%, and he improved this by a factor of 10 within a few years to a value of  $-1.60 \times 10^{-19}$  C. He also observed that all charges were multiples of the basic electron charge and that sudden changes could occur in which electrons were added or removed from the drops. For this very fundamental direct measurement of  $q_e$  and for his studies of the photoelectric effect, Millikan was awarded the 1923 Nobel Prize in Physics.

With the charge of the electron known and the charge-to-mass ratio known, the electron's mass can be calculated. It is

# **Equation:**

$$m=rac{q_e}{\left(rac{q_e}{m_e}
ight)}.$$

Substituting known values yields

# **Equation:**

$$m_e = rac{-1.60 imes 10^{-19} ext{ C}}{-1.76 imes 10^{11} ext{ C/kg}}$$

or

# **Equation:**

$$m_e = 9.11 \times 10^{-31} \text{ kg (electron's mass)},$$

where the round-off errors have been corrected. The mass of the electron has been verified in many subsequent experiments and is now known to an accuracy of better than one part in one million. It is an incredibly small mass and remains the smallest known mass of any particle that has mass. (Some particles, such as photons, are massless and cannot be brought to rest, but travel at the speed of light.) A similar calculation gives the masses of other particles, including the proton. To three digits, the mass of the proton is now known to be

# **Equation:**

$$m_p = 1.67 imes 10^{-27} \ \mathrm{kg} \ \ \mathrm{(proton's \ mass)},$$

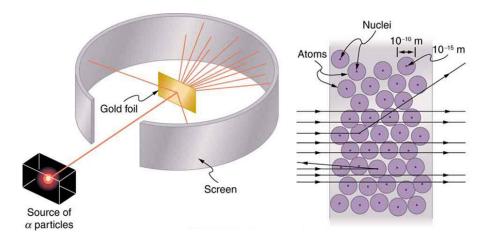
which is nearly identical to the mass of a hydrogen atom. What Thomson and Millikan had done was to prove the existence of one substructure of atoms, the electron, and further to show that it had only a tiny fraction of the mass of an atom. The nucleus of an atom contains most of its mass, and the nature of the nucleus was completely unanticipated.

Another important characteristic of quantum mechanics was also beginning to emerge. All electrons are identical to one another. The charge and mass of electrons are not average values; rather, they are unique values that all electrons have. This is true of other fundamental entities at the submicroscopic level. All protons are identical to one another, and so on.

# The Nucleus

Here, we examine the first direct evidence of the size and mass of the nucleus. In later chapters, we will examine many other aspects of nuclear physics, but the basic information on nuclear size and mass is so important to understanding the atom that we consider it here.

Nuclear radioactivity was discovered in 1896, and it was soon the subject of intense study by a number of the best scientists in the world. Among them was New Zealander Lord Ernest Rutherford, who made numerous fundamental discoveries and earned the title of "father of nuclear physics." Born in Nelson, Rutherford did his postgraduate studies at the Cavendish Laboratories in England before taking up a position at McGill University in Canada where he did the work that earned him a Nobel Prize in Chemistry in 1908. In the area of atomic and nuclear physics, there is much overlap between chemistry and physics, with physics providing the fundamental enabling theories. He returned to England in later years and had six future Nobel Prize winners as students. Rutherford used nuclear radiation to directly examine the size and mass of the atomic nucleus. The experiment he devised is shown in [link]. A radioactive source that emits alpha radiation was placed in a lead container with a hole in one side to produce a beam of alpha particles, which are a type of ionizing radiation ejected by the nuclei of a radioactive source. A thin gold foil was placed in the beam, and the scattering of the alpha particles was observed by the glow they caused when they struck a phosphor screen.



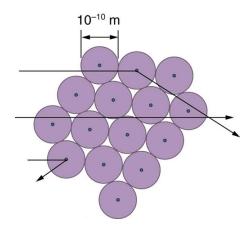
Rutherford's experiment gave direct evidence for the size and mass of the nucleus by scattering alpha particles from a thin gold foil. Alpha particles with energies of about 5 MeV are emitted from a radioactive source (which is a small metal container in which a specific amount of a radioactive material is sealed), are collimated into a beam, and fall upon the foil. The number of particles that penetrate the foil or scatter to various angles indicates that gold nuclei are very small and contain nearly all of the gold atom's mass. This is particularly indicated by the alpha particles that scatter to very large angles, much like a soccer ball bouncing off a goalie's head.

Alpha particles were known to be the doubly charged positive nuclei of helium atoms that had kinetic energies on the order of 5 MeV when emitted in nuclear decay, which is the disintegration of the nucleus of an unstable nuclide by the spontaneous emission of charged particles. These particles interact with matter mostly via the Coulomb force, and the manner in which they scatter from nuclei can reveal nuclear size and mass. This is analogous to observing how a bowling ball is scattered by an object you cannot see directly. Because the alpha particle's energy is so large compared with the typical energies associated with atoms (MeV versus eV), you would expect the alpha particles to simply crash through a thin foil much like a supersonic bowling ball would crash through a few dozen rows of bowling pins. Thomson had envisioned the atom to be a small sphere in which equal amounts of positive and negative charge were distributed evenly. The incident massive alpha particles would suffer only small deflections in such a model. Instead, Rutherford and his collaborators found that alpha particles occasionally were scattered to large angles, some even back in the direction from which they came! Detailed analysis using conservation of momentum and energy—particularly of the small number that came straight back implied that gold nuclei are very small compared with the size of a gold atom, contain almost all of the atom's mass, and are tightly bound. Since

the gold nucleus is several times more massive than the alpha particle, a head-on collision would scatter the alpha particle straight back toward the source. In addition, the smaller the nucleus, the fewer alpha particles that would hit one head on.

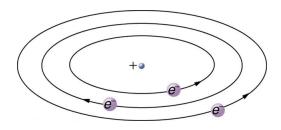
Although the results of the experiment were published by his colleagues in 1909, it took Rutherford two years to convince himself of their meaning. Like Thomson before him, Rutherford was reluctant to accept such radical results. Nature on a small scale is so unlike our classical world that even those at the forefront of discovery are sometimes surprised. Rutherford later wrote: "It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you. On consideration, I realized that this scattering backwards ... [meant] ... the greatest part of the mass of the atom was concentrated in a tiny nucleus." In 1911, Rutherford published his analysis together with a proposed model of the atom. The size of the nucleus was determined to be about  $10^{-15}$  m, or 100,000 times smaller than the atom. This implies a huge density, on the order of  $10^{15} \mathrm{\ g/cm}^3$ , vastly unlike any macroscopic matter. Also implied is the existence of previously unknown nuclear forces to counteract the huge repulsive Coulomb forces among the positive charges in the nucleus. Huge forces would also be consistent with the large energies emitted in nuclear radiation.

The small size of the nucleus also implies that the atom is mostly empty inside. In fact, in Rutherford's experiment, most alphas went straight through the gold foil with very little scattering, since electrons have such small masses and since the atom was mostly empty with nothing for the alpha to hit. There were already hints of this at the time Rutherford performed his experiments, since energetic electrons had been observed to penetrate thin foils more easily than expected. [link] shows a schematic of the atoms in a thin foil with circles representing the size of the atoms (about  $10^{-10}$  m) and dots representing the nuclei. (The dots are not to scale—if they were, you would need a microscope to see them.) Most alpha particles miss the small nuclei and are only slightly scattered by electrons. Occasionally, (about once in 8000 times in Rutherford's experiment), an alpha hits a nucleus head-on and is scattered straight backward.



An expanded view of the atoms in the gold foil in Rutherford's experiment. Circles represent the atoms (about  $10^{-10}$  m in diameter), while the dots represent the nuclei (about  $10^{-15}$  m in diameter). To be visible, the dots are much larger than scale. Most alpha particles crash through but are relatively unaffected because of their high energy and the electron's small mass. Some, however, head straight toward a nucleus and are scattered straight back. A detailed analysis gives the size and mass of the nucleus.

Based on the size and mass of the nucleus revealed by his experiment, as well as the mass of electrons, Rutherford proposed the **planetary model of the atom**. The planetary model of the atom pictures low-mass electrons orbiting a large-mass nucleus. The sizes of the electron orbits are large compared with the size of the nucleus, with mostly vacuum inside the atom. This picture is analogous to how low-mass planets in our solar system orbit the large-mass Sun at distances large compared with the size of the sun. In the atom, the attractive Coulomb force is analogous to gravitation in the planetary system. (See [link].) Note that a model or mental picture is needed to explain experimental results, since the atom is too small to be directly observed with visible light.



Rutherford's planetary model of the atom incorporates the characteristics of the nucleus, electrons, and the size of the atom. This model was the first to recognize the structure of atoms, in which low-mass electrons orbit a very small, massive nucleus in orbits much larger than the nucleus. The atom is mostly empty and is analogous to our planetary system.

Rutherford's planetary model of the atom was crucial to understanding the characteristics of atoms, and their interactions and energies, as we shall see in the next few sections. Also, it was an indication of how different nature is from the familiar classical world on the small, quantum mechanical scale. The discovery of a substructure to all matter in the form of atoms and molecules was now being taken a step further to reveal a substructure of atoms that was simpler than the 92 elements then known. We have continued to search for deeper substructures, such as those inside the nucleus, with some success. In later chapters, we will follow this quest in the discussion of quarks and other elementary particles, and we will look at the direction the search seems now to be heading.

### Note:

# PhET Explorations: Rutherford Scattering

How did Rutherford figure out the structure of the atom without being able to see it? Simulate the famous experiment in which he disproved the Plum Pudding model of the atom by observing alpha particles bouncing off atoms and determining that they must have a small core.

https://phet.colorado.edu/sims/html/rutherford-scattering/latest/rutherford-scattering\_en.html

# **Section Summary**

- Atoms are composed of negatively charged electrons, first proved to exist in cathode-ray-tube experiments, and a positively charged nucleus.
- All electrons are identical and have a charge-to-mass ratio of Equation:

$$rac{q_e}{m_e} = -1.76 imes 10^{11} \; {
m C/kg}.$$

 The positive charge in the nuclei is carried by particles called protons, which have a charge-to-mass ratio of Equation:

$$rac{q_p}{m_p} = 9.57 imes 10^7 ext{ C/kg}.$$

Mass of electron,Equation:

$$m_e = 9.11 \times 10^{-31}$$
 kg.

Mass of proton,Equation:

$$m_p = 1.67 \times 10^{-27} \text{ kg.}$$

• The planetary model of the atom pictures electrons orbiting the nucleus in the same way that planets orbit the sun.

# **Conceptual Questions**

## **Exercise:**

# **Problem:**

What two pieces of evidence allowed the first calculation of  $m_e$ , the mass of the electron?

- (a) The ratios  $q_e/m_e$  and  $q_p/m_p$ .
- (b) The values of  $q_e$  and  $E_B$ .
- (c) The ratio  $q_e/m_e$  and  $q_e$ .

Justify your response.

# **Exercise:**

# **Problem:**

How do the allowed orbits for electrons in atoms differ from the allowed orbits for planets around the sun? Explain how the correspondence principle applies here.

# **Problem Exercises**

# **Exercise:**

## **Problem:**

Rutherford found the size of the nucleus to be about  $10^{-15}$  m. This implied a huge density. What would this density be for gold?

# **Solution:**

$$6\times10^{20}~kg/m^3$$

# **Exercise:**

### **Problem:**

In Millikan's oil-drop experiment, one looks at a small oil drop held motionless between two plates. Take the voltage between the plates to be 2033 V, and the plate separation to be 2.00 cm. The oil drop (of density  $0.81~{\rm g/cm^3}$ ) has a diameter of  $4.0\times10^{-6}~{\rm m}$ . Find the charge on the drop, in terms of electron units.

### **Exercise:**

## **Problem:**

- (a) An aspiring physicist wants to build a scale model of a hydrogen atom for her science fair project. If the atom is 1.00 m in diameter, how big should she try to make the nucleus?
- (b) How easy will this be to do?

# **Solution:**

- (a)  $10.0 \ \mu m$
- (b) It isn't hard to make one of approximately this size. It would be harder to make it exactly  $10.0\ \mu m$ .

# Glossary

cathode-ray tube

a vacuum tube containing a source of electrons and a screen to view images

planetary model of the atom

the most familiar model or illustration of the structure of the atom

# Bohr's Theory of the Hydrogen Atom

- Describe the mysteries of atomic spectra.
- Explain Bohr's theory of the hydrogen atom.
- Explain Bohr's planetary model of the atom.
- Illustrate energy state using the energy-level diagram.
- Describe the triumphs and limits of Bohr's theory.

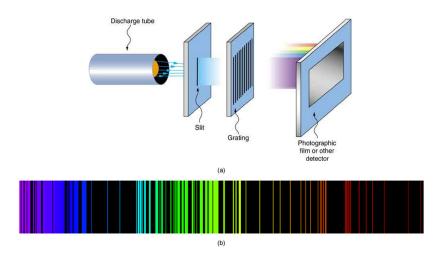
The great Danish physicist Niels Bohr (1885–1962) made immediate use of Rutherford's planetary model of the atom. ([link]). Bohr became convinced of its validity and spent part of 1912 at Rutherford's laboratory. In 1913, after returning to Copenhagen, he began publishing his theory of the simplest atom, hydrogen, based on the planetary model of the atom. For decades, many questions had been asked about atomic characteristics. From their sizes to their spectra, much was known about atoms, but little had been explained in terms of the laws of physics. Bohr's theory explained the atomic spectrum of hydrogen and established new and broadly applicable principles in quantum mechanics.



Niels Bohr, Danish physicist, used the planetary model of the atom to explain the atomic spectrum and size of the hydrogen atom. His many contributions to the development of atomic physics and quantum mechanics, his personal influence on many students and colleagues, and his personal integrity, especially in the face of Nazi oppression, earned him a prominent place in history. (credit: Unknown Author, via Wikimedia Commons)

# **Mysteries of Atomic Spectra**

As noted in Quantization of Energy, the energies of some small systems are quantized. Atomic and molecular emission and absorption spectra have been known for over a century to be discrete (or quantized). (See [link].) Maxwell and others had realized that there must be a connection between the spectrum of an atom and its structure, something like the resonant frequencies of musical instruments. But, in spite of years of efforts by many great minds, no one had a workable theory. (It was a running joke that any theory of atomic and molecular spectra could be destroyed by throwing a book of data at it, so complex were the spectra.) Following Einstein's proposal of photons with quantized energies directly proportional to their wavelengths, it became even more evident that electrons in atoms can exist only in discrete orbits.



Part (a) shows, from left to right, a discharge tube, slit, and diffraction grating producing a line spectrum. Part (b) shows the emission line spectrum for iron. The discrete lines imply quantized energy states for the atoms that produce them. The line spectrum for each element is unique, providing a powerful and much used analytical tool, and many line spectra were well known for many years before they could be explained with physics. (credit for (b): Yttrium91, Wikimedia Commons)

In some cases, it had been possible to devise formulas that described the emission spectra. As you might expect, the simplest atom—hydrogen, with its single electron—has a relatively simple spectrum. The hydrogen spectrum had been observed in the infrared (IR), visible, and ultraviolet (UV), and several series of spectral lines had been observed. (See [link].) These series are named after early researchers who studied them in particular depth.

The observed **hydrogen-spectrum wavelengths** can be calculated using the following formula:

## **Equation:**

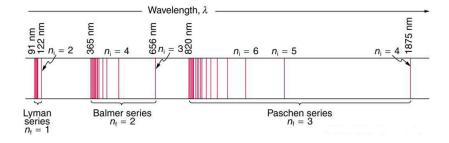
$$rac{1}{\lambda} = Rigg(rac{1}{n_{
m f}^2} - rac{1}{n_{
m i}^2}igg),$$

where  $\lambda$  is the wavelength of the emitted EM radiation and R is the **Rydberg constant**, determined by the experiment to be

## **Equation:**

$$R = 1.097 \times 10^7 / \mathrm{m} \ (\mathrm{or} \ \mathrm{m}^{-1}).$$

The constant  $n_{\rm f}$  is a positive integer associated with a specific series. For the Lyman series,  $n_{\rm f}=1$ ; for the Balmer series,  $n_{\rm f}=2$ ; for the Paschen series,  $n_{\rm f}=3$ ; and so on. The Lyman series is entirely in the UV, while part of the Balmer series is visible with the remainder UV. The Paschen series and all the rest are entirely IR. There are apparently an unlimited number of series, although they lie progressively farther into the infrared and become difficult to observe as  $n_{\rm f}$  increases. The constant  $n_{\rm i}$  is a positive integer, but it must be greater than  $n_{\rm f}$ . Thus, for the Balmer series,  $n_{\rm f}=2$  and  $n_{\rm i}=3,4,5,6,\ldots$  Note that  $n_{\rm i}$  can approach infinity. While the formula in the wavelengths equation was just a recipe designed to fit data and was not based on physical principles, it did imply a deeper meaning. Balmer first devised the formula for his series alone, and it was later found to describe all the other series by using different values of  $n_{\rm f}$ . Bohr was the first to comprehend the deeper meaning. Again, we see the interplay between experiment and theory in physics. Experimentally, the spectra were well established, an equation was found to fit the experimental data, but the theoretical foundation was missing.



A schematic of the hydrogen spectrum shows several series named for those who contributed most to their determination. Part of the Balmer series is in the visible spectrum, while the Lyman series is entirely in the UV, and the Paschen series and others are in the IR. Values of  $n_{\rm f}$  and  $n_{\rm i}$  are shown for some of the lines.

## **Example:**

# **Calculating Wave Interference of a Hydrogen Line**

What is the distance between the slits of a grating that produces a first-order maximum for the second Balmer line at an angle of  $15^{\circ}$ ?

# **Strategy and Concept**

For an Integrated Concept problem, we must first identify the physical principles involved. In this example, we need to know (a) the wavelength of light as well as (b) conditions for an interference maximum for the pattern from a double slit. Part (a) deals with a topic of the present chapter, while part (b) considers the wave interference material of <u>Wave Optics</u>.

## Solution for (a)

**Hydrogen spectrum wavelength**. The Balmer series requires that  $n_{\rm f}=2$ . The first line in the series is taken to be for  $n_{\rm i}=3$ , and so the second would have  $n_{\rm i}=4$ .

The calculation is a straightforward application of the wavelength equation. Entering the determined values for  $n_{\rm f}$  and  $n_{\rm i}$  yields

### **Equation:**

$$egin{array}{lll} rac{1}{\lambda} &=& R \Big(rac{1}{n_{
m f}^2} - rac{1}{n_{
m i}^2}\Big) \ &=& ig(1.097 imes 10^7 \ {
m m}^{-1}ig) \Big(rac{1}{2^2} - rac{1}{4^2}ig) \ &=& 2.057 imes 10^6 \ {
m m}^{-1}. \end{array}$$

Inverting to find  $\lambda$  gives

# **Equation:**

$$\lambda = \frac{1}{2.057 \times 10^6 \text{ m}^{-1}} = 486 \times 10^{-9} \text{ m}$$
  
= 486 nm.

### Discussion for (a)

This is indeed the experimentally observed wavelength, corresponding to the second (bluegreen) line in the Balmer series. More impressive is the fact that the same simple recipe predicts *all* of the hydrogen spectrum lines, including new ones observed in subsequent experiments. What is nature telling us?

# Solution for (b)

**Double-slit interference** (<u>Wave Optics</u>). To obtain constructive interference for a double slit, the path length difference from two slits must be an integral multiple of the wavelength. This condition was expressed by the equation

### **Equation:**

$$d\sin\theta = m\lambda$$
,

where d is the distance between slits and  $\theta$  is the angle from the original direction of the beam. The number m is the order of the interference; m=1 in this example. Solving for d and entering known values yields

### **Equation:**

$$d = \frac{(1)(486 \text{ nm})}{\sin 15^{\circ}} = 1.88 \times 10^{-6} \text{ m}.$$

# Discussion for (b)

This number is similar to those used in the interference examples of <u>Introduction to Quantum Physics</u> (and is close to the spacing between slits in commonly used diffraction glasses).

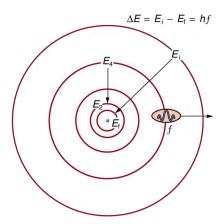
# **Bohr's Solution for Hydrogen**

Bohr was able to derive the formula for the hydrogen spectrum using basic physics, the planetary model of the atom, and some very important new proposals. His first proposal is that only certain orbits are allowed: we say that *the orbits of electrons in atoms are quantized*. Each orbit has a different energy, and electrons can move to a higher orbit by absorbing energy and drop to a lower orbit by emitting energy. If the orbits are quantized, the amount of energy absorbed or emitted is also quantized, producing discrete spectra. Photon absorption and emission are among the primary methods of transferring energy into and out of atoms. The energies of the photons are quantized, and their energy is explained as being equal to the change in energy of the electron when it moves from one orbit to another. In equation form, this is

### **Equation:**

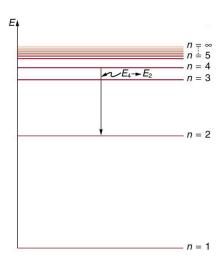
$$\Delta E = hf = E_{
m i} - E_{
m f}$$
.

Here,  $\Delta E$  is the change in energy between the initial and final orbits, and hf is the energy of the absorbed or emitted photon. It is quite logical (that is, expected from our everyday experience) that energy is involved in changing orbits. A blast of energy is required for the space shuttle, for example, to climb to a higher orbit. What is not expected is that atomic orbits should be quantized. This is not observed for satellites or planets, which can have any orbit given the proper energy. (See [link].)



The planetary model of the atom, as modified by Bohr, has the orbits of the electrons quantized. Only certain orbits are allowed, explaining why atomic spectra are discrete (quantized). The energy carried away from an atom by a photon comes from the electron dropping from one allowed orbit to another and is thus quantized. This is likewise true for atomic absorption of photons.

[link] shows an **energy-level diagram**, a convenient way to display energy states. In the present discussion, we take these to be the allowed energy levels of the electron. Energy is plotted vertically with the lowest or ground state at the bottom and with excited states above. Given the energies of the lines in an atomic spectrum, it is possible (although sometimes very difficult) to determine the energy levels of an atom. Energy-level diagrams are used for many systems, including molecules and nuclei. A theory of the atom or any other system must predict its energies based on the physics of the system.



An energy-level diagram plots energy vertically and is useful in visualizing the energy states of a system and the transitions between them. This diagram is for the hydrogen-atom electrons, showing a transition between two orbits having energies  $E_4$  and  $E_2$ .

Bohr was clever enough to find a way to calculate the electron orbital energies in hydrogen. This was an important first step that has been improved upon, but it is well worth repeating here, because it does correctly describe many characteristics of hydrogen. Assuming circular orbits, Bohr proposed that the **angular momentum** L **of an electron in its orbit is quantized**, that is, it has only specific, discrete values. The value for L is given by the formula

## **Equation:**

$$L=m_e {
m vr}_n=nrac{h}{2\pi}(n=1,2,3,\ldots),$$

where L is the angular momentum,  $m_e$  is the electron's mass,  $r_n$  is the radius of the n th orbit, and h is Planck's constant. Note that angular momentum is  $L=I\omega$ . For a small object at a radius r,  $I=mr^2$  and  $\omega=v/r$ , so that  $L=\left(mr^2\right)(v/r)=mvr$ . Quantization says that this value of mvr can only be equal to h/2, 2h/2, 3h/2, etc. At the time, Bohr himself did not know why angular momentum should be quantized, but using this assumption he was

able to calculate the energies in the hydrogen spectrum, something no one else had done at the time.

From Bohr's assumptions, we will now derive a number of important properties of the hydrogen atom from the classical physics we have covered in the text. We start by noting the centripetal force causing the electron to follow a circular path is supplied by the Coulomb force. To be more general, we note that this analysis is valid for any single-electron atom. So, if a nucleus has Z protons (Z=1 for hydrogen, 2 for helium, etc.) and only one electron, that atom is called a **hydrogen-like atom**. The spectra of hydrogen-like ions are similar to hydrogen, but shifted to higher energy by the greater attractive force between the electron and nucleus. The magnitude of the centripetal force is  $m_e v^2/r_n$ , while the Coulomb force is  $k(\mathrm{Zq}_e)(q_e)/r_n^2$ . The tacit assumption here is that the nucleus is more massive than the stationary electron, and the electron orbits about it. This is consistent with the planetary model of the atom. Equating these,

## **Equation:**

$$krac{Zq_e^2}{r_n^2} = rac{m_e v^2}{r_n} \; ext{(Coulomb} = ext{centripetal)}.$$

Angular momentum quantization is stated in an earlier equation. We solve that equation for v, substitute it into the above, and rearrange the expression to obtain the radius of the orbit. This yields:

# **Equation:**

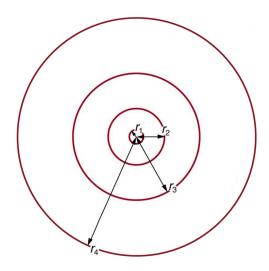
$$r_n = rac{n^2}{Z} a_{
m B}, ext{for allowed orbits} (n=1,\!2,\!3,\ldots),$$

where  $a_{\rm B}$  is defined to be the **Bohr radius**, since for the lowest orbit (n=1) and for hydrogen (Z=1),  $r_1=a_{\rm B}$ . It is left for this chapter's Problems and Exercises to show that the Bohr radius is

# **Equation:**

$$a_{
m B} = rac{h^2}{4\pi^2 m_e k q_e^2} = 0.529 imes 10^{-10} \ {
m m}.$$

These last two equations can be used to calculate the **radii of the allowed (quantized) electron orbits in any hydrogen-like atom**. It is impressive that the formula gives the correct size of hydrogen, which is measured experimentally to be very close to the Bohr radius. The earlier equation also tells us that the orbital radius is proportional to  $n^2$ , as illustrated in [link].



The allowed electron orbits in hydrogen have the radii shown. These radii were first calculated by Bohr and are given by the equation  $r_n = \frac{n^2}{Z} a_{\rm B}$ . The lowest orbit has the experimentally verified diameter of a hydrogen atom.

To get the electron orbital energies, we start by noting that the electron energy is the sum of its kinetic and potential energy:

# **Equation:**

$$E_n = KE + PE$$
.

Kinetic energy is the familiar  $\mathrm{KE}=(1/2)m_ev^2$ , assuming the electron is not moving at relativistic speeds. Potential energy for the electron is electrical, or  $\mathrm{PE}=q_eV$ , where V is the potential due to the nucleus, which looks like a point charge. The nucleus has a positive charge  $\mathrm{Zq}_e$ ; thus,  $V=\mathrm{kZq}_e/r_n$ , recalling an earlier equation for the potential due to a point charge. Since the electron's charge is negative, we see that  $\mathrm{PE}=-\mathrm{kZq}_e/r_n$ . Entering the expressions for  $\mathrm{KE}$  and  $\mathrm{PE}$ , we find

# **Equation:**

$$E_n=rac{1}{2}m_ev^2-krac{Zq_e^2}{r_n}.$$

Now we substitute  $r_n$  and v from earlier equations into the above expression for energy. Algebraic manipulation yields

# **Equation:**

$$E_n = -rac{Z^2}{n^2}E_0(n=1,2,3,...)$$

for the orbital **energies of hydrogen-like atoms**. Here,  $E_0$  is the **ground-state energy** (n=1) for hydrogen (Z=1) and is given by

# **Equation:**

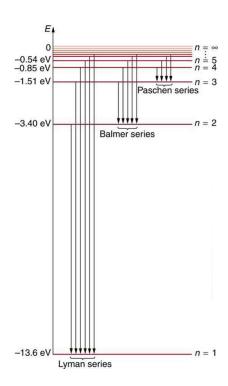
$$E_0 = rac{2\pi^2 q_e^4 m_e k^2}{h^2} = 13.6 ext{ eV}.$$

Thus, for hydrogen,

# **Equation:**

$$E_n = -rac{13.6 ext{ eV}}{n^2} (n=1,2,3,...).$$

[link] shows an energy-level diagram for hydrogen that also illustrates how the various spectral series for hydrogen are related to transitions between energy levels.



Energy-level diagram for

hydrogen showing the Lyman, Balmer, and Paschen series of transitions. The orbital energies are calculated using the above equation, first derived by Bohr.

Electron total energies are negative, since the electron is bound to the nucleus, analogous to being in a hole without enough kinetic energy to escape. As n approaches infinity, the total energy becomes zero. This corresponds to a free electron with no kinetic energy, since  $r_n$  gets very large for large n, and the electric potential energy thus becomes zero. Thus, 13.6 eV is needed to ionize hydrogen (to go from -13.6 eV to 0, or unbound), an experimentally verified number. Given more energy, the electron becomes unbound with some kinetic energy. For example, giving 15.0 eV to an electron in the ground state of hydrogen strips it from the atom and leaves it with 1.4 eV of kinetic energy.

Finally, let us consider the energy of a photon emitted in a downward transition, given by the equation to be

## **Equation:**

$$\Delta E = \mathrm{hf} = E_{\mathrm{i}} - E_{\mathrm{f}}$$
.

Substituting  $E_n = (-13.6 \text{ eV}/n^2)$ , we see that

## **Equation:**

$${
m hf} = (13.6~{
m eV}) \Bigg(rac{1}{n_{
m f}^2} - rac{1}{n_{
m i}^2}\Bigg).$$

Dividing both sides of this equation by hc gives an expression for  $1/\lambda$ :

### **Equation:**

$$rac{ ext{hf}}{ ext{hc}} = rac{f}{c} = rac{1}{\lambda} = rac{(13.6 ext{ eV})}{hc} igg(rac{1}{n_{ ext{f}}^2} - rac{1}{n_{ ext{i}}^2}igg).$$

It can be shown that

### **Equation:**

$$\left(rac{13.6~{
m eV}}{hc}
ight) = rac{(13.6~{
m eV})ig(1.602 imes10^{-19}~{
m J/eV}ig)}{ig(6.626 imes10^{-34}~{
m J\cdot s}ig)ig(2.998 imes10^8~{
m m/s}ig)} = 1.097 imes10^7~{
m m}^{-1} = R$$

is the **Rydberg constant**. Thus, we have used Bohr's assumptions to derive the formula first proposed by Balmer years earlier as a recipe to fit experimental data.

### **Equation:**

$$rac{1}{\lambda} = R igg(rac{1}{n_{
m f}^2} - rac{1}{n_{
m i}^2}igg)$$

We see that Bohr's theory of the hydrogen atom answers the question as to why this previously known formula describes the hydrogen spectrum. It is because the energy levels are proportional to  $1/n^2$ , where n is a non-negative integer. A downward transition releases energy, and so  $n_i$  must be greater than  $n_f$ . The various series are those where the transitions end on a certain level. For the Lyman series,  $n_f = 1$  — that is, all the transitions end in the ground state (see also  $[\underline{\text{link}}]$ ). For the Balmer series,  $n_f = 2$ , or all the transitions end in the first excited state; and so on. What was once a recipe is now based in physics, and something new is emerging—angular momentum is quantized.

# **Triumphs and Limits of the Bohr Theory**

Bohr did what no one had been able to do before. Not only did he explain the spectrum of hydrogen, he correctly calculated the size of the atom from basic physics. Some of his ideas are broadly applicable. Electron orbital energies are quantized in all atoms and molecules. Angular momentum is quantized. The electrons do not spiral into the nucleus, as expected classically (accelerated charges radiate, so that the electron orbits classically would decay quickly, and the electrons would sit on the nucleus—matter would collapse). These are major triumphs.

But there are limits to Bohr's theory. It cannot be applied to multielectron atoms, even one as simple as a two-electron helium atom. Bohr's model is what we call *semiclassical*. The orbits are quantized (nonclassical) but are assumed to be simple circular paths (classical). As quantum mechanics was developed, it became clear that there are no well-defined orbits; rather, there are clouds of probability. Bohr's theory also did not explain that some spectral lines are doublets (split into two) when examined closely. We shall examine many of these aspects of quantum mechanics in more detail, but it should be kept in mind that Bohr did not fail. Rather, he made very important steps along the path to greater knowledge and laid the foundation for all of atomic physics that has since evolved.

### Note:

PhET Explorations: Models of the Hydrogen Atom

How did scientists figure out the structure of atoms without looking at them? Try out different models by shooting light at the atom. Check how the prediction of the model

matches the experimental results.

https://archive.cnx.org/specials/d77cc1d0-33e4-11e6-b016-6726afecd2be/hydrogen-atom/#sim-hydrogen-atom

# **Section Summary**

• The planetary model of the atom pictures electrons orbiting the nucleus in the way that planets orbit the sun. Bohr used the planetary model to develop the first reasonable theory of hydrogen, the simplest atom. Atomic and molecular spectra are quantized, with hydrogen spectrum wavelengths given by the formula **Equation:** 

$$rac{1}{\lambda} = Rigg(rac{1}{n_{
m f}^2} - rac{1}{n_{
m i}^2}igg),$$

where  $\lambda$  is the wavelength of the emitted EM radiation and R is the Rydberg constant, which has the value

**Equation:** 

$$R = 1.097 \times 10^7 \text{ m}^{-1}$$
.

- The constants  $n_i$  and  $n_f$  are positive integers, and  $n_i$  must be greater than  $n_f$ .
- Bohr correctly proposed that the energy and radii of the orbits of electrons in atoms are quantized, with energy for transitions between orbits given by Equation:

$$\Delta E = hf = E_{\mathrm{i}} - E_{\mathrm{f}},$$

where  $\Delta E$  is the change in energy between the initial and final orbits and hf is the energy of an absorbed or emitted photon. It is useful to plot orbital energies on a vertical graph called an energy-level diagram.

 Bohr proposed that the allowed orbits are circular and must have quantized orbital angular momentum given by
 Equation:

$$L=m_e v r_n=nrac{h}{2\pi}(n=1,2,3\ldots),$$

where L is the angular momentum,  $r_n$  is the radius of the nth orbit, and h is Planck's constant. For all one-electron (hydrogen-like) atoms, the radius of an orbit is given by **Equation:** 

$$r_n = rac{n^2}{Z} a_{
m B} ext{(allowed orbits } n=1,2,3,...),$$

Z is the atomic number of an element (the number of electrons is has when neutral) and  $a_{\rm B}$  is defined to be the Bohr radius, which is

## **Equation:**

$$a_{
m B} = rac{h^2}{4\pi^2 m_e k q_e^2} = 0.529 imes 10^{-10} \ {
m m}.$$

• Furthermore, the energies of hydrogen-like atoms are given by **Equation:** 

$$E_n = -rac{Z^2}{n^2}E_0(n=1,2,3...),$$

where  $E_0$  is the ground-state energy and is given by **Equation:** 

$$E_0 = rac{2\pi^2 q_e^4 m_e k^2}{h^2} = 13.6 \; \mathrm{eV}.$$

Thus, for hydrogen, **Equation:** 

$$E_n = -rac{13.6 ext{ eV}}{n^2} (n, =, 1, 2, 3 ...).$$

• The Bohr Theory gives accurate values for the energy levels in hydrogen-like atoms, but it has been improved upon in several respects.

# **Conceptual Questions**

### **Exercise:**

#### **Problem:**

How do the allowed orbits for electrons in atoms differ from the allowed orbits for planets around the sun? Explain how the correspondence principle applies here.

#### **Exercise:**

## **Problem:**

Explain how Bohr's rule for the quantization of electron orbital angular momentum differs from the actual rule.

### **Exercise:**

### **Problem:**

What is a hydrogen-like atom, and how are the energies and radii of its electron orbits related to those in hydrogen?

# **Problems & Exercises**

### **Exercise:**

### **Problem:**

By calculating its wavelength, show that the first line in the Lyman series is UV radiation.

### **Solution:**

$$rac{1}{\lambda}=R\Big(rac{1}{n_{
m f}^2}-rac{1}{n_{
m i}^2}\Big)\Rightarrow \lambda=rac{1}{R}\left[rac{(n_{
m i}\cdot n_{
m f})^2}{n_{
m i}^2-n_{
m f}^2}
ight]; n_{
m i}=2,\, n_{
m f}=1,\,\,{
m so}\,\,{
m that}$$

$$\lambda=\left(\frac{m}{1.097\times10^7}\right)\left\lceil\frac{(2\times1)^2}{2^2-1^2}\right\rceil=1.22\times10^{-7}~m=122~nm$$
 , which is UV radiation.

### **Exercise:**

#### **Problem:**

Find the wavelength of the third line in the Lyman series, and identify the type of EM radiation.

### **Exercise:**

### **Problem:**

Look up the values of the quantities in  $a_{\rm B}=\frac{h^2}{4\pi^2m_ekq_e^2}$ , and verify that the Bohr radius  $a_{\rm B}$  is  $0.529\times 10^{-10}$  m.

## **Solution:**

$$a_{
m B}=rac{h^2}{4\pi^2 m_e k Z q_e^2}=rac{(6.626 imes 10^{-34}~{
m J}\cdot{
m s})^2}{4\pi^2 (9.109 imes 10^{-31}~{
m kg})(8.988 imes 10^9~{
m N}\cdot{
m m}^2/{
m C}^2)(1)(1.602 imes 10^{-19}~{
m C})^2}=0.529 imes 10^{-10}~{
m m}$$

#### **Exercise:**

**Problem:** Verify that the ground state energy  $E_0$  is 13.6 eV by using  $E_0 = \frac{2\pi^2 q_e^4 m_e k^2}{h^2}$ .

### **Exercise:**

### **Problem:**

If a hydrogen atom has its electron in the n=4 state, how much energy in eV is needed to ionize it?

### **Solution:**

0.850 eV

#### **Exercise:**

### **Problem:**

A hydrogen atom in an excited state can be ionized with less energy than when it is in its ground state. What is n for a hydrogen atom if 0.850 eV of energy can ionize it?

### **Exercise:**

### **Problem:**

Find the radius of a hydrogen atom in the n=2 state according to Bohr's theory.

### **Solution:**

$$2.12 \times 10^{-10} \; \mathrm{m}$$

### **Exercise:**

### **Problem:**

Show that  $(13.6~{\rm eV})/{\rm hc}=1.097\times 10^7~{\rm m}=R$  (Rydberg's constant), as discussed in the text.

### **Exercise:**

#### **Problem:**

What is the smallest-wavelength line in the Balmer series? Is it in the visible part of the spectrum?

# **Solution:**

365 nm

It is in the ultraviolet.

### **Exercise:**

#### **Problem:**

Show that the entire Paschen series is in the infrared part of the spectrum. To do this, you only need to calculate the shortest wavelength in the series.

### **Exercise:**

### **Problem:**

Do the Balmer and Lyman series overlap? To answer this, calculate the shortest-wavelength Balmer line and the longest-wavelength Lyman line.

### **Solution:**

No overlap

365 nm

122 nm

#### **Exercise:**

#### **Problem:**

- (a) Which line in the Balmer series is the first one in the UV part of the spectrum?
- (b) How many Balmer series lines are in the visible part of the spectrum?
- (c) How many are in the UV?

### **Exercise:**

### **Problem:**

A wavelength of  $4.653~\mu m$  is observed in a hydrogen spectrum for a transition that ends in the  $n_{\rm f}=5$  level. What was  $n_{\rm i}$  for the initial level of the electron?

### **Solution:**

7

# **Exercise:**

### **Problem:**

A singly ionized helium ion has only one electron and is denoted He<sup>+</sup>. What is the ion's radius in the ground state compared to the Bohr radius of hydrogen atom?

### **Exercise:**

### **Problem:**

A beryllium ion with a single electron (denoted  $Be^{3+}$ ) is in an excited state with radius the same as that of the ground state of hydrogen.

(a) What is n for the  $Be^{3+}$  ion?

(b) How much energy in eV is needed to ionize the ion from this excited state?

### **Solution:**

- (a) 2
- (b) 54.4 eV

#### **Exercise:**

### **Problem:**

Atoms can be ionized by thermal collisions, such as at the high temperatures found in the solar corona. One such ion is  $C^{+5}$ , a carbon atom with only a single electron.

- (a) By what factor are the energies of its hydrogen-like levels greater than those of hydrogen?
- (b) What is the wavelength of the first line in this ion's Paschen series?
- (c) What type of EM radiation is this?

### **Exercise:**

### **Problem:**

Verify Equations  $r_n=\frac{n^2}{Z}a_{\rm B}$  and  $a_{\rm B}=\frac{h^2}{4\pi^2m_ekq_e^2}=0.529\times 10^{-10}$  m using the approach stated in the text. That is, equate the Coulomb and centripetal forces and then insert an expression for velocity from the condition for angular momentum quantization.

### **Solution:**

$$rac{kZq_e^2}{r_n^2}=rac{m_eV^2}{r_n}$$
, so that  $r_n=rac{kZq_e^2}{m_eV^2}=rac{kZq_e^2}{m_e}rac{1}{V^2}$ . From the equation  $m_evr_n=nrac{h}{2\pi}$ , we can substitute for the velocity, giving:  $r_n=rac{kZq_e^2}{m_e}\cdotrac{4\pi^2m_e^2r_n^2}{n^2h^2}$  so that  $r_n=rac{n^2}{Z}rac{h^2}{4\pi^2m_ekq_e^2}=rac{n^2}{Z}a_{
m B}$ , where  $a_{
m B}=rac{h^2}{4\pi^2m_ekq_e^2}$ .

#### **Exercise:**

### **Problem:**

The wavelength of the four Balmer series lines for hydrogen are found to be 410.3, 434.2, 486.3, and 656.5 nm. What average percentage difference is found between these wavelength numbers and those predicted by  $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_1^2}\right)$ ? It is amazing how well a simple formula (disconnected originally from theory) could duplicate this phenomenon.

# Glossary

hydrogen spectrum wavelengths

the wavelengths of visible light from hydrogen; can be calculated by

$$rac{1}{\lambda} = R \Big(rac{1}{n_{
m f}^2} - rac{1}{n_{
m i}^2}\Big)$$

Rydberg constant

a physical constant related to the atomic spectra with an established value of  $1.097\times10^7\ m^{-1}$ 

double-slit interference

an experiment in which waves or particles from a single source impinge upon two slits so that the resulting interference pattern may be observed

energy-level diagram

a diagram used to analyze the energy level of electrons in the orbits of an atom

Bohr radius

the mean radius of the orbit of an electron around the nucleus of a hydrogen atom in its ground state

hydrogen-like atom

any atom with only a single electron

energies of hydrogen-like atoms

Bohr formula for energies of electron states in hydrogen-like atoms:

$$E_n = -rac{Z^2}{n^2} E_0(n=1,2,3,\ldots)$$

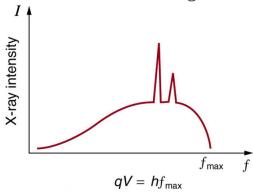
# X Rays: Atomic Origins and Applications

- Define x-ray tube and its spectrum.
- Show the x-ray characteristic energy.
- Specify the use of x rays in medical observations.
- Explain the use of x rays in CT scanners in diagnostics.

Each type of atom (or element) has its own characteristic electromagnetic spectrum. **X rays** lie at the high-frequency end of an atom's spectrum and are characteristic of the atom as well. In this section, we explore characteristic x rays and some of their important applications.

We have previously discussed x rays as a part of the electromagnetic spectrum in <u>Photon Energies and the Electromagnetic Spectrum</u>. That module illustrated how an x-ray tube (a specialized CRT) produces x rays. Electrons emitted from a hot filament are accelerated with a high voltage, gaining significant kinetic energy and striking the anode.

There are two processes by which x rays are produced in the anode of an x-ray tube. In one process, the deceleration of electrons produces x rays, and these x rays are called *bremsstrahlung*, or braking radiation. The second process is atomic in nature and produces *characteristic* x rays, so called because they are characteristic of the anode material. The x-ray spectrum in [link] is typical of what is produced by an x-ray tube, showing a broad curve of bremsstrahlung radiation with characteristic x-ray peaks on it.

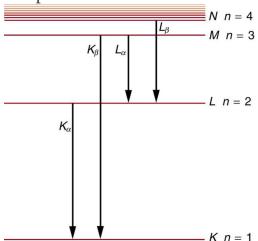


X-ray spectrum obtained when energetic electrons strike a material, such as in the anode of a CRT.
The smooth part of the spectrum is bremsstrahlung radiation, while the peaks are characteristic of the anode material. A different anode material would have characteristic x-ray peaks at different frequencies.

The spectrum in [link] is collected over a period of time in which many electrons strike the anode, with a variety of possible outcomes for each hit. The broad range of x-ray energies in the bremsstrahlung radiation indicates that an incident electron's energy is not usually converted entirely into photon energy. The highest-energy x ray produced is one for which all of the electron's energy was converted to photon energy. Thus the accelerating voltage and the maximum x-ray energy are related by conservation of energy. Electric potential energy is converted to kinetic energy and then to photon energy, so that  $E_{\rm max} = h f_{\rm max} = q_e V$ . Units of electron volts are convenient. For example, a 100-kV accelerating voltage produces x-ray photons with a maximum energy of 100 keV.

Some electrons excite atoms in the anode. Part of the energy that they deposit by collision with an atom results in one or more of the atom's inner electrons being knocked into a higher orbit or the atom being ionized. When the anode's atoms de-excite, they emit characteristic electromagnetic radiation. The most energetic of these are produced when an inner-shell vacancy is filled—that is, when an n=1 or n=2 shell electron has been excited to a higher level, and another electron falls into the vacant spot. A *characteristic x ray* (see <u>Photon Energies and the Electromagnetic Spectrum</u>) is electromagnetic (EM) radiation emitted by an atom when an inner-shell vacancy is filled. [<u>link</u>] shows a representative energy-level diagram that illustrates the labeling of characteristic x rays. X rays created

when an electron falls into an n=1 shell vacancy are called  $K_{\alpha}$  when they come from the next higher level; that is, an n=2 to n=1 transition. The labels  $K, L, M, \ldots$  come from the older alphabetical labeling of shells starting with K rather than using the principal quantum numbers  $1, 2, 3, \ldots$ . A more energetic  $K_{\beta}$  x ray is produced when an electron falls into an n=1 shell vacancy from the n=3 shell; that is, an n=3 to n=1 transition. Similarly, when an electron falls into the n=2 shell from the n=3 shell, an  $L_{\alpha}$  x ray is created. The energies of these x rays depend on the energies of electron states in the particular atom and, thus, are characteristic of that element: every element has it own set of x-ray energies. This property can be used to identify elements, for example, to find trace (small) amounts of an element in an environmental or biological sample.



A characteristic x ray is emitted when an electron fills an inner-shell vacancy, as shown for several transitions in this approximate energy level diagram for a multiple-electron atom.

Characteristic x rays are labeled according to the shell that had the vacancy and the shell from which

the electron came. A  $K_{\alpha}$  x ray, for example, is produced when an electron coming from the n=2 shell fills the n=1 shell vacancy.

### **Example:**

### **Characteristic X-Ray Energy**

Calculate the approximate energy of a  $K_{\alpha}$  x ray from a tungsten anode in an x-ray tube.

### **Strategy**

How do we calculate energies in a multiple-electron atom? In the case of characteristic x rays, the following approximate calculation is reasonable. Characteristic x rays are produced when an inner-shell vacancy is filled. Inner-shell electrons are nearer the nucleus than others in an atom and thus feel little net effect from the others. This is similar to what happens inside a charged conductor, where its excess charge is distributed over the surface so that it produces no electric field inside. It is reasonable to assume the inner-shell electrons have hydrogen-like energies, as given by  $E_n = -\frac{Z^2}{n^2} E_0(n=1,2,3,\ldots).$  As noted, a  $K_\alpha$  x ray is produced by an n=2 to n=1 transition. Since there are two electrons in a filled K shell, a vacancy would leave one electron, so that the effective charge would be Z-1 rather than Z. For tungsten, Z=74, so that the effective charge is 73.

#### Solution

 $E_n=-\frac{Z^2}{n^2}E_0 (n=1,2,3,...)$  gives the orbital energies for hydrogenlike atoms to be  $E_n=-(Z^2/n^2)E_0$ , where  $E_0=13.6$  eV. As noted, the effective Z is 73. Now the  $K_\alpha$  x-ray energy is given by

# **Equation:**

$$E_{K_lpha} = \Delta E = E_\mathrm{i} - E_\mathrm{f} = E_2 - E_1,$$

where

# **Equation:**

$$E_1 = -rac{Z^2}{1^2}E_0 = -rac{73^2}{1}igg(13.6~{
m eV}igg) = -72.5~{
m keV}$$

and

### **Equation:**

$$E_2 = -rac{Z^2}{2^2}E_0 = -rac{73^2}{4}igg(13.6~{
m eV}igg) = -18.1~{
m keV}.$$

Thus,

### **Equation:**

$$E_{K_{\alpha}} = -18.1 \text{ keV} - (-72.5 \text{ keV}) = 54.4 \text{ keV}.$$

#### Discussion

This large photon energy is typical of characteristic x rays from heavy elements. It is large compared with other atomic emissions because it is produced when an inner-shell vacancy is filled, and inner-shell electrons are tightly bound. Characteristic x ray energies become progressively larger for heavier elements because their energy increases approximately as  $Z^2$ . Significant accelerating voltage is needed to create these inner-shell vacancies. In the case of tungsten, at least 72.5 kV is needed, because other shells are filled and you cannot simply bump one electron to a higher filled shell. Tungsten is a common anode material in x-ray tubes; so much of the energy of the impinging electrons is absorbed, raising its temperature, that a high-melting-point material like tungsten is required.

# **Medical and Other Diagnostic Uses of X-rays**

All of us can identify diagnostic uses of x-ray photons. Among these are the universal dental and medical x rays that have become an essential part of medical diagnostics. (See [link] and [link].) X rays are also used to inspect

our luggage at airports, as shown in [link], and for early detection of cracks in crucial aircraft components. An x ray is not only a noun meaning highenergy photon, it also is an image produced by x rays, and it has been made into a familiar verb—to be x-rayed.



An x-ray image reveals fillings in a person's teeth. (credit: Dmitry G, Wikimedia Commons)



This x-ray image of a person's chest shows many details, including an artificial pacemaker. (credit: Sunzi99,

# Wikimedia Commons)



This x-ray image shows the contents of a piece of luggage.
The denser the material, the darker the shadow. (credit: IDuke, Wikimedia Commons)

The most common x-ray images are simple shadows. Since x-ray photons have high energies, they penetrate materials that are opaque to visible light. The more energy an x-ray photon has, the more material it will penetrate. So an x-ray tube may be operated at 50.0 kV for a chest x ray, whereas it may need to be operated at 100 kV to examine a broken leg in a cast. The depth of penetration is related to the density of the material as well as to the energy of the photon. The denser the material, the fewer x-ray photons get through and the darker the shadow. Thus x rays excel at detecting breaks in bones and in imaging other physiological structures, such as some tumors, that differ in density from surrounding material. Because of their high photon energy, x rays produce significant ionization in materials and

damage cells in biological organisms. Modern uses minimize exposure to the patient and eliminate exposure to others. Biological effects of x rays will be explored in the next chapter along with other types of ionizing radiation such as those produced by nuclei.

As the x-ray energy increases, the Compton effect (see Photon Momentum) becomes more important in the attenuation of the x rays. Here, the x ray scatters from an outer electron shell of the atom, giving the ejected electron some kinetic energy while losing energy itself. The probability for attenuation of the x rays depends upon the number of electrons present (the material's density) as well as the thickness of the material. Chemical composition of the medium, as characterized by its atomic number Z, is not important here. Low-energy x rays provide better contrast (sharper images). However, due to greater attenuation and less scattering, they are more absorbed by thicker materials. Greater contrast can be achieved by injecting a substance with a large atomic number, such as barium or iodine. The structure of the part of the body that contains the substance (e.g., the gastro-intestinal tract or the abdomen) can easily be seen this way.

Breast cancer is the second-leading cause of death among women worldwide. Early detection can be very effective, hence the importance of x-ray diagnostics. A mammogram cannot diagnose a malignant tumor, only give evidence of a lump or region of increased density within the breast. X-ray absorption by different types of soft tissue is very similar, so contrast is difficult; this is especially true for younger women, who typically have denser breasts. For older women who are at greater risk of developing breast cancer, the presence of more fat in the breast gives the lump or tumor more contrast. MRI (Magnetic resonance imaging) has recently been used as a supplement to conventional x rays to improve detection and eliminate false positives. The subject's radiation dose from x rays will be treated in a later chapter.

A standard x ray gives only a two-dimensional view of the object. Dense bones might hide images of soft tissue or organs. If you took another x ray from the side of the person (the first one being from the front), you would gain additional information. While shadow images are sufficient in many applications, far more sophisticated images can be produced with modern

technology. [link] shows the use of a computed tomography (CT) scanner, also called computed axial tomography (CAT) scanner. X rays are passed through a narrow section (called a slice) of the patient's body (or body part) over a range of directions. An array of many detectors on the other side of the patient registers the x rays. The system is then rotated around the patient and another image is taken, and so on. The x-ray tube and detector array are mechanically attached and so rotate together. Complex computer image processing of the relative absorption of the x rays along different directions produces a highly-detailed image. Different slices are taken as the patient moves through the scanner on a table. Multiple images of different slices can also be computer analyzed to produce three-dimensional information, sometimes enhancing specific types of tissue, as shown in [link]. G. Hounsfield (UK) and A. Cormack (US) won the Nobel Prize in Medicine in 1979 for their development of computed tomography.



A patient being positioned in a CT scanner aboard the hospital ship USNS Mercy. The CT scanner passes x rays through slices of the patient's body (or body part) over a range of directions.

The relative absorption of the x rays along different

directions is computer analyzed to produce highly detailed images. Three-dimensional information can be obtained from multiple slices. (credit: Rebecca Moat, U.S. Navy)



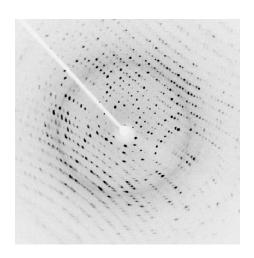
This three-dimensional image of a skull was produced by computed tomography, involving analysis of several x-ray slices of the head. (credit: Emailshankar, Wikimedia Commons)

# X-Ray Diffraction and Crystallography

Since x-ray photons are very energetic, they have relatively short wavelengths. For example, the 54.4-keV  $K_{\alpha}$  x ray of [link] has a

wavelength  $\lambda = \text{hc}/E = 0.0228$  nm. Thus, typical x-ray photons act like rays when they encounter macroscopic objects, like teeth, and produce sharp shadows; however, since atoms are on the order of 0.1 nm in size, x rays can be used to detect the location, shape, and size of atoms and molecules. The process is called **x-ray diffraction**, because it involves the diffraction and interference of x rays to produce patterns that can be analyzed for information about the structures that scattered the x rays. Perhaps the most famous example of x-ray diffraction is the discovery of the double-helix structure of DNA in 1953 by an international team of scientists working at the Cavendish Laboratory—American James Watson, Englishman Francis Crick, and New Zealand-born Maurice Wilkins. Using x-ray diffraction data produced by Rosalind Franklin, they were the first to discern the structure of DNA that is so crucial to life. For this, Watson, Crick, and Wilkins were awarded the 1962 Nobel Prize in Physiology or Medicine. There is much debate and controversy over the issue that Rosalind Franklin was not included in the prize.

[link] shows a diffraction pattern produced by the scattering of x rays from a crystal. This process is known as x-ray crystallography because of the information it can yield about crystal structure, and it was the type of data Rosalind Franklin supplied to Watson and Crick for DNA. Not only do x rays confirm the size and shape of atoms, they give information on the atomic arrangements in materials. For example, current research in high-temperature superconductors involves complex materials whose lattice arrangements are crucial to obtaining a superconducting material. These can be studied using x-ray crystallography.



X-ray diffraction from the crystal of a protein, hen egg lysozyme, produced this interference pattern. Analysis of the pattern yields information about the structure of the protein. (credit: Del45, Wikimedia Commons)

Historically, the scattering of x rays from crystals was used to prove that x rays are energetic EM waves. This was suspected from the time of the discovery of x rays in 1895, but it was not until 1912 that the German Max von Laue (1879–1960) convinced two of his colleagues to scatter x rays from crystals. If a diffraction pattern is obtained, he reasoned, then the x rays must be waves, and their wavelength could be determined. (The spacing of atoms in various crystals was reasonably well known at the time, based on good values for Avogadro's number.) The experiments were convincing, and the 1914 Nobel Prize in Physics was given to von Laue for his suggestion leading to the proof that x rays are EM waves. In 1915, the unique father-and-son team of Sir William Henry Bragg and his son Sir William Lawrence Bragg were awarded a joint Nobel Prize for inventing

the x-ray spectrometer and the then-new science of x-ray analysis. The elder Bragg had migrated to Australia from England just after graduating in mathematics. He learned physics and chemistry during his career at the University of Adelaide. The younger Bragg was born in Adelaide but went back to the Cavendish Laboratories in England to a career in x-ray and neutron crystallography; he provided support for Watson, Crick, and Wilkins for their work on unraveling the mysteries of DNA and to Max Perutz for his 1962 Nobel Prize-winning work on the structure of hemoglobin. Here again, we witness the enabling nature of physics—establishing instruments and designing experiments as well as solving mysteries in the biomedical sciences.

Certain other uses for x rays will be studied in later chapters. X rays are useful in the treatment of cancer because of the inhibiting effect they have on cell reproduction. X rays observed coming from outer space are useful in determining the nature of their sources, such as neutron stars and possibly black holes. Created in nuclear bomb explosions, x rays can also be used to detect clandestine atmospheric tests of these weapons. X rays can cause excitations of atoms, which then fluoresce (emitting characteristic EM radiation), making x-ray-induced fluorescence a valuable analytical tool in a range of fields from art to archaeology.

# **Section Summary**

- X rays are relatively high-frequency EM radiation. They are produced by transitions between inner-shell electron levels, which produce x rays characteristic of the atomic element, or by decelerating electrons.
- X rays have many uses, including medical diagnostics and x-ray diffraction.

# **Conceptual Questions**

#### **Exercise:**

#### **Problem:**

Explain why characteristic x rays are the most energetic in the EM emission spectrum of a given element.

#### **Exercise:**

#### **Problem:**

Why does the energy of characteristic x rays become increasingly greater for heavier atoms?

#### **Exercise:**

#### **Problem:**

Observers at a safe distance from an atmospheric test of a nuclear bomb feel its heat but receive none of its copious x rays. Why is air opaque to x rays but transparent to infrared?

#### **Exercise:**

#### **Problem:**

Lasers are used to burn and read CDs. Explain why a laser that emits blue light would be capable of burning and reading more information than one that emits infrared.

#### Exercise:

#### Problem:

Crystal lattices can be examined with x rays but not UV. Why?

#### **Exercise:**

#### **Problem:**

CT scanners do not detect details smaller than about 0.5 mm. Is this limitation due to the wavelength of x rays? Explain.

#### **Problem Exercises**

#### **Exercise:**

#### **Problem:**

(a) What is the shortest-wavelength x-ray radiation that can be generated in an x-ray tube with an applied voltage of 50.0 kV? (b) Calculate the photon energy in eV. (c) Explain the relationship of the photon energy to the applied voltage.

#### **Solution:**

- (a)  $0.248 \times 10^{-10} \text{ m}$
- (b) 50.0 keV
- (c) The photon energy is simply the applied voltage times the electron charge, so the value of the voltage in volts is the same as the value of the energy in electron volts.

#### **Exercise:**

#### **Problem:**

A color television tube also generates some x rays when its electron beam strikes the screen. What is the shortest wavelength of these x rays, if a 30.0-kV potential is used to accelerate the electrons? (Note that TVs have shielding to prevent these x rays from exposing viewers.)

#### **Exercise:**

#### **Problem:**

An x ray tube has an applied voltage of 100 kV. (a) What is the most energetic x-ray photon it can produce? Express your answer in electron volts and joules. (b) Find the wavelength of such an X–ray.

#### **Solution:**

(a) 
$$100 \times 10^3 \; \mathrm{eV}$$
,  $1.60 \times 10^{-14} \; \mathrm{J}$ 

(b) 
$$0.124 \times 10^{-10} \text{ m}$$

### **Exercise:**

### **Problem:**

The maximum characteristic x-ray photon energy comes from the capture of a free electron into a K shell vacancy. What is this photon energy in keV for tungsten, assuming the free electron has no initial kinetic energy?

#### **Exercise:**

#### **Problem:**

What are the approximate energies of the  $K_{\alpha}$  and  $K_{\beta}$  x rays for copper?

### **Solution:**

- (a) 8.00 keV
- (b) 9.48 keV

# Glossary

x rays

a form of electromagnetic radiation

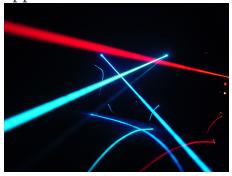
# x-ray diffraction

a technique that provides the detailed information about crystallographic structure of natural and manufactured materials

# Applications of Atomic Excitations and De-Excitations

- Define and discuss fluorescence.
- Define metastable.
- Describe how laser emission is produced.
- Explain population inversion.
- Define and discuss holography.

Many properties of matter and phenomena in nature are directly related to atomic energy levels and their associated excitations and de-excitations. The color of a rose, the output of a laser, and the transparency of air are but a few examples. (See [link].) While it may not appear that glow-in-the-dark pajamas and lasers have much in common, they are in fact different applications of similar atomic de-excitations.



Light from a laser is based on a particular type of atomic deexcitation. (credit: Jeff Keyzer)

The color of a material is due to the ability of its atoms to absorb certain wavelengths while reflecting or reemitting others. A simple red material, for example a tomato, absorbs all visible wavelengths except red. This is because the atoms of its hydrocarbon pigment (lycopene) have levels separated by a variety of energies corresponding to all visible photon energies except red. Air is another interesting example. It is transparent to visible light, because there are few energy levels that visible photons can

excite in air molecules and atoms. Visible light, thus, cannot be absorbed. Furthermore, visible light is only weakly scattered by air, because visible wavelengths are so much greater than the sizes of the air molecules and atoms. Light must pass through kilometers of air to scatter enough to cause red sunsets and blue skies.

# Fluorescence and Phosphorescence

The ability of a material to emit various wavelengths of light is similarly related to its atomic energy levels. [link] shows a scorpion illuminated by a UV lamp, sometimes called a black light. Some rocks also glow in black light, the particular colors being a function of the rock's mineral composition. Black lights are also used to make certain posters glow.



Objects glow in the visible spectrum when illuminated by an ultraviolet (black) light. Emissions are characteristic of the mineral involved, since they are related to its energy levels. In the case of scorpions, proteins near the surface of their skin give off the characteristic blue

glow. This is a colorful example of fluorescence in which excitation is induced by UV radiation while de-excitation occurs in the form of visible light. (credit: Ken Bosma, Flickr)

In the fluorescence process, an atom is excited to a level several steps above its ground state by the absorption of a relatively high-energy UV photon. This is called **atomic excitation**. Once it is excited, the atom can de-excite in several ways, one of which is to re-emit a photon of the same energy as excited it, a single step back to the ground state. This is called **atomic de-excitation**. All other paths of de-excitation involve smaller steps, in which lower-energy (longer wavelength) photons are emitted. Some of these may be in the visible range, such as for the scorpion in [link]. **Fluorescence** is defined to be any process in which an atom or molecule, excited by a photon of a given energy, and de-excites by emission of a lower-energy photon.

Fluorescence can be induced by many types of energy input. Fluorescent paint, dyes, and even soap residues in clothes make colors seem brighter in sunlight by converting some UV into visible light. X rays can induce fluorescence, as is done in x-ray fluoroscopy to make brighter visible images. Electric discharges can induce fluorescence, as in so-called neon lights and in gas-discharge tubes that produce atomic and molecular spectra. Common fluorescent lights use an electric discharge in mercury vapor to cause atomic emissions from mercury atoms. The inside of a fluorescent light is coated with a fluorescent material that emits visible light over a broad spectrum of wavelengths. By choosing an appropriate coating, fluorescent lights can be made more like sunlight or like the reddish glow of candlelight, depending on needs. Fluorescent lights are more efficient in converting electrical energy into visible light than incandescent filaments

(about four times as efficient), the blackbody radiation of which is primarily in the infrared due to temperature limitations.

This atom is excited to one of its higher levels by absorbing a UV photon. It can de-excite in a single step, re-emitting a photon of the same energy, or in several steps. The process is called fluorescence if the atom de-excites in smaller steps, emitting energy different from that which excited it. Fluorescence can be induced by a variety of energy inputs, such as UV, x-rays, and electrical discharge.

The spectacular Waitomo caves on North Island in New Zealand provide a natural habitat for glow-worms. The glow-worms hang up to 70 silk threads of about 30 or 40 cm each to trap prey that fly towards them in the dark. The fluorescence process is very efficient, with nearly 100% of the energy input turning into light. (In comparison, fluorescent lights are about 20% efficient.)

Fluorescence has many uses in biology and medicine. It is commonly used to label and follow a molecule within a cell. Such tagging allows one to study the structure of DNA and proteins. Fluorescent dyes and antibodies are usually used to tag the molecules, which are then illuminated with UV light and their emission of visible light is observed. Since the fluorescence of each element is characteristic, identification of elements within a sample can be done this way.

[link] shows a commonly used fluorescent dye called fluorescein. Below that, [link] reveals the diffusion of a fluorescent dye in water by observing it under UV light.



Fluorescein, shown here in powder form, is used to dye laboratory samples. (credit: Benjahbmm27, Wikimedia Commons)

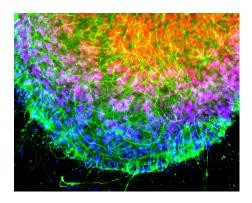


Here, fluorescent powder is added to a beaker of water. The mixture gives off a bright glow under ultraviolet light. (credit: Bricksnite, Wikimedia Commons)

#### Note:

### Nano-Crystals

Recently, a new class of fluorescent materials has appeared—"nanocrystals." These are single-crystal molecules less than 100 nm in size. The smallest of these are called "quantum dots." These semiconductor indicators are very small (2–6 nm) and provide improved brightness. They also have the advantage that all colors can be excited with the same incident wavelength. They are brighter and more stable than organic dyes and have a longer lifetime than conventional phosphors. They have become an excellent tool for long-term studies of cells, including migration and morphology. ([link].)



Microscopic image of chicken cells using nano-crystals of a fluorescent dye. Cell nuclei exhibit blue fluorescence while neurofilaments exhibit green. (credit:

Weerapong
Prasongchean,
Wikimedia Commons)

Once excited, an atom or molecule will usually spontaneously de-excite quickly. (The electrons raised to higher levels are attracted to lower ones by the positive charge of the nucleus.) Spontaneous de-excitation has a very short mean lifetime of typically about  $10^{-8}$  s. However, some levels have significantly longer lifetimes, ranging up to milliseconds to minutes or even hours. These energy levels are inhibited and are slow in de-exciting because their quantum numbers differ greatly from those of available lower levels. Although these level lifetimes are short in human terms, they are many orders of magnitude longer than is typical and, thus, are said to be metastable, meaning relatively stable. Phosphorescence is the deexcitation of a metastable state. Glow-in-the-dark materials, such as luminous dials on some watches and clocks and on children's toys and pajamas, are made of phosphorescent substances. Visible light excites the atoms or molecules to metastable states that decay slowly, releasing the stored excitation energy partially as visible light. In some ceramics, atomic excitation energy can be frozen in after the ceramic has cooled from its firing. It is very slowly released, but the ceramic can be induced to phosphoresce by heating—a process called "thermoluminescence." Since the release is slow, thermoluminescence can be used to date antiquities. The less light emitted, the older the ceramic. (See [link].)



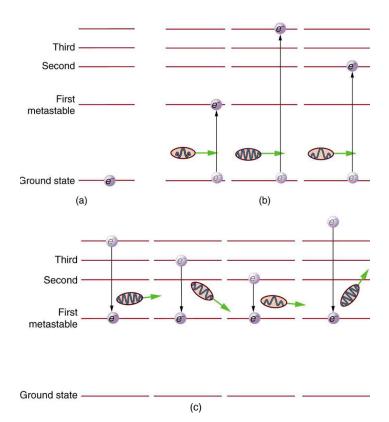
Atoms frozen in an excited state when this Chinese ceramic figure was fired can be stimulated to deexcite and emit EM radiation by heating a sample of the ceramic—a process called thermoluminescence . Since the states slowly de-excite over centuries, the amount of thermoluminescence decreases with age, making it possible to use this effect to date and authenticate antiquities. This figure dates from the 11<sup>th</sup> century. (credit: Vassil, Wikimedia Commons)

### Lasers

Lasers today are commonplace. Lasers are used to read bar codes at stores and in libraries, laser shows are staged for entertainment, laser printers produce high-quality images at relatively low cost, and lasers send prodigious numbers of telephone messages through optical fibers. Among other things, lasers are also employed in surveying, weapons guidance,

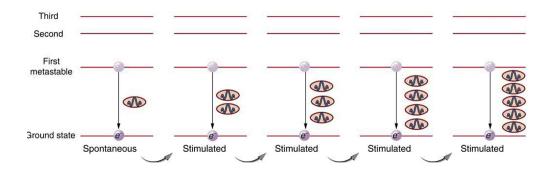
tumor eradication, retinal welding, and for reading music CDs and computer CD-ROMs.

Why do lasers have so many varied applications? The answer is that lasers produce single-wavelength EM radiation that is also very coherent—that is, the emitted photons are in phase. Laser output can, thus, be more precisely manipulated than incoherent mixed-wavelength EM radiation from other sources. The reason laser output is so pure and coherent is based on how it is produced, which in turn depends on a metastable state in the lasing material. Suppose a material had the energy levels shown in [link]. When energy is put into a large collection of these atoms, electrons are raised to all possible levels. Most return to the ground state in less than about  $10^{-8}$  s, but those in the metastable state linger. This includes those electrons originally excited to the metastable state and those that fell into it from above. It is possible to get a majority of the atoms into the metastable state, a condition called a **population inversion**.



(a) Energy-level diagram for an atom showing the first few states, one of which is metastable. (b) Massive energy input excites atoms to a variety of states. (c) Most states decay quickly, leaving electrons only in the metastable and ground state. If a majority of electrons are in the metastable state, a population inversion has been achieved.

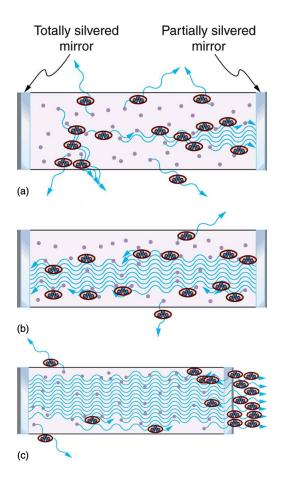
Once a population inversion is achieved, a very interesting thing can happen, as shown in [link]. An electron spontaneously falls from the metastable state, emitting a photon. This photon finds another atom in the metastable state and stimulates it to decay, emitting a second photon of the same wavelength and in phase with the first, and so on. Stimulated **emission** is the emission of electromagnetic radiation in the form of photons of a given frequency, triggered by photons of the same frequency. For example, an excited atom, with an electron in an energy orbit higher than normal, releases a photon of a specific frequency when the electron drops back to a lower energy orbit. If this photon then strikes another electron in the same high-energy orbit in another atom, another photon of the same frequency is released. The emitted photons and the triggering photons are always in phase, have the same polarization, and travel in the same direction. The probability of absorption of a photon is the same as the probability of stimulated emission, and so a majority of atoms must be in the metastable state to produce energy. Einstein (again Einstein, and back in 1917!) was one of the important contributors to the understanding of stimulated emission of radiation. Among other things, Einstein was the first to realize that stimulated emission and absorption are equally probable. The laser acts as a temporary energy storage device that subsequently produces a massive energy output of single-wavelength, in-phase photons.



One atom in the metastable state spontaneously decays to a lower level, producing a photon that goes on to stimulate another atom to de-excite. The second photon has exactly the same energy and wavelength as the first and is in phase with it. Both go on to stimulate the emission of other photons. A population inversion is necessary for there to be a net production rather than a net absorption of the photons.

The name **laser** is an acronym for light amplification by stimulated emission of radiation, the process just described. The process was proposed and developed following the advances in quantum physics. A joint Nobel Prize was awarded in 1964 to American Charles Townes (1915–), and Nikolay Basov (1922–2001) and Aleksandr Prokhorov (1916–2002), from the Soviet Union, for the development of lasers. The Nobel Prize in 1981 went to Arthur Schawlow (1921-1999) for pioneering laser applications. The original devices were called masers, because they produced microwaves. The first working laser was created in 1960 at Hughes Research labs (CA) by T. Maiman. It used a pulsed high-powered flash lamp and a ruby rod to produce red light. Today the name laser is used for all such devices developed to produce a variety of wavelengths, including microwave, infrared, visible, and ultraviolet radiation. [link] shows how a laser can be constructed to enhance the stimulated emission of radiation. Energy input can be from a flash tube, electrical discharge, or other sources, in a process sometimes called optical pumping. A large percentage of the original pumping energy is dissipated in other forms, but a population inversion must be achieved. Mirrors can be used to enhance stimulated

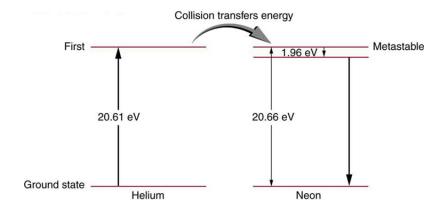
emission by multiple passes of the radiation back and forth through the lasing material. One of the mirrors is semitransparent to allow some of the light to pass through. The laser output from a laser is a mere 1% of the light passing back and forth in a laser.



Typical laser construction has a method of pumping energy into the lasing material to produce a population inversion. (a) Spontaneous emission begins with some photons escaping and others stimulating further emissions. (b) and (c)

Mirrors are used to enhance the probability of stimulated emission by passing photons through the material several times.

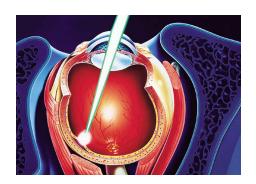
Lasers are constructed from many types of lasing materials, including gases, liquids, solids, and semiconductors. But all lasers are based on the existence of a metastable state or a phosphorescent material. Some lasers produce continuous output; others are pulsed in bursts as brief as  $10^{-14}$  s. Some laser outputs are fantastically powerful—some greater than  $10^{12}$  W —but the more common, everyday lasers produce something on the order of  $10^{-3}$  W. The helium-neon laser that produces a familiar red light is very common. [link] shows the energy levels of helium and neon, a pair of noble gases that work well together. An electrical discharge is passed through a helium-neon gas mixture in which the number of atoms of helium is ten times that of neon. The first excited state of helium is metastable and, thus, stores energy. This energy is easily transferred by collision to neon atoms, because they have an excited state at nearly the same energy as that in helium. That state in neon is also metastable, and this is the one that produces the laser output. (The most likely transition is to the nearby state, producing 1.96 eV photons, which have a wavelength of 633 nm and appear red.) A population inversion can be produced in neon, because there are so many more helium atoms and these put energy into the neon. Helium-neon lasers often have continuous output, because the population inversion can be maintained even while lasing occurs. Probably the most common lasers in use today, including the common laser pointer, are semiconductor or diode lasers, made of silicon. Here, energy is pumped into the material by passing a current in the device to excite the electrons. Special coatings on the ends and fine cleavings of the semiconductor material allow light to bounce back and forth and a tiny fraction to emerge as laser light. Diode lasers can usually run continually and produce outputs in the milliwatt range.



Energy levels in helium and neon. In the common helium-neon laser, an electrical discharge pumps energy into the metastable states of both atoms. The gas mixture has about ten times more helium atoms than neon atoms. Excited helium atoms easily deexcite by transferring energy to neon in a collision. A population inversion in neon is achieved, allowing lasing by the neon to

There are many medical applications of lasers. Lasers have the advantage that they can be focused to a small spot. They also have a well-defined wavelength. Many types of lasers are available today that provide wavelengths from the ultraviolet to the infrared. This is important, as one needs to be able to select a wavelength that will be preferentially absorbed by the material of interest. Objects appear a certain color because they absorb all other visible colors incident upon them. What wavelengths are absorbed depends upon the energy spacing between electron orbitals in that molecule. Unlike the hydrogen atom, biological molecules are complex and have a variety of absorption wavelengths or lines. But these can be determined and used in the selection of a laser with the appropriate wavelength. Water is transparent to the visible spectrum but will absorb light in the UV and IR regions. Blood (hemoglobin) strongly reflects red but absorbs most strongly in the UV.

Laser surgery uses a wavelength that is strongly absorbed by the tissue it is focused upon. One example of a medical application of lasers is shown in [link]. A detached retina can result in total loss of vision. Burns made by a laser focused to a small spot on the retina form scar tissue that can hold the retina in place, salvaging the patient's vision. Other light sources cannot be focused as precisely as a laser due to refractive dispersion of different wavelengths. Similarly, laser surgery in the form of cutting or burning away tissue is made more accurate because laser output can be very precisely focused and is preferentially absorbed because of its single wavelength. Depending upon what part or layer of the retina needs repairing, the appropriate type of laser can be selected. For the repair of tears in the retina, a green argon laser is generally used. This light is absorbed well by tissues containing blood, so coagulation or "welding" of the tear can be done.



A detached retina is burned by a laser designed to focus on a small spot on the retina, the resulting scar tissue holding it in place. The lens of the eye is used to focus the light, as is the device bringing the laser output to the eye.

In dentistry, the use of lasers is rising. Lasers are most commonly used for surgery on the soft tissue of the mouth. They can be used to remove ulcers, stop bleeding, and reshape gum tissue. Their use in cutting into bones and teeth is not quite so common; here the erbium YAG (yttrium aluminum garnet) laser is used.

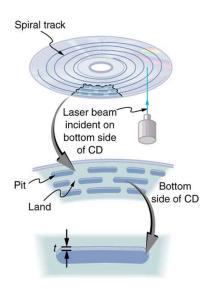
The massive combination of lasers shown in [link] can be used to induce nuclear fusion, the energy source of the sun and hydrogen bombs. Since lasers can produce very high power in very brief pulses, they can be used to focus an enormous amount of energy on a small glass sphere containing fusion fuel. Not only does the incident energy increase the fuel temperature significantly so that fusion can occur, it also compresses the fuel to great density, enhancing the probability of fusion. The compression or implosion is caused by the momentum of the impinging laser photons.



This system of lasers at Lawrence
Livermore Laboratory is used to ignite nuclear fusion. A tremendous burst of energy is focused on a small fuel pellet, which is imploded to the high density and temperature needed to make the fusion

reaction proceed.
(credit: Lawrence
Livermore National
Laboratory, Lawrence
Livermore National
Security, LLC, and the
Department of
Energy)

Music CDs are now so common that vinyl records are quaint antiquities. CDs (and DVDs) store information digitally and have a much larger information-storage capacity than vinyl records. An entire encyclopedia can be stored on a single CD. [link] illustrates how the information is stored and read from the CD. Pits made in the CD by a laser can be tiny and very accurately spaced to record digital information. These are read by having an inexpensive solid-state infrared laser beam scatter from pits as the CD spins, revealing their digital pattern and the information encoded upon them.



A CD has digital information

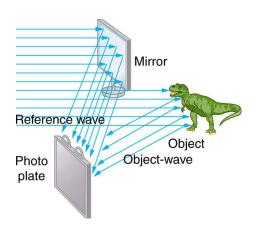
stored in the form of laser-created pits on its surface. These in turn can be read by detecting the laser light scattered from the pit. Large information capacity is possible because of the precision of the laser. Shorterwavelength lasers enable greater storage capacity.

Holograms, such as those in [link], are true three-dimensional images recorded on film by lasers. Holograms are used for amusement, decoration on novelty items and magazine covers, security on credit cards and driver's licenses (a laser and other equipment is needed to reproduce them), and for serious three-dimensional information storage. You can see that a hologram is a true three-dimensional image, because objects change relative position in the image when viewed from different angles.



Credit cards commonly
have holograms for logos,
making them difficult to
reproduce (credit:
Dominic Alves, Flickr)

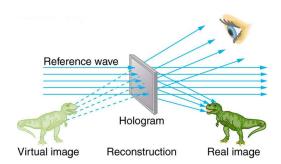
The name **hologram** means "entire picture" (from the Greek *holo*, as in holistic), because the image is three-dimensional. **Holography** is the process of producing holograms and, although they are recorded on photographic film, the process is quite different from normal photography. Holography uses light interference or wave optics, whereas normal photography uses geometric optics. [link] shows one method of producing a hologram. Coherent light from a laser is split by a mirror, with part of the light illuminating the object. The remainder, called the reference beam, shines directly on a piece of film. Light scattered from the object interferes with the reference beam, producing constructive and destructive interference. As a result, the exposed film looks foggy, but close examination reveals a complicated interference pattern stored on it. Where the interference was constructive, the film (a negative actually) is darkened. Holography is sometimes called lensless photography, because it uses the wave characteristics of light as contrasted to normal photography, which uses geometric optics and so requires lenses.



Production of a hologram. Single-wavelength coherent light from a laser produces a well-defined interference pattern on a piece of film. The laser beam is split by a partially silvered mirror, with part of the light illuminating the object and the remainder shining directly on the film.

Light falling on a hologram can form a three-dimensional image. The process is complicated in detail, but the basics can be understood as shown in [link], in which a laser of the same type that exposed the film is now used to illuminate it. The myriad tiny exposed regions of the film are dark and block the light, while less exposed regions allow light to pass. The film thus acts much like a collection of diffraction gratings with various spacings. Light passing through the hologram is diffracted in various directions, producing both real and virtual images of the object used to expose the film. The interference pattern is the same as that produced by the object. Moving

your eye to various places in the interference pattern gives you different perspectives, just as looking directly at the object would. The image thus looks like the object and is three-dimensional like the object.



A transmission hologram is one that produces real and virtual images when a laser of the same type as that which exposed the hologram is passed through it. Diffraction from various parts of the film produces the same interference pattern as the object that was used to expose it.

The hologram illustrated in [link] is a transmission hologram. Holograms that are viewed with reflected light, such as the white light holograms on credit cards, are reflection holograms and are more common. White light holograms often appear a little blurry with rainbow edges, because the diffraction patterns of various colors of light are at slightly different locations due to their different wavelengths. Further uses of holography include all types of 3-D information storage, such as of statues in museums and engineering studies of structures and 3-D images of human organs. Invented in the late 1940s by Dennis Gabor (1900–1970), who won the

1971 Nobel Prize in Physics for his work, holography became far more practical with the development of the laser. Since lasers produce coherent single-wavelength light, their interference patterns are more pronounced. The precision is so great that it is even possible to record numerous holograms on a single piece of film by just changing the angle of the film for each successive image. This is how the holograms that move as you walk by them are produced—a kind of lensless movie.

In a similar way, in the medical field, holograms have allowed complete 3-D holographic displays of objects from a stack of images. Storing these images for future use is relatively easy. With the use of an endoscope, high-resolution 3-D holographic images of internal organs and tissues can be made.

## **Section Summary**

- An important atomic process is fluorescence, defined to be any process in which an atom or molecule is excited by absorbing a photon of a given energy and de-excited by emitting a photon of a lower energy.
- Some states live much longer than others and are termed metastable.
- Phosphorescence is the de-excitation of a metastable state.
- Lasers produce coherent single-wavelength EM radiation by stimulated emission, in which a metastable state is stimulated to decay.
- Lasing requires a population inversion, in which a majority of the atoms or molecules are in their metastable state.

## **Conceptual Questions**

#### **Exercise:**

### **Problem:**

How do the allowed orbits for electrons in atoms differ from the allowed orbits for planets around the sun? Explain how the correspondence principle applies here.

### **Exercise:**

### **Problem:**

Atomic and molecular spectra are discrete. What does discrete mean, and how are discrete spectra related to the quantization of energy and electron orbits in atoms and molecules?

### **Exercise:**

### **Problem:**

Hydrogen gas can only absorb EM radiation that has an energy corresponding to a transition in the atom, just as it can only emit these discrete energies. When a spectrum is taken of the solar corona, in which a broad range of EM wavelengths are passed through very hot hydrogen gas, the absorption spectrum shows all the features of the emission spectrum. But when such EM radiation passes through room-temperature hydrogen gas, only the Lyman series is absorbed. Explain the difference.

## **Exercise:**

### **Problem:**

Lasers are used to burn and read CDs. Explain why a laser that emits blue light would be capable of burning and reading more information than one that emits infrared.

### **Exercise:**

### **Problem:**

The coating on the inside of fluorescent light tubes absorbs ultraviolet light and subsequently emits visible light. An inventor claims that he is able to do the reverse process. Is the inventor's claim possible?

### **Exercise:**

## **Problem:**

What is the difference between fluorescence and phosphorescence?

#### Exercise:

### **Problem:**

How can you tell that a hologram is a true three-dimensional image and that those in 3-D movies are not?

## **Problem Exercises**

### **Exercise:**

## **Problem:**

[link] shows the energy-level diagram for neon. (a) Verify that the energy of the photon emitted when neon goes from its metastable state to the one immediately below is equal to 1.96 eV. (b) Show that the wavelength of this radiation is 633 nm. (c) What wavelength is emitted when the neon makes a direct transition to its ground state?

## **Solution:**

- (a) 1.96 eV
- (b)  $(1240 \text{ eV} \cdot \text{nm})/(1.96 \text{ eV}) = 633 \text{ nm}$
- (c) 60.0 nm

### **Exercise:**

### **Problem:**

A helium-neon laser is pumped by electric discharge. What wavelength electromagnetic radiation would be needed to pump it? See [link] for energy-level information.

### **Exercise:**

### **Problem:**

Ruby lasers have chromium atoms doped in an aluminum oxide crystal. The energy level diagram for chromium in a ruby is shown in [link]. What wavelength is emitted by a ruby laser?

Third	3.0 eV
Second	2.3 eV
First	Metastable

Ground state  $\frac{}{\text{Ruby (Cr}^{3+} \text{in Al}_2 O_3 \, \text{crystal})} 0.0 \, \, \text{eV}$ 

Chromium atoms in an aluminum oxide crystal have these energy levels, one of which is metastable. This is the basis of a ruby laser. Visible light can pump the atom into an excited state above the metastable state to achieve a population inversion.

## **Solution:**

693 nm

### **Exercise:**

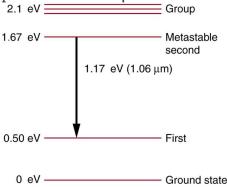
## **Problem:**

(a) What energy photons can pump chromium atoms in a ruby laser from the ground state to its second and third excited states? (b) What are the wavelengths of these photons? Verify that they are in the visible part of the spectrum.

### **Exercise:**

### **Problem:**

Some of the most powerful lasers are based on the energy levels of neodymium in solids, such as glass, as shown in [link]. (a) What average wavelength light can pump the neodymium into the levels above its metastable state? (b) Verify that the 1.17 eV transition produces  $1.06~\mu m$  radiation.



Neodymium atoms in glass have these energy levels, one of which is metastable. The group of levels above the metastable state is convenient for achieving a population inversion, since photons of many different energies can be absorbed by atoms in the ground state.

## **Solution:**

- (a) 590 nm
- (b)  $(1240 \text{ eV} \cdot \text{nm})/(1.17 \text{ eV}) = 1.06 \text{ } \mu\text{m}$

## **Glossary**

### metastable

a state whose lifetime is an order of magnitude longer than the most short-lived states

### atomic excitation

a state in which an atom or ion acquires the necessary energy to promote one or more of its electrons to electronic states higher in energy than their ground state

### atomic de-excitation

process by which an atom transfers from an excited electronic state back to the ground state electronic configuration; often occurs by emission of a photon

#### laser

acronym for light amplification by stimulated emission of radiation

## phosphorescence

the de-excitation of a metastable state

## population inversion

the condition in which the majority of atoms in a sample are in a metastable state

### stimulated emission

emission by atom or molecule in which an excited state is stimulated to decay, most readily caused by a photon of the same energy that is necessary to excite the state

## hologram

means *entire picture* (from the Greek word *holo*, as in holistic), because the image produced is three dimensional

## holography

the process of producing holograms

# fluorescence

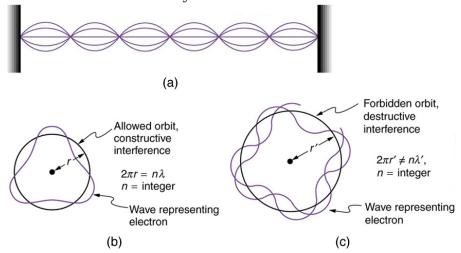
any process in which an atom or molecule, excited by a photon of a given energy, de-excites by emission of a lower-energy photon

## The Wave Nature of Matter Causes Quantization

- Explain Bohr's model of atom.
- Define and describe quantization of angular momentum.
- Calculate the angular momentum for an orbit of atom.
- Define and describe the wave-like properties of matter.

After visiting some of the applications of different aspects of atomic physics, we now return to the basic theory that was built upon Bohr's atom. Einstein once said it was important to keep asking the questions we eventually teach children not to ask. Why is angular momentum quantized? You already know the answer. Electrons have wave-like properties, as de Broglie later proposed. They can exist only where they interfere constructively, and only certain orbits meet proper conditions, as we shall see in the next module.

Following Bohr's initial work on the hydrogen atom, a decade was to pass before de Broglie proposed that matter has wave properties. The wave-like properties of matter were subsequently confirmed by observations of electron interference when scattered from crystals. Electrons can exist only in locations where they interfere constructively. How does this affect electrons in atomic orbits? When an electron is bound to an atom, its wavelength must fit into a small space, something like a standing wave on a string. (See [link].) Allowed orbits are those orbits in which an electron constructively interferes with itself. Not all orbits produce constructive interference. Thus only certain orbits are allowed—the orbits are quantized.



(a) Waves on a string have a wavelength related to the length of the string, allowing them to interfere constructively. (b) If we imagine the string bent into a closed circle, we get a rough idea of how electrons in circular orbits can interfere constructively. (c) If the wavelength does not fit into the circumference, the electron interferes destructively; it cannot exist in such an orbit.

For a circular orbit, constructive interference occurs when the electron's wavelength fits neatly into the circumference, so that wave crests always align with crests and wave troughs align with troughs, as shown in [link] (b). More precisely, when an integral multiple of the electron's wavelength equals the circumference of the orbit, constructive interference is obtained. In equation form, the *condition for constructive interference and an allowed electron orbit* is

## **Equation:**

$$n\lambda_n = 2\pi r_n (n=1,2,3 ...),$$

where  $\lambda_n$  is the electron's wavelength and  $r_n$  is the radius of that circular orbit. The de Broglie wavelength is  $\lambda = h/p = h/mv$ , and so here  $\lambda = h/m_ev$ . Substituting this into the previous condition for constructive interference produces an interesting result:

## **Equation:**

$$rac{\mathrm{nh}}{m_e v} = 2\pi r_n.$$

Rearranging terms, and noting that L = mvr for a circular orbit, we obtain the quantization of angular momentum as the condition for allowed orbits: **Equation:** 

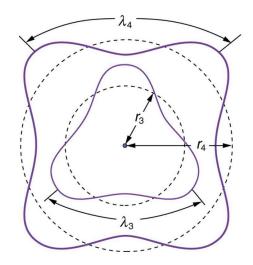
$$L = m_e {
m vr}_n = n rac{h}{2\pi} (n=1, \, 2, \, 3 \, ...).$$

This is what Bohr was forced to hypothesize as the rule for allowed orbits, as stated earlier. We now realize that it is the condition for constructive interference of an electron in a circular orbit. [link] illustrates this for n=3 and n=4.

## Note:

## Waves and Quantization

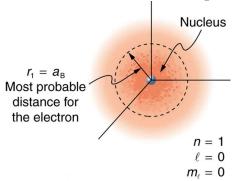
The wave nature of matter is responsible for the quantization of energy levels in bound systems. Only those states where matter interferes constructively exist, or are "allowed." Since there is a lowest orbit where this is possible in an atom, the electron cannot spiral into the nucleus. It cannot exist closer to or inside the nucleus. The wave nature of matter is what prevents matter from collapsing and gives atoms their sizes.



The third and fourth allowed circular orbits have three and four wavelengths,

respectively, in their circumferences.

Because of the wave character of matter, the idea of well-defined orbits gives way to a model in which there is a cloud of probability, consistent with Heisenberg's uncertainty principle. [link] shows how this applies to the ground state of hydrogen. If you try to follow the electron in some well-defined orbit using a probe that has a small enough wavelength to get some details, you will instead knock the electron out of its orbit. Each measurement of the electron's position will find it to be in a definite location somewhere near the nucleus. Repeated measurements reveal a cloud of probability like that in the figure, with each speck the location determined by a single measurement. There is not a well-defined, circular-orbit type of distribution. Nature again proves to be different on a small scale than on a macroscopic scale.



The ground state of a hydrogen atom has a probability cloud describing the position of its electron. The probability of finding the electron is proportional to the darkness of the cloud. The electron can be closer or farther than

the Bohr radius, but it is very unlikely to be a great distance from the nucleus.

There are many examples in which the wave nature of matter causes quantization in bound systems such as the atom. Whenever a particle is confined or bound to a small space, its allowed wavelengths are those which fit into that space. For example, the particle in a box model describes a particle free to move in a small space surrounded by impenetrable barriers. This is true in blackbody radiators (atoms and molecules) as well as in atomic and molecular spectra. Various atoms and molecules will have different sets of electron orbits, depending on the size and complexity of the system. When a system is large, such as a grain of sand, the tiny particle waves in it can fit in so many ways that it becomes impossible to see that the allowed states are discrete. Thus the correspondence principle is satisfied. As systems become large, they gradually look less grainy, and quantization becomes less evident. Unbound systems (small or not), such as an electron freed from an atom, do not have quantized energies, since their wavelengths are not constrained to fit in a certain volume.

#### Note:

## PhET Explorations: Quantum Wave Interference

When do photons, electrons, and atoms behave like particles and when do they behave like waves? Watch waves spread out and interfere as they pass through a double slit, then get detected on a screen as tiny dots. Use quantum detectors to explore how measurements change the waves and the patterns they produce on the screen.

<u>Quantum</u> <u>Wave</u>

## **Section Summary**

• Quantization of orbital energy is caused by the wave nature of matter. Allowed orbits in atoms occur for constructive interference of electrons in the orbit, requiring an integral number of wavelengths to fit in an orbit's circumference; that is,

## **Equation:**

$$n\lambda_n = 2\pi r_n (n = 1, 2, 3...),$$

where  $\lambda_n$  is the electron's de Broglie wavelength.

- Owing to the wave nature of electrons and the Heisenberg uncertainty principle, there are no well-defined orbits; rather, there are clouds of probability.
- Bohr correctly proposed that the energy and radii of the orbits of electrons in atoms are quantized, with energy for transitions between orbits given by

## **Equation:**

$$\Delta E = hf = E_{
m i} - E_{
m f},$$

where  $\Delta E$  is the change in energy between the initial and final orbits and hf is the energy of an absorbed or emitted photon.

- It is useful to plot orbit energies on a vertical graph called an energy-level diagram.
- The allowed orbits are circular, Bohr proposed, and must have quantized orbital angular momentum given by Equation:

$$L=m_{e}vr_{n}=nrac{h}{2\pi}(n=1,2,3\ ...),$$

where L is the angular momentum,  $r_n$  is the radius of orbit n, and h is Planck's constant.

# **Conceptual Questions**

## **Exercise:**

## **Problem:**

How is the de Broglie wavelength of electrons related to the quantization of their orbits in atoms and molecules?

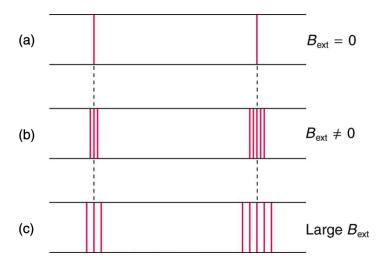
## Patterns in Spectra Reveal More Quantization

- State and discuss the Zeeman effect.
- Define orbital magnetic field.
- Define orbital angular momentum.
- Define space quantization.

High-resolution measurements of atomic and molecular spectra show that the spectral lines are even more complex than they first appear. In this section, we will see that this complexity has yielded important new information about electrons and their orbits in atoms.

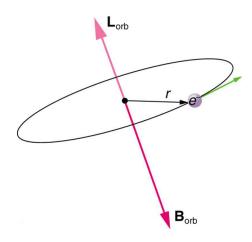
In order to explore the substructure of atoms (and knowing that magnetic fields affect moving charges), the Dutch physicist Hendrik Lorentz (1853–1930) suggested that his student Pieter Zeeman (1865–1943) study how spectra might be affected by magnetic fields. What they found became known as the **Zeeman effect**, which involved spectral lines being split into two or more separate emission lines by an external magnetic field, as shown in [link]. For their discoveries, Zeeman and Lorentz shared the 1902 Nobel Prize in Physics.

Zeeman splitting is complex. Some lines split into three lines, some into five, and so on. But one general feature is that the amount the split lines are separated is proportional to the applied field strength, indicating an interaction with a moving charge. The splitting means that the quantized energy of an orbit is affected by an external magnetic field, causing the orbit to have several discrete energies instead of one. Even without an external magnetic field, very precise measurements showed that spectral lines are doublets (split into two), apparently by magnetic fields within the atom itself.

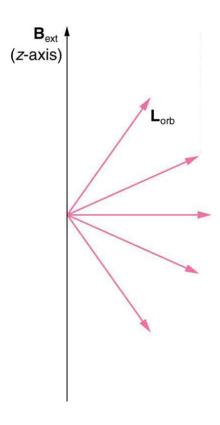


The Zeeman effect is the splitting of spectral lines when a magnetic field is applied. The number of lines formed varies, but the spread is proportional to the strength of the applied field. (a) Two spectral lines with no external magnetic field. (b) The lines split when the field is applied. (c) The splitting is greater when a stronger field is applied.

Bohr's theory of circular orbits is useful for visualizing how an electron's orbit is affected by a magnetic field. The circular orbit forms a current loop, which creates a magnetic field of its own,  $B_{\rm orb}$  as seen in [link]. Note that the **orbital magnetic field B\_{\rm orb}** and the **orbital angular momentum L\_{\rm orb}** are along the same line. The external magnetic field and the orbital magnetic field interact; a torque is exerted to align them. A torque rotating a system through some angle does work so that there is energy associated with this interaction. Thus, orbits at different angles to the external magnetic field have different energies. What is remarkable is that the energies are quantized—the magnetic field splits the spectral lines into several discrete lines that have different energies. This means that only certain angles are allowed between the orbital angular momentum and the external field, as seen in [link].



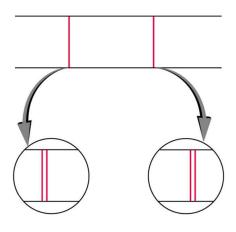
The approximate picture of an electron in a circular orbit illustrates how the current loop produces its own magnetic field, called  $\mathbf{B}_{\mathrm{orb}}$ . It also shows how  $\mathbf{B}_{\mathrm{orb}}$  is along the same line as the orbital angular momentum  $\mathbf{L}_{\mathrm{orb}}$ .



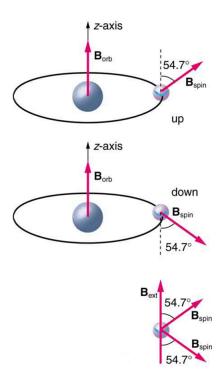
Only certain angles are allowed between the orbital angular momentum and an external magnetic field. This is implied by the fact that the Zeeman effect splits spectral lines into several discrete lines. Each line is associated with an angle between the external magnetic field and magnetic fields due to electrons and their orbits.

We already know that the magnitude of angular momentum is quantized for electron orbits in atoms. The new insight is that the *direction of the orbital angular momentum is also quantized*. The fact that the orbital angular momentum can have only certain directions is called **space quantization**. Like many aspects of quantum mechanics, this quantization of direction is totally unexpected. On the macroscopic scale, orbital angular momentum, such as that of the moon around the earth, can have any magnitude and be in any direction.

Detailed treatment of space quantization began to explain some complexities of atomic spectra, but certain patterns seemed to be caused by something else. As mentioned, spectral lines are actually closely spaced doublets, a characteristic called **fine structure**, as shown in [link]. The doublet changes when a magnetic field is applied, implying that whatever causes the doublet interacts with a magnetic field. In 1925, Sem Goudsmit and George Uhlenbeck, two Dutch physicists, successfully argued that electrons have properties analogous to a macroscopic charge spinning on its axis. Electrons, in fact, have an internal or intrinsic angular momentum called **intrinsic spin S**. Since electrons are charged, their intrinsic spin creates an **intrinsic magnetic field B\_{\rm int}**, which interacts with their orbital magnetic field  $\mathbf{B}_{orb}$ . Furthermore, *electron intrinsic spin is quantized in magnitude and direction*, analogous to the situation for orbital angular momentum. The spin of the electron can have only one magnitude, and its direction can be at only one of two angles relative to a magnetic field, as seen in [link]. We refer to this as spin up or spin down for the electron. Each spin direction has a different energy; hence, spectroscopic lines are split into two. Spectral doublets are now understood as being due to electron spin.



Fine structure. Upon close examination, spectral lines are doublets, even in the absence of an external magnetic field. The electron has an intrinsic magnetic field that interacts with its orbital magnetic field.



The intrinsic magnetic field  $\mathbf{B}_{\mathrm{int}}$ of an electron is attributed to its spin, **S**, roughly pictured to be due to its charge spinning on its axis. This is only a crude model, since electrons seem to have no size. The spin and intrinsic magnetic field of the electron can make only one of two angles with another magnetic field, such as that created by the electron's orbital

motion. Space is quantized for spin as well as for orbital angular momentum.

These two new insights—that the direction of angular momentum, whether orbital or spin, is quantized, and that electrons have intrinsic spin—help to explain many of the complexities of atomic and molecular spectra. In magnetic resonance imaging, it is the way that the intrinsic magnetic field of hydrogen and biological atoms interact with an external field that underlies the diagnostic fundamentals.

## **Section Summary**

- The Zeeman effect—the splitting of lines when a magnetic field is applied—is caused by other quantized entities in atoms.
- Both the magnitude and direction of orbital angular momentum are quantized.
- The same is true for the magnitude and direction of the intrinsic spin of electrons.

## **Conceptual Questions**

#### **Exercise:**

### **Problem:**

What is the Zeeman effect, and what type of quantization was discovered because of this effect?

## **Glossary**

## Zeeman effect

the effect of external magnetic fields on spectral lines

## intrinsic spin

the internal or intrinsic angular momentum of electrons

## orbital angular momentum

an angular momentum that corresponds to the quantum analog of classical angular momentum

## fine structure

the splitting of spectral lines of the hydrogen spectrum when the spectral lines are examined at very high resolution

## space quantization

the fact that the orbital angular momentum can have only certain directions

## intrinsic magnetic field

the magnetic field generated due to the intrinsic spin of electrons

## orbital magnetic field

the magnetic field generated due to the orbital motion of electrons

#### Quantum Numbers and Rules

- Define quantum number.
- Calculate angle of angular momentum vector with an axis.
- Define spin quantum number.

Physical characteristics that are quantized—such as energy, charge, and angular momentum—are of such importance that names and symbols are given to them. The values of quantized entities are expressed in terms of **quantum numbers**, and the rules governing them are of the utmost importance in determining what nature is and does. This section covers some of the more important quantum numbers and rules—all of which apply in chemistry, material science, and far beyond the realm of atomic physics, where they were first discovered. Once again, we see how physics makes discoveries which enable other fields to grow.

The energy states of bound systems are quantized, because the particle wavelength can fit into the bounds of the system in only certain ways. This was elaborated for the hydrogen atom, for which the allowed energies are expressed as  $E_n \propto 1/n^2$ , where  $n=1,2,3,\ldots$  We define n to be the principal quantum number that labels the basic states of a system. The lowest-energy state has n=1, the first excited state has n=2, and so on. Thus the allowed values for the principal quantum number are **Equation:** 

$$n = 1, 2, 3, \dots$$

This is more than just a numbering scheme, since the energy of the system, such as the hydrogen atom, can be expressed as some function of n, as can other characteristics (such as the orbital radii of the hydrogen atom).

The fact that the *magnitude of angular momentum is quantized* was first recognized by Bohr in relation to the hydrogen atom; it is now known to be true in general. With the development of quantum mechanics, it was found that the magnitude of angular momentum L can have only the values **Equation:** 

$$L=\sqrt{l(l+1)}rac{h}{2\pi}\quad (l=0,1,2,...,n-1),$$

where l is defined to be the **angular momentum quantum number**. The rule for l in atoms is given in the parentheses. Given n, the value of l can be any integer from zero up to n-1. For example, if n=4, then l can be 0, 1, 2, or 3.

Note that for n=1,l can only be zero. This means that the ground-state angular momentum for hydrogen is actually zero, not  $h/2\pi$  as Bohr proposed. The picture of circular orbits is not valid, because there would be angular momentum for any circular orbit. A more valid picture is the cloud of probability shown for the ground state of hydrogen in [link]. The electron actually spends time in and near the nucleus. The reason the electron does not remain in the nucleus is related to Heisenberg's uncertainty principle—the electron's energy would have to be much too large to be confined to the small space of the nucleus. Now the first excited state of hydrogen has n=2, so that l can be either 0 or 1, according to the rule in  $L=\sqrt{l(l+1)}\frac{h}{2\pi}$ . Similarly, for n=3,l can be 0, 1, or 2. It is often most convenient to state the value of l, a simple integer, rather than calculating the value of l from l=1,l for example, for l=1,l we see that

### **Equation:**

$$L = \sqrt{2(2+1)} rac{h}{2\pi} = \sqrt{6} rac{h}{2\pi} = 0.390 h = 2.58 imes 10^{-34} \, \mathrm{J\cdot s}.$$

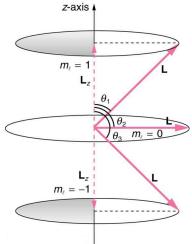
It is much simpler to state l = 2.

As recognized in the Zeeman effect, the *direction of angular momentum is quantized*. We now know this is true in all circumstances. It is found that the component of angular momentum along one direction in space, usually called the z-axis, can have only certain values of  $L_z$ . The direction in space must be related to something physical, such as the direction of the magnetic field at that location. This is an aspect of relativity. Direction has no meaning if there is nothing that varies with direction, as does magnetic force. The allowed values of  $L_z$  are

### **Equation:**

$$L_z = m_l rac{h}{2\pi} \quad (m_l = -l, -l+1, ..., \; -1, 0, 1, ... \; l-1, l),$$

where  $L_z$  is the z-component of the angular momentum and  $m_l$  is the angular momentum projection quantum number. The rule in parentheses for the values of  $m_l$  is that it can range from -l to l in steps of one. For example, if l=2, then  $m_l$  can have the five values -2, -1, 0, 1, and 2. Each  $m_l$  corresponds to a different energy in the presence of a magnetic field, so that they are related to the splitting of spectral lines into discrete parts, as discussed in the preceding section. If the z-component of angular momentum can have only certain values, then the angular momentum can have only certain directions, as illustrated in [link].



The component of a given angular momentum along the z-axis (defined by the direction of a magnetic field) can have only certain values; these are shown here for l=1, for which  $m_l=-1,0,$  and +1.

The direction of L is quantized in the sense that it can have only certain angles relative to the z-axis.

### **Example:**

### What Are the Allowed Directions?

Calculate the angles that the angular momentum vector  $\mathbf{L}$  can make with the z-axis for l=1, as illustrated in [link].

### Strategy

[link] represents the vectors  $\mathbf{L}$  and  $\mathbf{L}_z$  as usual, with arrows proportional to their magnitudes and pointing in the correct directions.  $\mathbf{L}$  and  $\mathbf{L}_z$  form a right triangle, with  $\mathbf{L}$  being the hypotenuse and  $\mathbf{L}_z$  the adjacent side. This means that the ratio of  $\mathbf{L}_z$  to  $\mathbf{L}$  is the cosine of the angle of interest. We can find  $\mathbf{L}$  and  $\mathbf{L}_z$  using  $L = \sqrt{l(l+1)} \frac{h}{2\pi}$  and  $L_z = m \frac{h}{2\pi}$ .

#### Solution

We are given l=1, so that  $m_l$  can be +1, 0, or –1. Thus L has the value given by  $L=\sqrt{l(l+1)}rac{h}{2\pi}$  .

### **Equation:**

$$L=rac{\sqrt{l(l+1)}h}{2\pi}=rac{\sqrt{2}h}{2\pi}$$

 $L_z$  can have three values, given by  $L_z=m_lrac{h}{2\pi}.$ 

### **Equation:**

$$L_z = m_l rac{h}{2\pi} = egin{array}{cccc} rac{h}{2\pi}, & m_l & = & +1 \ 0, & m_l & = & 0 \ -rac{h}{2\pi}, & m_l & = & -1 \end{array}$$

As can be seen in [link],  $\cos \theta = L_z/L$ , and so for  $m_l = +1$ , we have

#### **Equation:**

$$\cos heta_1 = rac{L_Z}{L} = rac{rac{h}{2\pi}}{rac{\sqrt{2}h}{2\pi}} = rac{1}{\sqrt{2}} = 0.707.$$

Thus,

### **Equation:**

$$\theta_1 = \cos^{-1} 0.707 = 45.0^{\circ}.$$

Similarly, for  $m_l = 0$ , we find  $\cos \theta_2 = 0$ ; thus,

### **Equation:**

$$\theta_2 = \cos^{-1}0 = 90.0^{\circ}.$$

And for  $m_l = -1$ ,

**Equation:** 

$$\cos heta_3 = rac{L_Z}{L} = rac{-rac{h}{2\pi}}{rac{\sqrt{2}h}{2\pi}} = -rac{1}{\sqrt{2}} = -0.707,$$

so that

**Equation:** 

$$heta_3 = \cos^{-1}(-0.707) = 135.0^{\circ}.$$

#### Discussion

The angles are consistent with the figure. Only the angle relative to the z-axis is quantized. L can point in any direction as long as it makes the proper angle with the z-axis. Thus the angular momentum vectors lie on cones as illustrated. This behavior is not observed on the large scale. To see how the correspondence principle holds here, consider that the smallest angle ( $\theta_1$  in the example) is for the maximum value of  $m_l = 0$ , namely  $m_l = l$ . For that smallest angle,

**Equation:** 

$$\cos heta = rac{L_z}{L} = rac{l}{\sqrt{l(l+1)}},$$

which approaches 1 as l becomes very large. If  $\cos \theta = 1$ , then  $\theta = 0^{\circ}$ . Furthermore, for large l, there are many values of  $m_l$ , so that all angles become possible as l gets very large.

## Intrinsic Spin Angular Momentum Is Quantized in Magnitude and Direction

There are two more quantum numbers of immediate concern. Both were first discovered for electrons in conjunction with fine structure in atomic spectra. It is now well established that electrons and other fundamental particles have *intrinsic spin*, roughly analogous to a planet spinning on its axis. This spin is a fundamental characteristic of particles, and only one magnitude of intrinsic spin is allowed for a given type of particle. Intrinsic angular momentum is quantized independently of orbital angular momentum. Additionally, the direction of the spin is also quantized. It has been found that the **magnitude of the intrinsic (internal) spin angular momentum**, S, of an electron is given by **Equation:** 

$$S=\sqrt{s(s+1)}rac{h}{2\pi}\quad (s=1/2 ext{ for electrons}),$$

where s is defined to be the **spin quantum number**. This is very similar to the quantization of L given in  $L = \sqrt{l(l+1)} \frac{h}{2\pi}$ , except that the only value allowed for s for electrons is 1/2.

The *direction of intrinsic spin is quantized*, just as is the direction of orbital angular momentum. The direction of spin angular momentum along one direction in space, again called the z-axis, can have only the values

### **Equation:**

$$S_z=m_srac{h}{2\pi}\quad \left(m_s=-rac{1}{2},+rac{1}{2}
ight)$$

for electrons.  $S_z$  is the *z*-component of spin angular momentum and  $m_s$  is the spin projection quantum number. For electrons, s can only be 1/2, and  $m_s$  can be either +1/2 or -1/2. Spin projection  $m_s$ =+1/2 is referred to as  $spin\ up$ , whereas  $m_s$  = -1/2 is called  $spin\ down$ . These are illustrated in [link].

#### Note:

### Intrinsic Spin

In later chapters, we will see that intrinsic spin is a characteristic of all subatomic particles. For some particles s is half-integral, whereas for others s is integral—there are crucial differences between half-integral spin particles and integral spin particles. Protons and neutrons, like electrons, have s=1/2, whereas photons have s=1, and other particles called pions have s=0, and so on.

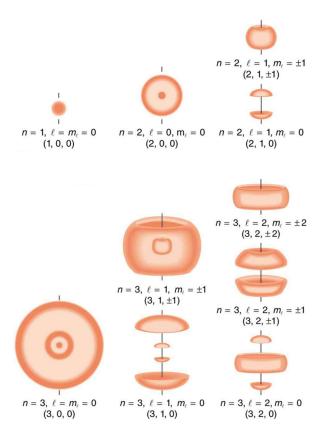
To summarize, the state of a system, such as the precise nature of an electron in an atom, is determined by its particular quantum numbers. These are expressed in the form  $(n, l, m_l, m_s)$  —see [link] For electrons in atoms, the principal quantum number can have the values n=1,2,3,... Once n is known, the values of the angular momentum quantum number are limited to l=1,2,3,...,n-1. For a given value of l, the angular momentum projection quantum number can have only the values  $m_l=-l,-l+1,...,-1,0,1,...,l-1,l$ . Electron spin is independent of n, l, and  $m_l$ , always having s=1/2. The spin projection quantum number can have two values,  $m_s=1/2$  or s=1/20 or s=1/21.

Name	Symbol	Allowed values
Principal quantum number	n	1,2,3,
Angular momentum	l	0,1,2,n-1
Angular momentum projection	$m_l$	$-l,\; -l\; +1,,\; -1,0,1,,l-1,l\; ({ m or}\; 0,\pm 1,\pm 2,,\; \pm l)$

Name	Symbol	Allowed values
Spin[footnote] The spin quantum number <i>s</i> is usually not stated, since it is always 1/2 for electrons	s	$1/2 ({ m electrons})$
Spin projection	$m_s$	$-1/2,\;+1/2$

#### **Atomic Quantum Numbers**

[link] shows several hydrogen states corresponding to different sets of quantum numbers. Note that these clouds of probability are the locations of electrons as determined by making repeated measurements—each measurement finds the electron in a definite location, with a greater chance of finding the electron in some places rather than others. With repeated measurements, the pattern of probability shown in the figure emerges. The clouds of probability do not look like nor do they correspond to classical orbits. The uncertainty principle actually prevents us and nature from knowing how the electron gets from one place to another, and so an orbit really does not exist as such. Nature on a small scale is again much different from that on the large scale.



Probability clouds for the electron in the ground state and several excited states of hydrogen. The nature of these states is determined by their sets of quantum numbers, here given as  $(n, l, m_l)$ . The ground state is (0, 0, 0); one of the possibilities for the second excited state is (3, 2, 1). The probability of finding the electron is indicated by the shade of color; the darker the coloring the greater the chance of finding the electron.

We will see that the quantum numbers discussed in this section are valid for a broad range of particles and other systems, such as nuclei. Some quantum numbers, such as intrinsic spin, are related to fundamental classifications of subatomic particles, and they obey laws that will give us further insight into the substructure of matter and its interactions.

### Note:

### PhET Explorations: Stern-Gerlach Experiment

The classic Stern-Gerlach Experiment shows that atoms have a property called spin. Spin is a kind of intrinsic angular momentum, which has no classical counterpart. When the z-component of the spin is

measured, one always gets one of two values: spin up or spin down.

https://phet.colorado.edu/sims/stern-gerlach/stern-gerlach en.html

### **Section Summary**

Quantum numbers are used to express the allowed values of quantized entities. The principal quantum number n labels the basic states of a system and is given by Equation:

$$n = 1, 2, 3, \dots$$

• The magnitude of angular momentum is given by **Equation:** 

$$L=\sqrt{l(l+1)}rac{h}{2\pi}\quad (l=0,1,2,...,n-1),$$

where l is the angular momentum quantum number. The direction of angular momentum is quantized, in that its component along an axis defined by a magnetic field, called the z-axis is given by

**Equation:** 

$$L_z = m_l rac{h}{2\pi} \quad (m_l = -l, -l+1, ..., \ -1, 0, 1, ... \ l-1, l),$$

where  $L_z$  is the z-component of the angular momentum and  $m_l$  is the angular momentum projection quantum number. Similarly, the electron's intrinsic spin angular momentum S is given by

**Equation:** 

$$S=\sqrt{s(s+1)}rac{h}{2\pi}\quad (s=1/2 ext{ for electrons}),$$

s is defined to be the spin quantum number. Finally, the direction of the electron's spin along the z-axis is given by

**Equation:** 

$$S_z=m_srac{h}{2\pi}\quadigg(m_s=-rac{1}{2},+rac{1}{2}igg),$$

where  $S_z$  is the z-component of spin angular momentum and  $m_s$  is the spin projection quantum number. Spin projection  $m_s = +1/2$  is referred to as spin up, whereas  $m_s = -1/2$  is called spin down. [link] summarizes the atomic quantum numbers and their allowed values.

### **Conceptual Questions**

### **Exercise:**

**Problem:** Define the quantum numbers n, l,  $m_l$ , s, and  $m_s$ .

**Exercise:** 

**Problem:** For a given value of *n*, what are the allowed values of *l*?

**Exercise:** 

#### **Problem:**

For a given value of l, what are the allowed values of  $m_l$ ? What are the allowed values of  $m_l$  for a given value of n? Give an example in each case.

#### **Exercise:**

#### **Problem:**

List all the possible values of s and  $m_s$  for an electron. Are there particles for which these values are different? The same?

#### **Problem Exercises**

#### **Exercise:**

#### **Problem:**

If an atom has an electron in the n=5 state with  $m_l=3$ , what are the possible values of l?

#### **Solution:**

l = 4, 3 are possible since l < n and  $|m_l| \le l$ .

#### **Exercise:**

**Problem:** An atom has an electron with  $m_l = 2$ . What is the smallest value of n for this electron?

### **Exercise:**

**Problem:** What are the possible values of  $m_l$  for an electron in the n=4 state?

#### **Solution:**

$$n = 4 \Rightarrow l = 3, 2, 1, 0 \Rightarrow m_l = \pm 3, \pm 2, \pm 1, 0$$
 are possible.

#### **Exercise:**

### **Problem:**

What, if any, constraints does a value of  $m_l = 1$  place on the other quantum numbers for an electron in an atom?

#### **Exercise:**

### **Problem:**

(a) Calculate the magnitude of the angular momentum for an l=1 electron. (b) Compare your answer to the value Bohr proposed for the n=1 state.

#### **Solution:**

- (a)  $1.49 \times 10^{-34} \, \text{J} \cdot \text{s}$
- (b)  $1.06 \times 10^{-34} \, \text{J} \cdot \text{s}$

#### **Exercise:**

### **Problem:**

(a) What is the magnitude of the angular momentum for an l=1 electron? (b) Calculate the magnitude of the electron's spin angular momentum. (c) What is the ratio of these angular momenta?

#### **Exercise:**

**Problem:** Repeat [link] for l = 3.

#### **Solution:**

- (a)  $3.66 \times 10^{-34} \, \mathrm{J \cdot s}$
- (b)  $s = 9.13 \times 10^{-35} \,\mathrm{J \cdot s}$

(c) 
$$\frac{L}{S} = \frac{\sqrt{12}}{\sqrt{3/4}} = 4$$

#### **Exercise:**

#### **Problem:**

(a) How many angles can L make with the z-axis for an l=2 electron? (b) Calculate the value of the smallest angle.

#### **Exercise:**

**Problem:** What angles can the spin S of an electron make with the z-axis?

### **Solution:**

$$\theta = 54.7^{\circ}, 125.3^{\circ}$$

### **Glossary**

#### quantum numbers

the values of quantized entities, such as energy and angular momentum

angular momentum quantum number

a quantum number associated with the angular momentum of electrons

## spin quantum number

the quantum number that parameterizes the intrinsic angular momentum (or spin angular momentum, or simply spin) of a given particle

### spin projection quantum number

quantum number that can be used to calculate the intrinsic electron angular momentum along the  $\boldsymbol{z}$  -axis

## z-component of spin angular momentum

component of intrinsic electron spin along the z-axis

## magnitude of the intrinsic (internal) spin angular momentum

given by 
$$S=\sqrt{s(s+1)}rac{h}{2\pi}$$

## z-component of the angular momentum

component of orbital angular momentum of electron along the *z*-axis

## The Pauli Exclusion Principle

- Define the composition of an atom along with its electrons, neutrons, and protons.
- Explain the Pauli exclusion principle and its application to the atom.
- Specify the shell and subshell symbols and their positions.
- Define the position of electrons in different shells of an atom.
- State the position of each element in the periodic table according to shell filling.

# **Multiple-Electron Atoms**

All atoms except hydrogen are multiple-electron atoms. The physical and chemical properties of elements are directly related to the number of electrons a neutral atom has. The periodic table of the elements groups elements with similar properties into columns. This systematic organization is related to the number of electrons in a neutral atom, called the **atomic number**, Z. We shall see in this section that the exclusion principle is key to the underlying explanations, and that it applies far beyond the realm of atomic physics.

In 1925, the Austrian physicist Wolfgang Pauli (see [link]) proposed the following rule: No two electrons can have the same set of quantum numbers. That is, no two electrons can be in the same state. This statement is known as the **Pauli exclusion principle**, because it excludes electrons from being in the same state. The Pauli exclusion principle is extremely powerful and very broadly applicable. It applies to any identical particles with half-integral intrinsic spin—that is, having s = 1/2, 3/2, ... Thus no two electrons can have the same set of quantum numbers.

# **Note:** Pauli Exclusion Principle

No two electrons can have the same set of quantum numbers. That is, no two electrons can be in the same state.

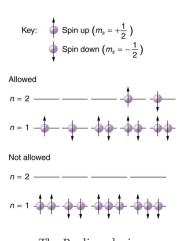


The Austrian physicist Wolfgang Pauli (1900–1958) played a major role in the development of quantum mechanics. He proposed the exclusion principle; hypothesized the existence of an important particle,

called the neutrino, before it was directly observed; made fundamental contributions to several areas of theoretical physics; and influenced many students who went on to do important work of their own. (credit: Nobel Foundation, via Wikimedia Commons)

Let us examine how the exclusion principle applies to electrons in atoms. The quantum numbers involved were defined in Quantum Numbers and Rules as n, l,  $m_l$ , s, and  $m_s$ . Since s is always 1/2 for electrons, it is redundant to list s, and so we omit it and specify the state of an electron by a set of four numbers  $(n, l, m_l, m_s)$ . For example, the quantum numbers (2, 1, 0, -1/2) completely specify the state of an electron in an atom.

Since no two electrons can have the same set of quantum numbers, there are limits to how many of them can be in the same energy state. Note that n determines the energy state in the absence of a magnetic field. So we first choose n, and then we see how many electrons can be in this energy state or energy level. Consider the n=1 level, for example. The only value l can have is 0 (see [link] for a list of possible values once n is known), and thus  $m_l$  can only be 0. The spin projection  $m_s$  can be either +1/2 or -1/2, and so there can be two electrons in the n=1 state. One has quantum numbers (1,0,0,+1/2), and the other has (1,0,0,-1/2). [link] illustrates that there can be one or two electrons having n=1, but not three.



The Pauli exclusion principle explains why some configurations of electrons are allowed while others are not. Since electrons cannot have the same set of quantum numbers, a maximum of two can be

In the n=1 level, and a third electron must reside in the higher-energy n=2 level. If there are two electrons in the n=1 level, their spins must be in opposite directions. (More precisely, their spin projections must differ.)

## **Shells and Subshells**

Because of the Pauli exclusion principle, only hydrogen and helium can have all of their electrons in the n=1 state. Lithium (see the periodic table) has three electrons, and so one must be in the n=2 level. This leads to the concept of shells and shell filling. As we progress up in the number of electrons, we go from hydrogen to helium, lithium, beryllium, boron, and so on, and we see that there are limits to the number of electrons for each value of n. Higher values of the shell n correspond to higher energies, and they can allow more electrons because of the various combinations of l,  $m_l$ , and  $m_s$  that are possible. Each value of the principal quantum number n thus corresponds to an atomic **shell** into which a limited number of electrons can go. Shells and the number of electrons in them determine the physical and chemical properties of atoms, since it is the outermost electrons that interact most with anything outside the atom.

The probability clouds of electrons with the lowest value of l are closest to the nucleus and, thus, more tightly bound. Thus when shells fill, they start with l=0, progress to l=1, and so on. Each value of l thus corresponds to a **subshell**.

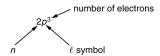
The table given below lists symbols traditionally used to denote shells and subshells.

Shell	Subshell	
n	l	Symbol
1	0	s
2	1	p
3	2	d
4	3	f

Shell	Subshell	
5	4	g
	5	h
	$6[\underline{\text{footnote}}]$ It is unusual to deal with subshells having $l$ greater than 6, but when encountered, they continue to be labeled in alphabetical order.	i

## Shell and Subshell Symbols

To denote shells and subshells, we write nl with a number for n and a letter for l. For example, an electron in the n=1 state must have l=0, and it is denoted as a 1s electron. Two electrons in the n=1 state is denoted as  $1s^2$ . Another example is an electron in the n=2 state with l=1, written as 2p. The case of three electrons with these quantum numbers is written  $2p^3$ . This notation, called spectroscopic notation, is generalized as shown in [link].



Counting the number of possible combinations of quantum numbers allowed by the exclusion principle, we can determine how many electrons it takes to fill each subshell and shell.

#### Example:

## **How Many Electrons Can Be in This Shell?**

List all the possible sets of quantum numbers for the n=2 shell, and determine the number of electrons that can be in the shell and each of its subshells.

## Strategy

Given n=2 for the shell, the rules for quantum numbers limit l to be 0 or 1. The shell therefore has two subshells, labeled 2s and 2p. Since the lowest l subshell fills first, we start with the 2s subshell possibilities and then proceed with the 2p subshell.

## Solution

It is convenient to list the possible quantum numbers in a table, as shown below.

n	$\ell$	m,	$m_s$	Subshell	Total in subshell	Total in shell
2	0	0	+1/2	0 -	2	
2	0	0	-1/2	≻ 2s	2	
2	1	1	+1/2			
2	1	1	-1/2			≻ 8
2	1	0	+1/2	- 2p	6	٥
2	1	0	-1/2	- 2ρ	0	
2	1	-1	+1/2			
2	1	-1	-1/2			

## Discussion

It is laborious to make a table like this every time we want to know how many electrons can be in a shell or subshell. There exist general rules that are easy to apply, as we shall now see.

The number of electrons that can be in a subshell depends entirely on the value of l. Once l is known, there are a fixed number of values of  $m_l$ , each of which can have two values for  $m_s$  First, since  $m_l$  goes from -l to l in steps of 1, there are 2l+1 possibilities. This number is multiplied by 2, since each electron can be spin up or spin down. Thus the maximum number of electrons that can be in a subshell is 2(2l+1).

For example, the 2s subshell in [link] has a maximum of 2 electrons in it, since 2(2l+1)=2(0+1)=2 for this subshell. Similarly, the 2p subshell has a maximum of 6 electrons, since 2(2l+1)=2(2+1)=6. For a shell, the maximum number is the sum of what can fit in the subshells. Some algebra shows that the maximum number of electrons that can be in a shell is  $2n^2$ .

For example, for the first shell n = 1, and so  $2n^2 = 2$ . We have already seen that only two electrons can be in the n = 1 shell. Similarly, for the second shell, n = 2, and so  $2n^2 = 8$ . As found in [link], the total number of electrons in the n = 2 shell is 8.

# **Example:**

## Subshells and Totals for n=3

How many subshells are in the n=3 shell? Identify each subshell, calculate the maximum number of electrons that will fit into each, and verify that the total is  $2n^2$ .

## Strategy

Subshells are determined by the value of l; thus, we first determine which values of l are allowed, and then we apply the equation "maximum number of electrons that can be in a subshell = 2(2l + 1)" to find the number of electrons in each subshell.

#### Solution

Since n=3, we know that l can be 0, 1, or 2; thus, there are three possible subshells. In standard notation, they are labeled the 3s, 3p, and 3d subshells. We have already seen that 2 electrons can be in an s state, and 6 in a p state, but let us use the equation "maximum number of electrons that can be in a subshell = 2(2l+1)" to calculate the maximum number in each:

#### **Equation:**

$$3s$$
 has  $l=0$ ; thus,  $2(2l+1)=2(0+1)=2$   
 $3p$  has  $l=1$ ; thus,  $2(2l+1)=2(2+1)=6$   
 $3d$  has  $l=2$ ; thus,  $2(2l+1)=2(4+1)=10$   
Total = 18  
(in the  $n=3$  shell)

The equation "maximum number of electrons that can be in a shell =  $2n^2$ " gives the maximum number in the n=3 shell to be

## **Equation:**

Maximum number of electrons = 
$$2n^2 = 2(3)^2 = 2(9) = 18$$
.

## Discussion

The total number of electrons in the three possible subshells is thus the same as the formula  $2n^2$ . In standard (spectroscopic) notation, a filled n=3 shell is denoted as  $3s^23p^63d^{10}$ . Shells do not fill in a simple manner. Before the n=3 shell is completely filled, for example, we begin to find electrons in the n=4 shell.

## **Shell Filling and the Periodic Table**

[link] shows electron configurations for the first 20 elements in the periodic table, starting with hydrogen and its single electron and ending with calcium. The Pauli exclusion principle determines the maximum number of electrons allowed in each shell and subshell. But the order in which the shells and subshells are filled is complicated because of the large numbers of interactions between electrons.

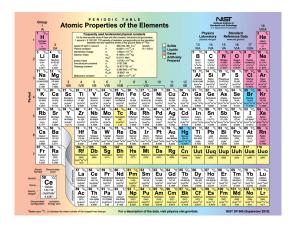
Element	Number of electrons (Z)	Ground state configuration					
Н	1	$1s^1$					
Не	2	$1s^2$					
Li	3	$1s^2$	$2s^1$				
Be	4	"	$2s^2$				
В	5	"	$2s^2$	$2p^1$			
С	6	"	$2s^2$	$2p^2$			
N	7	"	$2s^2$	$2p^3$			
0	8	"	$2s^2$	$2p^4$			
F	9	"	$2s^2$	$2p^5$			
Ne	10	"	$2s^2$	$2p^6$			
Na	11	"	$2s^2$	$2p^6$	$3s^1$		
Mg	12	"	"	"	$3s^2$		
Al	13	"	"	"	$3s^2$	$3p^1$	
Si	14	"	"	"	$3s^2$	$3p^2$	

Element	Number of electrons (Z)	Ground state configuration					
P	15	"	"	"	$3s^2$	$3p^3$	
S	16	"	"	"	$3s^2$	$3p^4$	
Cl	17	"	11	"	$3s^2$	$3p^5$	
Ar	18	"	11	"	$3s^2$	$3p^6$	
К	19	"	11	"	$3s^2$	$3p^6$	$4s^1$
Ca	20	"	11	"	"	"	$4s^2$

## Electron Configurations of Elements Hydrogen Through Calcium

Examining the above table, you can see that as the number of electrons in an atom increases from 1 in hydrogen to 2 in helium and so on, the lowest-energy shell gets filled first—that is, the n=1 shell fills first, and then the n=2 shell begins to fill. Within a shell, the subshells fill starting with the lowest l, or with the s subshell, then the s0, and so on, usually until all subshells are filled. The first exception to this occurs for potassium, where the s1 subshell begins to fill before any electrons go into the s2 subshell. The next exception is not shown in s3 subshell starts to fill before the s4 subshell. The reason for these exceptions is that s4 electrons have probability clouds that penetrate closer to the nucleus and, thus, are more tightly bound (lower in energy).

[link] shows the periodic table of the elements, through element 118. Of special interest are elements in the main groups, namely, those in the columns numbered 1, 2, 13, 14, 15, 16, 17, and 18.



Periodic table of the elements (credit:

# National Institute of Standards and Technology, U.S. Department of Commerce)

The number of electrons in the outermost subshell determines the atom's chemical properties, since it is these electrons that are farthest from the nucleus and thus interact most with other atoms. If the outermost subshell can accept or give up an electron easily, then the atom will be highly reactive chemically. Each group in the periodic table is characterized by its outermost electron configuration. Perhaps the most familiar is Group 18 (Group VIII), the noble gases (helium, neon, argon, etc.). These gases are all characterized by a filled outer subshell that is particularly stable. This means that they have large ionization energies and do not readily give up an electron. Furthermore, if they were to accept an extra electron, it would be in a significantly higher level and thus loosely bound. Chemical reactions often involve sharing electrons. Noble gases can be forced into unstable chemical compounds only under high pressure and temperature.

Group VII) contains the halogens, such as fluorine, chlorine, iodine and bromine, each of which has one less electron than a neighboring noble gas. Each halogen has 5 p electrons (a  $p^5$  configuration), while the p subshell can hold 6 electrons. This means the halogens have one vacancy in their outermost subshell. They thus readily accept an extra electron (it becomes tightly bound, closing the shell as in noble gases) and are highly reactive chemically. The halogens are also likely to form singly negative ions, such as  $C1^-$ , fitting an extra electron into the vacancy in the outer subshell. In contrast, alkali metals, such as sodium and potassium, all have a single s electron in their outermost subshell (an  $s^1$  configuration) and are members of Group 1 (Group I). These elements easily give up their extra electron and are thus highly reactive chemically. As you might expect, they also tend to form singly positive ions, such as  $Na^+$ , by losing their loosely bound outermost electron. They are metals (conductors), because the loosely bound outer electron can move freely.

Of course, other groups are also of interest. Carbon, silicon, and germanium, for example, have similar chemistries and are in Group 4 (Group IV). Carbon, in particular, is extraordinary in its ability to form many types of bonds and to be part of long chains, such as inorganic molecules. The large group of what are called transitional elements is characterized by the filling of the d subshells and crossing of energy levels. Heavier groups, such as the lanthanide series, are more complex—their shells do not fill in simple order. But the groups recognized by chemists such as Mendeleev have an explanation in the substructure of atoms.

#### Note:

## PhET Explorations: Stern-Gerlach Experiment

Build an atom out of protons, neutrons, and electrons, and see how the element, charge, and mass change. Then play a game to test your ideas!

https://phet.colorado.edu/sims/html/build-an-atom/latest/build-an-atom\_en.html

# **Section Summary**

- The state of a system is completely described by a complete set of quantum numbers. This set is written as  $(n, l, m_l, m_s)$ .
- The Pauli exclusion principle says that no two electrons can have the same set of quantum numbers; that is, no two electrons can be in the same state.
- This exclusion limits the number of electrons in atomic shells and subshells. Each value of *n* corresponds to a shell, and each value of *l* corresponds to a subshell.
- The maximum number of electrons that can be in a subshell is 2(2l+1).
- The maximum number of electrons that can be in a shell is  $2n^2$ .

# **Conceptual Questions**

## **Exercise:**

## **Problem:**

Identify the shell, subshell, and number of electrons for the following: (a)  $2p^3$ . (b)  $4d^9$ . (c)  $3s^1$ . (d)  $5g^{16}$ .

## **Exercise:**

## **Problem:**

Which of the following are not allowed? State which rule is violated for any that are not allowed. (a)  $1p^3$  (b)  $2p^8$ (c)  $3g^{11}$  (d)  $4f^2$ 

## **Problem Exercises**

## **Exercise:**

**Problem:** (a) How many electrons can be in the n = 4 shell?

(b) What are its subshells, and how many electrons can be in each?

## **Solution:**

(a) 32. (b) 2 in s, 6 in p, 10 in d, and 14 in f, for a total of 32.

## **Exercise:**

**Problem:** (a) What is the minimum value of 1 for a subshell that has 11 electrons in it?

(b) If this subshell is in the n = 5 shell, what is the spectroscopic notation for this atom?

## **Exercise:**

#### **Problem:**

(a) If one subshell of an atom has 9 electrons in it, what is the minimum value of l? (b) What is the spectroscopic notation for this atom, if this subshell is part of the n=3 shell?

## **Solution:**

- (a) 2
- (b)  $3d^9$

## **Exercise:**

## **Problem:**

- (a) List all possible sets of quantum numbers  $(n, l, m_l, m_s)$  for the n = 3 shell, and determine the number of electrons that can be in the shell and each of its subshells.
- (b) Show that the number of electrons in the shell equals  $2n^2$  and that the number in each subshell is 2(2l+1).

## **Exercise:**

## **Problem:**

Which of the following spectroscopic notations are not allowed? (a)  $5s^1$  (b)  $1d^1$  (c)  $4s^3$  (d)  $3p^7$  (e)  $5g^{15}$ . State which rule is violated for each that is not allowed.

#### **Solution:**

- (b) n > l is violated,
- (c) cannot have 3 electrons in s subshell since 3 > (2l+1) = 2
- (d) cannot have 7 electrons in p subshell since 7 > (2l+1) = 2(2+1) = 6

#### **Exercise:**

#### **Problem:**

Which of the following spectroscopic notations are allowed (that is, which violate none of the rules regarding values of quantum numbers)? (a)  $1s^1$ (b)  $1d^3$ (c)  $4s^2$  (d)  $3p^7$ (e)  $6h^{20}$ 

#### **Exercise:**

## **Problem:**

- (a) Using the Pauli exclusion principle and the rules relating the allowed values of the quantum numbers  $(n, l, m_l, m_s)$ , prove that the maximum number of electrons in a subshell is  $2n^2$ .
- (b) In a similar manner, prove that the maximum number of electrons in a shell is  $2n^2$ .

#### **Solution:**

- (a) The number of different values of  $m_l$  is  $\pm l$ ,  $\pm (l-1)$ , ...,0 for each l>0 and one for  $l=0 \Rightarrow (2l+1)$ . Also an overall factor of 2 since each  $m_l$  can have  $m_s$  equal to either +1/2 or  $-1/2 \Rightarrow 2(2l+1)$ .
- (b) for each value of l, you get 2(2l+1)

$$=0,1,2,...,(n-1)\Rightarrow 2\{[(2)(0)+1]+[(2)(1)+1]+....+[(2)(n-1)+1]\}=2[1+3+...+(2n-3)+1]$$

n term

to see that the expression in the box is  $=n^2$ , imagine taking (n-1) from the last term and adding it to first term =2[1+(n-1)+3+...+(2n-3)+(2n-1)-(n-1)]=2[n+3+....+(2n-3)+n]. Now take (n-3) from penultimate term and add to the second term  $2[n+n+...+n+n]=2n^2$ .

n terms

## **Exercise:**

# **Problem:**Integrated Concepts

Estimate the density of a nucleus by calculating the density of a proton, taking it to be a sphere 1.2 fm in diameter. Compare your result with the value estimated in this chapter.

# Exercise:

#### **Problem: Integrated Concepts**

The electric and magnetic forces on an electron in the CRT in [link] are supposed to be in opposite directions. Verify this by determining the direction of each force for the situation shown. Explain how you obtain the directions (that is, identify the rules used).

## **Solution:**

The electric force on the electron is up (toward the positively charged plate). The magnetic force is down (by the RHR).

## **Exercise:**

## **Problem:**

- (a) What is the distance between the slits of a diffraction grating that produces a first-order maximum for the first Balmer line at an angle of  $20.0^{\circ}$ ?
- (b) At what angle will the fourth line of the Balmer series appear in first order?
- (c) At what angle will the second-order maximum be for the first line?

## **Exercise:**

# **Problem:** Integrated Concepts

A galaxy moving away from the earth has a speed of 0.0100c. What wavelength do we observe for an  $n_i = 7$  to  $n_f = 2$  transition for hydrogen in that galaxy?

## **Solution:**

401 nm

#### **Exercise:**

## **Problem: Integrated Concepts**

Calculate the velocity of a star moving relative to the earth if you observe a wavelength of 91.0 nm for ionized hydrogen capturing an electron directly into the lowest orbital (that is, a  $n_i = \infty$  to  $n_f = 1$ , or a Lyman series transition).

#### **Exercise:**

# **Problem:** Integrated Concepts

In a Millikan oil-drop experiment using a setup like that in [link], a 500-V potential difference is applied to plates separated by 2.50 cm. (a) What is the mass of an oil drop having two extra electrons that is suspended motionless by the field between the plates? (b) What is the diameter of the drop, assuming it is a sphere with the density of olive oil?

## **Solution:**

(a) 
$$6.54 \times 10^{-16} \text{ kg}$$

(b) 
$$5.54 \times 10^{-7}$$
 m

## Exercise:

## **Problem:** Integrated Concepts

What double-slit separation would produce a first-order maximum at  $3.00^{\circ}$  for 25.0-keV x rays? The small answer indicates that the wave character of x rays is best determined by having them interact with very small objects such as atoms and molecules.

## **Exercise:**

## **Problem:** Integrated Concepts

In a laboratory experiment designed to duplicate Thomson's determination of  $q_e/m_e$ , a beam of electrons having a velocity of  $6.00 \times 10^7$  m/s enters a  $5.00 \times 10^{-3}$  T magnetic field. The beam moves perpendicular

to the field in a path having a 6.80-cm radius of curvature. Determine  $q_e/m_e$  from these observations, and compare the result with the known value.

#### **Solution:**

 $1.76 \times 10^{11}~C/kg$  , which agrees with the known value of  $1.759 \times 10^{11}~C/kg$  to within the precision of the measurement

#### Exercise:

## **Problem:**Integrated Concepts

Find the value of l, the orbital angular momentum quantum number, for the moon around the earth. The extremely large value obtained implies that it is impossible to tell the difference between adjacent quantized orbits for macroscopic objects.

#### Exercise:

#### **Problem:** Integrated Concepts

Particles called muons exist in cosmic rays and can be created in particle accelerators. Muons are very similar to electrons, having the same charge and spin, but they have a mass 207 times greater. When muons are captured by an atom, they orbit just like an electron but with a smaller radius, since the mass in  $a_{\rm B}=\frac{h^2}{4\pi^2 m_e k q_e^2}=0.529\times 10^{-10}~{\rm m}$  is 207  $m_e$ .

- (a) Calculate the radius of the n=1 orbit for a muon in a uranium ion (Z=92).
- (b) Compare this with the 7.5-fm radius of a uranium nucleus. Note that since the muon orbits inside the electron, it falls into a hydrogen-like orbit. Since your answer is less than the radius of the nucleus, you can see that the photons emitted as the muon falls into its lowest orbit can give information about the nucleus.

#### **Solution:**

- (a) 2.78 fm
- (b) 0.37 of the nuclear radius.

## Exercise:

## **Problem:**Integrated Concepts

Calculate the minimum amount of energy in joules needed to create a population inversion in a helium-neon laser containing  $1.00 \times 10^{-4}$  moles of neon.

## **Exercise:**

## **Problem: Integrated Concepts**

A carbon dioxide laser used in surgery emits infrared radiation with a wavelength of  $10.6~\mu m$ . In 1.00~ms, this laser raised the temperature of  $1.00~cm^3$  of flesh to  $100^{\circ}C$  and evaporated it.

(a) How many photons were required? You may assume flesh has the same heat of vaporization as water. (b) What was the minimum power output during the flash?

#### **Solution:**

(a)  $1.34 \times 10^{23}$ 

(b) 2.52 MW

## **Exercise:**

# **Problem:** Integrated Concepts

Suppose an MRI scanner uses 100-MHz radio waves.

- (a) Calculate the photon energy.
- (b) How does this compare to typical molecular binding energies?

#### **Exercise:**

## **Problem: Integrated Concepts**

- (a) An excimer laser used for vision correction emits 193-nm UV. Calculate the photon energy in eV.
- (b) These photons are used to evaporate corneal tissue, which is very similar to water in its properties. Calculate the amount of energy needed per molecule of water to make the phase change from liquid to gas. That is, divide the heat of vaporization in kJ/kg by the number of water molecules in a kilogram.
- (c) Convert this to eV and compare to the photon energy. Discuss the implications.

#### **Solution:**

- (a) 6.42 eV
- (b)  $7.27 \times 10^{-20} \text{ J/molecule}$
- (c) 0.454 eV, 14.1 times less than a single UV photon. Therefore, each photon will evaporate approximately 14 molecules of tissue. This gives the surgeon a rather precise method of removing corneal tissue from the surface of the eye.

#### Exercise:

## **Problem: Integrated Concepts**

A neighboring galaxy rotates on its axis so that stars on one side move toward us as fast as 200 km/s, while those on the other side move away as fast as 200 km/s. This causes the EM radiation we receive to be Doppler shifted by velocities over the entire range of  $\pm 200$  km/s. What range of wavelengths will we observe for the 656.0-nm line in the Balmer series of hydrogen emitted by stars in this galaxy. (This is called line broadening.)

#### **Exercise:**

## **Problem:** Integrated Concepts

A pulsar is a rapidly spinning remnant of a supernova. It rotates on its axis, sweeping hydrogen along with it so that hydrogen on one side moves toward us as fast as 50.0 km/s, while that on the other side moves away as fast as 50.0 km/s. This means that the EM radiation we receive will be Doppler shifted over a range of  $\pm 50.0$  km/s. What range of wavelengths will we observe for the 91.20-nm line in the Lyman series of hydrogen? (Such line broadening is observed and actually provides part of the evidence for rapid rotation.)

## **Solution:**

91.18 nm to 91.22 nm

#### Exercise:

## **Problem:** Integrated Concepts

Prove that the velocity of charged particles moving along a straight path through perpendicular electric and magnetic fields is v=E/B. Thus crossed electric and magnetic fields can be used as a velocity selector independent of the charge and mass of the particle involved.

#### **Exercise:**

#### **Problem:** Unreasonable Results

(a) What voltage must be applied to an X-ray tube to obtain 0.0100-fm-wavelength X-rays for use in exploring the details of nuclei? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

#### **Solution:**

- (a)  $1.24 \times 10^{11} \text{ V}$
- (b) The voltage is extremely large compared with any practical value.
- (c) The assumption of such a short wavelength by this method is unreasonable.

#### **Exercise:**

#### **Problem:** Unreasonable Results

A student in a physics laboratory observes a hydrogen spectrum with a diffraction grating for the purpose of measuring the wavelengths of the emitted radiation. In the spectrum, she observes a yellow line and finds its wavelength to be 589 nm. (a) Assuming this is part of the Balmer series, determine  $n_i$ , the principal quantum number of the initial state. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

## **Exercise:**

#### **Problem:** Construct Your Own Problem

The solar corona is so hot that most atoms in it are ionized. Consider a hydrogen-like atom in the corona that has only a single electron. Construct a problem in which you calculate selected spectral energies and wavelengths of the Lyman, Balmer, or other series of this atom that could be used to identify its presence in a very hot gas. You will need to choose the atomic number of the atom, identify the element, and choose which spectral lines to consider.

#### Exercise:

## **Problem:** Construct Your Own Problem

Consider the Doppler-shifted hydrogen spectrum received from a rapidly receding galaxy. Construct a problem in which you calculate the energies of selected spectral lines in the Balmer series and examine whether they can be described with a formula like that in the equation  $\frac{1}{\lambda} = R\left(\frac{1}{n_{\rm f}^2} - \frac{1}{n_{\rm i}^2}\right)$ , but with a different constant R.

# Glossary

## atomic number

the number of protons in the nucleus of an atom

# Pauli exclusion principle

a principle that states that no two electrons can have the same set of quantum numbers; that is, no two electrons can be in the same state

## shell

a probability cloud for electrons that has a single principal quantum number

# subshell

the probability cloud for electrons that has a single angular momentum quantum number  $\boldsymbol{l}$ 

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, t <sub>1/2</sub>
0	neutron	1	n	1.008 665	$oldsymbol{eta}^-$	10.37 min
1	Hydrogen	1	$^{1}\mathrm{H}$	1.007 825	99.985%	
	Deuterium	2	$^2\mathrm{H}~\mathrm{or}~\mathrm{D}$	2.014 102	0.015%	
	Tritium	3	$^3{ m H~or~T}$	3.016 050	$oldsymbol{eta}^-$	12.33 y
2	Helium	3	$^3{ m He}$	3.016 030	$1.38 \times 10^{-4}\%$	
		4	$^4{ m He}$	4.002 603	≈100%	
3	Lithium	6	$^6{ m Li}$	6.015 121	7.5%	
		7	$^7{ m Li}$	7.016 003	92.5%	
4	Beryllium	7	$^7{ m Be}$	7.016 928	EC	53.29 d
		9	$^9{ m Be}$	9.012 182	100%	
5	Boron	10	$^{10}\mathrm{B}$	10.012 937	19.9%	
		11	<sup>11</sup> B	11.009 305	80.1%	
6	Carbon	11	<sup>11</sup> C	11.011 432	EC, $\beta^+$	
		12	$^{12}\mathrm{C}$	12.000 000	98.90%	
		13	$^{13}\mathrm{C}$	13.003 355	1.10%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, t <sub>1/2</sub>
		14	$^{14}\mathrm{C}$	14.003 241	$eta^-$	5730 y
7	Nitrogen	13	$^{13}{ m N}$	13.005 738	$eta^+$	9.96 min
		14	$^{14}{ m N}$	14.003 074	99.63%	
		15	$^{15}{ m N}$	15.000 108	0.37%	
8	Oxygen	15	<sup>15</sup> O	15.003 065	EC, $\beta^+$	122 s
		16	<sup>16</sup> O	15.994 915	99.76%	
		18	<sup>18</sup> O	17.999 160	0.200%	
9	Fluorine	18	$^{18}{ m F}$	18.000 937	EC, $\beta^+$	1.83 h
		19	$^{19}{ m F}$	18.998 403	100%	
10	Neon	20	$^{20}{ m Ne}$	19.992 435	90.51%	
		22	$^{22}{ m Ne}$	21.991 383	9.22%	
11	Sodium	22	$^{22}\mathrm{Na}$	21.994 434	$eta^+$	2.602 y
		23	$^{23}\mathrm{Na}$	22.989 767	100%	
		24	$^{24}\mathrm{Na}$	23.990 961	$eta^-$	14.96 h
12	Magnesium	24	$^{24}{ m Mg}$	23.985 042	78.99%	
13	Aluminum	27	<sup>27</sup> Al	26.981 539	100%	
14	Silicon	28	$^{28}{ m Si}$	27.976 927	92.23%	2.62h

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		31	$^{31}{ m Si}$	30.975 362	$eta^-$	
15	Phosphorus	31	$^{31}\mathrm{P}$	30.973 762	100%	
		32	$^{32}\mathrm{P}$	31.973 907	$eta^-$	14.28 d
16	Sulfur	32	$^{32}{ m S}$	31.972 070	95.02%	
		35	$^{35}{ m S}$	34.969 031	$eta^-$	87.4 d
17	Chlorine	35	$^{35}\mathrm{Cl}$	34.968 852	75.77%	
		37	$^{37}\mathrm{Cl}$	36.965 903	24.23%	
18	Argon	40	$^{40}{ m Ar}$	39.962 384	99.60%	
19	Potassium	39	$^{39}{ m K}$	38.963 707	93.26%	
		40	$^{40}{ m K}$	39.963 999	0.0117%, EC, $\beta^-$	$1.28 imes10^9\mathrm{y}$
20	Calcium	40	$^{40}\mathrm{Ca}$	39.962 591	96.94%	
21	Scandium	45	$^{45}{ m Sc}$	44.955 910	100%	
22	Titanium	48	<sup>48</sup> Ti	47.947 947	73.8%	
23	Vanadium	51	$^{51}{ m V}$	50.943 962	99.75%	
24	Chromium	52	$^{52}{ m Cr}$	51.940 509	83.79%	
25	Manganese	55	$^{55}{ m Mn}$	54.938 047	100%	
26	Iron	56	$^{56}{ m Fe}$	55.934 939	91.72%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, t <sub>1/2</sub>
27	Cobalt	59	$^{59}\mathrm{Co}$	58.933 198	100%	
		60	<sup>60</sup> Co	59.933 819	$eta^-$	5.271 y
28	Nickel	58	<sup>58</sup> Ni	57.935 346	68.27%	
		60	$^{60}\mathrm{Ni}$	59.930 788	26.10%	
29	Copper	63	<sup>63</sup> Cu	62.939 598	69.17%	
		65	$^{65}\mathrm{Cu}$	64.927 793	30.83%	
30	Zinc	64	$^{64}{ m Zn}$	63.929 145	48.6%	
		66	$^{66}{ m Zn}$	65.926 034	27.9%	
31	Gallium	69	$^{69}{ m Ga}$	68.925 580	60.1%	
32	Germanium	72	$^{72}{ m Ge}$	71.922 079	27.4%	
		74	$^{74}{ m Ge}$	73.921 177	36.5%	
33	Arsenic	75	$^{75}\mathrm{As}$	74.921 594	100%	
34	Selenium	80	$^{80}\mathrm{Se}$	79.916 520	49.7%	
35	Bromine	79	$^{79}{ m Br}$	78.918 336	50.69%	
36	Krypton	84	$^{84}{ m Kr}$	83.911 507	57.0%	
37	Rubidium	85	$^{85}{ m Rb}$	84.911 794	72.17%	
38	Strontium	86	$^{86}{ m Sr}$	85.909 267	9.86%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, t <sub>1/2</sub>
		88	$^{88}\mathrm{Sr}$	87.905 619	82.58%	
		90	$^{90}{ m Sr}$	89.907 738	$eta^-$	28.8 y
39	Yttrium	89	$^{89}\mathrm{Y}$	88.905 849	100%	
		90	$^{90}\mathrm{Y}$	89.907 152	$eta^-$	64.1 h
40	Zirconium	90	$^{90}{ m Zr}$	89.904 703	51.45%	
41	Niobium	93	$^{93}{ m Nb}$	92.906 377	100%	
42	Molybdenum	98	$^{98}\mathrm{Mo}$	97.905 406	24.13%	
43	Technetium	98	$^{98}{ m Tc}$	97.907 215	$eta^-$	$4.2 imes10^6\mathrm{y}$
44	Ruthenium	102	$^{102}\mathrm{Ru}$	101.904 348	31.6%	
45	Rhodium	103	$^{103}\mathrm{Rh}$	102.905 500	100%	
46	Palladium	106	$^{106}\mathrm{Pd}$	105.903 478	27.33%	
47	Silver	107	$^{107}\mathrm{Ag}$	106.905 092	51.84%	
		109	$^{109}\mathrm{Ag}$	108.904 757	48.16%	
48	Cadmium	114	$^{114}\mathrm{Cd}$	113.903 357	28.73%	
49	Indium	115	$^{115}{ m In}$	114.903 880	95.7%, $eta^-$	$4.4 imes10^{14}\mathrm{y}$
50	Tin	120	$^{120}\mathrm{Sn}$	119.902 200	32.59%	
51	Antimony	121	$^{121}\mathrm{Sb}$	120.903 821	57.3%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, t <sub>1/2</sub>
52	Tellurium	130	$^{130}{ m Te}$	129.906 229	33.8%, <i>β</i> <sup>-</sup>	$2.5 imes10^{21} \mathrm{y}$
53	Iodine	127	$^{127}\mathrm{I}$	126.904 473	100%	
		131	$^{131}\mathrm{I}$	130.906 114	$eta^-$	8.040 d
54	Xenon	132	$^{132}\mathrm{Xe}$	131.904 144	26.9%	
		136	<sup>136</sup> Xe	135.907 214	8.9%	
55	Cesium	133	$^{133}\mathrm{Cs}$	132.905 429	100%	
		134	$^{134}\mathrm{Cs}$	133.906 696	EC, $\beta^-$	2.06 y
56	Barium	137	$^{137}\mathrm{Ba}$	136.905 812	11.23%	
		138	$^{138}\mathrm{Ba}$	137.905 232	71.70%	
57	Lanthanum	139	$^{139}{ m La}$	138.906 346	99.91%	
58	Cerium	140	<sup>140</sup> Ce	139.905 433	88.48%	
59	Praseodymium	141	$^{141}\mathrm{Pr}$	140.907 647	100%	
60	Neodymium	142	$^{142}\mathrm{Nd}$	141.907 719	27.13%	
61	Promethium	145	$^{145}\mathrm{Pm}$	144.912 743	EC, α	17.7 y
62	Samarium	152	$^{152}\mathrm{Sm}$	151.919 729	26.7%	
63	Europium	153	$^{153}\mathrm{Eu}$	152.921 225	52.2%	
64	Gadolinium	158	$^{158}\mathrm{Gd}$	157.924 099	24.84%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
65	Terbium	159	$^{159}{ m Tb}$	158.925 342	100%	
66	Dysprosium	164	$^{164}\mathrm{Dy}$	163.929 171	28.2%	
67	Holmium	165	$^{165}\mathrm{Ho}$	164.930 319	100%	
68	Erbium	166	$^{166}{ m Er}$	165.930 290	33.6%	
69	Thulium	169	$^{169}{ m Tm}$	168.934 212	100%	
70	Ytterbium	174	$^{174}{ m Yb}$	173.938 859	31.8%	
71	Lutecium	175	$^{175}\mathrm{Lu}$	174.940 770	97.41%	
72	Hafnium	180	$^{180}{ m Hf}$	179.946 545	35.10%	
73	Tantalum	181	$^{181}{ m Ta}$	180.947 992	99.98%	
74	Tungsten	184	$^{184}\mathrm{W}$	183.950 928	30.67%	
75	Rhenium	187	$^{187}\mathrm{Re}$	186.955 744	62.6%, β <sup>-</sup>	$4.6 imes10^{10}\mathrm{y}$
76	Osmium	191	$^{191}\mathrm{Os}$	190.960 920	$eta^-$	15.4 d
		192	$^{192}\mathrm{Os}$	191.961 467	41.0%	
77	Iridium	191	$^{191}{ m Ir}$	190.960 584	37.3%	
		193	$^{193}{ m Ir}$	192.962 917	62.7%	
78	Platinum	195	$^{195}\mathrm{Pt}$	194.964 766	33.8%	
79	Gold	197	$^{197}\mathrm{Au}$	196.966 543	100%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, t <sub>1/2</sub>
		198	$^{198}\mathrm{Au}$	197.968 217	$eta^-$	2.696 d
80	Mercury	199	$^{199}{ m Hg}$	198.968 253	16.87%	
		202	$^{202}{ m Hg}$	201.970 617	29.86%	
81	Thallium	205	$^{205}\mathrm{Tl}$	204.974 401	70.48%	
82	Lead	206	$^{206}\mathrm{Pb}$	205.974 440	24.1%	
		207	$^{207}\mathrm{Pb}$	206.975 872	22.1%	
		208	$^{208}\mathrm{Pb}$	207.976 627	52.4%	
		210	$^{210}\mathrm{Pb}$	209.984 163	$lpha,eta^-$	22.3 y
		211	$^{211}\mathrm{Pb}$	210.988 735	$eta^-$	36.1 min
		212	$^{212}\mathrm{Pb}$	211.991 871	$eta^-$	10.64 h
83	Bismuth	209	$^{209}\mathrm{Bi}$	208.980 374	100%	
		211	$^{211}{ m Bi}$	210.987 255	$lpha,eta^-$	2.14 min
84	Polonium	210	$^{210}\mathrm{Po}$	209.982 848	α	138.38 d
85	Astatine	218	$^{218}\mathrm{At}$	218.008 684	$lpha,eta^-$	1.6 s
86	Radon	222	$^{222}\mathrm{Rn}$	222.017 570	α	3.82 d
87	Francium	223	$^{223}{ m Fr}$	223.019 733	$lpha,eta^-$	21.8 min
88 Radium		226	$^{226}\mathrm{Ra}$	226.025 402	α	$1.60  imes 10^3 \mathrm{y}$

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, t <sub>1/2</sub>
89	Actinium	227	$^{227}\mathrm{Ac}$	227.027 750	$lpha,eta^-$	21.8 y
90	Thorium	228	$^{228}{ m Th}$	228.028 715	α	1.91 y
		232	$^{232}{ m Th}$	232.038 054	100%, α	$1.41 imes10^{10}\mathrm{y}$
91	Protactinium	231	$^{231}\mathrm{Pa}$	231.035 880	α	$3.28 imes10^4\mathrm{y}$
92	Uranium	233	$^{233}\mathrm{U}$	233.039 628	α	$1.59  imes 10^3 \mathrm{y}$
		235	$^{235}\mathrm{U}$	235.043 924	0.720%, $\alpha$	$7.04 imes10^8\mathrm{y}$
		236	$^{236}\mathrm{U}$	236.045 562	α	$2.34 imes10^7\mathrm{y}$
		238	$^{238}\mathrm{U}$	238.050 784	99.2745%, $\alpha$	$4.47 imes10^9\mathrm{y}$
		239	$^{239}\mathrm{U}$	239.054 289	$eta^-$	23.5 min
93	Neptunium	239	$^{239}{ m Np}$	239.052 933	$eta^-$	2.355 d
94	Plutonium	239	$^{239}\mathrm{Pu}$	239.052 157	α	$2.41  imes 10^4 \mathrm{y}$
95	Americium	243	$^{243}\mathrm{Am}$	243.061 375	$\alpha$ , fission	$7.37 imes10^3\mathrm{y}$
96	Curium	245	$^{245}\mathrm{Cm}$	245.065 483	α	$8.50 imes10^3\mathrm{y}$
97	Berkelium	247	$^{247}\mathrm{Bk}$	247.070 300	α	$1.38  imes 10^3 \mathrm{y}$
98	Californium	249	$^{249}\mathrm{Cf}$	249.074 844	α	351 y
99	Einsteinium	254	$^{254}\mathrm{Es}$	254.088 019	$lpha,eta^-$	276 d
100	Fermium	253	$^{253}{ m Fm}$	253.085 173	EC, $\alpha$	3.00 d

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, t <sub>1/2</sub>
101	Mendelevium	255	$^{255}\mathrm{Md}$	255.091 081	EC, α	27 min
102	Nobelium	255	$^{255}\mathrm{No}$	255.093 260	EC, α	3.1 min
103	Lawrencium	257	$^{257}{ m Lr}$	257.099 480	EC, α	0.646 s
104	Rutherfordium	261	$^{261}\mathrm{Rf}$	261.108 690	α	1.08 min
105	Dubnium	262	$^{262}\mathrm{Db}$	262.113 760	lpha, fission	34 s
106	Seaborgium	263	$^{263}\mathrm{Sg}$	263.11 86	lpha, fission	0.8 s
107	Bohrium	262	$^{262}\mathrm{Bh}$	262.123 1	α	0.102 s
108	Hassium	264	$^{264}\mathrm{Hs}$	264.128 5	α	0.08 ms
109	Meitnerium	266	$^{266}\mathrm{Mt}$	266.137 8	α	3.4 ms

Atomic Masses

# **Useful Information**

This appendix is broken into several tables.

- [link], Important Constants
- [link], Submicroscopic Masses
- [link], Solar System Data
- [link], Metric Prefixes for Powers of Ten and Their Symbols
- [link], The Greek Alphabet
- [link], SI units
- [link], Selected British Units
- [link], Other Units
- [link], Useful Formulae

Symbol	Meaning	Best Value	Approximate Value		
c	Speed of light in vacuum	$2.99792458 imes10^8{ m m/s}$	$3.00 imes10^8\mathrm{m/s}$		
G	Gravitational constant	$6.67408(31) imes10^{-11} m N\cdot m^2/kg^2$	$6.67 imes10^{-11} m N\cdot m^2/kg^2$		
$N_A$	Avogadro's number	$6.02214129(27) imes10^{23}$	$6.02  imes 10^{23}$		
k	Boltzmann's constant	$1.3806488(13) imes10^{-23}\mathrm{J/K}$	$1.38 imes10^{-23}{ m J/K}$		
R	Gas constant	$8.3144621(75) m J/mol\cdot K$	$8.31\mathrm{J/mol\cdot K}=1.99\mathrm{cal/mol\cdot K}$		
σ	Stefan- Boltzmann constant	$5.670373(21) imes10^{-8}\mathrm{W/m^2\cdot K}$	$5.67  imes 10^{-8}  { m W/m^2 \cdot K}$		
k	Coulomb force constant	$8.987551788  imes 10^9   ext{N} \cdot  ext{m}^2/ ext{C}^2$	$8.99 imes10^9{ m N\cdot m^2/C^2}$		
$q_e$	Charge on electron	$-1.602176565(35) imes10^{-19}\mathrm{C}$	$-1.60  imes 10^{-19}{ m C}$		
$\epsilon_0$	Permittivity of free space	$8.854187817 \times 10^{-12}  \mathrm{C^2/N \cdot m^2}$	$8.85 imes10^{-12}{ m C}^2/{ m N}\cdot{ m m}^2$		
$\mu_0$	Permeability of free space	$4\pi  imes 10^{-7}\mathrm{T\cdot m/A}$	$1.26 imes10^{-6}\mathrm{T\cdot m/A}$		
h	Planck's constant	$6.62606957(29) imes10^{-34} m J\cdot s$	$6.63 imes10^{-34} mJ\cdot s$		

Important Constants[footnote]

Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, an <a href="https://www.physics.nist.gov/cuu">www.physics.nist.gov/cuu</a> (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Nu are exact as defined.

Symbol	Meaning	Best Value	Approximate Value		
$m_e$	Electron mass $9.10938291(40)  imes 10^{-31} { m kg}$		$9.11 imes10^{-31}{ m kg}$		
$m_p$	Proton mass	$1.672621777(74) imes 10^{-27}{ m kg}$	$1.6726  imes 10^{-27} { m kg}$		
$m_n$	Neutron mass	$1.674927351(74) imes 10^{-27}{ m kg}$	$1.6749  imes 10^{-27} { m kg}$		
u	Atomic mass unit	$1.660538921(73) imes 10^{-27}{ m kg}$	$1.6605 imes10^{-27}\mathrm{kg}$		

# Submicroscopic Masses[footnote]

Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, <a href="www.physics.nist.gov/cuu">www.physics.nist.gov/cuu</a> (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

Sun	mass	$1.99  imes 10^{30} \mathrm{kg}$
	average radius	$6.96 imes10^8\mathrm{m}$
	Earth-sun distance (average)	$1.496\times10^{11}\mathrm{m}$
Earth	mass	$5.9736 imes10^{24}\mathrm{kg}$
	average radius	$6.376 imes10^6\mathrm{m}$
	orbital period	$3.16\times10^7\mathrm{s}$

Moon	mass	$7.35 imes10^{22}{ m kg}$
	average radius	$1.74  imes 10^6 \mathrm{m}$
	orbital period (average)	$2.36  imes 10^6 \mathrm{s}$
	Earth-moon distance (average)	$3.84  imes 10^8 \mathrm{m}$

Solar System Data

Prefix	Symbol	Value	Prefix	Symbol	Value
tera	Т	$10^{12}$	deci	d	$10^{-1}$
giga	G	$10^9$	centi	С	$10^{-2}$
mega	M	$10^6$	milli	m	$10^{-3}$
kilo	k	$10^3$	micro	$\mu$	$10^{-6}$
hecto	h	$10^2$	nano	n	$10^{-9}$
deka	da	$10^1$	pico	p	$10^{-12}$
_	_	$10^0 (=1)$	femto	f	$10^{-15}$

Metric Prefixes for Powers of Ten and Their Symbols

Alpha	A	$\alpha$	Eta	Н	η	Nu	N	ν	Tau	Т	au
Beta	В	β	Theta	Θ	$\theta$	Xi	Ξ	ξ	Upsilon	Υ	v
Gamma	Γ	$\gamma$	Iota	I	ι	Omicron	О	o	Phi	Φ	$\phi$
Delta	Δ	δ	Kappa	K	$\kappa$	Pi	П	$\pi$	Chi	X	χ

Epsilon	$\mathbf{E}$	arepsilon	Lambda	Λ	$\lambda$	Rho	P	ho	Psi	$\Psi$	$\psi$
Zeta	$\mathbf{Z}$	ζ	Mu	M	$\mu$	Sigma	Σ	$\sigma$	Omega	Ω	$\omega$

The Greek Alphabet

	Entity	Abbreviation	Name
Fundamental units	Length	m	meter
	Mass	kg	kilogram
	Time	S	second
	Current	A	ampere
Supplementary unit	Angle	rad	radian
Derived units	Force	$ m N = kg \cdot m/s^2$	newton
	Energy	$ m J = kg \cdot m^2/s^2$	joule
	Power	m W = J/s	watt
	Pressure	${ m Pa}={ m N/m^2}$	pascal
	Frequency	$\mathrm{Hz}=1/\mathrm{s}$	hertz
	Electronic potential	V = J/C	volt
	Capacitance	$\mathrm{F}=\mathrm{C}/\mathrm{V}$	farad
	Charge	$\mathrm{C} = \mathrm{s} \cdot \mathrm{A}$	coulomb
	Resistance	$\Omega={ m V/A}$	ohm

Entity	Abbreviation	Name
Magnetic field	$\mathrm{T}=\mathrm{N}/(\mathrm{A}\cdot\mathrm{m})$	tesla
Nuclear decay rate	$\mathrm{Bq}=1/\mathrm{s}$	becquerel

SI Units

Length	$1\mathrm{inch}\mathrm{(in.)} = 2.54\mathrm{cm}\mathrm{(exactly)}$
	$1{ m foot}({ m ft})=0.3048{ m m}$
	$1\mathrm{mile}\mathrm{(mi)} = 1.609\mathrm{km}$
Force	$1\mathrm{pound}\mathrm{(lb)}=4.448\mathrm{N}$
Energy	$1\mathrm{British}\ \mathrm{thermal}\ \mathrm{unit}\ (\mathrm{Btu}) = 1.055  imes 10^3\mathrm{J}$
Power	$1  \mathrm{horsepower}  (\mathrm{hp}) = 746  \mathrm{W}$
Pressure	$1{ m lb/in^2} = 6.895  imes 10^3{ m Pa}$

Selected British Units

Length	$1\mathrm{light\;year}(\mathrm{ly}) = 9.46 imes 10^{15}\mathrm{m}$
	$1\mathrm{astronomicalunit(au)} = 1.50  imes 10^{11}\mathrm{m}$
	$1  \mathrm{nautical \; mile} = 1.852  \mathrm{km}$
	$1\mathrm{angstrom}(\mathrm{\AA})=10^{-10}\mathrm{m}$
Area	$1{ m acre}({ m ac}) = 4.05 imes 10^3{ m m}^2$
	$1{ m squarefoot(ft^2)} = 9.29 imes 10^{-2}{ m m^2}$
	$1\mathrm{barn}(b) = 10^{-28}\mathrm{m}^2$
Volume	$1{ m liter}(L)=10^{-3}{ m m}^3$

	$1  ext{U.S. gallon (gal)} = 3.785  imes 10^{-3}  ext{m}^3$
Mass	$1\mathrm{solar\;mass}=1.99 imes10^{30}\mathrm{kg}$
	$1\mathrm{metric\ ton} = 10^3\mathrm{kg}$
	$1\mathrm{atomic}\ \mathrm{mass}\ \mathrm{unit}\ (u) = 1.6605  imes 10^{-27}\mathrm{kg}$
Time	$1\mathrm{year}(y) = 3.16 imes 10^7\mathrm{s}$
	$1{ m day}(d)=86{,}400{ m s}$
Speed	$1\mathrm{mile}\;\mathrm{per}\;\mathrm{hour}\;(\mathrm{mph}) = 1.609\mathrm{km/h}$
	$1 \ \mathrm{nautical \ mile \ per \ hour \ (naut)} = 1.852 \ \mathrm{km/h}$
Angle	$1\mathrm{degree}\left( angle ight)=1.745 imes10^{-2}\mathrm{rad}$
	$1\mathrm{minute}\ \mathrm{of}\ \mathrm{arc}\ ()=1/60\mathrm{degree}$
	$1\mathrm{second}$ of arc (") $=1/60\mathrm{minute}$ of arc
	$1\mathrm{grad} = 1.571  imes 10^{-2}\mathrm{rad}$
Energy	$1\mathrm{kiloton\;TNT}(\mathrm{kT}) = 4.2  imes 10^{12}\mathrm{J}$
	$1\mathrm{kilowatt\;hour}(\mathrm{kW}\cdot h) = 3.60 imes10^6\mathrm{J}$
	$1\mathrm{foodcalorie(kcal)}=4186\mathrm{J}$
	$1\mathrm{calorie}\mathrm{(cal)} = 4.186\mathrm{J}$
	$1\mathrm{electron\ volt}(\mathrm{eV}) = 1.60  imes 10^{-19}\mathrm{J}$
Pressure	$1\mathrm{atmosphere}\mathrm{(atm)} = 1.013  imes 10^5\mathrm{Pa}$
	$1\mathrm{millimeter}\;\mathrm{of}\;\mathrm{mercury}\;(\mathrm{mm}\mathrm{Hg}) = 133.3\mathrm{Pa}$
	$1\mathrm{torricelli}(\mathrm{torr}) = 1\mathrm{mm}\mathrm{Hg} = 133.3\mathrm{Pa}$
Nuclear decay rate	$1{ m curie}{ m (Ci)} = 3.70 imes 10^{10}{ m Bq}$

Other Units

Circumference of a circle with radius $r$ or diameter $d$	$C=2\pi r=\pi d$
Area of a circle with radius $r$ or diameter $d$	$A=\pi r^2=\pi d^2/4$
Area of a sphere with radius $r$	$A=4\pi r^2$

Volume of a sphere with radius	r
--------------------------------	---

 $V=(4/3)ig(\pi r^3ig)$ 

Useful Formulae

# Glossary of Key Symbols and Notation

In this glossary, key symbols and notation are briefly defined.

Symbol	Definition
any symbol	average (indicated by a bar over a symbol—e.g., $v$ is average velocity)
$^{\circ}\mathrm{C}$	Celsius degree
${ m ^{\circ}F}$	Fahrenheit degree
//	parallel
Т	perpendicular
$\propto$	proportional to
±	plus or minus

Symbol	Definition
0	zero as a subscript denotes an initial value
$\alpha$	alpha rays
$\alpha$	angular acceleration
$\alpha$	temperature coefficient(s) of resistivity
eta	beta rays
β	sound level
β	volume coefficient of expansion
$eta^-$	electron emitted in nuclear beta decay
$eta^+$	positron decay
γ	gamma rays

Symbol	Definition
$\gamma$	surface tension
$\gamma = 1/\sqrt{1-v^2/c^2}$	a constant used in relativity
Δ	change in whatever quantity follows
δ	uncertainty in whatever quantity follows
$\Delta E$	change in energy between the initial and final orbits of an electron in an atom
$\Delta E$	uncertainty in energy
$\Delta m$	difference in mass between initial and final products
$\Delta N$	number of decays that occur
$\Delta p$	change in momentum

Symbol	Definition
$\Delta p$	uncertainty in momentum
$\Delta\!\operatorname{PE}_{\operatorname{g}}$	change in gravitational potential energy
$\Delta  heta$	rotation angle
$\Delta s$	distance traveled along a circular path
$\Delta t$	uncertainty in time
$arDelta t_0$	proper time as measured by an observer at rest relative to the process
arDelta V	potential difference
$\Delta x$	uncertainty in position
$arepsilon_0$	permittivity of free space
$\eta$	viscosity

Symbol	Definition
$\theta$	angle between the force vector and the displacement vector
$\theta$	angle between two lines
$\theta$	contact angle
heta	direction of the resultant
$ heta_b$	Brewster's angle
$ heta_c$	critical angle
$\kappa$	dielectric constant
λ	decay constant of a nuclide
λ	wavelength
$\lambda_n$	wavelength in a medium

Symbol	Definition
$\mu_0$	permeability of free space
$\mu_{ m k}$	coefficient of kinetic friction
$\mu_{ m s}$	coefficient of static friction
$v_e$	electron neutrino
$\pi^+$	positive pion
$\pi^-$	negative pion
$\pi^0$	neutral pion
ρ	density
$ ho_{ m c}$	critical density, the density needed to just halt universal expansion
$ ho_{ m fl}$	fluid density

Symbol	Definition
$ ho_{ m obj}$	average density of an object
$ ho/ ho_{ m w}$	specific gravity
τ	characteristic time constant for a resistance and inductance $(RL)$ or resistance and capacitance $(RC)$ circuit
au	characteristic time for a resistor and capacitor $(RC)$ circuit
au	torque
Υ	upsilon meson
Φ	magnetic flux
$\phi$	phase angle
Ω	ohm (unit)
$\omega$	angular velocity

Symbol	Definition
A	ampere (current unit)
A	area
A	cross-sectional area
A	total number of nucleons
a	acceleration
$a_{ m B}$	Bohr radius
$a_{ m c}$	centripetal acceleration
$a_{ m t}$	tangential acceleration
$\mathbf{AC}$	alternating current
AM	amplitude modulation

Symbol	Definition
$\operatorname{atm}$	atmosphere
В	baryon number
B	blue quark color
B	antiblue (yellow) antiquark color
b	quark flavor bottom or beauty
B	bulk modulus
B	magnetic field strength
$\mathrm{B}_{\mathrm{int}}$	electron's intrinsic magnetic field
$\mathrm{B}_{\mathrm{orb}}$	orbital magnetic field
BE	binding energy of a nucleus—it is the energy required to completely disassemble it into separate protons and neutrons

Symbol	Definition
$\mathrm{BE}/A$	binding energy per nucleon
$_{ m Bq}$	becquerel—one decay per second
C	capacitance (amount of charge stored per volt)
C	coulomb (a fundamental SI unit of charge)
$C_{ m p}$	total capacitance in parallel
$C_{ m s}$	total capacitance in series
$^{\mathrm{CG}}$	center of gravity
CM	center of mass
c	quark flavor charm
c	specific heat

Symbol	Definition
c	speed of light
Cal	kilocalorie
cal	calorie
$COP_{ m hp}$	heat pump's coefficient of performance
$COP_{ m ref}$	coefficient of performance for refrigerators and air conditioners
$\cos heta$	cosine
$\cot heta$	cotangent
$\csc heta$	cosecant
D	diffusion constant
d	displacement

Symbol	Definition
d	quark flavor down
dB	decibel
$d_{ m i}$	distance of an image from the center of a lens
$d_{ m o}$	distance of an object from the center of a lens
DC	direct current
$oldsymbol{E}$	electric field strength
arepsilon	emf (voltage) or Hall electromotive force
$\operatorname{emf}$	electromotive force
$oldsymbol{E}$	energy of a single photon
E	nuclear reaction energy

Symbol	Definition
$oldsymbol{E}$	relativistic total energy
$oldsymbol{E}$	total energy
$E_0$	ground state energy for hydrogen
$E_0$	rest energy
EC	electron capture
$E_{ m cap}$	energy stored in a capacitor
Eff	efficiency—the useful work output divided by the energy input
$\mathrm{Eff}_{\mathrm{C}}$	Carnot efficiency
$E_{ m in}$	energy consumed (food digested in humans)
$E_{ m ind}$	energy stored in an inductor

Symbol	Definition
$E_{ m out}$	energy output
e	emissivity of an object
$e^{+}$	antielectron or positron
${ m eV}$	electron volt
F	farad (unit of capacitance, a coulomb per volt)
F	focal point of a lens
F	force
F	magnitude of a force
F	restoring force
$F_{ m B}$	buoyant force

Symbol	Definition
$F_{ m c}$	centripetal force
$F_{ m i}$	force input
$\mathbf{F}_{ ext{net}}$	net force
$F_{ m o}$	force output
FM	frequency modulation
f	focal length
f	frequency
$f_0$	resonant frequency of a resistance, inductance, and capacitance (RLC) series circuit
$f_0$	threshold frequency for a particular material (photoelectric effect)

Symbol	Definition
$f_1$	fundamental
$f_2$	first overtone
$f_3$	second overtone
$f_{ m B}$	beat frequency
$f_{ m k}$	magnitude of kinetic friction
$f_{ m s}$	magnitude of static friction
G	gravitational constant
G	green quark color
G	antigreen (magenta) antiquark color

Symbol	Definition
g	acceleration due to gravity
g	gluons (carrier particles for strong nuclear force)
h	change in vertical position
h	height above some reference point
h	maximum height of a projectile
h	Planck's constant
hf	photon energy
$h_{ m i}$	height of the image
$h_{ m o}$	height of the object
I	electric current

Symbol	Definition
I	intensity
I	intensity of a transmitted wave
I	moment of inertia (also called rotational inertia)
$I_0$	intensity of a polarized wave before passing through a filter
$I_{ m ave}$	average intensity for a continuous sinusoidal electromagnetic wave
$I_{ m rms}$	average current
J	joule
$J/\Psi$	Joules/psi meson
K	kelvin
k	Boltzmann constant

Symbol	Definition
k	force constant of a spring
$K_{lpha}$	x rays created when an electron falls into an $n=1$ shell vacancy from the $n=3$ shell
$K_eta$	x rays created when an electron falls into an $n=2$ shell vacancy from the $n=3$ shell
kcal	kilocalorie
KE	translational kinetic energy
$\mathrm{KE} + \mathrm{PE}$	mechanical energy
$\mathrm{KE}_e$	kinetic energy of an ejected electron
$\mathrm{KE}_{\mathrm{rel}}$	relativistic kinetic energy
$\mathrm{KE}_{\mathrm{rot}}$	rotational kinetic energy
KE	thermal energy

Symbol	Definition
kg	kilogram (a fundamental SI unit of mass)
L	angular momentum
L	liter
L	magnitude of angular momentum
L	self-inductance
$\ell$	angular momentum quantum number
$L_{lpha}$	x rays created when an electron falls into an $n=2$ shell from the $n=3$ shell
$L_e$	electron total family number
$L_{\mu}$	muon family total number
$L_{ au}$	tau family total number

Symbol	Definition
$L_{ m f}$	heat of fusion
$L_{ m f}  { m and}  L_{ m v}$	latent heat coefficients
$ m L_{orb}$	orbital angular momentum
$L_{ m s}$	heat of sublimation
$L_{ m v}$	heat of vaporization
$L_z$	z - component of the angular momentum
M	angular magnification
M	mutual inductance
m	indicates metastable state
m	magnification

Symbol	Definition
m	mass
m	mass of an object as measured by a person at rest relative to the object
m	meter (a fundamental SI unit of length)
m	order of interference
m	overall magnification (product of the individual magnifications)
$m\Big(^A\mathrm{X}\Big)$	atomic mass of a nuclide
MA	mechanical advantage
$m_{ m e}$	magnification of the eyepiece
$m_e$	mass of the electron
$m_\ell$	angular momentum projection quantum number

Symbol	Definition
$m_n$	mass of a neutron
$m_{ m o}$	magnification of the objective lens
mol	mole
$m_p$	mass of a proton
$m_{ m s}$	spin projection quantum number
N	magnitude of the normal force
N	newton
N	normal force
N	number of neutrons
n	index of refraction

Symbol	Definition
n	number of free charges per unit volume
$N_{ m A}$	Avogadro's number
$N_{ m r}$	Reynolds number
${f N}\cdot{f m}$	newton-meter (work-energy unit)
$\mathbf{N}\cdot\mathbf{m}$	newtons times meters (SI unit of torque)
OE	other energy
P	power
P	power of a lens
P	pressure
р	momentum

Symbol	Definition
p	momentum magnitude
p	relativistic momentum
$\mathbf{p}_{\mathrm{tot}}$	total momentum
$\mathbf{p}_{ ext{tot}}^{'}$	total momentum some time later
$P_{ m abs}$	absolute pressure
$P_{ m atm}$	atmospheric pressure
$P_{ m atm}$	standard atmospheric pressure
PE	potential energy
$\mathrm{PE}_{\mathrm{el}}$	elastic potential energy
$\mathrm{PE}_{\mathrm{elec}}$	electric potential energy

Symbol	Definition
$\mathrm{PE}_{\mathrm{s}}$	potential energy of a spring
$P_{ m g}$	gauge pressure
$P_{ m in}$	power consumption or input
$P_{ m out}$	useful power output going into useful work or a desired, form of energy
Q	latent heat
Q	net heat transferred into a system
Q	flow rate—volume per unit time flowing past a point
+Q	positive charge
-Q	negative charge

Symbol	Definition
q	electron charge
$q_p$	charge of a proton
q	test charge
QF	quality factor
R	activity, the rate of decay
R	radius of curvature of a spherical mirror
R	red quark color
R	antired (cyan) quark color
R	resistance
R	resultant or total displacement

Symbol	Definition
R	Rydberg constant
R	universal gas constant
r	distance from pivot point to the point where a force is applied
r	internal resistance
$r_{\perp}$	perpendicular lever arm
r	radius of a nucleus
r	radius of curvature
r	resistivity
r or rad	radiation dose unit
rem	roentgen equivalent man

Symbol	Definition
rad	radian
RBE	relative biological effectiveness
RC	resistor and capacitor circuit
${ m rms}$	root mean square
$r_n$	radius of the <i>n</i> th H-atom orbit
$R_{ m p}$	total resistance of a parallel connection
$R_{ m s}$	total resistance of a series connection
$R_{ m s}$	Schwarzschild radius
S	entropy
S	intrinsic spin (intrinsic angular momentum)

Symbol	Definition
S	magnitude of the intrinsic (internal) spin angular momentum
S	shear modulus
S	strangeness quantum number
8	quark flavor strange
S	second (fundamental SI unit of time)
S	spin quantum number
S	total displacement
$\sec  heta$	secant
$\sin  heta$	sine
$oldsymbol{s}_z$	z-component of spin angular momentum

Symbol	Definition
T	period—time to complete one oscillation
T	temperature
$T_{ m c}$	critical temperature—temperature below which a material becomes a superconductor
T	tension
Т	tesla (magnetic field strength $B$ )
t	quark flavor top or truth
t	time
$t_{1/2}$	half-life—the time in which half of the original nuclei decay
an heta	tangent
U	internal energy

Symbol	Definition
u	quark flavor up
u	unified atomic mass unit
u	velocity of an object relative to an observer
u'	velocity relative to another observer
V	electric potential
V	terminal voltage
V	volt (unit)
V	volume
V	relative velocity between two observers
v	speed of light in a material

Symbol	Definition
v	velocity
V	average fluid velocity
$V_{ m B}-V_{ m A}$	change in potential
$\mathbf{v}_{\mathrm{d}}$	drift velocity
$V_{ m p}$	transformer input voltage
$V_{ m rms}$	rms voltage
$V_{ m s}$	transformer output voltage
$\mathbf{v}_{\mathrm{tot}}$	total velocity
$v_{ m w}$	propagation speed of sound or other wave
$\mathbf{v}_{\mathrm{w}}$	wave velocity

Symbol	Definition
W	work
W	net work done by a system
W	watt
w	weight
$w_{ m fl}$	weight of the fluid displaced by an object
$W_{ m c}$	total work done by all conservative forces
$W_{ m nc}$	total work done by all nonconservative forces
$W_{ m out}$	useful work output
X	amplitude
X	symbol for an element

Symbol	Definition
$_{A}^{Z}X_{N}$	notation for a particular nuclide
x	deformation or displacement from equilibrium
x	displacement of a spring from its undeformed position
x	horizontal axis
$X_{ m C}$	capacitive reactance
$X_{ m L}$	inductive reactance
$x_{ m rms}$	root mean square diffusion distance
y	vertical axis
Y	elastic modulus or Young's modulus
Z	atomic number (number of protons in a nucleus)

Symbol	Definition
Z	impedance